N1

$$I_2 = \int_0^\infty \frac{\sin^2 \lambda x}{x^2} dx$$

$$\frac{\partial I}{\partial \lambda} = \int_0^\infty \frac{2\sin\lambda x \cos\lambda x \cdot x}{x^2} dx = \int_0^\infty \frac{\sin 2\lambda x}{x} dx = \int_0^\infty \frac{\sin 2\lambda x}{2\lambda x} d(2\lambda x) = \frac{\pi}{2}$$

$$I = \frac{\pi}{2}\lambda + C$$

$$I = \int_0^\infty \frac{\sin^2 \lambda x}{x^2} dx = \lambda \int_0^\infty \frac{\sin^2(\lambda x)}{(\lambda x)^2} d(\lambda x) = \frac{\pi}{2}\lambda$$

$$\boxed{I = \frac{\pi}{2}\lambda}$$

N2

$$I = \int_0^a x^2(a^2-x^2)^{1/2} dx = a \int_0^a x^2\left(1-\frac{x^2}{a^2}\right)^{1/2} dx = a \int_0^1 u a^2 (1-u)^{1/2} \frac{a\,du}{2\sqrt{u}} \ominus$$

$u = \frac{x^2}{a^2}$ $\qquad x = a\sqrt{u}$

$du = \frac{1}{a^2} 2x\,dx$

$dx = \frac{a\,du}{2\sqrt{u}}$

$$\ominus \frac{a^2}{2} \int_0^1 u^{1/2}(1-u)^{1/2} du \ominus$$

$$\ominus \frac{a^2}{2} B\left(\frac{3}{2}; \frac{3}{2}\right) = \frac{a^2}{2} \frac{\Gamma(\frac{3}{2})\Gamma(\frac{3}{2})}{\Gamma(3)} =$$

$$= \frac{a^2 \pi}{16}$$

N3

$$I(\alpha) = \int_0^\infty \frac{e^{-\alpha x^2} - e^{-\beta x^2}}{x} dx$$

$$I'(\alpha) = -\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha} e^{-\alpha x^2}\Big|_0^{+\infty} = -\frac{1}{2\alpha}$$

$I'(\beta) = \frac{1}{2\beta}$ $\qquad I = -\frac{1}{2}\ln\alpha + \theta(\beta)$

$I'(\beta) = \theta'(\beta) \quad \Rightarrow \theta(\beta) = \frac{1}{2}\ln\beta + C$

$$I = \frac{1}{2}\ln\frac{\beta}{\alpha} + C$$

Пусть $\alpha = \beta$

$I = 0 \Rightarrow C = 0$

N5

$$I(m)=\int_0^\infty \frac{\cos x}{x^m}dx$$

$$\frac{1}{x^m}=\frac{1}{\Gamma(m)}\int_0^\infty t^{m-1}e^{-tx}dt \qquad \varphi=-m+1$$

$$I(m)=\operatorname{Re}\int_0^\infty \frac{e^{ix}}{x^m}dx=\operatorname{Re}\int_0^\infty x^{(-m+1)-1}e^{-x(-i)}dx=\operatorname{Re}\frac{\Gamma(-m+1)}{(-i)^{-m+1}}=$$

$$=\operatorname{Re}\frac{\Gamma(\varphi)}{(-i)^\varphi}=\operatorname{Re}\Gamma(\varphi)e^{\varphi\frac{\pi}{2}i}=\Gamma(\varphi)\cos\left(\frac{\pi}{2}\varphi\right)=$$

$$=\Gamma(1-m)\cos\left(\frac{\pi}{2}(1-m)\right)$$

N4

$$I(a)=\int_0^\infty \frac{e^{-ax}}{1+x^2}dx$$

$$I'(a)=\int_0^\infty \frac{-xe^{-ax}}{1+x^2}dx$$

$$I''(a)=\int_0^\infty \frac{x^2e^{-ax}}{1+x^2}dx=\int_0^\infty \frac{(x^2+1-1)e^{-ax}}{1+x^2}dx=\underbrace{\int_0^\infty e^{-ax}dx}_{=\frac{1}{a}}-\underbrace{\int_0^\infty \frac{e^{-ax}}{1+x^2}dx}_{=I(a)}$$

$$I''(a)+I(a)=\frac{1}{a}$$

1) Общее решение

$$I''(a)+I(a)=0$$

$$\lambda^2+1=0$$

$$\lambda=\pm i$$

$$I(a)=C_1e^{ix}+C_2e^{-ix}$$

2) Частное решение..