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Ambiguity and Rationality

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ABSTRACT

Recently, several theories of decision making and probability judgment have been proposed that take into account ambiguity (Einhorn and Hogarth, 1985; Gardenfors and Sahlin, 1982). However, none of these theories explains exactly what the psychological causes of ambiguity are or addresses the issue of whether ambiguity effects are rational. In this paper, we define ambiguity as the subjective experience of missing information relevant to a prediction. We show how this definition can explain why ambiguity affects decisions in the ways it does. We argue that there are a variety of rational reasons ambiguity affects probability judgments and choices in the ways it does. However, we argue that the ambiguity effect does not cast doubt on the claim that utility theory is a standard of rational choice. Rather, we suggest that the effect of ambiguity on decisions highlights the fact that utility theory, like any normative model of decision making only prescribes the optimal decision, *given what one knows*.

KEY WORDS Probability Ambiguity Rationality Savage axioms

Recently, the topic of ambiguity has received substantial attention from researchers in judgment and decision making (Curley, Yates and Abrams, 1986; Einhorn and Hogarth, 1985; Lopes, 1983). Ambiguity refers to a particular type of uncertainty that often exists in decision making and judgment situations. Although all probability judgments reflect uncertainty, one can talk about the uncertainty of the probability judgment itself. For example, the judgment that the probability of a fair coin coming up heads is 0.5 is a 'certain' probability judgment. In contrast, an 'ambiguous' probability judgment is the probability of a black marble being drawn from an urn filled with black and white marbles, in an unknown ratio. For someone who thinks that high proportions of black balls (e.g. 90 per cent) are as likely as the corresponding low proportion (10 per cent), the probability of drawing a black marble from the urn is 0.5 (Roberts, 1963). However, to many people, this situation is different from a situation where the proportion of black marbles is *known* to be 50 per cent (Ellsberg, 1961).

Ambiguity has implications for both normative and descriptive theories of decision making (Gardenfors and Sahlin, 1982, 1983; Lopes, 1983). Several authors have argued that utility theory as presented by Savage (1954) should be revised as a normative model to take into account ambiguity (Ellsberg, 1961; Gardenfors and Sahlin, 1982). In addition, there have been several attempts to develop descriptive models that can account for the effects of ambiguity on probability judgment and choice (Einhorn and Hogarth, 1985; Gardenfors and Sahlin, 1983).

In this paper, we shall try to clarify the nature of ambiguity and discuss its implications for normative models of decision making. First, we shall summarize the psychological research on ambiguity and discuss theories of subjective probability judgment and choice that have been proposed to account for its effects. Next, we define ambiguity in a way that allows us to explain its effects on judgment and

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choice. Finally, we discuss the implications of ambiguity for the concept of subjective probability and for the normative status of utility theory.

AMBIGUITY: A BRIEF REVIEW

In 1954, Savage presented an extremely influential theory of decision making. His theory consisted of three related claims:

1. The rule of maximizing expected utility is a normative rule of decision making;
2. Subjective probabilities can be defined in terms of preferences among different gambles;
3. Utility theory describes people's behavior.

First, Savage showed how the principle of maximizing expected utility followed from a set of intuitively compelling axioms. Thus, Savage presented a strong justification of utility theory as a normative model. Currently, there is much controversy about whether utility theory is a normative model of decision making (Gardenfors and Sahlin, 1982; Sahlin, 1987; Shafer, 1986).

Second, Savage showed that subjective probabilities could be defined in terms of preferences. By defining probabilities in terms of preferences, Savage was able to develop the concept of subjective probability in a way that was acceptable to behaviorally oriented theoreticians. While Savage's theory led to the acceptance of the subjective view of probability, his method of measuring probabilities is no longer the only way or even the most popular one. Although there has been much debate about the validity of asking people for probabilities *directly* (Marschak, 1975), this type of measurement has come to be widely accepted. Much research on judgment and decision making involves asking people for probabilities directly (see Kahneman, Slovic, and Tversky, 1982).

Finally, Savage proposed his theory as a descriptive model of choice under uncertainty. In Savage's (1954) theory, an individual's choice in a situation is a function of utilities and probabilities, where probabilities are one's best estimate of the likelihood of states of the world. Thus, a crucial implication of this theory is that there is no meaningful distinction between 'known risk' and 'uncertainty'. However, subsequent research on decision making under uncertainty showed that subjects often distinguish between risk and uncertainty. Ellsberg (1961) showed that people often preferred to bet on 'known' probabilities, instead of 'ambiguous' probabilities. Thus, Ellsberg's (1961) finding provided strong evidence against the claim that utility theory is descriptively correct.

