书面作业 第5次 参考解答或提示

第1部分 基础

无

第2部分 理论

无

第3部分 综合应用（T2 可选做）

T1. 形式化并证明下列自然语言推理是有效的. 个体域均为全总域.

(1) 所有的玫瑰和蔷薇都是芳香带刺的, 因此, 所有的玫瑰都是带刺的.

(2) 三角函数都是周期函数; 一些三角函数是连续函数; 所以, 一些周期函数是连续函数.

(3) 每个科学工作者都是勤奋的, 每个既勤奋又聪明的人在他的事业中都将获得成功, 王大志是科学工作者并且是聪明的, 所以, 王大志在他的事业中将获得成功.

(4) 如果一个人长期吸烟或酗酒, 那么他身体绝不会健康; 如果一个人身体不健康, 那么他就不能参加体育比赛;有人参加了体育比赛. 所以, 有人不长期酗酒.

(5) 小王是一年级生理科生; 每个非文科的一年级生都有辅导员; 凡小王的辅导员都是理科生; 所有的理科生都不是文科生. 所以, 至少有一个不是文科生的辅导员.

(1) 设 $S(x)$:x是三角函数; $T(x)$:x是周期函数; $P(x)$:x是连续函数, 则推理可以形式化为:

$\forall x(S(x) \rightarrow T(x)), \exists x(S(x) \wedge P(x)) \Rightarrow \exists x(T(x) \wedge P(x))$.

证明:

(1) $\exists x(S(x) \wedge P(x))$ P

(2) $S(c) \wedge P(c)$ ES(1)

(3) $\forall x (S(x) \rightarrow T(x))$ P

(4) $S(c) \rightarrow T(c)$ US(3)

(5) $S(c)$ T,I(2)

(6) $T(c)$ T,I(4)(5)

(7) $P(c)$ T,I(2)

(8) $T(c) \wedge P(c)$ T,I(6)(7)

(9) $\exists x(T(x) \wedge P(x))$ EG(8)

(2) 设 $P(x)$:x是玫瑰; $Q(x)$:x是蔷薇; $R(x)$:x是芳香的; $S(x)$:x是带刺的, 则推理可以形式化为:

$\forall x((P(x) \vee Q(x)) \rightarrow (R(x) \wedge S(x))) \Rightarrow \forall x(P(x) \rightarrow S(x))$

证明: 可以直接证明, 也可以用CP规则, 以及反证法.

直接法

(1) ∀x((P(x)∨Q(x))→(R(x)∧S(x)))   P

(2) P(x)∨Q(x)→R(x)∧S(x)   US(1)

(3) (¬P(x)∧¬Q(x))∨(R(x)∧S(x))   T,I(2)

(4) (¬P(x)∨S(x))∧(¬P(x)∨R(x))∧(¬Q(x)∨S(x))∧(¬Q(x)∨S(x))   T,I(3)

(5) ¬P(x)∨S(x)   T,I(4)

(6) P(x)→S(x)   T,I(5)

∀x(P(x)→S(x)) UG,5

CP规则

(1) ∀x((P(x)∨Q(x))→(R(x)∧S(x)))   P

(2) P(x)∨Q(x)→R(x)∧S(x)   US(1)

(3) P(x)   P(附加，for: P(x)→S(x)）

(4) P(x)∨Q(x)   T,I(3)

(5) R(x)∧S(x)   T,I(2)(4)

(6) S(x)   T,I(5)

(7) P(x)→S(x)   CP(3)(6)

(8) ∀x(P(x)→S(x))   UG(7)

反证法(略).

