DEVELOPING CLOSURES FOR TURBULENT FLOW OF VISCOELASTIC FENE-P FLUIDS

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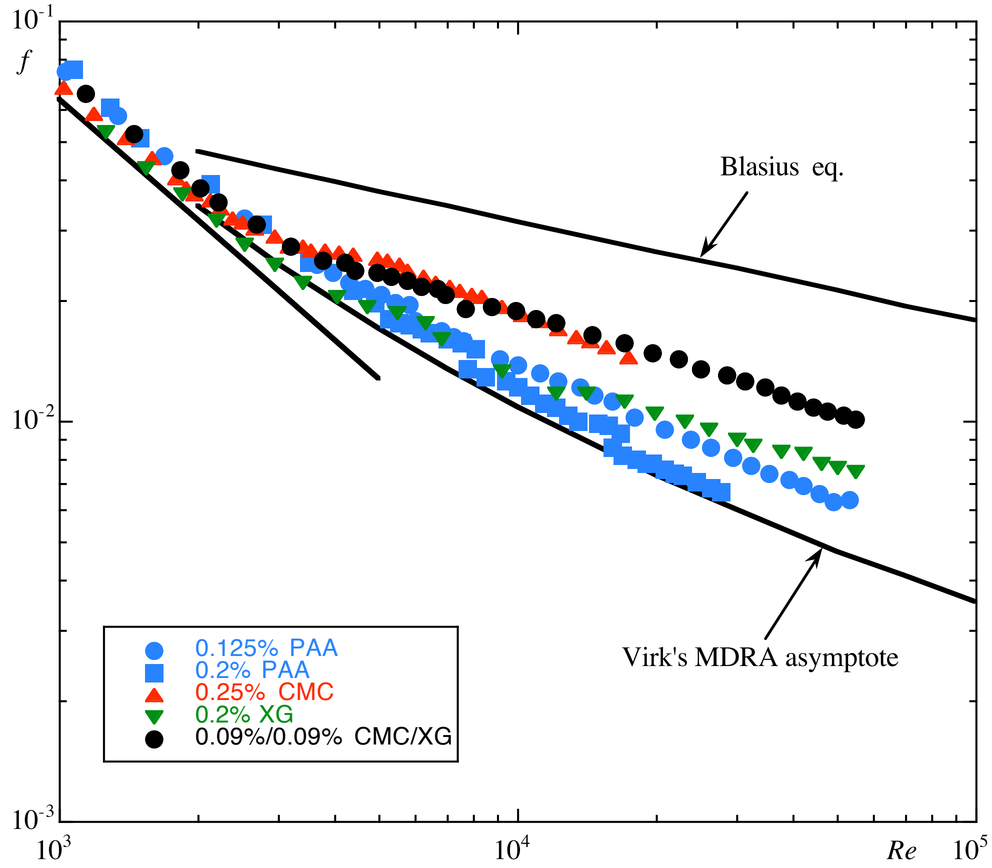
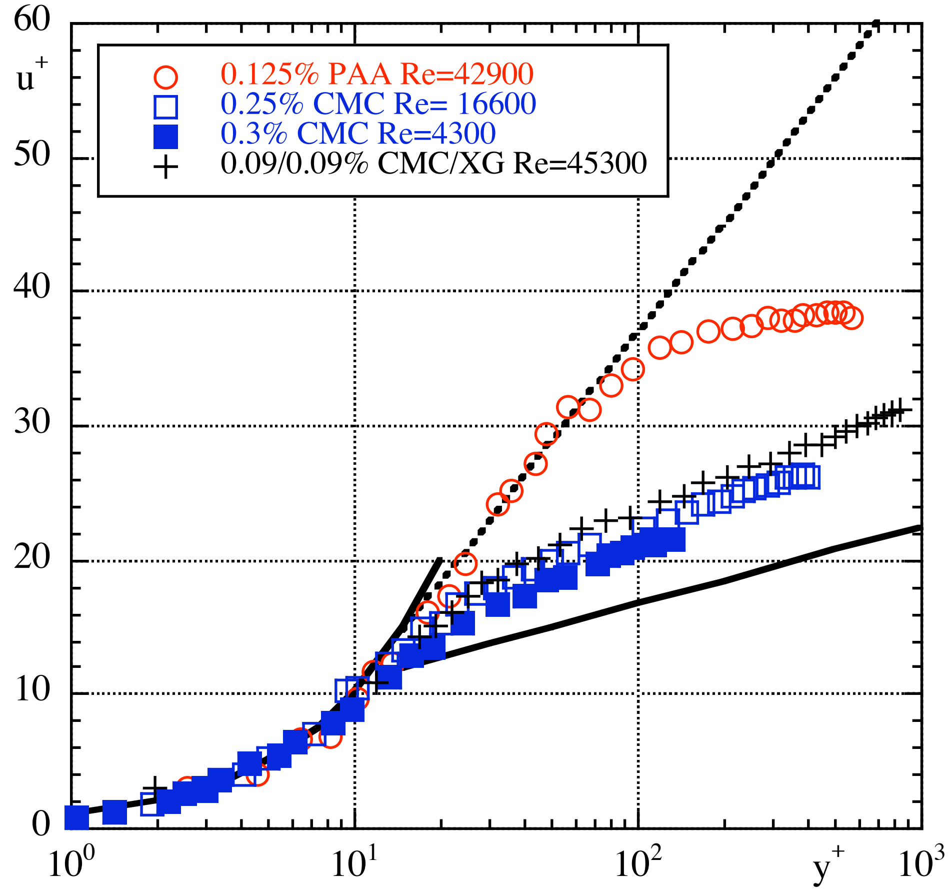
Conference on "Whither rheology ?"
2nd-4th April 2007
Lake Vyrnwy Hotel, Mid Wales, UK

Relevance: drag reduction in turbulent pipe flow





Escudier et al JNNFM (1999)

