Homework 2

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Q1.

H3

Q1. Height h, vertical velocity $v=\dot{h}$, trust T, mass m, gravity $g>0$

$$\therefore \quad m\ddot{h} = T - mg \Rightarrow \dot{v} = \frac{1}{m}T - g, \quad \dot{h} = v$$

$$x = \begin{bmatrix} h \\ v \end{bmatrix}, \quad u = T$$

$$\therefore \quad \dot{x} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_c} x + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{B_c} u + \underbrace{\begin{bmatrix} 0 \\ -g \end{bmatrix}}_{b_c}$$

Use ZOH

$$\therefore \quad h_{k+1} = h_k + v_k \Delta t + \frac{1}{2}\left(\frac{1}{m}u_k - g\right)\Delta t^2$$

$$V_{k+1} = V_k + \left(\frac{1}{m}V_k - g\right)\Delta t$$

$$\therefore \quad x_{k+1} = A x_k + B u_k + d$$

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\Delta t^2}{2m} \\ \frac{\Delta t}{m} \end{bmatrix}, \quad d = \begin{bmatrix} -\frac{1}{2} g \Delta t^2 \\ -g\Delta t \end{bmatrix}$$

Measurement model

$$z_k = C x_k + v_k, \quad C = [1 \quad 0]$$

Noises and covariances

$$u_k = T_k + n_k^{(u)}, \quad n_k^{(u)} \sim N(0, \Sigma_u), \quad \Sigma_u = \sigma_T^2 (1\times1)$$

$$w_k = B n_k^{(u)}, \quad Q = \mathrm{cov}(w_k) = B \Sigma_u B^T$$

$$v_k \sim N(0, \Sigma_z), \quad \Sigma_z = \sigma_n^2 (1\times1)$$

Final canonical model

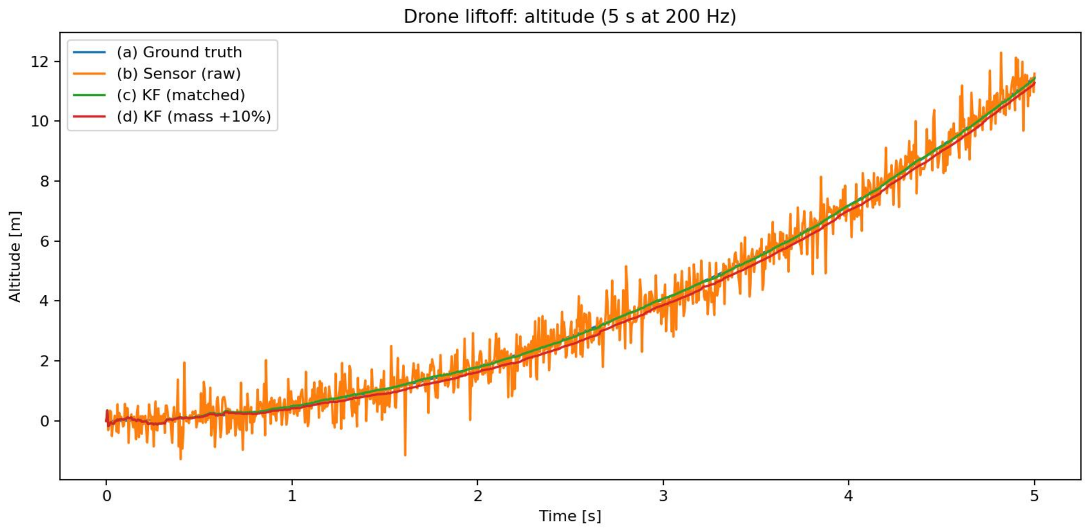
$$x_{k+1} = A x_k + B u_k + d + w_k, \quad w_k \sim N(0, Q)$$

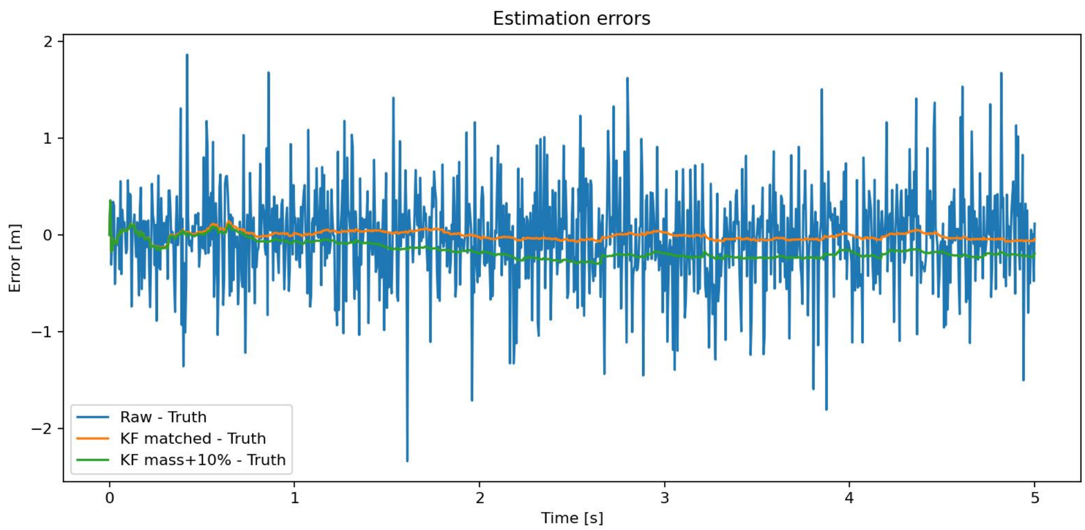
$$z_k = C x_k + v_k, \quad v_k \sim N(0, \Sigma_z)$$

with A, B, C, d as above, $\Sigma_u = \sigma_T^2$, $\Sigma_z = \sigma_n^2$ and $Q = B \Sigma_u B^T$

$\because x \in R^2 \quad \therefore P$ is 2×2

Q2.





If the filter assumes the drone is heavier than it really is (mass +10%), it will think the thrust causes less acceleration. This makes the predicted altitude lower than the real one, so the filter output stays a bit below the true altitude. The sensor updates help reduce the error, but cannot fully remove it, especially when the sensor noise is large.

Answer: A wrong mass value causes the Kalman filter to underestimate altitude and produce a small bias, but the result is still smoother and more accurate than using the raw sensor data.