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Comparative Ignorance and the Ellsberg Paradox

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Abstract

We investigate the evaluation of known (where probability is known) and unknown (where probability is unknown) bets in comparative and non-comparative contexts. A series of experiments support the finding that ambiguity avoidance persists in both comparative and non-comparative conditions. The price difference between known and unknown bets is, however, larger in a comparative evaluation than in separate evaluation. Our results are consistent with Fox and Tversky's (1995) Comparative Ignorance Hypothesis, but we find that the strong result obtained by Fox and Tversky is more fragile and the complete disappearance of ambiguity aversion in non-comparative condition may not be as robust as Fox and Tversky had supposed.

Keywords: ambiguity, Ellsberg Paradox, comparative ignorance

1. Introduction

The Ellsberg Paradox demonstrates that people prefer to bet on known rather than unknown probabilities (Ellsberg, 1961). Essentially, the example given by Ellsberg illustrates that people prefer to bet on the outcome of an urn that contains 50 yellow and 50 white balls rather than the outcome of an urn that contains 100 yellow and white balls, but in an unknown proportion. In the former bet, the probability of drawing a yellow or white ball is precisely 0.5, while in the latter bet this probability is ambiguous or vague. Presumably, people like the specificity of probability to vagueness in probability. The distinction between known and unknown probabilities dates back at least to Knight (1921), with his risk versus uncertainty dichotomy. Keynes (1921) also argues that people will be more willing to act in situations in which the probability is supported by a larger weight of evidence.

Several researchers, including Becker and Brownson (1964), Slovic and Tversky (1974) and MacCrimmon and Larsson (1979) found strong support for ambiguity avoidance.

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Ambiguity about probabilities was manipulated both as a second order ditsribution (Curley and Yates, 1985; Kahn and Sarin, 1988; Boiney, 1993) and as a lack of familiarity with an event, such as Pierce Industries stock price will go up or down (MacCrimmon, 1965). The ambiguity avoidance was confirmed with sophisticated subjects (Hogarth and Kunreuther, 1989) as well as in experimental market settings (Sarin and Weber, 1993). Though ambiguity avoidance for modest probabilities and gains is generally observed, several studies (see Viscusi and Chesson, 1999) have demonstrated both ambiguity-avoiding and ambiguity-seeking behavior. Camerer and Weber (1992) provide a review of the literature on decisions under ambiguity.

In an important study, Heath and Tversky (1991) demonstrated that people prefer to bet on events about which they feel more knowledgeable or competent. Heath and Tversky (1991) found, for example, that people prefer to bet on events such as "football" or "politics," which supposedly have vague or unknown probabilities, over matched chance events. Fox and Tversky (1995) extended these results by showing that the perception of knowledge can be manipulated by a suggested comparison to others who are more knowledgeable.

The Ellsberg Paradox was first discovered when the two bets were evaluated jointly. Many experiments were conducted by varying the Ellsberg's problem (e.g. manipulating the degree of vagueness, source of preference, etc.) to study the ambiguity behavior. A remarkable result that Fox and Tversky (1995) obtain is that the ambiguity aversion disappears in a non-comparative context in which a person evaluates either an ambiguous or an unambiguous bet separately. Thus, in their view the Ellsberg phenomenon is an inherently comparative effect and it does not arise in an independent or separate evaluation of uncertain prospects. In this paper, we examine whether ambiguity aversion disappears in a non-comparative context.

Our finding is that the strong result obtained by Fox and Tversky (1995) is more fragile and the complete disappearance of ambiguity aversion in non-comparative condition may not be as robust as Fox and Tversky had supposed. Our findings are, however, consistent with their comparative ignorance hypothesis which implies that in a comparative condition the knowledge difference between the known and the unknown bet becomes more salient and thus the comparison accentuates the relative difference between the worths of the known and the unknown bets.

