№1

$$\rho(z) = b - \frac{a^2}{z}, \quad \frac{a^2}{b} \le z \le \infty$$

$$\rho^2 = \left(b - \frac{a^2}{z}\right)^2 = b^2 - 2\frac{ba^2}{z} + \frac{a^2}{z^2}a^2 = b^2 - 2ba\sqrt{tg\frac{\chi}{2}} + tg\frac{\chi}{2}a^2 =$$

$$tg\frac{\chi}{2} = \frac{d\rho}{dz} = \frac{a^2}{z^2} \qquad = \left(b - \sqrt{tg\frac{\chi}{2}}\,a\right)^2$$

$$|d\rho^2| = \frac{2|b - \sqrt{tg\frac{\chi}{2}}\,a|\,a\,d\chi}{2\sqrt{tg\frac{\chi}{2}}\cos^2\frac{\chi}{2}\cdot 2} = \frac{a|b - \sqrt{tg\frac{\chi}{2}}\,a|\,d\chi}{2\cos^2\frac{\chi}{2}\sqrt{tg\frac{\chi}{2}}}$$

$$\underline{d\sigma} = \frac{\rho(\chi)}{\sin\chi}\left|\frac{d\rho(\chi)}{d\chi}\right|d\Omega = \frac{d\Omega}{2\sin\chi}|d\rho^2| =$$

$$= \frac{d\Omega}{2\sin\chi}\,\frac{a|b - \sqrt{tg\frac{\chi}{2}}\,a|}{2\cos^2\frac{\chi}{2}\sqrt{tg\frac{\chi}{2}}} = \frac{a|b - \sqrt{tg\frac{\chi}{2}}\,a|}{4\sin\chi\cos^2\frac{\chi}{2}\sqrt{tg\frac{\chi}{2}}} \quad - 0 < \chi < \chi_m$$

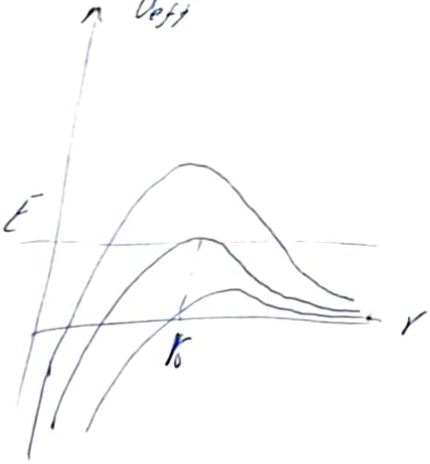
$$\rho \to 0 \qquad \frac{b}{a} = \sqrt{tg\frac{\chi}{2}}$$

$$\chi_m = 2\,arctg\,\frac{b^2}{a^2}$$

$$d\sigma = \begin{cases} \dfrac{a|b - \sqrt{tg\frac{\chi}{2}}\,a|}{4\sin\chi\cos^2\frac{\chi}{2}\sqrt{tg\frac{\chi}{2}}} & 0 < \chi < \chi_m \\ 0 & \chi > \chi_m \end{cases} \qquad \chi_m = 2\,arctg\,\frac{b^2}{a^2}$$

№3

$$U(r) = -\frac{a}{r^n} \qquad n \geq 2$$



$$U_{eff}(r) = \frac{-a}{r^n} + \frac{E\rho^2}{r^2}$$

$$U'_{eff} = n\frac{a}{r_0^{n+1}} - \frac{2E\rho^2}{r^3} = 0$$

$$\frac{an}{2E\rho^2} = r_0^{n-2}$$

$$r_0 = \left[\frac{an}{2E\rho^2}\right]^{\frac{1}{n-2}}$$

$$U(r_0) = E = -a\left[\frac{an}{2E\rho^2}\right]^{\frac{-n}{n-2}} + E\rho^2\left[\frac{an}{2E\rho^2}\right]^{\frac{-2}{n-2}} =$$

$$= -a\left[\frac{an}{2E}\right]^{\frac{-n}{n-2}} \rho^{\frac{+2n}{n-2}} + E\left[\frac{an}{2E}\right]^{\frac{-2}{n-2}} \rho^{2 + \frac{+4}{n-2}} =$$