- Reduction of shear Reynolds stress (DR)
- Increase of normal streamwise Reynolds stress
- Dampening of normal radial and tangential Reynolds stress

Deficit of Reynolds stress

Time-average governing equations: turbulent flow & FENE-P

^- instantaneous
Overbar or capital letter- time-average
` or small letter- fluctuations

Continuity: $$\frac{\partial U_i}{\partial x_i} = 0$$

Momentum balance:

$$\rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial \overline{p}}{\partial x_i} + \eta_s \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k}\left(\rho \overline{u_i u_k}\right) + \frac{\partial \overline{\tau}_{ik,p}}{\partial x_k}$$

$$\overline{\tau}_{ij} = 2\eta_s S_{ij} + \overline{\tau}_{ij,p}$$

Rheological constitutive equation: **FENE-P**

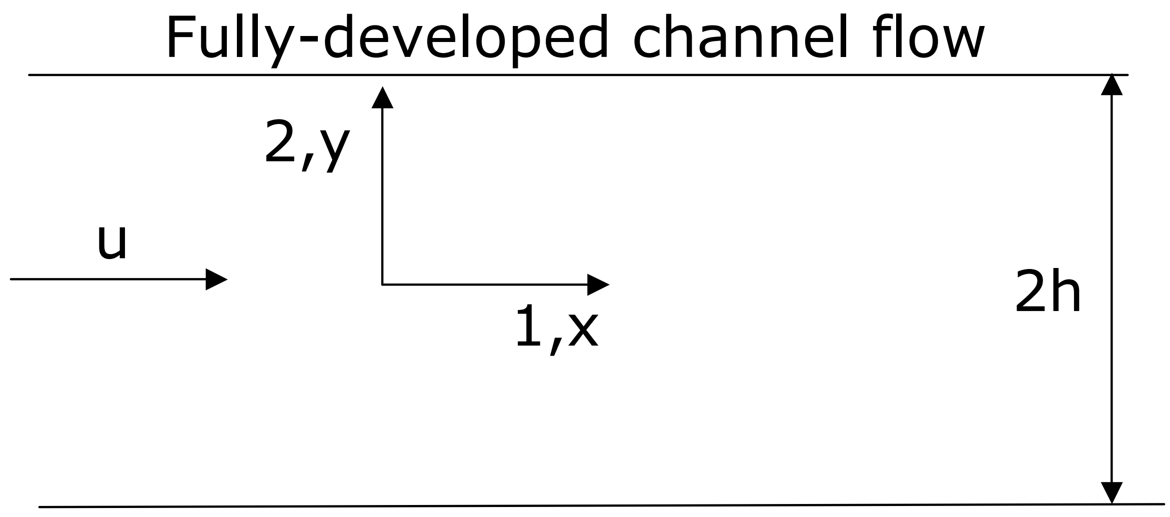
$$\hat{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f\left(\hat{C}_{kk}\right)\hat{C}_{ij} - f(L)\delta_{ij} \right]$$

$$f\left(\hat{C}_{kk}\right)\hat{C}_{ij} + \lambda\left(\frac{\partial \hat{C}_{ij}}{\partial t} + \hat{U}_k \frac{\partial \hat{C}_{ij}}{\partial x_k} - \hat{C}_{jk}\frac{\partial \hat{U}_i}{\partial x_k} - \hat{C}_{ik}\frac{\partial \hat{U}_j}{\partial x_k} \right) = f(L)\delta_{ij}$$

$$\left(\frac{\partial \hat{C}_{ij}}{\partial t} + \hat{U}_k \frac{\partial \hat{C}_{ij}}{\partial x_k} - \hat{C}_{jk}\frac{\partial \hat{U}_i}{\partial x_k} - \hat{C}_{ik}\frac{\partial \hat{U}_j}{\partial x_k} \right) = \overset{\nabla}{\hat{C}}_{ij} = -\frac{\hat{\tau}_{ij,p}}{\eta_p}$$

DNS case: LDR

DNS, DR=18% (LDR)



$$We_\tau = 25, Re_\tau = 395$$

$$\beta = 0.9, L^2 = 900$$

$$We_\tau = \frac{\lambda u_\tau^2}{\nu_0} \qquad Re_\tau = \frac{h u_\tau}{\nu_0}$$

Reynolds decomposition of conformation tensor

$$\hat{B} = B + b' \quad \text{where} \quad \overline{b'} = 0$$

Function: $f\left(C_{kk}\right) = \dfrac{L^2 - 3}{L^2 - C_{kk}}$

Time average polymeric stress

$$\overline{\tau}_{ij,p} = \frac{\eta_p}{\lambda}\left[f\left(C_{kk}\right)C_{ij} - f\left(L\right)\delta_{ij}\right] + \frac{\eta_p}{\lambda}\overline{f\left(C_{kk}+c_{kk}\right)c_{ij}}$$

