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Introduction

A team of conservation biologists is researching two populations of organisms, trying to ensure that the populations can sustain themselves for a long time, without either going extinct or overwhelming the ecosystem. The team has hired you as a mathematical analyst to model the populations and make recommendations about how to stabilize their growth. This job is not all about just crunching numbers! Realistic solutions to the issues facing the populations will require some imagination and research.

In Project 2, you will build on your knowledge of matrix population models to study real-world populations. Before you start this project, you may find it useful to review Project 1 and the feedback you received on it.

Research different populations of the same species from [COMADRE or COMPADRE](), databases which compile matrix population models developed by scientists. Matrices in the database are labeled with values of "λ." One of your populations should have $\lambda < 1$ and one should have $\lambda > 1$. These λs are the largest positive eigenvalue (the so-called "dominant eigenvalue") of the transition matrix, and their significance will become clear as you perform your analysis. If you need a refresher on how matrix population models work, refer back to [Project 1: Population Analysis]().

Be sure to consult with your instructor about your choice, though (you will have to solidify your choice by the end of Checkpoint 2). The model has to be complex enough to yield an interesting analysis, but not so complex as to make calculations intractable.

Checkpoints

This project consists of material that you will learn throughout Units 3, 4, and 2. You will have three checkpoints for this project; you must complete each checkpoint before you can move on to the next.

[Project 2 Checkpoint 1]()

It is recommended that you complete Checkpoint 1 at the beginning of Unit 4. For this checkpoint, you are going to provide information about how you intend to present your project.

[Project 2 Checkpoint 2]()

For Checkpoint 2, you will provide your instructor will information about your progress on the project by answering questions. It is recommended that you complete this checkpoint at the beginning of Unit 5. Please wait for feedback from your instructor before moving on to Checkpoint 3.

[Project 2 Submission (Checkpoint 3)]()

For Checkpoint 3, you will submit your completed project and complete a self-reflection survey. This checkpoint should be completed at the end of Unit 5.

Instructions

Your project should consist of five parts: Introduction, Mathematical Methods, Analysis, Conclusion, and References. Refer to the **Project Resources** page for helpful information on data analysis, mathematical writing, and evaluating internet sources.

Introduction

Describe the populations you are studying, how they are modeled, and what questions or hypotheses guided your investigation. If your populations are affected by specific environmental factors (for example, overfishing or habitat loss), summarize that context in this section.

Mathematical Methods

In this section, provide detailed descriptions explaining each of the following topics to an audience that is familiar with linear algebra but not with matrix population models. You may find it useful to use specific examples of population models from outside your study — you can invent these, find them on COMPADRE, or use matrices from Project 1.

1. Matrix Population Models: What are population vectors and transition matrices? How do you make predictions with the model? Are transition matrices always invertible?

2. Long-Term Growth Rate: Give a numerical example showing how to estimate a population's long-term growth rate $G = \lim_{t\to\infty} \frac{n(t+1)}{n(t)}$, where $n(t)$ is the total population at time t. Then describe how to compute G exactly, with an explanation of your method. What does the value of G mean for the stability of the population? How is G related to the eigenvalues, determinant, and rank of a transition matrix?

3. Long-Term Population Distribution: How can you estimate the long-term proportions of, say, seedlings to adults in a model? Give a numerical example. Does the long-term distribution of the population depend on what initial population you feed into the model? To answer this last question, consider the following:

   > Call a population vector *population-proportional* if all its components stay in proportion under transformation by the transition matrix. Consider the set S of linear combinations of these population-proportional population vectors. Is S a subspace? How does the transition matrix affect S? Can you say anything about the dimension of S? Construct a 4×4 transition matrix that has two "qualitatively different" kinds of population-proportional populations and another which has only one. (Tip: it may help to draw a flowchart of the transitions between life stages.)

4. Linear Transformations and Rank: If we understand the transition matrix as representing a linear transformation, what space does it transform? What is the significance of the rank of the transition matrix in terms of understanding population structure and long-term population dynamics?

5. Sensitivity Analysis: A sensitivity analysis can tell you how much a population's long-term growth rate depends on the various entries of the transition matrix. The sensitivity can be computed approximately as $\frac{\Delta G}{\Delta a}$, where ΔG is the change in the growth rate when matrix entry a is changed by a small amount (and the other entries stay fixed). Perform a sensitivity analysis on the transition matrix of your model from Project 1 to illustrate the method. If you're feeling adventurous, you can use calculus to compute these sensitivity ratios exactly for some matrices.

Analysis

Using the mathematical techniques presented in the previous section, analyze the two populations you selected earlier. Present evidence for all the claims you make. Your analysis should include all of the following, and may address additional questions you have posed yourself:

1. What proportions do you expect to find the different segments of the populations in, in the long run? Why? Does the initial population you feed into the model matter?

2. Which life-stage transitions have the biggest impact when altered?

3. Based on your sensitivity analysis, which life stages should interventions focus on to be most efficient? Propose measures to make the two populations more stable. This part may involve some additional research — for example, if you want to make sure that more acorns get eaten before germinating, you could learn about which species in the population's area eat acorns. Consider also whether stability is a

reasonable goal for the population, and whether conservation goals may be better reached by population growth or decline.

Conclusion

In this section, summarize your findings and recommendations. Then reflect on the model, addressing the following:

1. What phenomena relevant to the populations does your model leave out? (i.e. interaction with other organisms, climate change, etc.) If these factors were considered, how might they affect your conclusions?
2. What are the benefits and limitations of modeling populations in this way — put differently, what does this model allow you to "see" that you couldn't otherwise? What does the model "hide"?
3. Is the usefulness of this model dependent on computing power? How might the model be made more effective with the use of computers?
4. How might this model be used as part of a solution to problems of global importance? How might the use of this model exacerbate those problems?

References (Bibliography)

If writing a report in Microsoft Word, you can include this directly at the end of the report. If you are presenting your project as a PowerPoint, video, or some other format, you will need to upload your bibliography as a separate Microsoft Word document. Microsoft Word can manage your sources and create a bibliography for you or you can choose to create one yourself using any standard citation structure such as MLA, APA, or Chicago-style.

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