2. Evaluation of known versus unknown probability

We report the results of four experiments that provide evidence on whether ambiguity avoidance disappears in a non-comparative setting. Unless stated, all statistical tests are t-tests for the difference in means and we report the selected p-values. Subjects who did not respond or violated dominance (reported a price of \$0 or \$100) are excluded from analysis. Subjects were undergraduates or MBA students from UCLA. None of the subjects participated in more than one study.

Study 1

We replicated Fox and Tversky (1996)'s Study 1 with two major differences: the elicitation method used and the motivation mode. In Fox and Tversky's (1996) Study 1, the willingness to pay was elicited, whereas we elicited the willingness to accept. Our subjects had a chance to play their selected choice, whereas subjects in Fox and Tversky's study received course credit. In our study, the Becker, Degroot, and Marshak's (1964) procedure was used to elicit the willingness to accept the bets and at the end of the experiment three students were randomly selected to play the game for real money. The following Ellsberg's two-color problem was presented to 130 undergraduate students at UCLA.

There are two bags, labeled as Bag A and Bag B on the table. Bag A is filled with exactly 50 yellow balls and 50 white balls. Bag B is filled with 100 balls that are yellow and white but you do not know their relative proportion.

	Bag A		Bag B
50	yellow balls	?	yellow balls
50	white balls	?	white balls
100	total balls	100	total balls

First, you are to guess a color (white or yellow). Next, without looking, you are to draw a ball from either Bag A or Bag B. If you draw the ball matching the color you guessed, then you will win $100; otherwise you win nothing. What is the smallest amount of money that you would accept rather than play this game with Bag A? with Bag B?

The questionnaire contained no other items except the Ellsberg's two-color problem similar to that of Fox and Tversky's (1996) Study 1 (as shown). The experiment was conducted in one classroom where the subjects were assigned at random to three groups of approximately equal size. Group 1 ($n = 45$) priced the clear bet; Group 2 ($n = 43$) priced the vague bet; and Group 3 ($n = 42$) priced both clear and vague bets (the order in which the two bets were presented was counterbalanced). One subject from Group 3 was eliminated because he violated dominance. We informed the students that three of them would get the chance to play the game for real money and the Becker, Degroot and Marschak (1964) payment procedure was described to them at the beginning of the experiment. They did not know that the experiment contained three conditions. Thus, a subject only priced whatever bet was stated on the questionnaire without knowing what bet(s) the other student was evaluating. After all responses were collected, one subject from each group was randomly chosen to play the game for real money. The results are presented in Table 1.

Under comparative condition, we found that the clear bet is priced higher on average than the vague bet ($p < 0.01$). Under non-comparative condition, again the clear bet is priced higher than the vague bet ($p < 0.05$). This interaction is not significant ($z = 0.86$, n.s.).

Table 1. Mean willingness to accept for clear and vague bets under comparative and non-comparative conditions (Study 1)

	Comparative condition	Non-comparative condition
Clear Bet	\$53.46, $n = 41$, SE = 2.95	\$43.62, $n = 45$, SE = 2.98
Vague Bet	\$38.39, $n = 41$, SE = 3.07	\$34.17, $n = 43$, SE = 3.30

Figure 1 shows that the price differential between clear and vague bets is smaller in the non-comparative condition (\$9.45) than in the comparative condition (\$15.07). The ambiguity effect is thus reduced in the non-comparative condition. The main conclusion of our Study 1 is that ambiguity aversion does not disappear, but is significantly reduced in the context when the bets are evaluated separately.

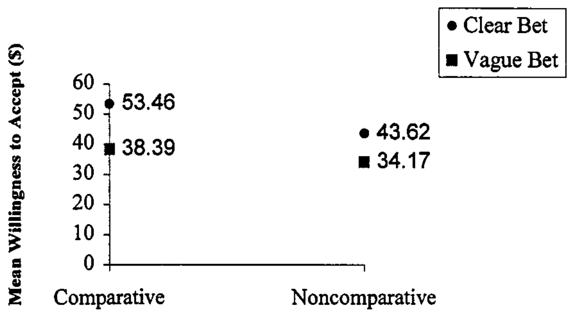

Figure 1. Mean willingness to accept for clear and vague bets under comparative and non-comparative conditions (Study 1).