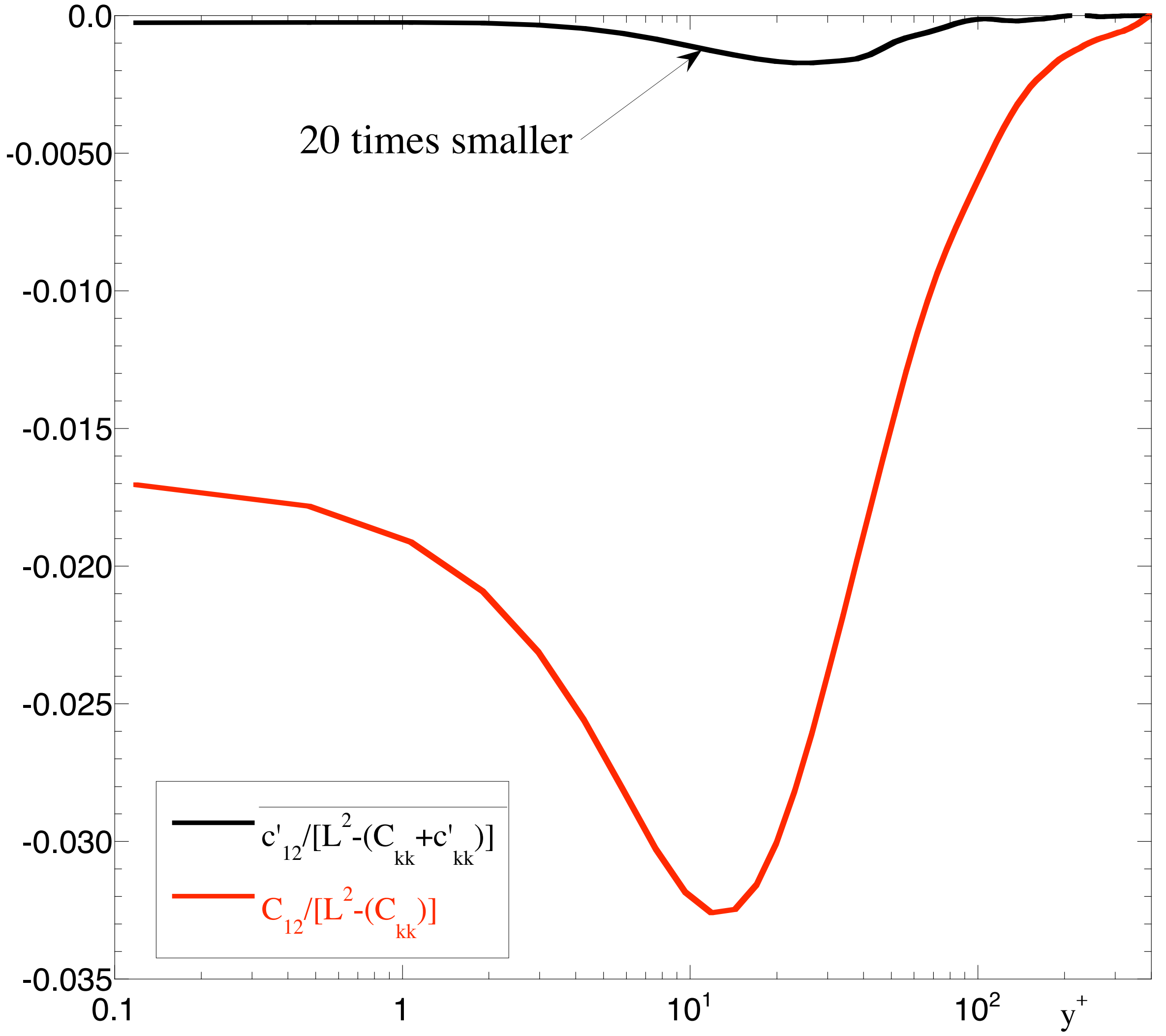
$\overline{f'c_{ij}}$ **Neglected** (see slide)

Time average conformation tensor equation

$$\lambda \overset{\nabla}{C}_{ij} + \lambda\left[\overline{u_k \frac{\partial c_{ij}}{\partial x_k}} - \left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)\right] = -\left[f\left(C_{kk}\right)C_{ij} - f\left(L\right)\delta_{ij} + \overline{f\left(C_{kk}+c_{kk}\right)c_{ij}}\right]$$

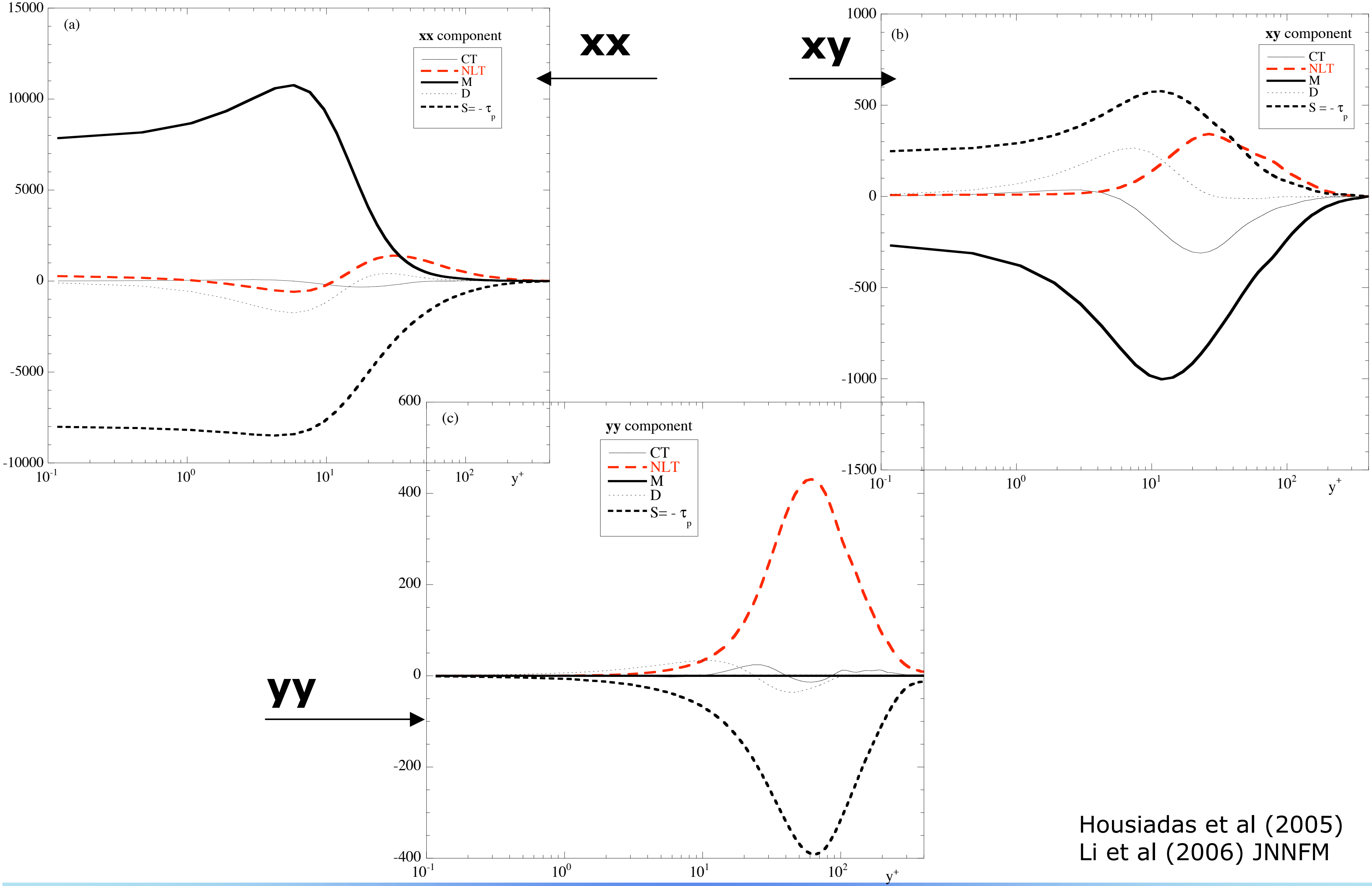
$$\boxed{\overset{\nabla}{C}_{ij} + \underbrace{\overline{u_k \frac{\partial c_{ij}}{\partial x_k}}}_{CT_{ij}} - \underbrace{\left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)}_{NLT_{ij}} = -\frac{\overline{\tau}_{ij,p}}{\eta_p}}$$

