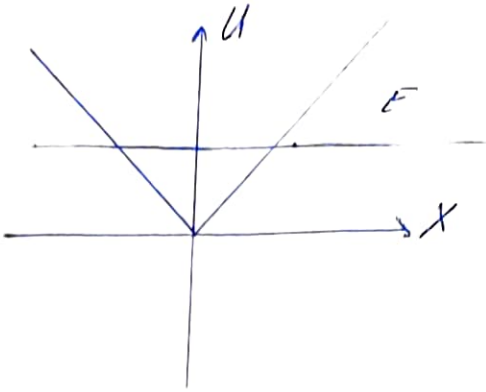
№1

(1) $U(x) = F|x|$



$$\frac{T(E)}{\sqrt{2m}} = \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - F|x|}}$$

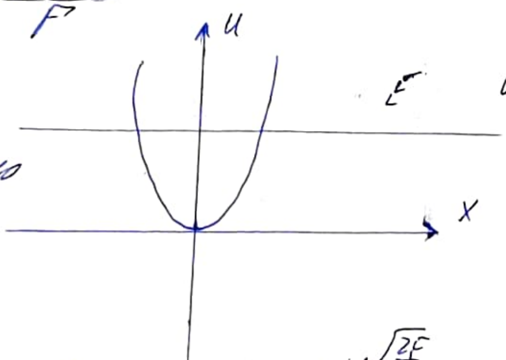
При $E>0$ Движение финитно

$|x|$ - чётная ф-ция

$$\frac{T(E)}{\sqrt{2m}} = 2\int_0^{\frac{E}{F}} \frac{dx}{\sqrt{E - Fx}} = \frac{2}{\sqrt{F}}\int_0^{\frac{E}{F}} \frac{dx}{\sqrt{\frac{E}{F} - x}} = \frac{-4}{\sqrt{F}}\left(\sqrt{\frac{E}{F} - \frac{E}{F}} - \sqrt{\frac{E}{F}}\right) =$$

$$= 4\frac{\sqrt{E}}{F} \Rightarrow T(E) = \frac{4\sqrt{2mE}}{F}$$

(2) $U(x) = \frac{kx^2}{2}$



$U(x) = \frac{kx^2}{2}$ - Парабола ↑ чётная ф-ция

При $E>0$ Движение финитно

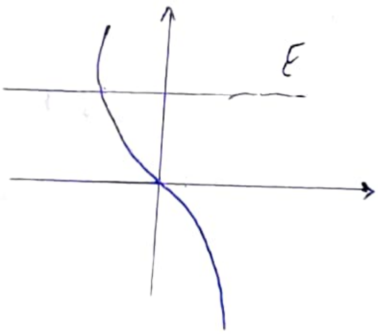
$$\frac{T(E)}{\sqrt{2m}} = \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - \frac{kx^2}{2}}} =$$

$$= 2\int_0^{x_2} \frac{dx}{\sqrt{\frac{k}{2}}\sqrt{E\frac{2}{k} - x^2}} = 2\sqrt{\frac{2}{k}} \arcsin\left(\frac{x}{\sqrt{\frac{2E}{k}}}\right)\Bigg|_0^{\sqrt{\frac{2E}{k}}} = 2\sqrt{\frac{2}{k}}\,\frac{\pi}{2}$$