Study 2

In this study, we examine the effect of ambiguity in the non-comparative setting alone. We did not employ the comparative condition here because the presence of ambiguity avoidance has been demonstrated in our Study 1, as well as in numerous other studies where both clear and vague bets are evaluated jointly. Further, in this study, the subjects indicate their willingness to pay rather than the willingness to accept as in Study 1. The study was conducted both with MBA subjects as well as undergraduate subjects. The instructions and the questionnaire given to both MBA and undergraduate subjects were identical.

MBA subjects were recruited from two separate sections. Students in one section ($n = 31$) were asked to price the vague bet; whereas, students in the other section ($n = 27$) were asked to price the clear bet. Specifically, they were asked to indicate the maximum amount that they were willing to pay to play the game with Bag A or Bag B, as described in Study 1. Each section was given the description of either the clear or the vague bet, but not both. The respective bag filled with white and yellow

Ping-Pong balls was actually shown to the subjects. Thus, the subjects had the visual aid; however, they did not play the game with real money. Since students who evaluate clear and vague bets are in separate classrooms and are shown only one bag (Bag A or Bag B), the comparison between clear and vague bets cannot be easily induced.

Undergraduate students were recruited from one class ($n = 110$) where they were randomly divided into two groups of approximately equal size. Each group of undergraduate students was asked to evaluate only one bet (either clear or vague bet). The subjects indicated the maximum amount that they were willing to pay to play the game with the bag described to them. The bag was not shown to them, so they only had the description and not the visual aid. Four undergraduate responses were discarded because of non-response (2 subjects) or dominance violation (2 subjects). Subjects did not play the game with real money. Since all undergraduate subjects are placed in the same classroom, there is a possibility that, in spite of our instructions, some students may peek at another student's questionnaire, thus inducing comparison. The results of our study with MBA and undergraduate subjects, along with the results of Fox and Tversky's (1995) Study 1, are reported in Table 2.

The key finding of our study is that both MBA and undergraduate students price the clear bet higher than the vague bet ($p < 0.01$). These results are therefore consistent with the hypothesis that people prefer the known uncertainty and exhibit an aversion to the unknown uncertainty. It is well documented that willingness to pay is typically observed to be much lower than willingness to accept. Therefore, it is not surprising that the average prices observed in this study are lower than in our Study 1.

Table 2. The mean willingness to pay (WTP) for clear and vague bets under the non-comparative condition (Study 2)

	WTP (MBA)	WTP (Undergraduate)	WTP (Fox and Tversky Study 1)
Clear Bet	\$32.89, $n = 27$, SE = 3.6	\$28.21, $n = 52$, SE=2.7	\$17.94, $n = 35$, SE = 2.5
Vague Bet	\$14.32, $n = 31$, SE = 3.33	\$12.24, $n = 54$, SE=2.27	\$18.42, $n = 39$, SE = 2.87
Subjects	MBA at UCLA	Undergrads at UCLA	Undergrads at Stanford

Study 3

In the previous two studies, the mean probability was 0.5. Ellsberg introduced a three-color problem where the mean probabilities are $1/3$ and $2/3$. The purpose of this study was to test ambiguity avoidance for the three-color problem. In this study, we placed 111 first-year undergraduate students in one classroom where we have three groups performing the comparative and non-comparative tasks. Due to the nature of the experimental design, we did not show the bags to the subjects. Similar to Study 1, we described the Becker, Degroot and Marshak payment procedure and informed them that three persons will be randomly selected to play the game. The subjects were asked to state the

Table 3. Ellsberg's three-color problem (Study 3)

	(10 balls)	(20 balls)	
Bet	Red ball ($)	White ball ($)	Yellow ball ($)
f_1 Clear	100	0	0
g_1 Vague	0	100	0
f_2 Clear	0	100	100
g_2 Vague	100	0	100

minimum amount they are willing to accept for the clear and vague bets displayed in Table 3.

