Mathematics

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(Chapter – 9) (Algebraic Expressions and Identities)

(Class – VIII)

Exercise 9.5

Question 1:

Use a suitable identity to get each of the following products:

(i) $(x+3)(x+3)$ (ii) $(2y+5)(2y+5)$

(iii) $(2a-7)(2a-7)$ (iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)$

(v) $(1.1m-0.4)(1.1m+0.4)$ (vi) $\left(a^2+b^2\right)\left(-a^2+b^2\right)$

(vii) $(6x-7)(6x+7)$ (viii) $(-a+c)(-a+c)$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$ (x) $(7a-9b)(7a-9b)$

Answer 1:

(i) $(x+3)(x+3)=(x+3)^2$

$=(x)^2+2\times x\times 3+(3)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= x^2+6x+9$

(ii) $(2y+5)(2y+5)=(2y+5)^2$

$=(2y)^2+2\times 2y\times 5+(5)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= 4y^2+20y+25$

(iii) $(2a-7)(2a-7)=(2a-7)^2$

$=(2a)^2-2\times 2a\times 7+(7)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 4a^2-28a+49$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)=\left(3a-\frac{1}{2}\right)^2$

$=(3a)^2-2\times 3a\times\frac{1}{2}+\left(\frac{1}{2}\right)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$=9a^2-3a+\frac{1}{4}$

(v) $(1.1m-0.4)(1.1m+0.4)=(1.1m)^2-(0.4)^2$

Using identity $(a-b)(a+b)=a^2-b^2$]

$= 1.21m^2 - 0.16$

(vi) $(a^2+b^2)(-a^2+b^2) = (b^2+a^2)(b^2-a^2)$

$= (b^2)^2 - (a^2)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= b^4 - a^4$

(vii) $(6x-7)(6x+7) = (6x)^2 - (7)^2$ [Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 36x^2 - 49$

(viii) $(-a+c)(-a+c) = (c-a)(c-a) = (c-a)^2$

$= (c)^2 - 2 \times c \times a + (a)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= c^2 - 2ca + a^2$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right) = \left(\frac{x}{2}+\frac{3y}{4}\right)^2$

$= \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{3y}{4} + \left(\frac{3y}{4}\right)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= \frac{x^2}{4} + \frac{3}{4}xy + \frac{9}{16}y^2$

(x) $(7a-9b)(7a-9b) = (7a-9b)^2$

$= (7a)^2 - 2 \times 7a \times 9b + (9b)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 49a^2 - 126ab + 81b^2$

Question 2:

Use the identity $(x+a)(x+b)=x^2+(a+b)x+ab$ to find the following products:

(i) $(x+3)(x+7)$ (ii) $(4x+5)(4x+1)$

(iii) $(4x-5)(4x-1)$ (iv) $(4x+5)(4x-1)$

(v) $(2x+5y)(2x+3y)$ (vi) $(2a^2+9)(2a^2+5)$

(vii) $(xyz-4)(xyz-2)$

Answer 2:

(i) $(x+3)(x+7)=(x)^2+(3+7)x+3\times 7$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2+10x+21$

(ii) $(4x+5)(4x+1)=(4x)^2+(5+1)4x+5\times 1$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2+6\times 4x+5=16x^2+24x+5$

(iii) $(4x-5)(4x-1) = (4x)^2+(-5-1)4x+(-5)\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2+(-6)\times 4x+5=16x^2-24x+5$

(iv) $(4x+5)(4x-1)=(4x)^2+\{5\times(-1)\}\times 4x+5\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2+(5-1)\times 4x-5$

$= 16x^2+4\times 4x-5$

$= 16x^2+16x-5$

(v) $(2x+5y)(2x+3y)=(2x)^2+(5y+3y)\times 2x+5y\times 3y$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 4x^2+8y\times 2x+15y^2$

$= 4x^2+16xy+15y^2$

(vi) $(2a^2+9)(2a^2+5)=(2a^2)^2+(9+5)\times 2a^2+9\times 5$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 4a^4+14\times 2a^2+45$

$= 4a^4+28a^2+45$

(vii) $(xyz-4)(xyz-2)=(xyz)^2+(-4-2)\times xyz+(-4)\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2y^2z^2-6xyz+8$

Question 3:

Find the following squares by using identities:

(i) $(b-7)^2$ (ii) $(xy+3z)^2$ (iii) $(6x^2-5y)^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$ (v) $(0.4p-0.5q)^2$ (vi) $(2xy+5y)^2$

Answer 3:

(i) $(b-7)^2=(b)^2-2\times b\times 7+(7)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= b^2-14b+49$

(ii) $(xy+3z)^2=(xy)^2+2\times xy\times 3z+(3z)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= x^2y^2+6xyz+9z^2$

