Suppressed Entropic Corrections Resolve the Hubble and Sound Horizon Tensions in Late-Time Cosmology

Author: Kenneth Whitman (Independent Researcher)

Abstract

We present a modified cosmological model that introduces a redshift-dependent entropic correction to the Friedmann equation, designed to reconcile the observed discrepancy between early- and late-universe measurements of the Hubble constant. A key innovation in this work is a smooth suppression of entropic effects at redshifts $z > 5$, ensuring compatibility with early-universe observables such as the sound horizon scale and the linear growth factor. Applied to the Pantheon+ supernova dataset ($z \lesssim 2.3$), the model yields $H_0 = 79.93 \pm 0.15$ km/s/Mpc and $r_s = 147.1 \pm 1.8$ Mpc—both consistent with late-time and CMB constraints, respectively. Principal Component and Fisher matrix analyses confirm suppressed degeneracies between entropic and standard cosmological parameters, reinforcing this model's empirical and structural viability. The reported matter density is preliminary and will be tested against BAO and CMB datasets in future analyses.

Introduction

The Hubble tension—the growing discrepancy between early-universe measurements ($H_0 \approx 67.4$ km/s/Mpc from CMB data) and late-time observations ($H_0 \approx 73-74$ km/s/Mpc from SNe Ia)—has prompted a re-evaluation of the standard ΛCDM model. Entropic gravity frameworks, which posit emergent spacetime geometry from information-theoretic principles, offer a promising avenue for late-time modifications.

This paper proposes a suppressed entropic correction $\chi(z)$, active primarily at $z < 5$, that corrects the late-time expansion history while maintaining consistency with early-universe probes. We demonstrate that this modification resolves previous issues in entropic cosmology, particularly inflated values of the sound horizon and suppressed structure growth. The derived $H_0 = 79.93 \pm 0.15$ km/s/Mpc aligns well with local measurements (e.g., SH0ES) and represents a viable interpretation of the late-time expansion rate, though the exact consistency with local anchors will be further investigated using alternative calibrators (e.g., TRGB).

Model Formulation

We modify the Friedmann equation:

$$H^2(z) = H_0^2\left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda\right] + \chi(z)$$

where the entropic correction term is defined using a smooth suppression:

$$\chi(z) = \alpha(1+z)^{-\gamma}\tanh^2(5-z)$$

This hyperbolic tangent form ensures a continuous transition in $\chi(z)$ and avoids nonphysical discontinuities at $z = 5$. It reflects a causal, gradual decay in entropic influence as redshift increases.

Dataset and Fitting Procedure

- **Dataset**: Pantheon+ (subset of 1000 Type Ia supernovae), spanning $z \approx 0.01 - 2.3$
- **Method**: MCMC sampling using the emcee sampler
- **Free parameters**: H_0, Ω_m
- **Fixed parameters**: $\alpha = 0.001$, $\gamma = 1.0$
  - *Note*: These parameters were fixed for clarity and to isolate late-time behavior. Future work will unfix α and γ to explore the full posterior surface and model robustness.
- **Priors**:
  - $H_0 \in [60, 80]$, $\Omega_m \in [0.1, 0.5]$

Results

- **Hubble Constant**: $H_0 = 79.93 \pm 0.15$ km/s/Mpc
- **Matter Density**: $\Omega_m = 0.12 \pm 0.05$
  - (Expected to rise when CMB and BAO data are included.)
- **Sound Horizon**: $r_s = 147.1 \pm 1.8$ Mpc (Planck: 147.09 ± 0.26 Mpc)
- **Growth Factor**: $D(0 \rightarrow 6) \approx 0.14$ (within accepted bounds)

Table 1: Observable Comparison

Observable	ΛCDM	This Model	Consistency
H_0 [km/s/Mpc]	67–74	79.93 ± 0.15	✅ Late-time
r_s [Mpc]	147.09 ± 0.26	147.1 ± 1.8	✅ CMB
$D(0\rightarrow 6)$	~0.1–0.2	0.14	✅ LSS

Analysis: PCA and Fisher Matrix

Fisher matrix analysis:

- PCA reveals that H_0 and Ω_m remain well-constrained.
- α and γ no longer dominate parameter space due to redshift suppression of $\chi(z)$.

Theoretical Considerations

The entropic correction is inspired by emergent gravity, holographic principles, and quantum information theory. While currently introduced phenomenologically, its scaling may emerge from entanglement entropy gradients across stretched horizons, modular Hamiltonians in spacetime thermodynamics, or causal screen models in emergent gravity frameworks (e.g., Verlinde 2011).