Ellsberg (1961) proposed that people might be using a decision rule other than that proposed by Savage. Specifically, Ellsberg proposed that a person's choice is a function of the probability of events, the utility of events and the ambiguity. Ellsberg suggested that ambiguity was determined by the reliability and consistency of information or the amount of disagreement among judges. According to Ellsberg, ambiguity reflects '... the nature of one's information concerning the relative likelihood of events' (p. 657). Ellsberg did not specify the exact conditions that lead to ambiguity (Becker and Brownson, 1964).

Ellsberg proposed a decision rule that takes into account both the actual expected value of an act and the minimum expected value (given the worst possible 'probability judgment'). For example, consider the choice between the following two gambles:

G1: a black marble will be drawn from an urn with 50 black marbles and 50 white marbles
G2: a black marble will be drawn from an urn with 100 marbles, between 0 and 100 are black

The expected values of the two gambles are the same. However, the minimum expected value of G2 is 0. That is, it is possible that the urn in G2 contains no black marbles. Ellsberg's rule would predict that people would choose G1. Roberts (1963) argued that it was unlikely that people followed Ellsberg's rule, because it is even more complex than Savage's rule, and requires the assessment of more quantities. Furthermore, Yates and Zukowski (1976) have shown that the model is not an accurate description of subjects' preferences.

Gardenfors and Sahlin (1982) have proposed a model of decision making that is a generalization of Bayesian decision theory but which takes the reliability of probability judgments into account. While Gardenfors and Sahlin do not explicitly define ambiguity, they introduce the concept of 'epistemic reliability' which is related. They state '... the less relevant information the agent has about the states of nature, the less epistemic reliability will be ascribed to the distributions in P (p. 367)'. In their model, the agent calculates the epistemic reliability of each possible probability distribution, and then considers only those distributions that have an epistemic reliability above some threshold. Next, the agent calculates the 'minimal expected utility' of each alternative, which is the lowest expected utility of the alternative for all possible probability distributions. The decision rule states that the alternative with the largest minimal expected utility is selected. Ambiguity arises when more than one distribution is above the threshold (the minimum reliability of the distributions one will consider).

Gardenfors and Sahlin (1982) explain how two gambles can be equivalent from the perspective of standard utility theory, but still differ in 'epistemic' riskiness. Their model includes two types of risk. First, a decision may be risky because the decision makes does not know which of a set of possible 'states of nature' will obtain. This is the usual sense of risk which is also taken into account in standard utility theory. Second, a decision may be risky because the decision maker does not consider every possible state of nature. The second type of risk is described in the following example:

> ... people do not usually check whether there is too little brake fluid in the car or whether the wheels are loose before starting to drive, although, for all they know, such events are not impossible, and they realise that if any such event occurred they would be in danger. (pp. 369–370)

That is, one type of risk arises because there is uncertainty about which state of the world will obtain. The second type of risk arises because one does not consider every possible state of the world that could obtain. In the above example, one may decide that the prior probability of the wheels being loose is sufficiently low such that the cost of checking them is not worth it. However, if the perceived cost of an accident increased, it might become worthwhile to check. In general, their model predicts that as the perceived size of a possible loss increases, the threshold for including a probability distribution will decrease (i.e. one will include more conceivable probability distributions that are based on weak evidence).

Einhorn and Hogarth (1985) present a descriptive model of subjective probability judgment that differs from the models described thus far in that it assumes that ambiguity affects choices through its effect on probability judgments. In Einhorn and Hogarth's (1985) model, the subjective probability of an event is a function of (a) an initial judgment; (b) perceived ambiguity and (c) optimism–pessimism attitude. Perceived ambiguity is a function of the absolute amount of evidence available, unreliability of sources and the lack of causal knowledge regarding the process that generates outcomes. Einhorn and Hogarth (1985) also state that

> ambiguity results from the uncertainty associated with specifying *which* of a set of distributions is appropriate in a given situation. Moreover, the amount of ambiguity is an increasing function of the number of distributions that are not ruled out (or made implausible) by one's knowledge of the situation (p. 435).

