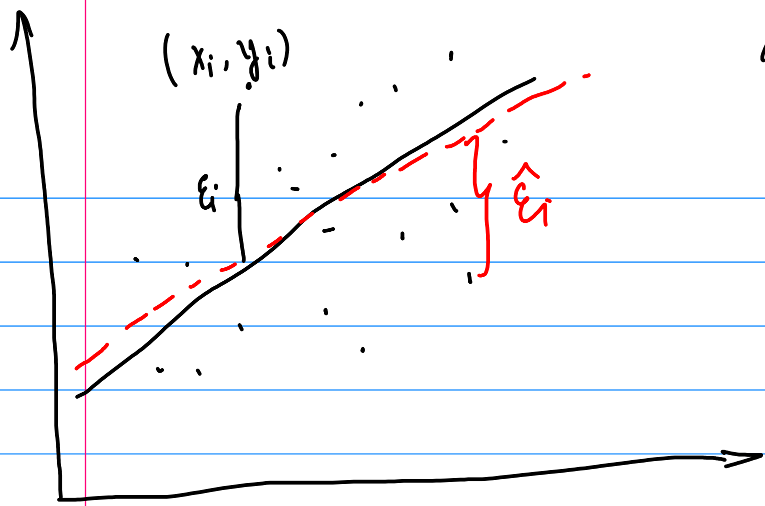

$y_i = \alpha + \beta x_i + \varepsilon_i \quad \varepsilon_i \sim N(0, \sigma_\varepsilon^2)$

βx_i — fixed deterministic; ε_i — error

$y_i = \hat{\alpha} + \hat{\beta} x + \hat{\varepsilon}_i$

$\hat{\varepsilon}_i$ — residuals

$y = \alpha + \beta x$ - true regression line

$y = \hat{\alpha} + \hat{\beta} x$ - estimated regression line

estimators (funct.)

$$\hat{\beta} = \frac{\hat{Cov}(x,y)}{\hat{Var}(x)} = \frac{\cancel{\frac{1}{n-1}} \sum (x_i - \bar{x})(y_i - \bar{y})}{\cancel{\frac{1}{n-1}} \sum (x_i - \bar{x})^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

$\circledast$ estimate (number)

Properties of estimators:

1) Unbiasedness $\quad E(\hat{\theta}) = \theta$

2) Consistency $\quad \underset{n \to \infty}{\text{plim}}\ \hat{\theta} = \theta$

$$\lim_{n \to \infty} P_r\left(|\hat{\theta} - \theta| < \varepsilon\right) = 1$$

- unbiasdeness (at least asympt.)
- $\lim_{n \to \infty} Var(\hat{\theta}) = 0$

3) Efficiency $\quad Var(\hat{\theta}) \leq Var(\tilde{\theta}), \ \forall \tilde{\theta} \in Cl_{LUE}$

Classical Linear Regression Assumption

1) Correct specification : Endogeneity: simultaneity, omitted var., measurement error (inconsistency)

$$y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + \varepsilon_i$$

2) $x_1, \dots, x_k$ - deterministic (fixed), not linearly dependent (no perfect multicollinearity)

3) $E\varepsilon_i = 0$

4) $var(\varepsilon_i) = \sigma^2_\varepsilon$ Heteroscedasticity (efficiency)

5) $cov(\varepsilon_i, \varepsilon_j) = 0$ Autocorrelation (efficiency)

GMT: if 1-5 are true =>

$$\hat{\beta}_{OLS} = \begin{bmatrix} \hat{\beta}_0 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} - \text{BLUE} \quad (\text{Hansen, 2022} \quad \text{BUE})$$

Pr.1. X, Y are independent $\Rightarrow$ $\rho_{X,Y} = 0$?

$$P(X,Y) = P(X)P(Y)$$ independency

$$E(X|Y) = E(X) \qquad E(Y|X) = E(Y)$$ unpredictability

$$E(XY) = E(X)E(Y)$$ uncorrelatedness

$$\rho_{x,y} = \frac{E(XY) - E(X)E(Y)}{\sigma_x \cdot \sigma_y} = 0$$

P2.

X		
	-1	$1/3$
	0	$1/3$
	1	$1/3$

$$Y \equiv X^2$$

Pr. 3 $X \sim \text{law}(\mu, \sigma^2)$

$\{X_1, \dots X_n\}$

$$\hat{\mu} = \frac{n+2}{n^2+3n+1} \sum X_i$$

$\bar{X}$ - consistent estimator

1) $$E(\hat{\mu}) = \frac{n^2+2n}{n^2+3n+1} E\left(\frac{\sum X_i}{n}\right) = \frac{n^2+2n}{n^2+3n+1} \cdot \mu$$

$$\lim_{n\to\infty} E(\hat{\mu}) = \mu$$ asympt. unbiased

2) $$\text{Var}(\hat{\mu}) = \left(\frac{n+2}{n^2+3n+1}\right)^2 \text{Var}(\sum X_i) = \left(\frac{n+2}{n^2+3n+1}\right)^2 n \cdot \sigma^2$$

$$\lim_{n\to\infty} \text{Var}(\hat{\mu}) = 0 \quad \Rightarrow \quad \text{consistent}$$