$$\Downarrow$$

$$T(E) = 2\pi\sqrt{\frac{m}{k}}$$

(3) $U(x) = -ax^3$

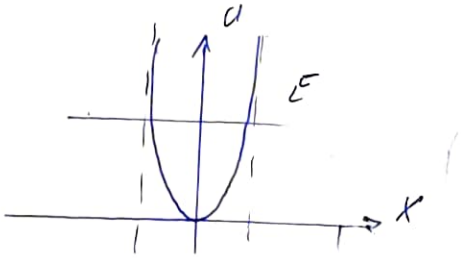


Движение всегда инфинитно

$\Downarrow$

Периода нет

(4) $U(x) = U_0\,tg^2(ax)$ Ф-ция чётная



При $E>0$ Движение финитно

$$\frac{T(E)}{\sqrt{2m}} = 2\int_0^{\#} \frac{dx}{\sqrt{E - U_0\,tg^2(ax)}} =$$

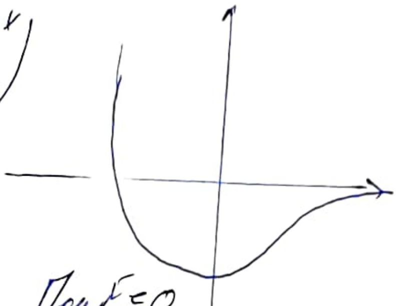
$$= 2\int_0^{\#} \frac{dx}{\left(E + U_0 - \frac{U_0}{\cos^2(ax)}\right)^{1/2}} = 2\int_0^{\#} \frac{\cos(ax)dx}{\left(\cos^2(ax)(E+U_0) - U_0\right)^{1/2}} = \frac{2}{a}\int_0^{\#} \frac{d(ax)\cos(ax)}{\left((1-\sin^2 ax)(E+U_0) - U_0\right)^{1/2}} =$$

$$= \frac{2}{a}\int_0^{\#}\frac{dt}{((1-t^2)(E+U_0)-U_0)^{1/2}} = \frac{2}{a}\int_0^{\#}\frac{dx}{(E-(E+U_0)t^2)^{1/2}} = \frac{2}{a\sqrt{E+U_0}}\int_0^{\#}\frac{dt}{\left(\frac{E}{E+U_0}-t^2\right)^{1/2}}$$

$$= \frac{2}{a\sqrt{E+U_0}}\arcsin\left(\frac{t}{\sqrt{\frac{E}{E+U_0}}}\right)\Bigg|_0^{\sqrt{\frac{E}{E+U_0}}} = \frac{2}{a\sqrt{E+U_0}}\cdot\frac{\Pi}{2}$$

$$T(E) = \frac{\sqrt{2m}\,\Pi}{a\sqrt{E+U_0}}$$

(5) $U(x) = U_0(e^{-2ax} - 2e^{-ax}) = U_0\left(\frac{1-2e^{ax}}{e^{2ax}}\right)$



$$\frac{T(E)}{\sqrt{2m}} = \int_{\#_1}^{\#_2}\frac{dx}{\sqrt{-|E| - U_0\left(\frac{1-2e^{ax}}{e^{2ax}}\right)}} =$$

$$= \int_{\#_1}^{\#_2}\frac{e^{ax}\,dx}{(-|E|e^{2ax}+2U_0e^{ax}-U_0)^{1/2}} = \frac{1}{a}\int_{\#_1}^{\#_2}\frac{dt}{(-|E|t^2+2U_0t-U_0)^{1/2}} =$$

При $E<0$ движение финитно

$$= \frac{1}{a\sqrt{|E|}}\int_{\#_1}^{\#_2}\frac{dt}{\left(-t^2+\frac{2U_0}{|E|}t-\frac{U_0}{|E|}\right)^{1/2}} = \frac{1}{a\sqrt{|E|}}\int_{\#_1}^{\#_2}\frac{dt}{\left(\frac{U_0^2}{|E|^2}-\frac{U_0}{E}-\left(t-\frac{U_0}{E}\right)^2\right)^{1/2}} = \frac{1}{a\sqrt{|E|}}\arcsin\left(\frac{t-\frac{U_0}{E}}{\sqrt{\frac{U_0^2}{E^2}-\frac{U_0}{E}}}\right)\Bigg|_{\#_1}^{\#_2} =$$