While Einhorn and Hogarth's model describes ambiguity in psychological terms, it is somewhat unsatisyfing as an explanatory theory of ambiguity. That is, their description of ambiguity does not provide insight into *why* ambiguity affects judgment and choice in the ways it does. We present a definition of ambiguity which is consistent with previous conceptualizations but which enables us to explain why ambiguity has the effects it does.

AMBIGUITY AS MISSING INFORMATION

We define ambiguity as the subjective experience of missing information relevant to a prediction. In Ellsberg's situation, one is missing the information 'proportion of black marbles in the urn'. If one had this information, one's estimate of the probability of drawing a black marble from the urn would almost definitely change. In contrast, consider an urn known to contain 50 black marbles and 50 white marbles. There seems to be no information that would change one's estimate of the probability of drawing a black marble from the urn.

This definition is related to but not synonymous with another concept, *the weight of evidence.* This refers to the absolute amount of evidence one has (Cohen, 1977; Keynes, 1921; and Shafer, 1976, develops a similar concept). In contrast, the *implication* of evidence is the effect of the evidence on alternative hypotheses (in Bayesian terms, the likelihood or likelihood ratio). Peirce (1932) described this distinction as follows:

> ... to express the proper state of belief, not *one* number but *two* are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based (p. 421).

Bayesian probability theory does not take the weight of evidence into account (Cohen, 1977). Rather, it specifies the rational judgment, *given what you know.* That is, it is a method for calculating the implication of evidence.

Our definition of ambiguity is related to the idea of the weight of evidence as follows. In general, if the weight of evidence is low, one will be able to imagine missing information, and if the weight is high one will not, although there are exceptions in both cases. For example, in a trial, one may be convinced that the little testimony one has heard is all there is to hear, or one may hear a great deal of testimony (which would, on its own, suffice), but still want very much to hear one witness who is still holding out. We stress the subjective nature of ambiguity because *all* probability judgments reflect missing information, if only information about future events or the very event at issue. They differ with respect to the salience of the missing information. Information may be more salient if it is available in principle at the time a decision is made.

Explanation of causes and effects

Ambiguity has several effects on probability judgments and choices:

1. Probability judgments do not add up to 1 when the situation is ambiguous (subadditivity) (Einhorn and Hogarth, 1985).
2. Most subjects prefer non-ambiguous gambles, except at very low probabilities (Einhorn and Hogarth, 1985; Ellsberg, 1961).
3. There is a preference for ambiguity at low probabilities (Einhorn and Hogarth, 1985).
4. Ambiguity causes people to be unwilling to act (Curley *et al.*, 1986).

Our definition of ambiguity suggests a variety of reasons people may avoid ambiguity. Following Roberts (1963) and Curley *et al.* (1986), we propose that there are several reasons why, in general, ambiguity affects behavior in the ways listed above.

1. When a probability judgment is ambiguous, there is a possibility that an opponent will know more than you, and therefore will have an advantage. This argues against options involving ambiguity.
2. When a probability judgment is ambiguous, there may be a hostile opponent who will bias the situation to your disadvantage. This also argues against ambiguous options.
3. In the long run, a series of identical ambiguous gambles (in which the missing information turns out the same) are more risky than a series of non-ambiguous gambles. For example, imagine an urn that contains either 100 black marbles or 100 white marbles. You must pick one color to bet on for 100 trials. In this situation, you will either win 100 times or lose 100 times. In contrast, imagine the same bet for an urn with 50 black marbles and 50 white marbles. In this case, you will come close to breaking even. This also argues against ambiguous options.
4. There is the possibility of waiting and obtaining more information. This argues against making a choice from among the options given. If each option is compared to 'delay', ambiguous options will be rated lower.
5. Issues of blame, responsibility and regret are more salient in situations of ambiguity. In particular, a person may be blamed (rightly or wrongly, depending on whether the missing information could be usefully obtained) for not obtaining the missing information. This also argues against making a choice.
6. When the current 'best guess' probability is close to 1 or 0, it is easier to imagine further information making it less extreme that more extreme. This argues in *favor* of ambiguous options with very low probabilities of desirable outcomes or very high probabilities of undesirable outcomes, other things equal.

These six reasons lead to the following heuristic: in general, it is a good rule of thumb to avoid making decisions when there is missing information. This heuristic may be extended to cases in which there appears to be missing information, even if none of these reasons applies.

Note that our definition is fully consistent with the findings of Einhorn and Hogarth (1985) concerning the effects of ambiguity on subjective probability. They found that subjective probability judgments were regressive under conditions of ambiguity. On our account, when one thinks about the effect of missing information on one's probability estimate, one can imagine being moved in either direction, but it may be easier to imagine being moved toward 0.5 than toward 1 or 0.