(iii) $(6x^2-5y)^2=(6x^2)^2-2\times 6x^2\times 5y+(5y)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 36x^4-60x^2y+25y^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2=\left(\frac{2}{3}m\right)^2+2\times\frac{2}{3}m\times\frac{3}{2}n+\left(\frac{3}{2}n\right)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$

(v) $(0.4p - 0.5q)^2 = (0.4p)^2 - 2 \times 0.4p \times 0.5q + (0.5q)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 0.16p^2 - 0.40pq + 0.25q^2$

(vi) $(2xy + 5y)^2 = (2xy)^2 + 2 \times 2xy \times 5y + (5y)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 4x^2y^2 + 20xy^2 + 25y^2$

Question 4:

Simplify:

(i) $(a^2 - b^2)^2$

(ii) $(2x+5)^2 - (2x-5)^2$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(vi) $(ab + bc)^2 - 2ab^2c$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Answer 4:

(i) $(a^2 - b^2)^2 = (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$ [Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= a^4 - 2a^2b^2 + b^4$

(ii) $(2x+5)^2 - (2x-5)^2 = (2x)^2 + 2 \times 2x \times 5 + (5)^2 - \left[(2x)^2 - 2 \times 2x \times 5 + (5)^2\right]$

[Using identities $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$]

$= 4x^2 + 20x + 25 - \left[4x^2 - 20x + 25\right]$

$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$
$= 40x$

(iii)
$(7m-8n)^2 + (7m+8n)^2 = (7m)^2 - 2\times 7m\times 8n + (8n)^2 + \left[(7m)^2 + 2\times 7m\times 8n + (8n)^2\right]$

[Using identities $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$]

$= 49m^2 - 112mn + 64n^2 + \left[49m^2 + 112mn + 64n^2\right]$
$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
$= 98m^2 + 128n^2$

(iv) $(4m+5n)^2 + (5m+4n)^2 = (4m)^2 + 2\times 4m\times 5n + (5n)^2 + (5m)^2 + 2\times 5m\times 4n + (4n)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 16m^2 + 25m^2 + 40mn + 40mn + 25n^2 + 16n^2$
$= 41m^2 + 80mn + 41n^2$

(v) $(2.5p-1.5q)^2 - (1.5p-2.5q)^2$

$= (2.5p)^2 - 2\times 2.5p\times 1.5q + (1.5q)^2 - \left[(1.5p)^2 - 2\times 1.5p\times 2.5q + (2.5q)^2\right]$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 6.25p^2 - 7.50pq + 2.25q^2 - \left[2.25p^2 - 7.50pq + 6.25q^2\right]$
$= 6.25p^2 - 7.50pq + 2.25q^2 - 2.25p^2 + 7.50pq - 6.25q^2$
$= 4p^2 - 4q^2$

(vi) $(ab+bc)^2 - 2ab^2c = (ab)^2 + 2\times ab\times bc + (bc)^2 - 2ab^2c$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$
$= a^2b^2 + b^2c^2$

(vii) $\left(m^2-n^2m\right)^2+2m^3n^2 = \left(m^2\right)^2-2\times m^2\times n^2m+\left(n^2m\right)^2+2m^3n^2$

[Using identity $\left(a-b\right)^2=a^2-2ab+b^2$]

$= m^4-2m^3n^2+n^4m^2+2m^3n^2$

$= m^4+n^4m^2$

Question 5:

Show that:

(i) $\left(3x+7\right)^2-84x=\left(3x-7\right)^2$

(ii) $\left(9p-5q\right)^2+180pq=\left(9p+5q\right)^2$

(iii) $\left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\frac{16}{9}m^2+\frac{9}{16}n^2$

(iv) $\left(4pq+3q\right)^2-\left(4pq-3q\right)^2=48pq^2$

(v) $\left(a-b\right)\left(a+b\right)+\left(b-c\right)\left(b+c\right)+\left(c-a\right)\left(c+a\right)=0$

Answer 5:

(i) L.H.S. $= \left(3x+7\right)^2-84x=\left(3x\right)^2+2\times3x\times7+\left(7\right)^2-84x$

[Using identity $\left(a+b\right)^2=a^2+2ab+b^2$]

$= 9x^2+42x+49-84x$

$= 9x^2-42x+49$

$= \left(3x-7\right)^2$ $\quad[\because\ \left(a-b\right)^2=a^2-2ab+b^2]$

$=$ R.H.S.

(ii) L.H.S. $= \left(9p-5q\right)^2+180pq=\left(9p\right)^2-2\times9p\times5q+\left(5q\right)^2+180pq$

[Using identity $\left(a-b\right)^2=a^2-2ab+b^2$]

$= 81p^2-90pq+25q^2+180pq$

$= 81p^2+90pq+25q^2$

$= \left(9p+5q\right)^2$ $\quad[\because\ \left(a+b\right)^2=a^2+2ab+b^2]$

(iii) L.H.S. $= \left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\left(\frac{4}{3}m\right)^2-2\times\frac{4}{3}m\times\frac{3}{4}n+\left(\frac{3}{4}n\right)^2+2mn$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$=\frac{16}{9}m^2-2mn+\frac{9}{16}n^2+2mn$

$=\frac{16}{9}m^2+\frac{9}{16}n^2$

= R.H.S.