$$= -a\left[\frac{an}{2E}\right]^{\frac{-n}{n-2}} \rho^{\frac{+2n}{n-2}} + E\left[\frac{an}{2E}\right]^{\frac{-2}{n-2}} \rho^{\frac{2n}{n-2}} = \rho^{\frac{2n}{n-2}}\left\{\left[\frac{an}{2}\right]^{\frac{-2}{n-2}} E^{1+\frac{2}{n-2}} - \left[\frac{an}{2E}\right]^{\frac{-n}{n-2}} a^{1-\frac{n}{n-2}}\right\} = \rho^{\frac{2n}{n-2}}\left\{\left[\frac{an}{2}\right]^{\frac{-2}{n-2}} E^{\frac{n}{n-2}} - \left[\frac{2E}{n}\right]^{\frac{n}{n-2}} a^{-\frac{2}{n-2}}\right\} =$$

$$= \rho^{\frac{2n}{n-2}} E^{\frac{n}{n-2}} a^{\frac{-2}{n-2}} \left\{\left[\frac{2}{n}\right]^{\frac{2}{n-2}} - \left[\frac{2}{n}\right]^{\frac{n}{n-2}}\right\} = E$$

$$\rho^{\frac{2n}{n-2}} = \frac{E^{1-\frac{n}{n-2}} a^{\frac{2}{n-2}}}{\left\{\left[\frac{2}{n}\right]^{\frac{2}{n-2}} - \left[\frac{2}{n}\right]^{\frac{n}{n-2}}\right\}} = \left(\frac{a}{E}\right)^{\frac{2}{n-2}} \frac{n^{\frac{n}{n-2}}}{2^{\frac{2}{n-2}} \cdot n - 2^{\frac{n}{n-2}}}$$

$$\rho_0 = \left(\frac{a}{E}\right)^{1/n}\left[\frac{n^{\frac{n}{n-2}}}{2^{\frac{2}{n-2}} n - 2^{\frac{n}{n-2}}}\right]^{\frac{n-2}{2n}} = \left(\frac{a}{E}\right)^{1/n} \frac{n^{1/2}}{\left(2^{\frac{2}{n-2}}[n-2]\right)^{\frac{n-2}{2n}}} =$$

$$= \left(\frac{a}{E}\right)^{1/n} \frac{n^{1/2}}{2^{1/n}[n-2]^{\frac{1}{2}-\frac{1}{n}}} = \sqrt{\frac{n}{n-2}}\left[\frac{a}{2E}(n-2)\right]^{1/n}$$

$$\ell > r_0 \Rightarrow \sigma = \pi \rho^2(\ell) = \pi \ell^2 \left(1 + \frac{a}{E\ell^n}\right)$$

$$E = -\frac{a}{\ell^n} + \frac{E\rho^2}{\ell^2}$$

$$\frac{E\rho^2}{\ell^2} = E + \frac{a}{\ell^n}$$

$$\rho^2 = \ell^2\left(1 + \frac{a}{E\ell^n}\right)$$

$\ell < r_0$ – Падение на центр

$$\sigma = \pi \rho_0^2 = \frac{\pi n}{n-2}\left[\frac{a}{2E}(n-2)\right]^{2/n}$$

Ответ:
$$\sigma = \begin{cases} \frac{\pi n}{n-2}\left[\frac{a}{2E}(n-2)\right]^{2/n} & \ell < r_0 \\ \ell^2\left(1 + \frac{a}{E\ell^n}\right) & \ell > r_0 \end{cases}$$

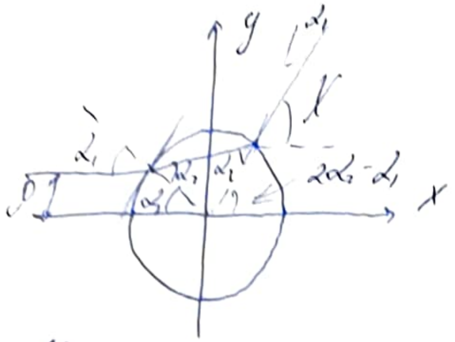
$$r_0 = \left[\frac{an}{2E\rho_0^2}\right]^{\frac{1}{n-2}}$$

№2

$$U(r)=\begin{cases}U_0 & r<R\\ 0 & r>R\end{cases} \Rightarrow U(r)=U_0\,\theta(R-r)$$

$$F=-\frac{\partial U(r)}{\partial r}=-U_0\,\delta(R-r)$$

Когда частица входит в поле, то на неё скачком действует сила (только при $R=r$)



