$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$\hat{\alpha}, \hat{\beta}$ - OLS

$\tilde{\beta}$ - OLS ($\alpha = 0$ is assumed)

$$\tilde{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$Var(\tilde{\beta}) = Var\left(\frac{\sum x_i y_i}{\sum x_i^2}\right) = Var\left(\frac{\sum x_i(\alpha + \beta x_i + \varepsilon_i)}{\sum x_i^2}\right) =$$

$$\frac{Var\left(\sum x_i \alpha + \sum x_i^2 \beta + \sum x_i \varepsilon_i\right)}{(\sum x_i^2)^2} = \frac{\sum x_i^2 Var(\varepsilon_i^2)}{(\sum x_i)^2} = \frac{\sigma^2}{\sum x_i^2}$$

$$Var(\hat{\beta}) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2} = \frac{\sigma^2}{\sum x_i^2 - n\bar{x}^2}$$

$$Var(\tilde{\beta}) < Var(\hat{\beta})$$

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$Var((X'X)^{-1}X'y) =$$

$$\left\{\begin{array}{l} Var(AX) = A\,Var(X)A' \\ Var(y) = \sigma^2 \cdot I \end{array}\right.$$

$$(X'X)^{-1} X' Var(y) X (X'X)^{-1} = \sigma^2 (X'X)^{-1}$$

$$\hat{Var}(\hat{\beta}) = \begin{bmatrix} \hat{Var}(\hat{\alpha}) & \hat{Cov}(\hat{\alpha},\hat{\beta}) \\ \hat{Cov}(\hat{\alpha},\hat{\beta}) & \hat{Var}(\hat{\beta}) \end{bmatrix} = \begin{bmatrix} \frac{\sigma^2 \sum x_i^2}{n\sum(x_i - \bar{x})^2} & \frac{-\bar{x}\sigma^2}{\sum(x_i - \bar{x})^2} \\ \frac{-\bar{x}\sigma^2}{\sum(x_i - \bar{x})^2} & \frac{\sigma^2}{\sum(x_i - \bar{x})^2} \end{bmatrix}$$

$$\left\{ \hat{Var}(\hat{\beta}_j) = \frac{\sigma_\varepsilon^2}{TSS_j(1-R_j)} \right\}$$

$0 \in CI$

$\Updownarrow$

$H_0: \beta = 0$

$$\hat{Var}(\hat{\beta}) = \frac{\hat{\sigma}_\varepsilon^2}{TSS_j(1-R_j)}$$

$TSS_j = \sum (x_{j,i} - \bar{x}_j)^2$

R_j — R^2 from $x_j \mid X_{-j}$

1) $\sigma_\varepsilon^2 \uparrow$ (noisy data) $\Rightarrow Var(\hat{\beta}_j) \uparrow$

2) $\hat{Var}(X) \uparrow \Rightarrow TSS_j \uparrow \Rightarrow Var(\hat{\beta}_j) \downarrow$

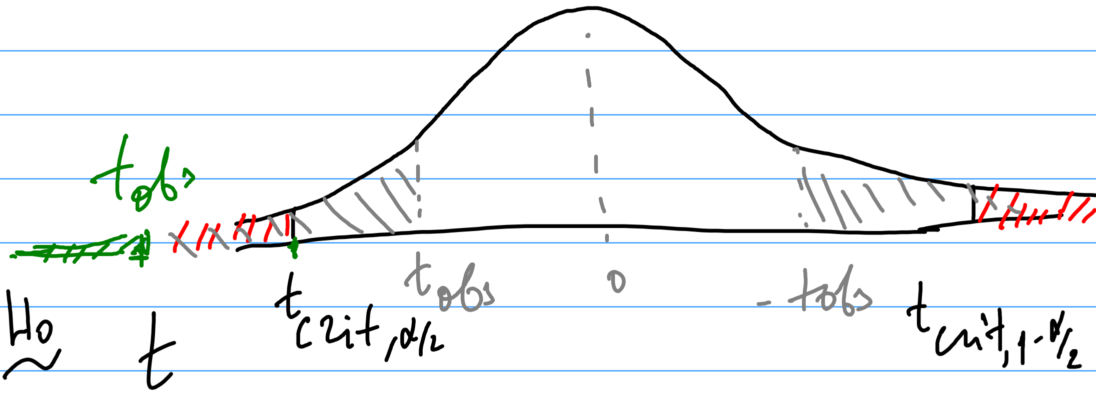
$n\# \uparrow \Rightarrow TSS_j \uparrow \nearrow$

3) R_j (regressors correlated, multicollinearity) $\approx 1 \Rightarrow$

$(1 - R_j) \approx 0 \Rightarrow Var(\hat{\beta}_j) \uparrow$

$H_0: \beta = 0$

$H_a: \beta \neq 0$



$$t_{obs} = \frac{\hat{\beta} - 0}{se(\hat{\beta})} \overset{H_0}{\sim} t_{n-k}$$

n: # obs

k: # est coefficient

1) $t_{obs} > |t_{crit}| \Rightarrow H_0$ is rejected

$t_{obs} < |t_{crit}| \Rightarrow H_0$ is not rejected

2) CI: $\left(\hat{\beta} \pm t_{\alpha/2, n-k} \cdot se(\hat{\beta})\right)$

3) p-value - min significance level under which H_0 is not rejected

$$p\text{-value} = 2 \cdot \min\{P(t \geq t_{obs} | H_0), P(t \leq t_{obs} | H_0)\}$$

p-value $< \alpha \Rightarrow H_0$ is rej

p-value $> \alpha \Rightarrow H_0$ is not rej