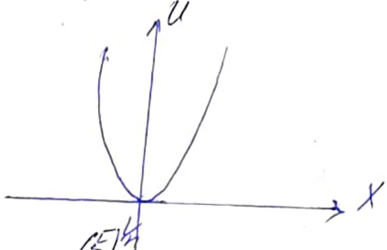
$\theta = t - \frac{U_0}{|E|}$

$$= \frac{1}{a\sqrt{|E|}}\left(\arcsin(1) - \arcsin(-1)\right) = \frac{\Pi}{a\sqrt{|E|}} \Rightarrow T(E) = \frac{\Pi\sqrt{2m}}{a\sqrt{|E|}} = \frac{\Pi}{a}\sqrt{\frac{2m}{|E|}}$$

(6) $U(x) = \lambda|x|^n$

Ф-ция чётная

$E \geq 0$ Движение финитное



$$\frac{T}{\sqrt{2m}} = \int_{\#_1}^{\#_2}\frac{dx}{(E-\lambda|x|^n)^{1/2}} = 2\int_0^{\left(\frac{E}{\lambda}\right)^{1/n}}\frac{dx}{(E-\lambda|x|^n)^{1/2}} = \left[\begin{matrix}|x|^n = t & \\ dx\, n|x|^{n-1} = dt & n t^{\frac{n-1}{n}}dx = dt\end{matrix}\right] =$$

$$= \frac{2}{\sqrt{E}}\int_0^{E/\lambda}\frac{dt}{n t^{1-1/n}}\left(1-\frac{\lambda}{E}t\right)^{-1/2} = \left[\begin{matrix}du = \frac{\lambda}{E}dt \\ t = u\frac{E}{\lambda}\end{matrix}\right] = \frac{2\left(\frac{E}{\lambda}\right)}{n\sqrt{E}}\int_0^1\left(\frac{E}{\lambda}\right)^{\frac{1}{n}-1}u^{\frac{1}{n}-1}(1-u)^{-1/2}du$$

$$= \frac{2\cancel{E}E^{1/n}\cancel{\lambda}}{nE^{1/2}\cancel{\lambda}\lambda^{1/n}\cancel{E}}\int u^{1/n-1}(1-u)^{-1/2}du = \frac{2E^{1/n-1/2}}{n\lambda^{1/n}}B\left(\frac{1}{n};\frac{1}{2}\right)$$

$$T = \frac{2\sqrt{2m}E^{1/n-1/2}}{n\lambda^{1/n}}B\left(\frac{1}{n};\frac{1}{2}\right)$$

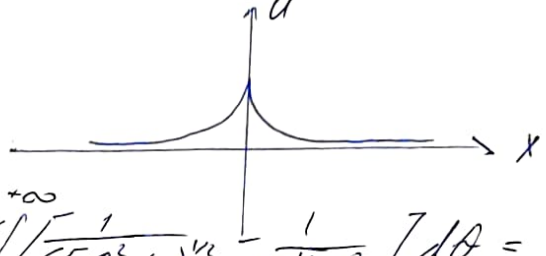
№2

$U(x) = U_0 e^{-\frac{|x|}{a}}$ – Ф-ция чётная

$\tau = \sqrt{\frac{m}{2}}\, 2\int_0^{+\infty}\left[\frac{1}{(E-U_0e^{-|x|/a})^{1/2}} - \frac{1}{E^{1/2}}\right]dx = \sqrt{2m}\int_0^{\infty}\left[\frac{e^{\frac{x}{2a}}}{(Ee^{x/a}-U_0)} - \frac{1}{E^{1/2}}\right]dx =$

$\theta = e^{|x|/2a}$ $\qquad dx = \frac{2a\,d\theta}{e^{|x|/2a}}$

$d\theta = \frac{1}{2a}e^{|x|/2a}dx$ $\qquad \gamma = 2a\sqrt{2m}$



$= \gamma\int_1^{+\infty}\left[\frac{\theta}{(E\theta^2-U_0)^{1/2}} - \frac{1}{E^{1/2}}\right]\frac{d\theta}{\theta} = \gamma\int_1^{+\infty}\left[\frac{1}{(E\theta^2-U_0)^{1/2}} - \frac{1}{E^{1/2}\theta}\right]d\theta =$

