

N1

$$(X-1)^3 = \epsilon X$$

$$X-1 = \epsilon^{1/3} X^{1/3}$$

$$X_0 = 1$$

$$X_1 = \frac{\epsilon^{1/3} G(X_0)}{F'(X_0)} = \epsilon^{1/3}$$

$$X_2 = \frac{\epsilon^{1/3} G'(X_0) X_1 - \frac{F''(X_0)}{2} X_1^2}{F'(X_0)}$$

$$= \frac{1}{3} \epsilon^{4/3}$$

$$X = 1 + \epsilon^{1/3} + \frac{1}{3} \epsilon^{4/3}$$

N2

$$\ln x = e^{-2x}$$

$$1) L \gg 1$$

$$X_0 \approx 1$$

$$X = e^{e^{-2x}}$$

$$X_1 = e^{e^{-2X_0}}$$

$$X_2 = e^{e^{-2e^{e^{-2X_0}}}}$$

$$X_2 = e^{e^{-2e^{e^{-2}}}} \approx e^{e^{-2}}$$

$$\frac{1}{e^{e^{-2}} - e^{e^{-2X_1}}} \approx e^{e^{-2}}$$

$$\text{Order: } X = e^{e^{-2}}$$

$$L \gg 1$$

$$2) L \ll 1$$

$$\ln \ln x = -2x$$

$$X_0 \approx e$$

$$X_1 = \frac{-2 G(X_0)}{F'(X_0)} = -2e^2$$

$$X_2 = \frac{-2 G'(X_0) X_1 - \frac{F''(X_0)}{2} X_1^2}{F'(X_0)}$$

$$= (2e^2 + \frac{1}{2} 2e^4) e = 2e^3$$

$$X = e^{-2e^2 + 2e^3}$$

$$\text{Order: } X = e^{-2e^2 + 2e^3}$$

$$L \ll 1$$

N3

$$\tanh x = \arctan x$$

$$\tan(\tanh x) = x$$

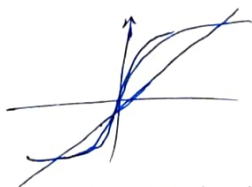
$$L \gg 1$$

$$X_0 \approx 1$$

$$X_1 = \tan(\tanh(X_0)) \approx 1$$

$$X_2 = \tan(\tanh(\tan(\tanh(X_1)))) = \tan(\tanh(1))$$

$$X = \tan(\tanh(1)) L \gg 1$$



$$0 < 2-1 < 1 \quad 1 < 2 < 2$$

$$\varepsilon = 2-1 < 1$$

$$\tanh 2x = \operatorname{arctan} x$$

$$2x - \frac{1}{3} 2^3 x^3 = x \quad |:2$$

$$x - \frac{1}{3} 2^2 x^3 = \frac{x}{2}$$

$$1 - \frac{1}{3} 2^2 x^2 - \frac{1}{2} = 0$$

$$\frac{1}{3} 2^2 x^2 = 1 - \frac{1}{2}$$

$$\frac{1}{3} 2^2 x^2 \approx \frac{\varepsilon}{\varepsilon+1}$$

$$x \approx \pm \sqrt{\frac{3\varepsilon}{\varepsilon+1}}$$

$2 = 1$ - одно решение $(0;0)$

$0 < 2 < 1$ - 1 решение $(0;0)$

$2 > 1$ - несколько решений

N4

$$1) x = \operatorname{arctan} cx \quad c \gg 1$$

$$x_0 \sim 1$$

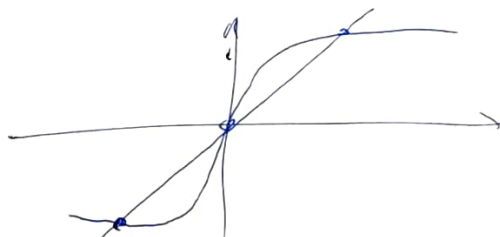
$$x_1 = \operatorname{arctan} cx_0$$

$$x_2 = \operatorname{arctan}[c \operatorname{arctan} cx] \approx \operatorname{arctan}[c]$$

$$|\operatorname{arctan}(c) - \operatorname{arctan} cx_0| < \operatorname{arctan} cx_0$$

$$x = \operatorname{arctan}(C) \cdot k \quad k \in \mathbb{Z}$$

$$x \approx \frac{\pi}{2}$$



$$2) C < 1 \quad \tan x = cx$$

$$1) \tan x = 0$$

$$x = \pi k, k \in \mathbb{Z}$$

$$2) f'(x_0) x_1 = C \cdot G(x_1)$$

$$x_1 = C \pi k, k \in \mathbb{Z}$$

$$3) x = x_0 + x_1 + x_2$$

$$f'(x_0) x_2 + \frac{f''}{2}(x_0) x_1^2 = C G'(x_0) x_1$$

$$\frac{1}{\cos^2 \pi k} x_2 + 2 \frac{\sin \pi k}{2 \cos^3 \pi k} x_1^2 = C x_1$$

1)

$$x_2 = C x_1 = C^2 \pi k, k \in \mathbb{Z}$$

$$x = \pi k + C \pi k + C^2 \pi k, k \in \mathbb{Z}$$