(3) 令M(x)：x是人；K(x)：x是科学工作者；Q(x)：x勤奋；T(x)：x聪明；S(x)：x将获得成功；a：王大志，则推理可以形式化为：

∀x((M(x)∧K(x))→Q(x)), ∀x((M(x)∧Q(x)∧T(x))→S(x)), M(a)∧K(a)∧T(a)⇒S(a)

证明：

1) M(a)∧K(a)∧T(a)   P

2) ∀x((M(x)∧K(x))→Q(x))   P

3) (M(a)∧K(a))→Q(a)   US,(2)

4) M(a)∧K(a)   T,I,(1)

5) Q(a)   T,I,(2),(4)

6) M(a)∧T(a)   T,I,(1)

7) M(a)∧Q(a)∧T(a)   T,I,(5),(6)

8) ∀x((M(x)∧Q(x)∧T(x))→S(x))   P

9) (M(a)∧Q(a)∧T(a))→S(a)   US,(8)

10) S(a)   T,I,(7),(9)

(4) 令M(x)：x是人；C(x)：x长期吸烟；K(x)：x长期酗酒；J(x)：x身体健康；P(x)：x能参加体育比赛，则推理可以形式化为：

∀x((M(x)∧(C(x)∨K(x)))→¬J(x)), ∀x((M(x)∧¬J(x))→¬P(x)), ∃x(M(x)∧P(x)) ⇒ ∃x(M(x)∧¬K(x))

证明：

1)	∃x(M(x)∧P(x))	P
2)	M(c)∧P(c)	ES,(1)
3)	∀x((M(x)∧¬J(x))→¬P(x))	P
4)	(M(c)∧¬J(c))→¬P(c)	US,(3)
5)	P(c)	T,I,(2)
6)	¬(M(c)∧¬J(c))	T,I,(4),(5)
7)	¬M(c)∨J(c)	R,E,(6)
8)	M(c)	T,I,(2)
9)	J(c)	T,I,(7),(8)
10)	∀x((M(x)∧(C(x)∨K(x)))→¬J(x))	P
11)	(M(c)∧(C(c)∨K(c)))→¬J(c)	US,(10)
12)	¬(M(c)∧(C(c)∨K(c)))	T,I,(9),(11)
13)	¬M(c)∨(¬C(c)∧¬K(c))	R,E,(12)
14)	¬C(c)∧¬K(c)	T,I,(8),(13)
15)	¬K(c)	T,I,(14)
16)	M(c)∧¬K(c)	T,I,(8),(15)
17)	∃x(M(x)∧¬K(x))	EG,(16)

（5）令a：小王；S(x)：x是一年级生；L(x)：x是理科生；W(x)：x是文科生；F(x,y)：x是y的辅导员，则推理可以形式化为：

∀x(S(x)∧¬W(x)→∃yF(y,x)), S(a), L(a), ∀x(F(x, a)→L(x)), ∀x(L(x)→¬W(x)) ⇒ ∃x∃y(¬W(x)∧F(x, y)).

证明：

(1)	∀x(L(x)→¬W(x))	P
(2)	L(a)→¬W(a)	US,(1)
(3)	L(a)	P
(4)	¬W(a)	T,I,(2),(3)
(5)	S(a)	P
(6)	S(a)∧¬W(a)	T,I,(4),(5)
(7)	∀x(S(x)∧¬W(x)→∃yF(y,x))	P
(8)	S(a)∧¬W(a)→∃yF(y, a)	US,(7)
(9)	∃yF(y, a)	T,I,(6),(8)
(10)	F(c, a)	ES,(9)
(11)	∀x(F(x, a)→L(x))	P

(12) F(c, a)→L(c)   US,(11)

(13) L(c)   T,I,(10),(12)

(14) L(c)→¬W(c)   US,(1)

(15) ¬W(c)   T,I,(13),(14)

(16) ¬W(c)∧F(c, a)   T,I,(10),(15)

(17) ∃y(¬W(c)∧F(c, y))   EG,(16)

(18) ∃x∃y(¬W(x)∧F(x, y))   EG,(17)

因此，该推理是有效的.