Simplifying assumptions: justification from DNS



$$f\left(C_{kk}\right)C_{12} \gg \overline{f' c_{12}} = \overline{f\left(C_{kk}+c'_{kk}\right)c'_{12}} = \left(L^2-3\right)\overline{\frac{c'_{12}}{L^2-\left(C_{kk}+c'_{kk}\right)}}$$

Polymer stress: DNS



Housiadas et al (2005)
Li et al (2006) JNNFM

Modeling requirements

$$\frac{\overline{\tau}_{ij,p}}{\eta_p} = -\underbrace{\overset{\nabla}{C}_{ij}}_{M_{ij}} - \underbrace{\cancel{\overline{u_k \frac{\partial c_{ij}}{\partial x_k}}}}_{CT_{ij}} + \underbrace{\left(\overline{c_{kj}\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik}\frac{\partial u_j}{\partial x_k}}\right)}_{NLT_{ij}}$$

NLT_{ij}: originates in Oldroyd derivative
not negligible, must be modeled

CT_{ij}: originates in advective term, it is negligible
no need for modeling

DNS: Housiadas et al (2005), Li et al (2006) JNNFM

What about:

1) Reynolds stresses?

2) Turbulent kinetic energy ?

3) Dissipation of turbulent kinetic energy or of Reynolds stresses ?

Transport equation for the Reynolds stresses and *k*

$$\boxed{\rho\frac{\partial\overline{u_iu_j}}{\partial t}+\rho U_k\frac{\partial\overline{u_iu_j}}{\partial x_k}=P_{ij}+Q_{ij}+Q_{ij}^V+D_{ij,N}+\Pi_{ij}-\rho\varepsilon_{ij}^N-\rho\varepsilon_{ij}^V}$$

$$Q_{ij}^V=\frac{\partial}{\partial x_k}\left(\overline{u_i\tau'_{jk,p}}+\overline{u_j\tau'_{ik,p}}\right)$$

Viscoelastic turbulent transport due to fluctuations polymeric stresses

$$\varepsilon_{ij}^V=\frac{1}{\rho}\left(\overline{\tau'_{jk,p}\frac{\partial u_i}{\partial x_k}}+\overline{\tau'_{ik,p}\frac{\partial u_j}{\partial x_k}}\right)$$

Viscoelastic work of polymer chains: dissipation of energy plus stored free energy
(<0 ou >0)

$$\rho\frac{Dk}{Dt}+\rho\overline{u_iu_k}\frac{\partial U_i}{\partial x_k}=-\rho\overline{u_i\frac{\partial k'}{dx_i}}-\frac{\partial\overline{p'u_i}}{\partial x_i}+\eta_s\frac{\partial^2 k}{\partial x_i\partial x_i}-\eta_s\underbrace{\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k}}}_{\varepsilon^N}+\underbrace{\frac{\partial\overline{\tau'_{ik,p}u_i}}{\partial x_k}}_{Q^V}-\underbrace{\overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}}}_{\varepsilon^V}$$