The subjects were randomly divided into three groups. Group 1 ($n = 33$) evaluated all four bets jointly. One subject was discarded from this group because he violated dominance. Group 2 ($n = 37$) evaluated the two clear bets: f_1 and f_2. Four subjects were discarded because their responses were internally inconsistent. Two subjects responded to f_1, but did not respond to f_2. Group 3, ($n = 41$) evaluated the two vague bets: g_1 and g_2. Three subjects were discarded because their responses were internally inconsistent. The order of the bets was counterbalanced. Three randomly chosen subjects, one from each group, played the game with real money. The results are reported in Table 4.

The results show that under the comparative condition, the average price for the clear bet is higher than that for the corresponding vague bet for both $p = 1/3$ and $p = 2/3$ cases ($p < 0.01$). In the non-comparative setting, the average price for the clear bet is higher than that for the vague bet for $p = 2/3$ ($p < 0.01$); however, for $p = 1/3$ ambiguity avoidance cannot be statistically established ($p < 0.15$). The price differentials between clear and vague bets are shown in Figure 2. This study is a replication of Fox and Tversky's (1995) Study 3. Fox and Tversky (1995) also found ambiguity avoidance at the higher probability level, but not at the lower probability level. The three-color problem is inherently comparative, as the description of each bet requires a discussion of both vague and clear probability events.

Table 4. Mean willingness to accept for clear and vague bets under comparative and non-comparative conditions (Study 3)

	Comparative condition	Non-comparative condition
f_1 Clear Bet (1/3)	\$33.17, $n = 32$, SE = 2.72	\$30.12, $n = 33$, SE = 3.33
g_1 Vague Bet (1/3)	\$25.05, $n = 32$, SE = 2.56	\$25.59, $n = 38$, SE = 2.72
f_2 Clear Bet (2/3)	\$51.77, $n = 32$, SE = 3.79	\$50.13, $n = 31$, SE = 4.50
g_2 Vague Bet (2/3)	\$38.00, $n = 32$, SE = 3.47	\$39.84, $n = 38$, SE = 2.97

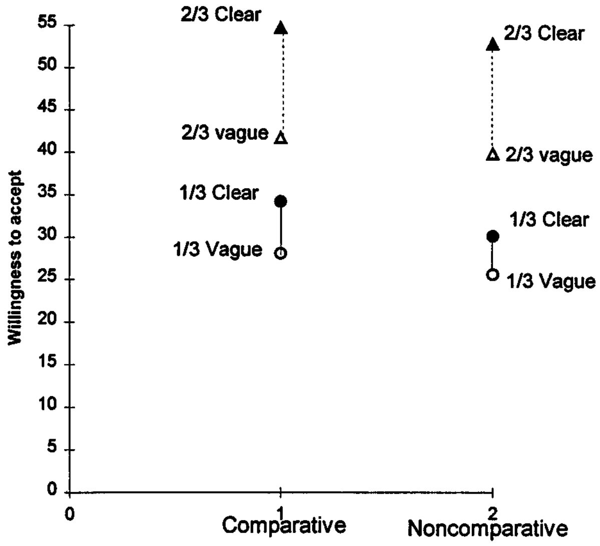

Figure 2. Three-color problem under the comparative and non-comparative conditions (Study 3).

Study 4

In this study, 119 MBA students from 2 separate sections were randomly divided into four groups (2 groups per section). Each group read the description of only one of the four bets in the Ellsberg three-color problem. Recall that in Study 3 subjects evaluate two bets (f_1 and f_2 or g_1 and g_2) even in the non-comparative condition. The two groups in one section evaluated clear bets (f_1 or f_2). The two groups in the other section evaluated vague bets (g_1 and g_2). Since the subjects who evaluated the clear bet were in a separate classroom than those who evaluated the vague bet, an inadvertent comparison is not possible. Thus, comparative ignorance cannot be induced by one subject peeking at the questionnaire of another subject. The subjects were asked to state the maximum amount of money they would be willing to pay for the bet. One response was discarded because of dominance violation. The results are summarized in Table 5.

