$y_i = \alpha + \beta x_i + \varepsilon_i$ $\qquad$ $\hat{\alpha}, \hat{\beta}$ - OLS

$\tilde{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$ $\qquad$ (assume $\alpha = 0$)

$Var(\tilde{\beta}) < Var(\hat{\beta})$

if $\alpha = 0$ $\qquad$ $\tilde{\beta}$ - more efficient

if $\alpha \neq 0$ $\qquad$ $\hat{\beta}$ — // -

Variance of $\hat{\beta}$

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

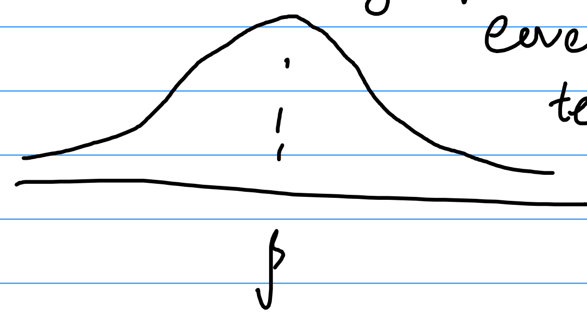
$$Var(\hat{\beta}) = \frac{\sigma^2}{\Sigma (X_i - \bar{X})^2} \qquad \hat{\sigma}_{\varepsilon}^2 \text{ instead}$$

$$Var(\hat{\beta}) = \frac{1}{n} \frac{\frac{1}{n-2} \cdot \Sigma (X_i - \bar{X})^2 \varepsilon_i^2}{\left[\Sigma (X_i - \bar{X})^2 \right]^2}$$

α % - significance level test

$H_0: \beta = 0$

$H_a: \beta \neq 0$



$$t = \frac{\hat{\beta} - \beta}{se(\hat{\beta})} \sim t_{n-k}$$

n ← # obs

k ← # est coefficients

$\updownarrow$

CI : $\left[\hat{\beta} \pm t_{\alpha/2,\, n-k} \cdot se(\hat{\beta}) \right]$

$(1-\alpha)$% confidence level

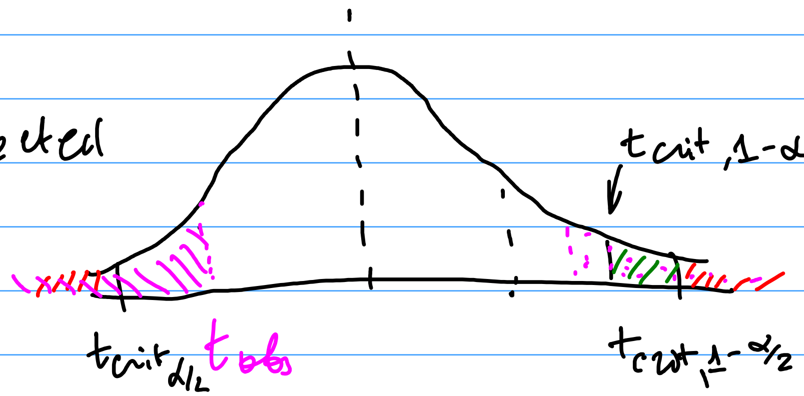
$H_0: \beta = 0 \iff 0 \in CI$

$t > |t_{crit}| \iff H_0$ is rejected $\iff$ p-value $< \alpha$

$t < |t_{crit}| \iff H_0$ is not rejected $\iff$ p-value $> \alpha$

p-value - min conf. level under which hyp. is rejected

$H_1: \beta \neq 0$



$$\text{p-value} = 2\min\{P\{t_{obs} \geq t \mid H_0\}, P\{t_{obs} \leq t \mid H_0\}\}$$

α — 1% ; 5% $\leftarrow$ standard level (reject) $\nearrow$ reject ; 10%

$\hat{\beta} = (X'X)^{-1}X'y$ $\quad$ $Var(AX) = A\,Var(X)\,A'$

$Var(\hat{\beta}) = Var(\underbrace{(X'X)^{-1}X'}y) =$

$(X'X)^{-1}X'\,Var(\varepsilon)\,X(X'X)^{-1} =$

$\sigma^2 (X'X)^{-1}$

$Var(\hat{\beta}) = \sigma^2 (X'X)^{-1}$

$$Var(\hat{\beta}_j) = \frac{\sigma^2}{TSS_j \cdot (1 - R_j^2)}$$

$TSS_j = \sum (x_j - \bar{x}_j)^2$

R_j^2 : R^2 from $x_j \mid x_{-j}$

1) $\sigma_\varepsilon^2 \downarrow \Rightarrow Var(\hat{\beta}) \downarrow$

2) $n \uparrow \Rightarrow TSS_j \uparrow \Rightarrow Var(\hat{\beta}) \downarrow$

$Var(x_j) \uparrow \Rightarrow Var(\hat{\beta}_j) \downarrow$

3) x - correlated (multicollinearity) $\Rightarrow$

$R_j \approx 1 \Rightarrow 1 - R_j \approx 0 \Rightarrow Var(\hat{\beta}_j)$