Transport equation of ε^N

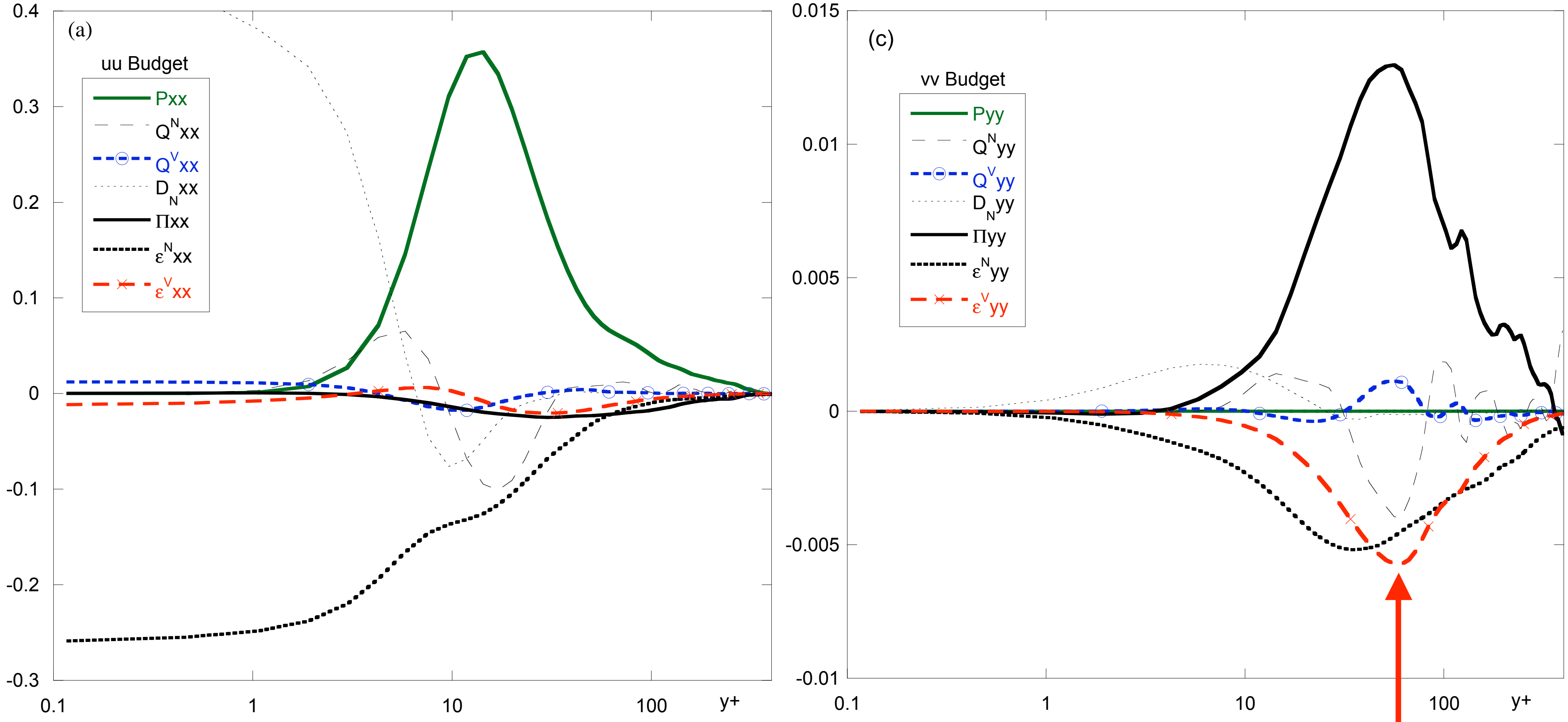
$$2\nu_s \overline{\frac{\partial u_i}{\partial x_m}\frac{\partial}{\partial x_m}\left(\rho\frac{Du_i}{Dt}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m}\frac{\partial}{\partial x_m}\left(\rho u_k\frac{\partial U_i}{\partial x_k}\right)} + 2\nu_s \overline{\frac{\partial u_i}{\partial x_m}\frac{\partial}{\partial x_m}\left(\rho\frac{\partial u_i u_k}{\partial x_k}\right)}$$

$$+2\nu_s \overline{\frac{\partial u_i}{\partial x_m}\frac{\partial}{\partial x_m}\left(\frac{\partial p'}{\partial x_i}\right)} - 2\rho\nu_s^2 \overline{\frac{\partial u_i}{\partial x_m}\frac{\partial}{\partial x_m}\left(\frac{\partial^2 u_i}{\partial x_k^2}\right)} - 2\nu_s \overline{\frac{\partial u_i}{\partial x_m}\frac{\partial}{\partial x_m}\left(\frac{\partial \tau'_{ik,p}}{\partial x_k}\right)} = 0$$

New term

As for Newtonian fluids, the whole equation will be approximated

Budget of Reynolds stress



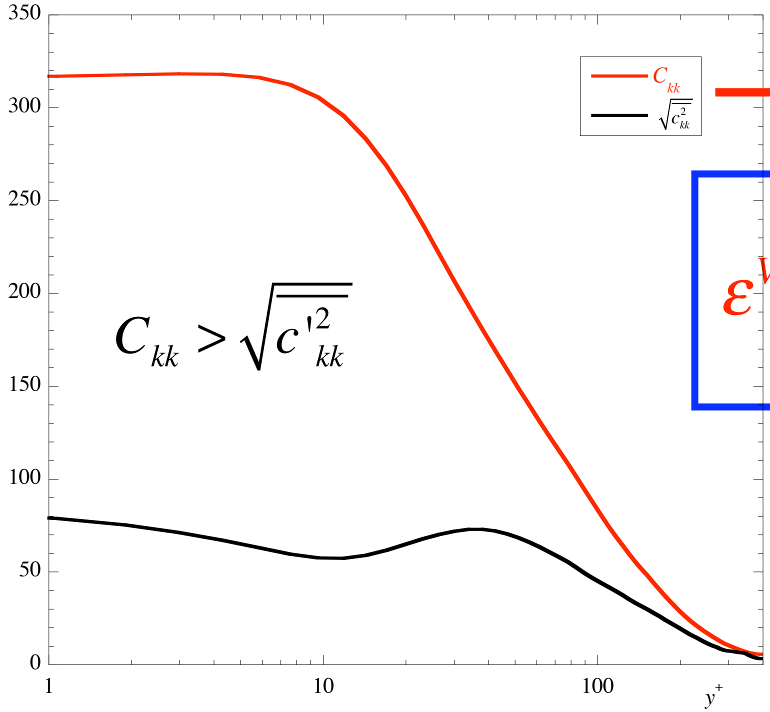
- Need to model viscoelastic stress work
- Need to model pressure strain (effect of elasticity)- Advanced mod.
- Viscoelastic turbulent transport is not so important