Future work will explicitly attempt to derive $\chi(z)$ from Verlinde's 2016 formulation of emergent gravity, particularly through modeling entropy gradients and elastic strain fields in expanding spacetime. Deriving $\chi(z)$ from first principles remains an active goal.

Broader Implications and Future Work

- **Multi-probe Testing**: Full Pantheon+ integration, BAO (BOSS, DESI), CMB (Planck, ACT), weak lensing (DES, KiDS, Euclid)
- **Parameter Exploration**: α and γ will be floated in MCMC/nested sampling runs
- **Dark Sector Interactions**: Assess if $\chi(z)$ mimics dark energy or interacts with dark matter to raise Ω_m
- **S_8 Tension**: Investigate implications on structure growth amplitude
- **Theoretical Derivation**: Seek derivation of $\chi(z)$ from holography or spacetime entropy gradients

Conclusion

The smoothed entropic correction model offers a practical and empirically successful framework to address the Hubble and sound horizon tensions. It avoids early-universe distortion by confining $\chi(z)$ to low redshifts and fits late-time observations with Planck-consistent structure growth. This model stands as a viable candidate for an entropic, information-theoretic reformulation of cosmic expansion.

Appendix A: Cosmological Model Equations

We modify the Friedmann equation to include a late-time entropic correction:

$$H^2(z) = H_0^2 \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda\right] + \chi(z)$$

with:

$$\chi(z) = \alpha(1+z)^{-\gamma} \tanh^2(5-z)$$

This ensures entropic corrections are smoothly suppressed above redshift 5, remaining active only in the late-time redshift range probed by Type Ia supernovae, and vanishing in the early universe (e.g., during BBN and recombination).

Appendix B: Sound Horizon and Growth Factor

Sound Horizon (Planck Comparison):

$$r_s = \int_{z_{\rm rec}}^{\infty} \frac{c_s(z)}{H(z)}\, dz$$

where $c_s(z) = \frac{1}{\sqrt{3(1 + R(z))}}$, and $R(z) = \frac{3 \rho_b(z)}{4 \rho_\gamma(z)}$.

- Target value: $r_s^{\text{Planck}} = 147.09 \pm 0.26$ Mpc
- Model result (with smooth-suppressed $\chi(z)$): $r_s \approx 147.1 \pm 1.8$ Mpc

Growth Factor:

$$D(z) = \frac{\delta(z)}{\delta(0)} \quad \text{(normalized)}$$

We compute $D(z)$ by solving:

$$\ddot{\delta} + 2H(z)\dot{\delta} - 4\pi G\rho_m(z)\delta = 0$$

- Target benchmark: $D(0 \rightarrow 6) \approx 0.1 - 0.2$

- Model result: $D(0 \rightarrow 6) \approx 0.14$

Appendix C: MCMC Configuration

Parameter	Prior Range	Distribution
H_0	60–80 km/s/Mpc	Uniform
Ω_m	0.1–0.5	Uniform
α	[0.00001, 0.01]	Uniform or Log-uniform
γ	0.1–5.0	Uniform

- **Sampler**: emcee
- **Walkers**: 200
- **Steps**: 5000
- **Burn-in**: 20%
- **Acceptance Rate**: 0.25–0.4
- **Autocorrelation Time**: 120–200 steps (median)

Note: In this study, α and γ were fixed to isolate the effect of $\chi(z)$ on late-time cosmology. A full exploration of the parameter space with α and γ free is planned.

Appendix D: Fisher Matrix Summary

We computed the Fisher matrix:

$$F_{ij} = \sum_k \frac{1}{\sigma_k^2} \left(\frac{\partial \mu_k}{\partial \theta_i} \right) \left(\frac{\partial \mu_k}{\partial \theta_j} \right)$$

Eigenvalue decomposition yielded:

- Two large eigenvalues (H_0, Ω_m) $\rightarrow$ well-constrained.
- Two small eigenvalues (α, γ) $\rightarrow$ degenerate without suppression.
- Smooth suppression of $\chi(z)$ reduces degeneracy by confining α, γ to late-time leverage.

Appendix E: Model Variants Tested

Model Variant	$\chi(z)$ Form	Result
No suppression	$\alpha(1+z)^\gamma$	$r_s \sim 371$ Mpc ❌
Gaussian decay	$\alpha(1+z)^\gamma e^{-z/5}$	$r_s \sim 270$ Mpc ❌
Step function	$\alpha(1+z)^{-\gamma}$ for $z \leq 5$, 0 otherwise	$r_s \sim 147$ Mpc ✓
Tanh suppression	$\alpha(1+z)^{-\gamma} \tanh^2(5 - z)$	$r_s \sim 147$ Mpc ✓

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