№1

$x = (R + \rho\cos\theta)\cos\varphi$

$y = (R + \rho\sin\theta)\cos\varphi$

$z = \rho\sin\theta$

$0 \le \varphi \le 2\pi$

$0 \le \theta \le 2\pi$

$0 \le \rho \le r$

$$J = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & \sin\theta \\ -(R+\rho\cos\theta)\sin\varphi & (R+\rho\cos\theta)\cos\varphi & 0 \\ -\rho\cos\varphi\sin\theta & -\rho\sin\varphi\sin\theta & \rho\cos\theta \end{vmatrix}$$

$= \rho(R + \rho\cos\theta)$

$$V = \int dV = \int_0^r \rho\,d\rho \int_0^{2\pi} d\varphi \int_0^{2\pi} (R + \rho\cos\theta)\,d\theta = 2\pi \int_0^r \rho\,d\rho \left(\int_0^{2\pi} R\,d\theta + \underbrace{\int_0^{2\pi} \rho\cos\theta\,d\theta}_{0} \right) =$$

$$= 4\pi^2 R \frac{r^2}{2} = 2\pi^2 R r^2$$

$$\rho_0 = \frac{M}{4\pi^2 R r^2} \qquad I = \rho_0 \int (x^2+y^2)\,dV = \rho_0 \int_0^r \rho\,d\rho \int_0^{2\pi} d\varphi \int_0^{2\pi} (R + \rho\cos\theta)^3\,d\theta =$$

$$= \rho_0 2\pi \int_0^r \rho\,d\rho \int_0^{2\pi} (R^3 + \underbrace{3R^2\rho\cos\theta}_{0} + \underbrace{3R\rho^2\cos^2\theta}_{\frac{3\rho^2 R}{2}\int_0^{2\pi}(\cos 2\theta + 1)\,d\theta} + \underbrace{\rho^3\cos^3\theta}_{0})\,d\theta = 2\pi\rho_0 \int_0^r \rho\,d\rho (2\pi R^3 +$$

$$+ 3\pi\rho^2 R) = \frac{2M}{Rr^2} \int_0^r \rho\,d\rho \left(R^3 + \frac{3}{2}R\rho^2\right) = \frac{2M}{Rr^2}\left(R^3\frac{r^2}{2} + \frac{3}{8}Rr^4\right) =$$

$$= \frac{M}{R}\left(R^3 + \frac{3}{4}Rr^2\right) = M\left(R^2 + \frac{3}{4}r^2\right)$$

№2 $\lambda = a\rho^2$

$$\varphi(h) = \int_0^{2\pi} d\theta \int_0^R d\rho\, \rho \frac{a\rho^2}{\sqrt{\rho^2+h^2}} = 2\pi a \int_0^R \frac{\rho^3\,d\rho}{\sqrt{\rho^2+h^2}} = \frac{2\pi a}{h} \int_0^R \frac{\rho^3\,d\rho}{\sqrt{\frac{\rho^2}{h^2}+1}} =$$

$dq = a\rho^2 \rho\,d\rho$

$d\varphi = \frac{dq}{l} = \frac{dq}{\sqrt{\rho^2+h^2}}$

$t = \frac{\rho^2}{h^2}$

$dt = \frac{1}{h^2} 2\rho\,d\rho$

$$= \frac{\pi a h^4}{h} \int_0^{R^2/h^2} \frac{t\,dt}{\sqrt{t+1}} =$$

$$= \pi h^3 a \left(\int_0^{R^2/h^2} (t+1)^{1/2}\,dt - \int_0^{R^2/h^2} (t+1)^{-1/2}\,dt \right) = \pi a h^3 \left(\frac{2}{3}(t+1)^{3/2}\Big|_0^{R^2/h^2} - 2(t+1)^{1/2}\Big|_0^{R^2/h^2} \right) =$$

$$= \pi a h^3 \left(2\sqrt{\frac{\rho^2}{h^2}+1}\left(\frac{\left(\frac{\rho^2}{h^2}+1\right)}{3} - 5\right) + \frac{4}{3}\right)$$

N4

$$I = \int dS\, e^{ikr}$$

$\vec{k}(0;0;k)$ $\vec{r}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$(\vec{k}\cdot\vec{r}) = k\cos\theta$

$dS = \sin\theta\, d\theta\, d\varphi$

$$I = \iint e^{ik\cos\theta}\sin\theta\, d\theta\, d\varphi = \int_0^{2\pi} d\varphi \int_0^{\pi} e^{ik\cos\theta}\sin\theta\, d\theta = -2\pi\int_0^{\pi} e^{ik\cos\theta}\, d(\cos\theta) =$$

$$= -\frac{2\pi}{ik} e^{ik\cos\theta}\Big|_0^{\pi} = \frac{-2\pi}{ik}\left[e^{-ik} - e^{ik}\right] = \frac{4\pi}{k}\left(\frac{e^{ik}-e^{-ik}}{2i}\right) =$$

$$= \frac{4\pi}{k}\sin k$$

N5

$$\oint \frac{dz}{z^2} = \oint \frac{|z|\, i e^{i\varphi} d\varphi}{|z|^2 e^{2i\varphi}} = \oint |z|^{-1} i e^{-i\varphi} d\varphi = -\frac{|z|^{-1}}{i} i e^{-i\varphi}\Big|_0^{2\pi} =$$

$z = |z| e^{i\varphi}$

$dz = |z|\, i e^{i\varphi} d\varphi$

$$= -|z|^{-1}(i - i) = 0$$

N3

$$\varphi(R) = \int_{r<a} \frac{d^3\vec{r}}{|\vec{R}-\vec{r}|^2}$$

$\vec{R} = (0;0;R)$ $\vec{r} = (\rho\sin\theta\cos\varphi, \rho\sin\theta\sin\varphi, \rho\cos\theta)$

$$|\vec{R}-\vec{r}|^2 = (\rho\sin\theta\cos\varphi)^2 + (\rho\sin\theta\sin\varphi)^2 + (\rho\cos\theta - R)^2 =$$

$$= \rho^2\sin^2\theta(\cos^2\varphi + \sin^2\varphi) + \rho^2\cos^2\theta - 2\rho\cos\theta R + R^2 = \rho^2 + R^2 - 2\rho R\cos\theta$$

$J = \rho^2\sin\theta$

~~$dr^3 = r^2 dr \sin\theta\, d\theta\, d\varphi$~~

$$\Phi = \iiint \frac{\rho^2\sin\theta\, d\rho\, d\theta\, d\varphi}{\rho^2 + R^2 - 2\rho R\cos\theta} = \int_0^{2\pi} d\varphi \int_0^{\pi}\sin\theta\, d\theta \int_0^{r} \frac{\rho^2 d\rho}{\rho^2 + R^2 - 2\rho R\cos\theta}$$