Viscoelastic work

$$\varepsilon^V \equiv \frac{1}{\rho}\overline{\tau'_{ik,p}\frac{\partial u_i}{\partial x_k}} = \frac{\eta_p}{\rho\lambda}\left[\overline{C_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}} + \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}\right]$$

Assumptions & DNS:

$$\overline{C_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}} < \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)\frac{\partial u_i}{\partial x_k}}$$



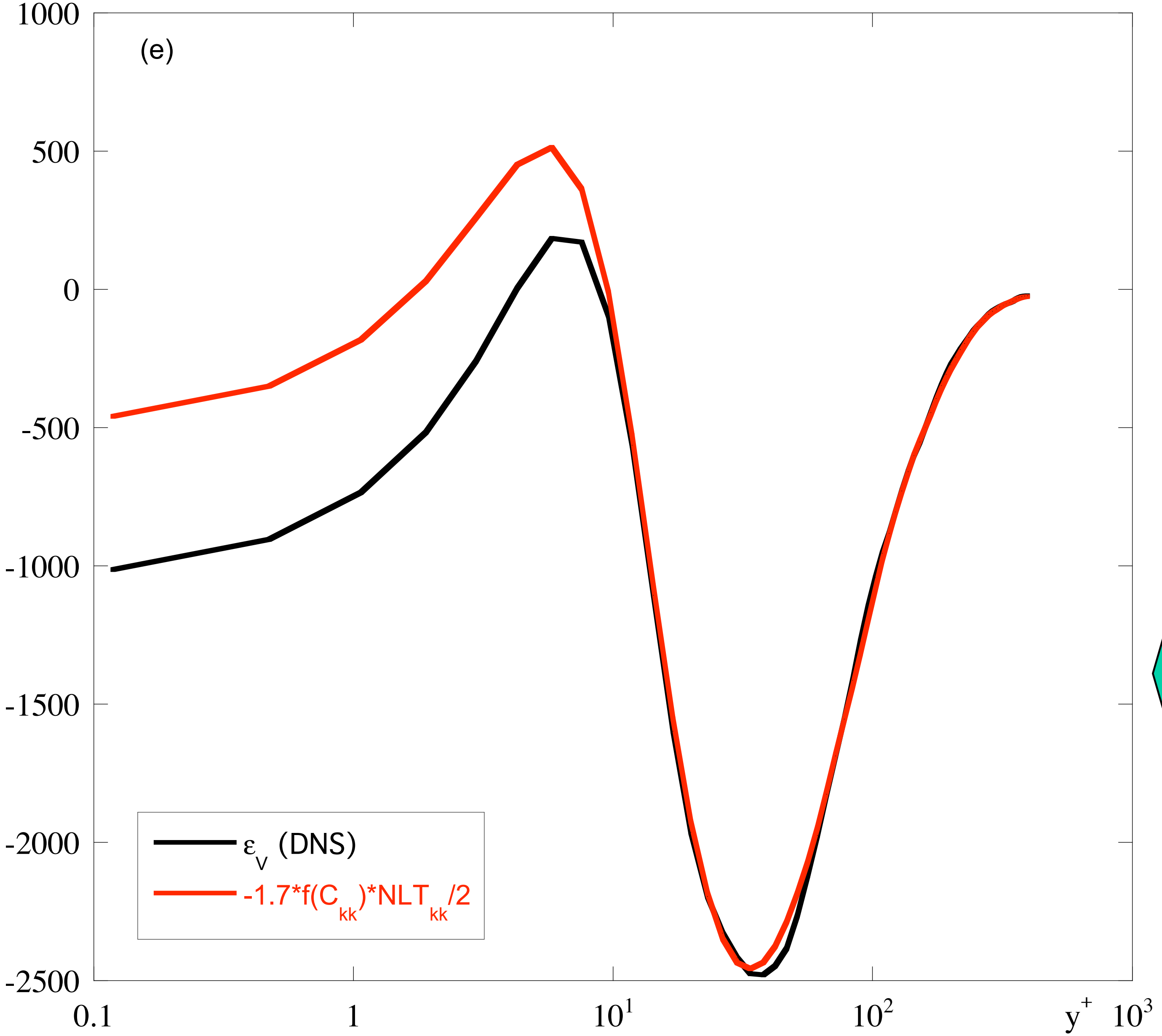
$$\boxed{\varepsilon^V \approx \frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\overline{c_{ik}\frac{\partial u_i}{\partial x_k}} = C_{\varepsilon^V}\frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\frac{NLT_{nn}}{2}}$$

Needs model

$$C_{\varepsilon^V} = 1.076$$

(from DNS- next slide)

Performance of the viscoelastic work model



$$\varepsilon^{V} > 0$$

Negative sign:
different definitions

$$\varepsilon^{V+}\left(\mathrm{Re}_{\tau_0}\right)^2$$

versus

$$f\left(C_{kk}\right)NLT_{ij}^{*}$$

Modeling NLT_{ij} 1

Key ideas:

1) Write down exact equation- complex 4 lines long
2) Assumptions, physical insight, trial-and-error
3) Do *a priori* testing of each term
4) Select appropriate combination and dimensional homogeneity
5) Try in code - under investigation

$$\overline{u_i u_m \frac{\partial c_{kj}}{\partial x_m}} + \overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} \approx Coef \times \overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m}$$