$= \gamma\frac{1}{\sqrt{E}}\int_1^{+\infty}\left[\frac{1}{(\theta^2-\frac{U_0}{E})^{1/2}} - \frac{1}{\theta}\right]d\theta = \frac{\gamma}{\sqrt{E}}\left[\ln\left(\theta+\sqrt{\theta^2-\frac{U_0}{E}}\right) - \ln\theta\right]\Big|_1^{+\infty} =$

$= \frac{\gamma}{\sqrt{E}}\ln\left(1+\sqrt{1-\frac{U_0}{E\theta^2}}\right)\Big|_1^{+\infty} = \frac{\gamma}{\sqrt{E}}\left[\ln 2 - \ln\left(1+\sqrt{1-\frac{U_0}{E}}\right)\right] =$

$= \frac{\gamma}{\sqrt{E}}\ln\left(\frac{2}{1+\sqrt{\frac{E-U_0}{E}}}\right) = \frac{\gamma}{\sqrt{E}}\ln\left(\frac{2\sqrt{E}}{\sqrt{E}+\sqrt{E-U_0}}\right) = 2a\sqrt{\frac{2m}{E}}\ln\left(\frac{2\sqrt{E}}{\sqrt{E}+\sqrt{E-U_0}}\right)$

№3

$U(x) = -\frac{\beta x^2}{2} + \frac{\lambda x^4}{4}$ $\qquad m\ddot{x} = -\frac{\partial U(x)}{\partial x} = \beta x - \lambda x^3$

$x(0) = \sqrt{\frac{2\beta}{\lambda}}$ $\quad \dot{x}(0) = 0$

$m\ddot{x} = \beta x - \lambda x^3$

$\ddot{x} - \frac{\beta}{m}x = -\frac{\lambda}{m}x^3$ – Линейное неоднородное ДУ 2-го порядка

1) Общее решение:

$\ddot{x} - \frac{\beta}{m}x = 0$

$\lambda^2 = \frac{\beta}{m}$

$\lambda = \pm\frac{\beta}{m}$

$x_0(t) = C_1e^{-\frac{\beta}{m}t} + C_2e^{\frac{\beta}{m}t}$

2) Частное решение:

$\tilde{x}(t) = Ax^3+Bx^2+Cx+D$

$\dot{\tilde{x}}(t) = 3Ax^2+2Bx+C$

$\ddot{\tilde{x}}(t) = 6Ax+2B$

Подставляя в ДУ имеем:

$6Ax+2B-\frac{\beta}{m}(Ax^3+Bx^2+Cx+D) = -\frac{\lambda}{m}x^3$

$-\frac{\beta}{m}Ax^3 - \frac{\beta}{m}Bx^2 - \frac{\beta}{m}Cx + 6Ax + 2B - \frac{\beta}{m}D = -\frac{\lambda}{m}x^3$

$\begin{cases} B = 0 \\ -\frac{\beta}{m}A = -\frac{\lambda}{m} \\ D = 0 \end{cases}$ $\qquad -\frac{\beta}{m}C + 6A = 0$ $\qquad A = \frac{\lambda}{\beta}$

$C = 6\frac{\lambda m}{\beta^2}$

$$\tilde{x}(t) = \frac{\lambda}{\delta}x^3 - 6\frac{\lambda m}{\delta^2}x$$

$$x(t) = C_1 e^{-\frac{\delta}{m}t} + C_2 e^{\frac{\delta}{m}t} + \frac{\lambda}{\delta}x^3 - 6\frac{\lambda m}{\delta^2}x$$

$$\dot{x}(t) = -\frac{\delta}{m}C_1 e^{-\frac{\delta}{m}t} + \frac{\delta}{m}C_2 e^{\frac{\delta}{m}t} + 3\frac{\lambda}{\delta}x^3 - 6\frac{\lambda m}{\delta^2}$$

$$\begin{cases} \dot{x}(0) = -\frac{\delta}{m}C_1 + \frac{\delta}{m}C_2 - 6\frac{\lambda m}{\delta^2} = 0 \\ x(0) = C_1 + C_2 = \sqrt{\frac{2\delta}{\lambda}} \end{cases}$$

