

Econometrics, 2022-2023

Class teacher: Ksenia Kasianova.

Class 2: Simple regression analysis.

Problem 1

Derive regression coefficients in a simple regression model

- (a) with intercept
- (b) without intercept.

Regress $Y_i|1, X_i$. True or False:

- (a) $\frac{1}{n} \sum_{i=1}^n \hat{u}_i = 0$,
- (b) $\frac{1}{n} \sum_{i=1}^n \hat{Y}_i = \frac{1}{n} \sum_{i=1}^n Y_i$,
- (c) $\frac{1}{n} \sum_{i=1}^n X_i \hat{u}_i = 0$,
- (d) $\frac{1}{n} \sum_{i=1}^n \hat{Y}_i \hat{u}_i = 0$,
- (e) $TSS = ESS + RSS$.
- (f) How the above analysis changes if you regress $Y_i|1$? $Y_i|X_i$?

Define R^2 . How it is related to

- (a) the residual sum of squares,
- (b) the correlation between the actual and fitted values of the dependent variable, $r_{Y, \hat{Y}}$.
- (c) How would you measure goodness of fit, if you had to choose among RSS, R^2 and $r_{Y, \hat{Y}}$? Why?

Note: Watch β vs $\hat{\beta}$ vs b , residuals vs errors, etc. Provide interpretations and geometric intuition when possible.

Problem 2

Demonstrate that the sample correlation between fitted values of the dependent variable and the residuals equals to zero in a simple regression model. (This result generalizes to the multiple regression case.)

Problem 3

Suppose the data is generated by the model $y_i = \alpha + \beta x_i + \epsilon_i, i = 1, \dots, n$, satisfying the conditions of classic linear regression. $\hat{\alpha}, \hat{\beta}$ – are the OLS estimators. The estimator $\tilde{\beta}$ is obtained using ordinary least squares method with the additional (generally speaking, incorrect) assumption that $\alpha = 0$.

- Find the OLS estimator $\tilde{\beta}$. Under what conditions is it an unbiased estimator of the parameter β ?
- Find the variance of $\tilde{\beta}$, compare it with the variance of $\hat{\beta}$.
- Discuss which of the two estimates is better.

Problem 4

A researcher has international cross-sectional data on aggregate wages, W , aggregate profits, P , and aggregate income, Y , for a sample of n countries. By definition,

$$Y_i = W_i + P_i$$

The regressions

$$\hat{W}_i = \hat{\alpha}_1 + \hat{\alpha}_2 Y_i$$

$$\hat{P}_i = \hat{\beta}_1 + \hat{\beta}_2 Y_i$$

are fitted using OLS regression analysis. Show that the regression coefficients will automatically satisfy the following equations:

$$\hat{\alpha}_2 + \hat{\beta}_2 = 1$$

$$\hat{\alpha}_1 + \hat{\beta}_1 = 0$$

Explain intuitively why this should be so.