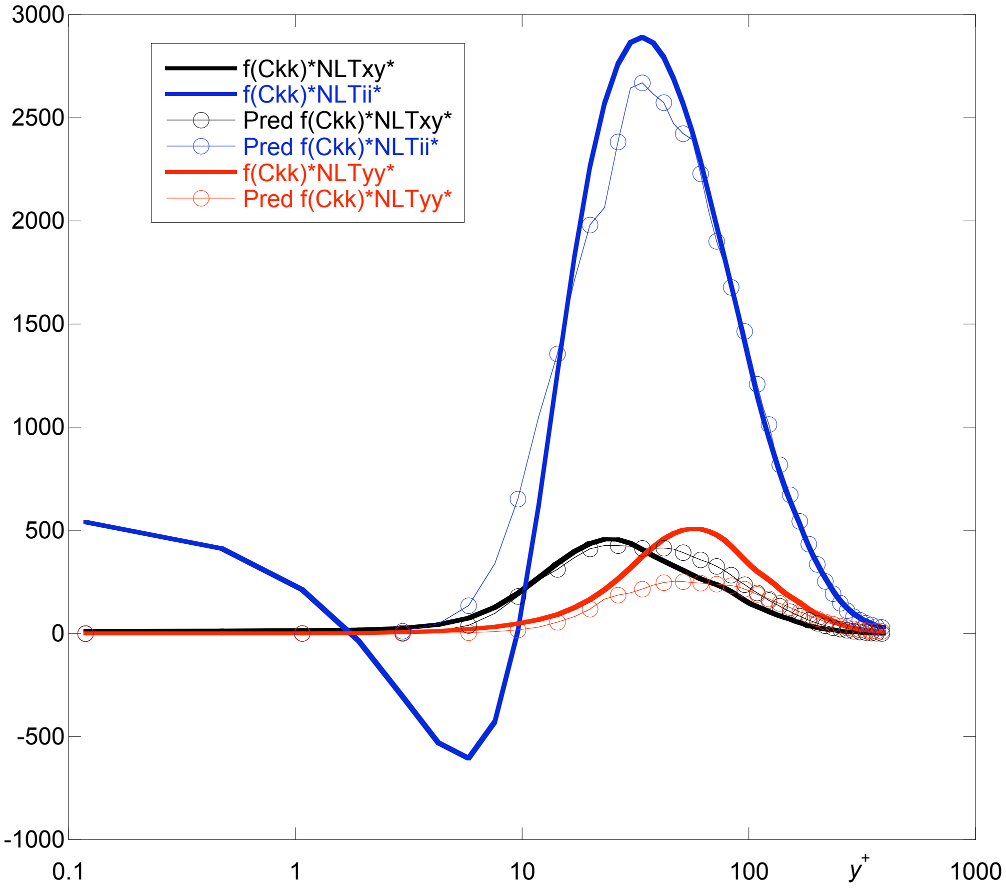
$$f(C_{mm})\frac{NLT_{ij}}{\lambda} = function\left(S_{ij}, W_{ij}, C_{ij}, \varepsilon_{ij}^{N}, \frac{\partial \overline{u_i u_j}}{\partial x_k}, \frac{\partial C_{ij}}{\partial x_k}, \frac{\partial NLT_{ij}}{\partial x_n}, M_{ij}, \overline{u_i u_j}\right)$$

$$\boxed{f(C_{mm})\frac{NLT_{ij}}{\lambda} = f_{\mu_1}\left[C_{\gamma_1}\frac{\partial \overline{u_k u_n}}{\partial x_n}\frac{\partial C_{ij}}{\partial x_k} + \boxed{\frac{C_{E_3} u_\tau^2}{\nu_0^2} C_{kk}\overline{u_i u_j} + \frac{C_{\alpha_{14}}}{\nu_0}\left(\overline{u_i u_k} W_{kn} C_{nj} + \overline{u_j u_k} W_{kn} C_{ni} + \overline{u_k u_i} W_{jn} C_{nk}\right)}\right]}$$

