№1

$$I = \int_{+\sigma} d\sigma\,(x - 3y + 6z) = \iint f(x,y,z)\sqrt{z_x'^2 + z_y'^2 + 1}\,dx\,dy =$$

$$x - y + 2z + 4 = 0$$

$$2z = y - x - 4 \qquad \frac{\partial z}{\partial x} = -\frac{1}{2} \qquad \sqrt{\frac{1}{4} + \frac{1}{4} + 1} =$$

$$z = \frac{y-x}{2} - 2 \qquad \frac{\partial z}{\partial y} = \frac{1}{2} \qquad = \sqrt{\frac{1}{2} + 1} = \sqrt{\frac{3}{2}}$$

$$= \iint \left(x - 3y + 6\left(\frac{y-x}{2} - 2\right)\right)\sqrt{\frac{3}{2}}\,dx\,dy =$$

$$= \sqrt{\frac{3}{2}}\iint (x - 3y + 3y - 3x - 12)\,dx\,dy = \sqrt{\frac{3}{2}}\iint (-2x - 12)\,dx\,dy =$$

$$= \sqrt{\frac{3}{2}}\int_{-4}^{0}(-2x-12)\,dx\int_{0}^{x+4} dy = \sqrt{\frac{3}{2}}\int_{-4}^{0}(-2x-12)(x+4)\,dx \;\textcircled{=}$$

$$\cancel{x-3y} \qquad x - y = -4$$

$$y = 0 \quad x = -4 \qquad x - y = -4$$

$$x = 0 \quad y = 4 \qquad y = x + 4$$

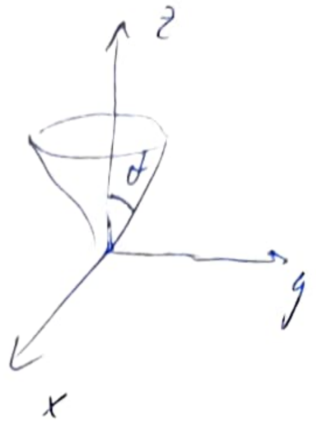
$$\textcircled{=}\ \sqrt{\frac{3}{2}}(-2)\int_{-4}^{0}(x+6)(x+4)\,dx = -\sqrt{6}\int_{-4}^{0}(x^2 + 10x + 24)\,dx =$$

$$= -\sqrt{6}\left(\frac{x^3}{3} + \frac{10x^2}{2} + 24x\right)\Bigg|_{-4}^{0} = -\sqrt{6}\left(\frac{x^3}{3} + 5x^2 + 24x\right)\Bigg|_{-4}^{0} =$$

$$= -\sqrt{6}\left(-\frac{64}{3} + 5\cdot 16 - 24\cdot 4\right) = \frac{112\sqrt{6}}{3}$$

N2

$F = 3xyz\vec{i} + y\sin x\,\vec{k}$




$z = \sqrt{x^2+y^2}$

$x^2+y^2+z^2=4$

$\oint d\vec{S}\vec{F} = \int dV(\vec{\nabla}\cdot\vec{F}) = \int 3yz\,dV$

$\vec{\nabla}\cdot\vec{F} = 3yz$

Цилиндрическая система координат:

$\begin{cases} z = r\cos\theta \\ x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \end{cases}$ $\quad dV = r^2\sin\theta\,dr\,d\theta\,d\varphi$

$V = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2} r^2\sin\theta\,dr\,d\theta\,d\varphi\; 3r\sin\theta\sin\varphi\, r\cos\theta =$

$= \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{4}} d\theta \int_0^{2} dr\; 3r^4\sin^2\theta\cos\theta\sin\varphi = 0$

N3

$F = r^2\vec{r}$

в шаре $F = R^3$ $\quad S = 4\pi R^2 \Rightarrow \Phi = 4\pi R^5$

$(\vec{n}\vec{F}) = R^3$

$\vec{\nabla}\cdot\vec{F} = (3x^2+y^2+z^2) + (x^2+3y^2+z^2) + (x^2+y^2+3z^2) = 5(x^2+y^2+z^2) = 5r^2$

$\Phi = \int dV(\vec{\nabla}\vec{F}) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \int_0^{R} 5r^2 dr\, r^2\sin\theta = 2\pi\cdot 2\cdot 5\frac{R^5}{5} = 4\pi R^5$

$\Downarrow$

Th Гаусса выполняется