T2. Consider the following problem. We know that horses are faster than dogs and that there is a greyhound that is faster than every rabbit. We know that Harry is a horse and that Ralph is a rabbit. Our job is to derive the fact that Harry is faster than Ralph (可以应用直接证明方法与归结法. 注意要增加两个公认的事实：Greyhound是Dog，速度可以传递比较的).

Problem translated in FOPL:

∀x ∀y ((Horse(x) ∧ Dog(y)) → Faster(x,y))

∃y (Greyhound(y) ∧ (∀z Rabbit(z) → Faster(y,z)))

Horse(Harry)

Rabbit(Ralph)

Derive the following fact:

Faster(Harry, Ralph)

Added axioms to represent commonsense knowledge:

∀y (Greyhound(y) → Dog(y))

∀x ∀y ∀z ((Faster(x,y) ∧ Faster(y,z)) → Faster(x,z))

Proving using Proof Theory and a set of inference rules

1. ∀x ∀y Horse(x) ∧ Dog(y) → Faster(x, y)   Premise

2. ∃y Greyhound(y) ∧ (∀z Rabbit(z)→ Faster(y,z))   Premise

3. ∀y Greyhound(y) → Dog(y)   Premise

4. ∀x∀y∀z Faster(x, y) ∧ Faster(y, z) → Faster(x, z)   Premise

5. Horse(Harry)   Premise

6. Rabbit(Ralph)   Premise

7. Greyhound(Greg) ∧ (∀z Rabbit(z) → Faster(Greg,z))   ES (2)

8. Greyhound(Greg)   T,I (7)

9. ∀z Rabbit(z) → Faster(Greg,z))   T,I (7)

10. Rabbit(Ralph) → Faster(Greg,Ralph)   US (9)

11. Faster(Greg,Ralph)   T,I (6)(10)

12. Greyhound(Greg) → Dog(Greg)   US (3)

13. Dog(Greg)   T,I (12)(8)

14. Horse(Harry) ∧ Dog(Greg) → Faster(Harry, Greg)   US (1)

15. Horse(Harry) ∧ Dog(Greg)   T,I (5)(13)

16. Faster(Harry, Greg)   T,I (14)(15)

17. Faster(Harry, Greg) ∧ Faster(Greg, Ralph) → Faster(Harry,Ralph)   US (4)

18. Faster(Harry, Greg) ∧ Faster(Greg, Ralph)   T,I (11)(16)

19. Faster(Harry,Ralph)   T,I (17)(19)

QED.

Using Resolution(归结法) to determine logical entailment

1. $\{\neg \text{Horse}(x), \neg \text{Dog}(y), \text{Faster}(x,y)\}$ Premise

2. $\{\text{Greyhound}(\text{Greg})\}$ Premise

3. $\{\neg \text{Rabbit}(z), \text{Faster}(\text{Greg}, z)\}$ Premise

4. $\{\neg \text{Greyhound}(y), \text{Dog}(y)\}$ Premise

5. $\{\neg \text{Faster}(x,y),\ \neg \text{Faster}(y,z), \text{Faster}(x,z)\}$ Premise

6. $\{\text{Horse}(\text{Harry})\}$ Premise

7. $\{\text{Rabbit}(\text{Ralph})\}$ Premise

8. $\{\neg \text{Faster}(\text{Harry}, \text{Ralph})\}$ Negated Goal

9. $\{\text{Dog}(\text{Greg})\}$ (2)(4)归结

10. $\{\neg \text{Dog}(y),\ \text{Faster}(\text{Harry}, y)\}$ (6)(1)归结

11. $\{\text{Faster}(\text{Harry}, \text{Greg})\}$ (9)(10)归结

12. $\{\text{Faster}(\text{Greg}, \text{Ralph})\}$ (7)(3)归结

13. $\{\neg \text{Faster}(\text{Greg}, z), \text{Faster}(\text{Harry}, z)\}$ (11)(5)归结

14. $\{\text{Faster}(\text{Harry}, \text{Ralph})\}$ (12)(13)归结

15. $\{\}$ (14)(8)归结

QED.