

1. Use function `random()` from the `random` Python module to estimate the value of  $\pi$ . Your algorithm must be based on the given random generator (you may not use any alternative algorithm for generating the value of  $\pi$ ) and you may not use any other package/library.
2. Covariance matrix for a multivariate random variable is defined as  $\Sigma_{xx} = E((x - \mu_x)(x - \mu_x)^T)$ . The cross covariance matrix for two multivariate random variables is defined as  $K_{xy} = E((x - \mu_x)(y - \mu_y)^T)$ .
  - a. Express the covariance of a sum of two multivariate random variables in terms of covariance and cross-covariance.
  - b. Prove that the cross-covariance of two independent multivariate random variables is a zero-matrix.
  - c. What is the corollary of the above two results?
3. A robot has 10 (unreliable) sensors that independently detect the presence of an object in its intake mechanism. The probability of detecting the object for each sensor is 0.1.
  - a. Write the distribution of the random variable that represents how many sensors have detected the presence of an object?
  - b. What is the probability that at least one sensor has detected the presence of an object?
4. A two-dimensional normally-distributed random variable has the uncertainty ellipse centered at (3, 4) and one principal axis of the ellipse forms a 30-degree angle with the x-axis of the coordinate system. The variance associated with that axis is 1. The variance associated with the other axis is 0.25. Write the probability density function of this random variable.
5. Suppose there is a village in which every family wants to have a boy. If they get a boy, they stop having more children, if they get a girl, they continue to have more children. What is the average number of children per family?