№1

$$U(\vec{r}_1, \vec{r}_2) = \frac{e_1 e_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$L = \frac{m_1 \dot{\vec{r}}_1^2}{2} + \frac{m_2 \dot{\vec{r}}_2^2}{2} - \frac{e_1 e_2}{|\vec{r}_1 - \vec{r}_2|} - e_1 \varphi_1 - e_2 \varphi_2$$

$$\left.\begin{aligned} \vec{R} &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \\ \vec{\rho} &= \vec{r}_1 - \vec{r}_2 \end{aligned}\right\} \Rightarrow \vec{R} = \frac{m_1 \vec{r}_1 + m_2 (\vec{r}_1 - \vec{\rho})}{m_1 + m_2}$$

$$r_1 (m_1 + m_2) = \vec{R}(m_1 + m_2) + m_2 \vec{\rho}$$

$$\vec{r}_1 = \vec{R} + \frac{m_2}{m_1 + m_2} \vec{\rho}$$

$$\vec{r}_2 = \vec{R} - \frac{m_1}{m_1 + m_2} \vec{\rho}$$

$$\dot{\vec{r}}_1 = \dot{\vec{R}} + \frac{m_2}{m_1 + m_2} \dot{\vec{\rho}}$$

$$\dot{\vec{r}}_2 = \dot{\vec{R}} - \frac{m_1}{m_1 + m_2} \dot{\vec{\rho}}$$

$$L = \frac{m_1}{2} \left(\dot{\vec{R}} + \frac{m_2}{m_1 + m_2} \dot{\vec{\rho}}\right)^2 + \frac{m_2}{2} \left(\dot{\vec{R}} - \frac{m_1}{m_1 + m_2} \dot{\vec{\rho}}\right) - e_1 \varphi_1 - e_2 \varphi_2 - \frac{e_1 e_2}{|\vec{\rho}|}$$

$$L = \frac{m_1}{2} \left(\dot{\vec{R}}^2 + \frac{2 m_2}{m_1 + m_2} \dot{\vec{R}} \dot{\vec{\rho}} + \left(\frac{m_2}{m_1 + m_2}\right)^2 \dot{\vec{\rho}}^2\right) + \frac{m_2}{2} \left(\dot{\vec{R}}^2 - \frac{2 m_1}{m_1 + m_2} \dot{\vec{R}} \dot{\vec{\rho}} + \right.$$

$$\left. + \left(\frac{m_1}{m_1 + m_2}\right)^2 \dot{\vec{\rho}}^2\right) - e_1 \varphi_1 - e_2 \varphi_1 - \frac{e_1 e_2}{|\vec{\rho}|}$$

$$L = \frac{m_1 + m_2}{2} \dot{\vec{R}}^2 + \frac{m_1}{2} \left(\frac{m_2}{m_1 + m_2}\right)^2 \dot{\vec{\rho}}^2 + \frac{m_2}{2} \left(\frac{m_1}{m_1 + m_2}\right)^2 \dot{\vec{\rho}}^2 - e_1 \varphi_1 - e_2 \varphi_2 - \frac{e_1 e_2}{|\vec{\rho}|}$$

$$L = \underbrace{\frac{m_1 + m_2}{2} \dot{\vec{R}}^2}_{1} + \underbrace{\frac{m_1 m_2}{2 (m_1 + m_2)} \dot{\vec{\rho}}^2 - e_1 \varphi_1 - e_2 \varphi_2 - \frac{e_1 e_2}{|\vec{\rho}|}}_{\text{Лагранжиан } L(\rho, \dot{\rho}) \text{ от параметров частиц}}$$

Лагранжиан ц.м

№4

① $\vec{J} = m[\vec{r}\vec{v}] - \frac{eq}{cr}\vec{r}$

$\frac{d\vec{J}}{dt} = m[\dot{\vec{r}}\vec{v}] + m[\vec{r}\dot{\vec{v}}] - \frac{eq}{cr}\dot{\vec{r}}$

$m\dot{\vec{v}} = \frac{e}{c}[\vec{v}\vec{B}]$ $\Rightarrow m\dot{\vec{v}} = \frac{eq}{cr^3}[\vec{v}\vec{r}]$

$\vec{B} = \frac{q}{r^3}\vec{r}$

Умножим скалярно на $\vec{r}$

$m\vec{r}\dot{\vec{v}} = \frac{eq}{cr^3}\vec{r}\cdot[\vec{v}\vec{r}]$

смешанное произведение

$\vec{r}[\vec{v}\vec{r}] = \vec{v}[\vec{r}\vec{r}] = 0$

$(\vec{r}\cdot\vec{v}) = 0$

$\mathcal{E} = \frac{mv^2}{2}$

$\frac{d\vec{J}}{dt} = [\vec{r}\times m\dot{\vec{v}}] - \frac{eq}{cr}\dot{\vec{r}} =$

$= \underbrace{[\vec{r}[\vec{v}\vec{r}]}_{\text{"bac-cab"}}\frac{eq}{cr^3} - \frac{eq}{cr}\dot{\vec{r}} =$

$= \{\vec{v}(\vec{r}\cdot\vec{r}) - \vec{r}(\vec{r}\cdot\vec{v})\}\frac{eq}{cr^3} - \frac{eq}{cr}\vec{v} =$

$\vec{v}r^2$ $\quad 0$

$= \frac{eq}{cr^3}\vec{v}r^2 - \frac{eq}{cr}\vec{v} = 0$