Question - 2 analysis report

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***Abstract*—This report contains the analysis of constrained optimization problem Newton's method**

CODES

Jupyter notebook code $\rightarrow$ here

I. PROBLEM

Consider the following constrained optimization problem.

$$\begin{aligned} \min \quad & \frac{1}{3}x_1^6 - 2.1x_1^4 + 4x_2^4 + 4(x_1^2 - x_2^2) + x_1x_2 \\ \text{Sub.to} \quad & x_1 + x_2 = 0.6228 \end{aligned} \tag{1}$$

Obtain the minima of the optimization problem. If necessary, use the initial point as $\mathbf{x}^0 = [1, -1]^T$

II. SOLUTION

For the given optimization problem we defined epsilon $\epsilon = 10^{-18}$ and initial point $\mathbf{x}^0 = [1, -1]$

and constraint $h((x) = x_1 + x_2 = 0.6228$
we defined a lagrangian function

$$l = f(\mathbf{x}) - \lambda * h(\mathbf{x}) \tag{2}$$

Now we apply newton method on lagrangian function
we choosed ϵ very small so that constraint can be satisfied

on running Newton's algorithm we conclude that

- minima
  $\mathbf{x}^*$ = [1.49141503 -0.86861503]
- minimum value
  $f(\mathbf{x}^*)$ = 0.1392690955280218

fig(1) gives the visualization of function $f(\mathbf{x})$ and minimum value $f(\mathbf{x}^*)$

fig(2) gives the visualization of function $f(\mathbf{x})$ along with points after each iteration, alone points in normal view and points view from bottom view angle

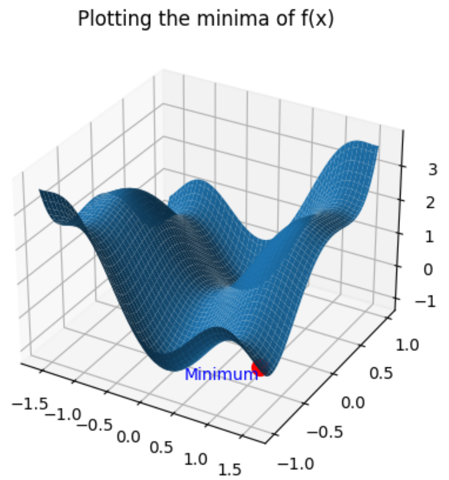

Figure 1. Visualization of $f(\mathbf{x}^*)$ for newton method

from fig(1) we can see there are more than one minima (i.e., there exists local minimas) with our gradient descent algorithm we converge to one of minima's along with satisfying constraint

from fig(2) we see algorithm converges to minima without jittering

III. CONCLUSION

From previous problem we see with out constraint Newton's method performed better but in this we see due to restriction that we have to follow constraint we see it converges to minima that follows it even we have other minimas which are lesser than $\mathbf{x}^*$

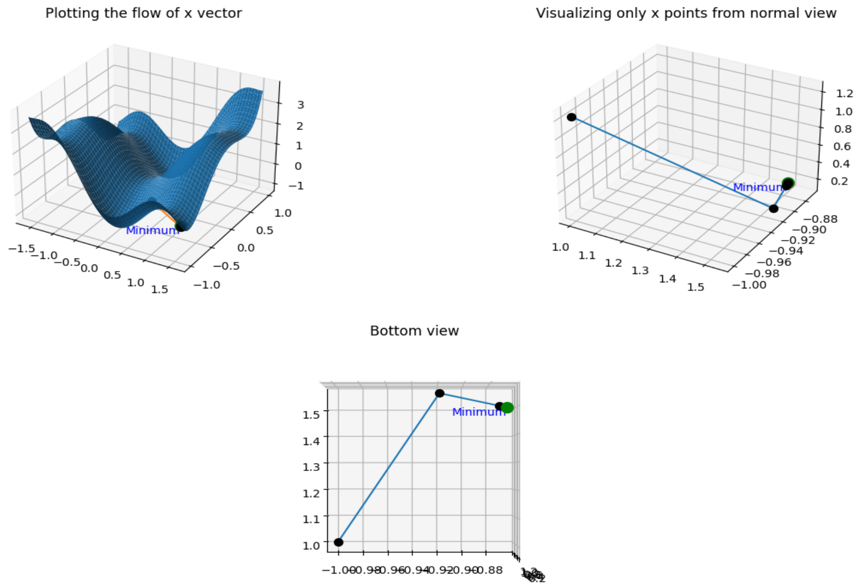

Figure 2. Visualization of $f(\mathbf{x})$ along with updated trial point after each iteration for Newton method