Table 5. The mean willingness to pay (WTP) for clear and vague bets under the non-comparative condition (Study 4)

	Non-comparative condition
Clear Bet (1/3)	\$25.20, $n = 25$, SE = 2.33
Vague Bet (1/3)	\$14.90, $n = 35$, SE = 2.88
Clear Bet (2/3)	\$44.00, $n = 27$, SE = 4.46
Vague Bet (2/3)	\$27.40, $n = 31$, SE = 3.79

The average price of the clear bet is significantly higher than that of the vague bet for both the low probability and the high probability bets ($p < 0.01$). Thus, our results support ambiguity avoidance even in non-comparative setting. In Study 3, the average price for the less probable ($p = 1/3$) vague bet was not significantly lower than the clear bet (\$25.59 versus \$30.12) in the non-comparative condition. In Study 4, however, the vague bet is priced significantly lower than the clear bet (\$14.90 versus \$25.20). It is possible that in Study 4, the subjects who are MBA students were able to make a spontaneous comparison of the unknown bag with the known bag even though they were given the description of only one bag (either clear or vague). Alternatively, when the noise (variation in prices among subjects) to signal (mean price) ratio is high, sometimes the differences cannot be established at a high significance level.

3. Comparative versus non-comparative

The results of our studies, reported in Section 2, suggest that people prefer the known uncertainty to the unknown uncertainty. The prices for clear bets (known probability) are higher than those for vague bets (unknown probability) in all cases. In some cases the price differential between clear and vague bets is statistically not significant at the usual stringent levels of 0.05.

The key finding that emerges from our experiments is that comparison enhances the difference in prices between clear and vague bets. In absence of a direct comparison (non-comparative condition) this difference is smaller but it does not disappear. This reduction in price differential between the clear and vague bets in the non-comparative condition is not evidence against ambiguity avoidance. We conjecture that when one uses two clear bets—one with probability equal to 1/3 and the other with probability equal to 2/3—the difference in observed prices will be larger for the comparative condition than that for the non-comparative condition. This is investigated in Study 5.

Study 5

The following hypothetical problem was presented to 127 first-year MBA students from two sections at UCLA.

> *Imagine that there is a bag on the table (Bag A) filled with exactly 20 white balls and 10 yellow balls, and a second bag (Bag B) filled with exactly 10 white balls and 20 yellow balls. You will draw a single ball from one of the bags without looking. If the ball you draw is white you win \$100. If it is yellow, you receive nothing.*

	Bag A		Bag B
20	white balls	10	white balls
10	yellow balls	20	yellow balls
30	total balls	30	total balls

Suppose that you are offered a ticket to play this game. What is the most that you would pay for a ticket to play such a game for each of the bags?

1. *The most that I would be willing to pay for a ticket to Bag A (i.e. drawing a ball from Bag A)?* $______
2. *The most that I would be willing to pay for a ticket to Bag B (i.e. drawing a ball from Bag B)?* $______

Students in one section ($n = 63$) evaluated both bets (comparative task) and the order of the bets was counterbalanced. Students in the other section ($n = 64$) performed the non-comparative task where approximately half of the subjects evaluated Bag A and the other half evaluated Bag B. The results are displayed in Table 6.