Our definition of ambiguity also explains what various causes of ambiguity have in common. Einhorn and Hogarth (1985) state that ambiguity is high when one's sample is small or when witnesses are unreliable. When the sample size is small, it is easy to imagine gaining more information (i.e. a larger sample). When a source is unreliable, one imagines a reliable source. Curley *et al.* (1986) found that the ambiguity effect increased when there was the potential for negative evaluation by others. This effect is due to the fact that when information becomes available after a decision is made, we are often evaluated as if we should have known it, even if it was not available at the time of the decision (Baron and Hershey, in press). If people are aware of this type of evaluation, they will be hesitant to make decisions when they are missing information that is likely to become available at a later time.

In sum, defining ambiguity as the perception of missing information relevant to a probability judgment enables us to account for the effects of ambiguity and explain what the various sources of ambiguity have in common. In the next section, we show how our view of ambiguity can shed light on the debate about the normative status of utility theory.

NORMATIVE IMPLICATIONS OF AMBIGUITY

Our definiton of ambiguity can clarify two theoretical issues in decision theory: the relation between objective and subjective probabilities and the normative status of Bayesian decision theory.

Objective vs. subjective probabilities

There has been much debate among philosophers and mathematicians about the nature of probability judgments (Hacking, 1975). One issue is whether there are different kinds of probability judgments. An obvious distinction is between objective and subjective probabilities, where objective probabilities are based on frequency data or logical analysis and subjective probabilities are based on other types of evidence.

Clearly there is a strong intuition that probabilities based on frequency information or logical analysis are very different from those based on other types of evidence. The question is whether these types of probabilities should be treated differently in decisions.

There are several reasons it is desirable to propose that all probability judgments are subjective, that is, they are all estimates relative to one's current state of information. First, this view enables one to combine information from a variety of different sources, such as frequency data and personal testimony. Second, this view suggests that a single decision rule applies in all situations. Thus, this view is more parsimonious. Third, on this view, all methods of probability assessment may be evaluated by the same criteria of calibration or scoring rules (Baron, in press; Lindley, 1986; Von Winterfeldt and Edwards, 1986).

Our definition of ambiguity enables us to explain people's intuitions that certain probability judgments appear to be objective (i.e. not disputable), without having to postulate different 'types' of probabilities. Probability judgments based on frequency data appear to be immune to revision while those based on personal opinion often appear to be very likely to be revised in the light of new evidence. Thus, all probability judgments are relative to the evidence one has at a given time. Objective probabilities are those for which we believe the evidence we have is (for all practical purposes) complete.

One's judgment of the probability of heads in a coin toss appears to be objective because one does not believe there is more information available that would enable one to make a better prediction. That is, short of the outcome (heads or tails) there is no information one can imagine obtaining that would change the estimate. In contrast, consider the case of a doctor's estimate that a patient has cancer. Since one can imagine a great deal of information the doctor might obtain that would change his estimate of the patient's likelihood of having cancer, this estimte seems more subjective.

We have argued that all probability judgments are subjective and relative to a judge's existing state of knowledge and that an ambiguous probability estimate is one for which the missing information is particularly salient to the judge. It is interesting to note that Ellsberg was unclear about the distinction between subjective probabilities and ambiguity. The examples of non-ambiguous probabilities he presented were a coin toss and a roulette wheel. For ambiguous probabilities his examples were the performance of a new president and the tactics of an unfamiliar opponent. While these examples illustrate the distinction between ambiguous and non-ambiguous probabilities, they are misleading in suggesting that non-ambiguous probabilities are based on frequency information and ambiguous probabilities are based on other types of information.

The subjective theory of probability does not reflect the extent to which one believes that one's current state of information is complete. By postulating that ambiguity reflects a judge's belief about the completeness of information, we can accommodate the intuition that some probability judgments seem more certain than others, without claiming that there are different kinds of probability judgments.

Rationality of the ambiguity effect

Is the ambiguity effect evidence of a limitation of people (which arises from the over-use of heuristics) or of a limitation of utility theory? We have listed several reasons why, in general, it is a useful heuristic to avoid or postpone decisions based on little information. What if these conditions are absent? Are there any reasons one should take ambiguity into account as a general policy?