$\chi=2(\alpha_2-\alpha_1)$ $\quad \sin\alpha_1=\frac{\rho}{R}$

$\frac{\chi}{2}=\alpha_2-\alpha_1$ $\quad p_0\sin\alpha_1=p_0'\sin\alpha_2$

$$\frac{p_0'}{p_0}=\frac{\sin\alpha_1}{\sin\alpha_2}=\frac{\sqrt{2m(E-U_0)}}{\sqrt{2mE}}=\sqrt{1-\frac{U_0}{E}}=\varphi$$

$$\sin\alpha_2=\frac{\sin\alpha_1}{\varphi}=\frac{\rho}{R\varphi}$$

$\Downarrow$

$$\frac{\chi}{2}=\arcsin\left(\frac{\rho}{R\varphi}\right)-\arcsin\left(\frac{\rho}{R}\right) \longrightarrow \cos\frac{\chi}{2}=\sqrt{1-\left(\frac{\rho}{R\varphi}\right)^2}\cdot$$

$$\cdot\sqrt{1-\left(\frac{\rho}{R}\right)^2}+\frac{\rho^2}{R^2\varphi}$$

$$\frac{\chi_{max}}{2}=\frac{\pi}{2}-\arcsin\varphi \qquad \frac{\rho}{R\varphi}\to 1$$

$\Downarrow$

$$\chi_{max}=2\arccos\varphi$$

$$\left(\cos\frac{\chi}{2}-\frac{\rho^2}{R^2\varphi}\right)^2=\left(1-\left(\frac{\rho}{R\varphi}\right)^2\right)\left(1-\left(\frac{\rho}{R}\right)^2\right)$$

$$\cos^2\frac{\chi}{2}-2\frac{\rho^2}{R^2\varphi}\cos\frac{\chi}{2}+\frac{\rho^4}{R^4\varphi^2}=$$

$$=1-\left(\frac{\rho}{R}\right)^2-\left(\frac{\rho}{R\varphi}\right)^2+\frac{\rho^4}{R^4\varphi^2}$$

$$\rho^2\left(\frac{1}{R^2}+\frac{1}{R^2\varphi^2}-2\frac{1}{R^2\varphi}\cos\frac{\chi}{2}\right)=1-\cos^2\frac{\chi}{2}$$

$$\rho^2=\frac{1-\cos^2\frac{\chi}{2}}{\frac{1}{R^2}+\frac{1}{R^2\varphi^2}-2\frac{1}{R^2\varphi}\cos\frac{\chi}{2}}=\frac{R^2\varphi^2\sin^2\frac{\chi}{2}}{\varphi^2+1-2\varphi\cos\frac{\chi}{2}}$$

$$\sigma=\pi\rho^2=\frac{\pi R^2\varphi^2\sin^2\frac{\chi}{2}}{1+\varphi^2-2\varphi\cos\frac{\chi}{2}}, \quad \varphi=\sqrt{1-\frac{U_0}{E}}$$

$$\rho = \frac{l\psi \sin\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2}}$$

$$\frac{d\rho}{d\chi} = l\psi \, \frac{\frac{1}{2}\cos\frac{\chi}{2}\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2} - \frac{2\psi\sin^2\frac{\chi}{2}\cdot\frac{1}{2}}{2\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2}}}{1+\psi^2-2\psi\cos\frac{\chi}{2}} =$$

$$d\sigma = 2\pi\rho(\chi)\left|\frac{d\rho(\chi)}{d\chi}\right| d\chi = 2\pi\frac{l\psi}{2}\left[\frac{\cos\frac{\chi}{2}\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2}}{1+\psi^2-2\psi\cos\frac{\chi}{2}} - \frac{\psi\sin^2\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2}}\right] \cdot \frac{l\psi\sin\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2}}\, d\chi =$$

$$= \frac{\pi(l\psi)^2 \sin\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{1/2}} \, \frac{d\Omega}{2\pi\sin\chi} \, \frac{\cos\frac{\chi}{2}\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right] - \psi\sin^2\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^{3/2}} =$$