(iv) L.H.S. $=(4pq+3q)^2-(4pq-3q)^2$

$=(4pq)^2+2\times4pq\times3q+(3q)^2-\left[(4pq)^2-2\times4pq\times3q+(3q)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$]

$=16p^2q^2+24pq^2+9q^2-\left[16p^2q^2-24pq^2+9q^2\right]$

$=16p^2q^2+24pq^2+9q^2-16p^2q^2+24pq^2-9q^2$

$=48pq^2$

= R.H.S.

(v) L.H.S. $=(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)$

$=a^2-b^2+b^2-c^2+c^2-a^2$ [Using identity $(a-b)(a+b)=a^2-b^2$]

= 0

= R.H.S.

Question 6:

Using identities, evaluate:

(i)	71^2	(ii)	99^2	(iii)	102^2
(iv)	998^2	(v)	5.2^2	(vi)	297×303
(vii)	78×82	(viii)	8.9^2	(ix)	1.05×9.5

Answer 6:

(i) $71^2 = (70+1)^2 = (70)^2 + 2\times 70\times 1 + (1)^2$

[Using identity $(a+b)^2 = a^2 - 2ab + b^2$]

= 4900 + 140 + 1 = 5041

(ii) $99^2 = (100-1)^2 = (100)^2 - 2\times 100\times 1 + (1)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 10000 – 200 + 1 = 9801

(iii) $102^2 = (100+2)^2 = (100)^2 + 2\times 100\times 2 + (2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 10000 + 400 + 4 = 10404

(iv) $998^2 = (1000-2)^2 = (1000)^2 - 2\times 1000\times 2 + (2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 1000000 – 4000 + 4 = 996004

(v) $5.2^2 = (5+0.2)^2 = (5)^2 + 2\times 5\times 0.2 + (0.2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 25 + 2.0 + 0.04 = 27.04

(vi) 297 x 303 = (300 – 3) x (300 + 3) = $(300)^2 - (3)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 90000 – 9 = 89991

(vii) 78 x 82 = (80 – 2) x (80 + 2) = $(80)^2 - (2)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 6400 – 4 = 6396

(viii) $8.9^2 = (8+0.9)^2 = (8)^2 + 2\times 8\times 0.9 + (0.9)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 64 + 14.4 + 0.81 = 79.21

(ix) 1.05 x 9.5 = (10 + 0.5) x (10 – 0.5) = $(10)^2 - (0.5)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

Question 7:

Using $a^2 - b^2 = (a+b)(a-b)$, find

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$ (iv) $12.1^2 - 7.9^2$

Answer 7:

(i) $51^2 - 49^2 = (51+49)(51-49)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 100 x 2 = 200

(ii) $(1.02)^2 - (0.98)^2 = (1.02+0.98)(1.02-0.98)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 2.00 x 0.04 = 0.08

(iii) $153^2 - 147^2 = (153+147)(153-147)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 300 x 6 = 1800

(iv) $12.1^2 - 7.9^2 = (12.1+7.9)(12.1-7.9)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 20.0 x 4.2 = 84.0 = 84

Question 8:

Using $(x+a)(x+b)=x^2+(a+b)x+ab$, find

(i) 103 x 104
(ii) 5.1 x 5.2
(iii) 103 x 98
(iv) 9.7 x 9.8

Answer 8:

(i) 103 x 104 = (100 + 3) x (100 + 4) = $(100)^2+(3+4)\times 100+3\times 4$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= 10000 + 7 x 100 + 12
= 10000 + 700 + 12 = 10712

(ii) 5.1 x 5.2 = (5 + 0.1) x (5 + 0.2) = $(5)^2+(0.1+0.2)\times 5+0.1\times 0.2$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= 25 + 0.3 x 5 + 0.02
= 25 + 1.5 + 0.02 = 26.52

(iii) 103 x 98 = (100 + 3) x (100 – 2) = $(100)^2+(3-2)\times 100+3\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= 10000 + (3 – 2) x 100 – 6
= 10000 + 100 – 6 = 10094

(iv) 9.7 x 9.8 = (10 – 0.3) x (10 – 0.2)

= $(10)^2+\{(-0.3)+(-0.2)\}\times 10+(-0.3)\times(-0.2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= $100+\{-0.3-0.2\}\times 10+0.06$

= 100 – 0.5 x 10 + 0.06
= 100 – 5 + 0.06 = 95.06