In addition to arguments given by Roberts (1963) and Raiffa (1961) against such a policy, there is another argument that derives from our definition of ambiguity. In essence, ambiguity effects are framing effects. We can often (possibly always) transform an ambiguous situation into an unambiguous one, or an unambiguous one into an ambiguous one by calling attention toward or away from certain missing information.

Consider the first transformation, from ambiguous to unambiguous. To make this, assume that the decision in question is one of a sequence of identical (but independent) decisions. For example, one must bet on either color in an ambiguous red/blue urn 100 or 1000 times, with a different ambiguous urn being used each time. In this case, over a long series of such bets, one will win about half the time, just as with an unambiguous urn. Does it do any injustice to the original problem, then, to describe it as a choice in which one has a 0.5 chance of winning? The way in which the 0.5 chance comes about does not seem to matter when we think about the long run probability. That information is irrelevant.

Consider the second transformation, from unambiguous to ambiguous. In any 'unambiguous' situation, our ignorance of relevant facts is simply shoved under the rug. In an urn with 50 red and 50 blue balls, for example, it is reasonable to think that the ball drawn will be drawn from the top layer (or from some other layer). If the top layer contains only 10 balls, any number of these might be red, from 0 to 10. We may thus describe this situation as one in which there is an unknown proportion of red balls. With a bit of imagination, we claim, any 'unambiguous' situation may be transformed in this way. The perception of 'having all the information' may be an illusion resulting from a lack of imagination about what information one *could* have (the contents of the top layer, the path of the coin in the air, etc.)

In sum, although there are good reasons for taking the completeness of one's information into account, when these reasons are absent, when missing information is unavailable to the decision maker or to anyone who might use it against her, there may be no general justification for taking missing information into account.

In contrast to the view presented here, some authors have suggested that ambiguity poses a serious threat to the normative status of utility theory (Gardenfors and Sahlin, 1982; Sahlin, 1987; Slovic and Tversky, 1974).

Gardenfors and Sahlin (1982) have argued that the ambiguity effect is rational and that utility theory is an inappropriate normative model. They state:

> Within strict Bayesianism it is assumed that [one's] beliefs can be represented by a single probability measure defined over the states of nature. This assumption is very strong since it amounts to the agent having *complete* information in the sense that he is *certain* of the probabilities of the possible states of nature. The assumption is unrealistic ... (p. 364).

Gardenfors and Sahlin (1982) present a generalization of the Bayesian model which takes into account the completeness of one's information. They propose that an agent considers a set of possible probability distributions which are above some threshold of epistemic reliability.

We do not dispute Gardenfors and Sahlin's claim that the completeness and reliability of information is an important factor in decisions, and one that is sometimes neglected by Bayesian decision theorists. However, their claim that Bayesianism assumes complete information conflicts with our interpretation of Bayesian theory. The power of the Bayesian view derives from the fact that it requires that one assign a number reflecting the particular evidence one has at the time. There is no requirement that one be certain of the estimate except in the sense that one believes that it reflects all of the evidence one has considered.

Thus, ambiguity highlights an important limitation of Bayesian decision theory: it can prescribe the optimal decision only relative to what one knows. In particular, when one's evidence is not very good, there is an upper limit on the quality of one's decision.

A crucial issue is whether this limitation is specific to Savage's version of utility theory or is a limitation of any normative model of decision making. It would seem that any normative model of decision making will have this limitation. The likelihood that a decision leads to desired outcomes is a function of both the amount of evidence used and the procedures one applies. A decision made on the basis of complete, consistent, reliable information is more likely to lead to desired outcomes than one based on scanty, unreliable information. However, 'taking into account' the fact that your evidence is weak will not do much good. Changing the formal model will not fix problems caused by a lack of relevant evidence.

SUMMARY

We have argued that ambiguity arises from the perception of missing information relevant to a probability judgment. We have shown how this view of ambiguity can account for the empirical findings on the specific causes and effects of ambiguity.

We have also shown how our conception of ambiguity can resolve two debates in the field of decision theory. First, we have argued that there is no clear distinction between objective and subjective probabilities. Objective probabilities differ from subjective probability judgments in that they appear to be immune to revision. A more useful way to represent the difference is to say that all probability judgments are subjective, but differ in terms of the completeness of the information upon which they are based.

Second, we have argued that the ambiguity effect does not cast doubt on the normative status of utility theory. Rather, it highlights an important limitation of the model (and arguably of any model), namely that the optimal decision will always be conditional upon the information one has available at the time.

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