$$= \frac{l^2\psi^2 \sin\frac{\chi}{2}\, d\Omega}{2\sin\chi} \cdot \frac{\cos\frac{\chi}{2} + \psi^2\cos\frac{\chi}{2} - 2\psi\cos\frac{\chi}{2}\cos\frac{\chi}{2} - \psi\sin^2\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^2} =$$

$$= \frac{l^2\psi^2\sin\frac{\chi}{2}}{2\sin\chi}\, d\Omega \, \frac{\cos\frac{\chi}{2} + \psi^2\cos\frac{\chi}{2} - 2\psi\cos^2\frac{\chi}{2} - \psi + \psi\cos^2\frac{\chi}{2}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^2} =$$

$$= \frac{l^2\psi^2\sin\frac{\chi}{2}}{2\sin\chi}\, d\Omega \, \frac{\cos\frac{\chi}{2} + \psi^2\cos\frac{\chi}{2} - \psi\cos^2\frac{\chi}{2} - \psi}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^2} =$$

$$= \frac{l^2\psi^2}{4\cos\frac{\chi}{2}}\, d\Omega \, \frac{\cos\frac{\chi}{2} + \psi^2\cos\frac{\chi}{2} - \psi\cos^2\frac{\chi}{2} - \psi}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^2} =$$

$$= \frac{l^2\psi^2}{4} \, \frac{1+\psi^2-\psi\cos\frac{\chi}{2} - \frac{\psi}{\cos\frac{\chi}{2}}}{\left[1+\psi^2-2\psi\cos\frac{\chi}{2}\right]^2}\, d\Omega$$

$d\sigma = 0 \quad \chi_m < \chi$

$0 < \chi < \chi_m$

$\chi_m = 2\arccos\psi$

№5 $\varphi_0 = \int\limits_{r_{min}}^{\infty} \frac{\rho \, dr}{r^2\sqrt{1-\frac{\rho^2}{r^2}-\frac{2U}{mv_\infty^2}}} = \int\limits_{r_{min}}^{\infty} \frac{\rho\, dr}{r^2\left[1-\frac{\rho^2}{r^2}-\frac{2\beta}{mv_\infty^2 r^2}\right]^{1/2}} =$

$\xi = \frac{1}{r}$

1) $\beta > 0$

$U = \frac{\beta}{r^2}$

$= \int\limits_{1/\xi_{min}}^{0} \frac{-\rho\, d\xi}{\left[1-\rho^2\xi^2-\frac{2\beta}{mv_\infty^2}\xi^2\right]^{1/2}} = \int\limits_{\#}^{0} \frac{-\rho\, d\xi}{\left[1-\left(\rho^2+\frac{2\beta}{mv_\infty^2}\right)\xi^2\right]^{1/2}}$

$= \frac{-\rho \arcsin}{\sqrt{\rho^2+\frac{2\beta}{mv_\infty^2}}}\left(\frac{\rho\,\xi}{\sqrt{\rho^2+\frac{2\beta}{mv_\infty^2}}}\right)$

$= \int\limits_{0}^{\#} \frac{+\rho\, d\xi}{\left[1-\left(\rho^2+\frac{2\beta}{mv_\infty^2}\right)\xi^2\right]^{1/2}} = \int\limits_{0}^{\#} \frac{\rho\, d\xi}{\sqrt{\rho^2+\frac{2\beta}{mv_\infty^2}}\left[\frac{1}{\rho^2+\frac{2\beta}{mv_\infty^2}}-\xi^2\right]^{1/2}} =$

$= \int\limits_{0}^{\#} \frac{\cancel{\rho}}{\cancel{\rho}\sqrt{1+\frac{2\beta}{mv_\infty^2\rho^2}}}\frac{d\xi}{\left[\psi^2-\xi^2\right]^{1/2}} =$

$= \frac{1}{\sqrt{1+\frac{2\beta}{mv_\infty^2\rho^2}}} \arcsin\left(\frac{\xi}{\psi}\right)\Bigg|_0^{\#} = \frac{1}{\sqrt{1+\frac{2\beta}{mv_\infty^2\rho^2}}}\,\frac{\pi}{2}$

$\varphi_0 = \frac{\pi}{2}\frac{1}{\sqrt{1+\frac{2\beta}{mv_\infty^2\rho^2}}}$ $\quad \chi = |\pi - 2\varphi_0| =$

$= \left|\pi\left(1-\frac{1}{\sqrt{1+\frac{2\beta}{mv_\infty^2\rho^2}}}\right)\right|$

