$y_i = \alpha + \beta X_i + \varepsilon_i$

$\hat{\alpha}, \hat{\beta}$ - OLS

$$\tilde{\beta} = \frac{\sum X_i y_i}{\sum X_i^2}$$

$\tilde{\beta}$ - OLS (assume $\alpha = 0$)

$\hat{\beta} = (X'X)^{-1} X' y$

$Var(\hat{\beta}) = \sigma_\varepsilon^2 \cdot (X'X)^{-1} =$

$$= \begin{bmatrix} Var(\hat{\alpha}) & Cov(\hat{\alpha}, \hat{\beta}) \\ Cov(\hat{\alpha}, \hat{\beta}) & Var(\hat{\beta}) \end{bmatrix}$$

$Var(AX) = A \, Var(X) \, A^T$

$Var(\underbrace{(X'X)^{-1} X'}_{A} y) =$

$((X'X)^{-1} X')^T$

$\shortparallel$

$X((X'X)^{-1})^T$

$\shortparallel$

$X((X'X)^T)^{-1}$

$(X'X)^{-1} X' \, \overset{\sigma^2 \cdot I}{\overset{\shortparallel}{Var(y)}} \, X (X'X)^{-1} =$

$\sigma^2 \cdot (X'X)^{-1}$

$$Var(\hat{\beta}_j) = \frac{\sigma_\varepsilon^2}{TSS_j(1-R_j)}$$

$TSS_j = \sum (x_i - \bar{x})^2$

R_j — R^2 from

$X_j \mid X_{-j}$

$X_j \mid X_1, \dots, X_{j-1}, X_{j+1}, \dots, X_k$

1) noisy data $\Rightarrow \sigma_\varepsilon^2 \uparrow \Rightarrow Var(\hat{\beta}_j) \uparrow$

2) $Var(X) \uparrow \Rightarrow TSS \uparrow \Rightarrow Var(\hat{\beta}_j) \downarrow$

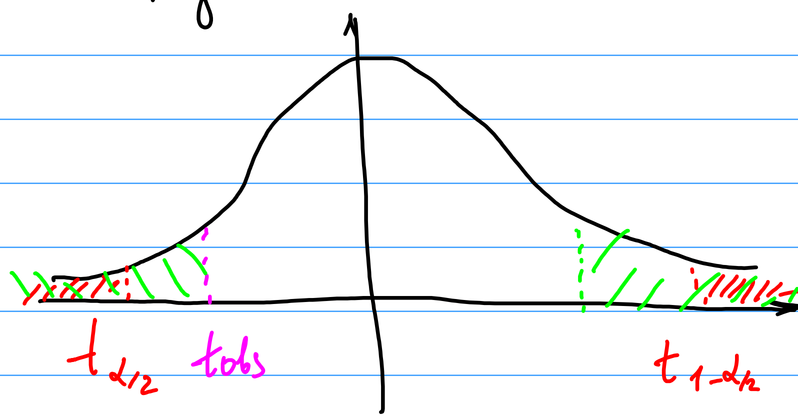
\# obs $\uparrow \Rightarrow Var(\hat{\beta}_j) \downarrow$

3) multicollinearity $\Rightarrow R_j \approx 1 \Rightarrow (1 - R_j) \approx 0 \Rightarrow Var(\hat{\beta}_j)$

Hypothesis Testing

$H_0: \beta = 0$

$H_a: \beta \neq 0$



$$t_{obs} = \frac{\hat{\beta} - 0}{se(\hat{\beta})} \overset{H_0}{\sim} t_{n-k}$$

\#obs   \# est. coefficients

1) $t_{obs} > |t_{crit, \alpha/2}| \Rightarrow H_0$ is rejected

$t_{obs} < |t_{crit, \alpha/2}| \Rightarrow H_0$ is not rejected

2) CI: $\{\hat{\beta} \pm t_{\alpha/2, n-k} \cdot se(\hat{\beta})\}$

3) p_value – min probability / conf. level under which H_0 is not rejected

$$\text{p-value} = 2 \cdot \min\{P_1(t \le t_{obs} \mid H_0);\ P_2(t \ge t_{obs} \mid H_0)\}$$

$p\text{-value} < \alpha \Rightarrow H_0$ is rejected

$p\text{-value} > \alpha \Rightarrow H_0$ is not rejected