← Подстановка начальных условий

$$\begin{cases} \frac{\delta}{m}(C_2 - C_1) = \frac{6\lambda m}{\delta^2} \\ C_1 = \sqrt{\frac{2\delta}{\lambda}} - C_2 \end{cases}$$

$$\frac{\delta}{m}\left(2C_2 - \sqrt{\frac{2\delta}{\lambda}}\right) = \frac{6\lambda m}{\delta^2}$$

$$2C_2 - \sqrt{\frac{2\delta}{\lambda}} = \frac{6\lambda m^2}{\delta^3}$$

$$C_2 = \frac{3\lambda m^2}{\delta^3} + \sqrt{\frac{\delta}{2\lambda}}$$

$$C_1 = \sqrt{\frac{2\delta}{\lambda}} - \frac{3\lambda m^2}{\delta^3} - \sqrt{\frac{\delta}{2\lambda}}$$

$$x(t) = \left(\sqrt{\frac{2\delta}{\lambda}} - \frac{3\lambda m^2}{\delta^3} - \sqrt{\frac{\delta}{2\lambda}}\right)e^{-\frac{\delta}{m}t} + \left(\frac{3\lambda m^2}{\delta^3} + \sqrt{\frac{\delta}{2\lambda}}\right)e^{\frac{\delta}{m}t} +$$
$$+ \frac{\lambda}{\delta}x^3 - \frac{6\lambda m}{\delta^2}x$$

№4

№4

$$x = x'\cos(\Omega t) - y'\sin(\Omega t)$$
$$y = x'\sin(\Omega t) + y'\cos(\Omega t)$$
$$\mathcal{L} = \frac{m\dot{r}^2}{2} - U(r)$$

1) Обобщённый импульс

$$P_k = \sum_{i=1}^{s} \frac{\partial f_i}{\partial Q_k} p_i \qquad p'_x = p_x\cos(\Omega t) - p_y\sin(\Omega t)$$
$$p'_y = p_x\sin(\Omega t) + p_y\cos(\Omega t)$$

2) Лагранжиан

$$\mathcal{L} = \frac{m}{2}\sum_{i=1}^{s}\left(\frac{\partial f_i}{\partial t} + \sum_{j=1}^{s}\frac{\partial f_i}{\partial Q_j}\dot{Q}_j\right)^2 + U(Q) = \frac{m}{2}\Big[\big(-x'\Omega\sin(\Omega t) - y'\Omega\cos(\Omega t) +$$
$$+ \dot{x}'\cos(\Omega t) - \dot{y}'\sin(\Omega t)\big)^2 + \big(x'\Omega\cos(\Omega t) - y'\Omega\sin(\Omega t) + \dot{x}'\sin(\Omega t) + \dot{y}'\cos(\Omega t)\big)^2\Big]$$

$$+ U(Q) = \frac{m}{2}\Big[\big(-[\bar{y}\times\bar{\Omega}] + \dot{x}'\cos(\Omega t) - \dot{y}'\sin(\Omega t)\big)^2 +$$
$$\big([\bar{x}\times\bar{\Omega}] + \dot{x}'\sin(\Omega t) + \dot{y}'\cos(\Omega t)\big)^2\Big] + U(Q)$$

3) Энергия

$$E' = E - \sum_{i=1}^{s}\frac{\partial f_i}{\partial t}p_i = E - p_x\big(-x'\Omega\sin(\Omega t) - y'\Omega\cos\Omega t\big) -$$
$$- p_y\big(x'\Omega\cos\Omega t - y'\Omega\sin\Omega t\big) = E + p_x[\bar{y}\times\bar{\Omega}] - p_y[\bar{x}\times\bar{\Omega}] =$$