Table 6. Mean willingness to pay for bets with probability 1/3 and 2/3 under comparative and non-comparative conditions (Study 5)

	Comparative condition	Non-comparative condition
Bet with 1/3 probability	\$17.49, $n = 63$, SE = 1.64	\$20.87, $n = 30$, SE = 2.69
Bet with 2/3 probability	\$35.47, $n = 63$, SE = 3.28	\$27.41, $n = 34$, SE = 3.99

In the comparative condition, the mean price of the 2/3 bet is larger than that of the 1/3 bet ($p < 0.01$). In the non-comparative condition, the 2/3 bet is priced higher than the 1/3 bet, but the difference in the mean prices is not statistically significant. The interaction is significant ($z = 1.894$, $p < 0.05$). The price differential between the 1/3 probability and the 2/3 probability bets is shown in Figure 3. The results support our conjecture that in separate evaluations prices sometimes cannot be discriminated. Therefore, the claim that two options are equal in attractiveness because their prices are not significantly different in separate evaluations may be too strong.

Fox and Tversky (1995, Study 5) did not find the difference between bets contingent on future temperature in Palo Alto under the comparative condition to be larger than that under the non-comparative condition. In their study, however, Bet A (\$25.77) strongly

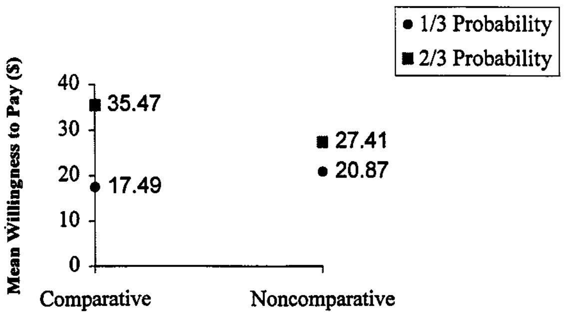

Figure 3. Mean willingness to pay for 1/3 and 2/3 probability bets under comparative and non-comparative conditions (Study 5).

dominates Bet B ($6.42); therefore, a comparison does not enhance the difference in attractiveness.

4. Discussion

The key finding that emerges from our experiments is that the clear bet is priced higher than the vague bet under both comparative and non-comparative conditions. The comparison, however, enhances the difference in prices between clear and vague bets. In the absence of a direct comparison (non-comparative condition) this difference is smaller, but it does not disappear. This reduction in price differential between the clear and vague bets in the non-comparative condition is not evidence against ambiguity avoidance. Our results do not support the strong conclusion of Fox and Tversky (1995) that ambiguity aversion disappears in separate evaluations. Why is then the price difference larger in the comparative condition than in the non-comparative condition? We believe that several factors explain this interaction. The first contributing factor is that the information advantage of the known bet over the unknown bet is made vivid in a comparative context. In a separate evaluation, the vagueness becomes a secondary criterion and is therefore not significantly weighted. This observation is supported by Fox and Tversky's comparative ignorance hypothesis. The second factor is the Evaluability Hypothesis (Hsee, 1996; Hsee et al., 1999) which says that ignorance is an implicit attribute of uncertain prospects, which is easier to evaluate in comparative contexts than in separate evaluations. Finally, discriminability between two options will be more pronounced in a joint evaluation than in separate evaluations when one option serves as an easy reference point to evaluate the other option. Notice that when two options are far apart (Rolls-Royce and Ford Pinto), then one option may not serve as a reference point to evaluate the other option. Because of discriminability, when two bets are close in attractiveness then the difference in their relative prices will be larger in a joint evaluation than in separate evaluations. Similarly, the subjective temperature difference between two pails of water—one at 70°F and the other at 75°F—would seem larger in a simultaneous comparison than in a comparison inferred through separate evaluations. In our Study 5, where there is no comparative ignorance effect, the difference in the mean price of a 2/3 bet and a 1/3 bet is significantly larger under a comparative condition than under a non-comparative condition. This result is consistent with discriminability. These three factors are intertwined and in our studies it was not possible to tease apart their individual contribution to prices. The cumulative influence of comparative ignorance, evaluability, and discriminability is that ambiguity avoidance is generally observed in a comparative setting and sometimes also in settings that are not explicitly comparative.

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