$d\sigma = 2\pi\rho(\chi)\left|\frac{d\rho(\chi)}{d\chi}\right| d\chi$

$d\sigma = \frac{\rho(\chi)}{\sin\chi}\left|\frac{d\rho}{d\chi}\right| d\Omega$

$$\chi = \pi\left(1 - \frac{1}{\sqrt{1 + \frac{2\beta}{m v_\infty \rho^2}}}\right)$$

$$\frac{\chi}{\pi} = 1 - \frac{1}{\sqrt{}}$$

$$\frac{1}{\sqrt{}} = 1 - \frac{\chi}{\pi} = \frac{\pi - \chi}{\pi}$$

$$\frac{\pi}{\pi - \chi} = \sqrt{1 + \frac{2\beta}{m v_\infty \rho^2}}$$

$$\left(\frac{\pi}{\pi - \chi}\right)^2 = 1 + \frac{2\beta}{m v_\infty \rho^2}$$

$$\left(\frac{\pi}{\pi - \chi}\right)^2 - 1 = \frac{2\beta}{m v_\infty \rho^2}$$

$$\rho^2 = \frac{2\beta}{m v_\infty \left(\frac{\pi}{\pi - \chi} - 1\right)\left(\frac{\pi}{\pi - \chi} + 1\right)} = \frac{2\beta}{m v_\infty \left(\frac{\pi - \pi + \chi}{\pi - \chi}\right)\left(\frac{\pi + \pi - \chi}{\pi - \chi}\right)}$$

$$= \frac{2\beta}{m v_\infty \frac{\chi(2\pi - \chi)}{(\pi - \chi)^2}} = \frac{2\beta(\pi - \chi)^2}{m v_\infty \chi(2\pi - \chi)}$$

$$(\rho^2(\chi))' = 2\rho(\chi)\frac{d\rho}{d\chi}$$

$$d\sigma = \frac{\rho(\chi)}{\sin\chi}\left|\frac{d\rho}{d\chi}\right| do =$$

$$= \frac{do}{2\sin\chi} \cdot \left(\frac{2\beta(\pi - \chi)^2}{m v_\infty \chi(2\pi - \chi)}\right)' = \frac{do}{2\sin\chi}$$

$$\frac{2\beta(\pi - \chi)^2}{m v_\infty (2\pi\chi - \chi^2)} = \frac{2\beta}{m v_\infty}\left\{\frac{-2(\pi - \chi)(2\pi\chi - \chi^2) - (\pi - \chi)^2(2\pi - 2\chi)}{(2\pi\chi - \chi^2)^2}\right. =$$

$$= \frac{2\beta}{m v_\infty}\left\{\frac{-2(2\pi^2\chi - \pi\chi^2 - 2\pi\chi^2 + \chi^3) - 2(\pi - \chi)^3}{\chi^2(2\pi - \chi)^2}\right\} =$$

$$= \frac{2\beta}{m v_\infty} \left\{ \frac{-2\left[2\pi^2 \chi - 3\pi\chi^2 + \chi^3 + (\pi-\chi)^3\right]}{\chi^2 (2\pi-\chi)^2} \right\} =$$

$$= \frac{2\beta}{m v_\infty} \left\{ \frac{-2\left[2\pi^2 \chi \cancel{- 3\pi\chi^2} \cancel{+ \chi^3} + \pi^3 - 3\pi^2\chi \cancel{+ 3\pi\chi^2} \cancel{- \chi^3}\right]}{\chi^2 (2\pi-\chi)^2} \right\} =$$

$$= \frac{2\beta}{m v_\infty} \left\{ \frac{-2\left[\pi^3 - \pi^2\chi\right]}{\chi^2 (2\pi-\chi)^2} \right\} = \frac{-4\pi^2\beta}{m v_\infty} \frac{\pi-\chi}{\chi^2(2\pi-\chi)^2}$$

$$d\sigma = \frac{d\Omega}{2\sin\chi} \frac{\overset{2}{\cancel{4}}\pi^2\beta}{m v_\infty} \frac{\pi-\chi}{\chi^2(2\pi-\chi)^2} = \frac{2\pi^2\beta}{m v_\infty} \frac{\pi-\chi}{\chi^2(2\pi-\chi)^2} \frac{d\Omega}{\sin\chi}$$