Model for NLT_{ij} 2

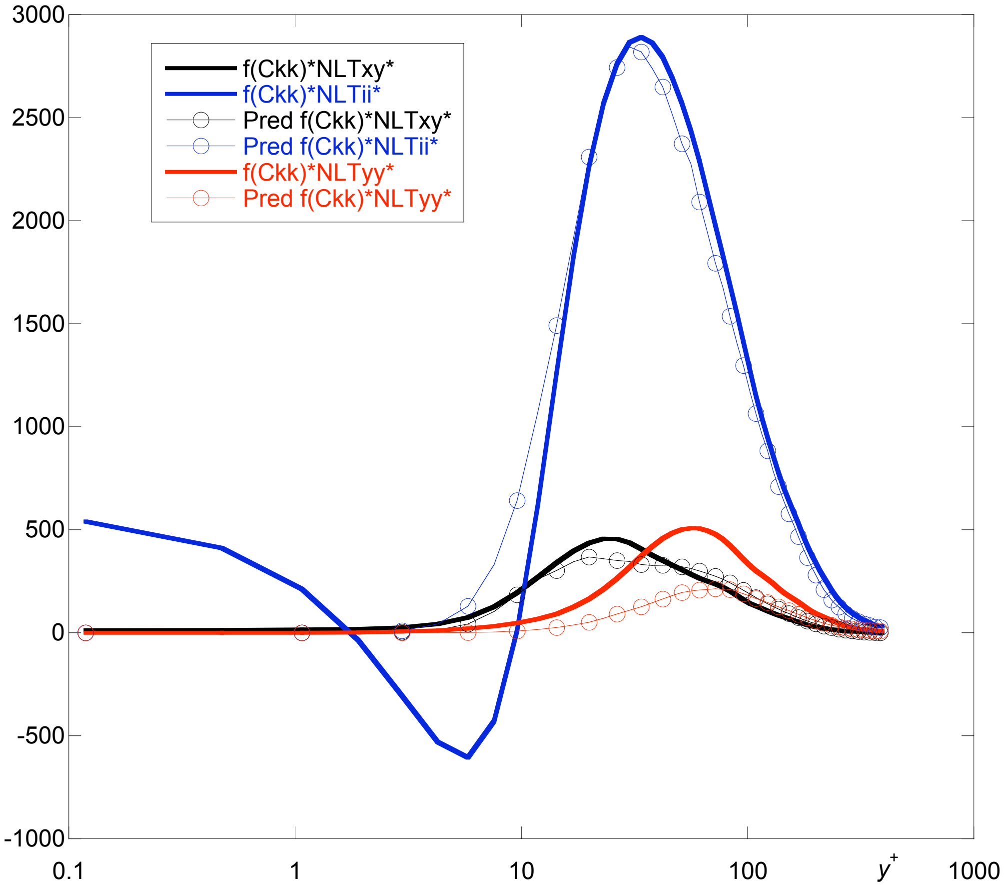
Blue model

$$C_{E_3} = 0.0004; C_{\gamma_1} = 3; C_{\alpha_{14}} = 0.00015$$



Red model

$$C_{E_3} = 0.00035; C_{\gamma_1} = 0; C_{\alpha_{14}} = 0.00015$$



$$f_{\mu_1} = \left(1 - \exp\left(-y^+/26.5\right)\right)^2$$

Viscoelastic turbulent transport

$$Q^V \equiv \frac{\partial \overline{\tau'_{ik,p} u_i}}{\partial x_k} = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)u_i} + \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)u_i} \right]$$

$$C_{kk} > \sqrt{\overline{c'^2_{kk}}}$$

$$f\left(\hat{C}_{mm}\right) = \frac{L^2-3}{L^2-\left(C_{mm}+c_{mm}\right)}$$

Weak coupling between c_{kk} and c_{ij}, u_i

$$C_{ik}\overline{f\left(C_{mm}+c_{mm}\right)u_i} < \overline{c_{ik} f\left(C_{mm}+c_{mm}\right)u_i}$$

Neglect of this term is irrelevant because non-neglected term is modeled

$$Q^V = \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[f\left(C_{mm}\right) CU_{iik} \right]$$

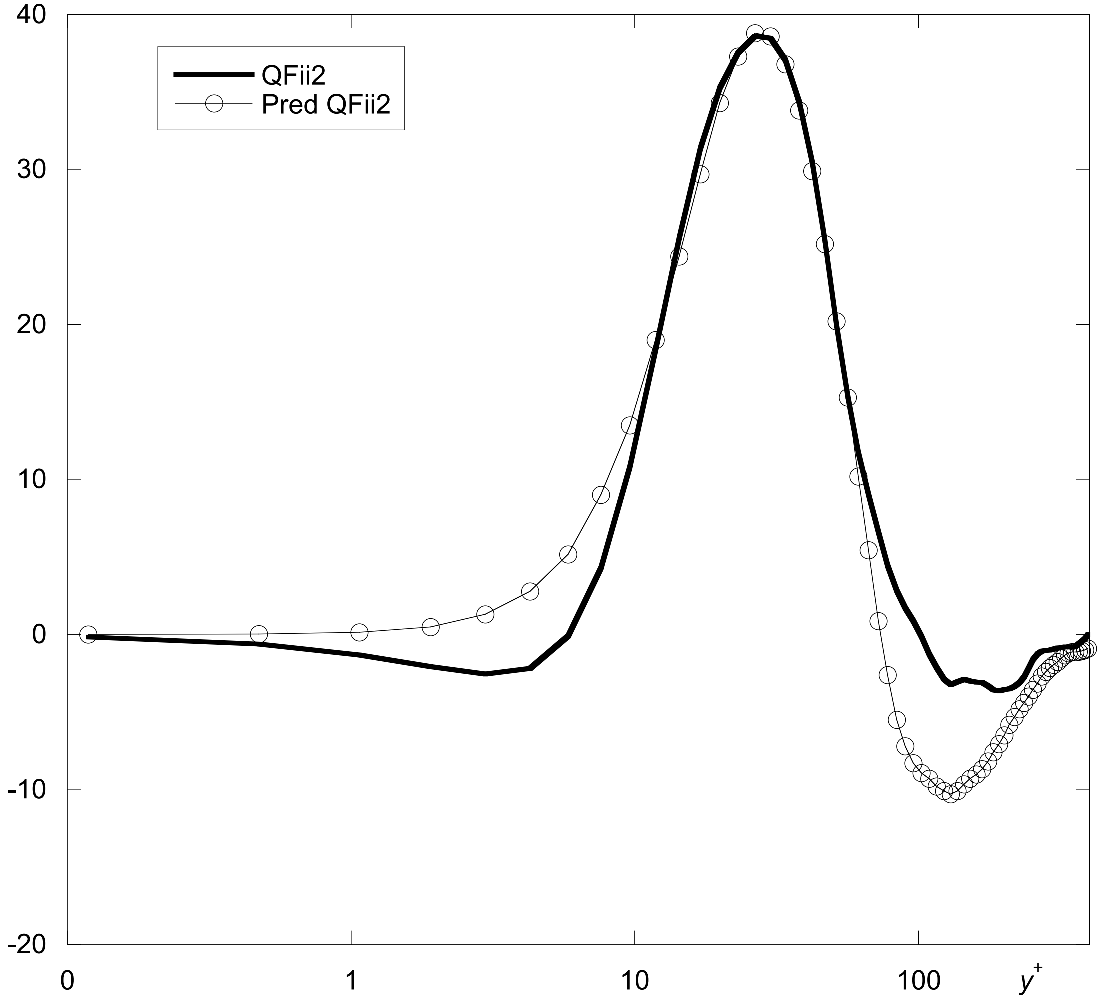
General case

Needs model $\left(CU_{ijk}\right)$

Model for CU_{ijk}

- Same modelling approach as with NLT_{ij}

$$\boxed{\frac{f(C_{mm})CU_{ijk}}{\lambda} = f_{\mu_2}\left[-C_{\beta_1}\left(\overline{u_i u_m}\frac{\partial C_{kj}}{\partial x_m} + \overline{u_j u_m}\frac{\partial C_{ik}}{\partial x_m}\right) - \frac{C_{\beta_7}}{\lambda} f(C_{mm})\left[\pm\sqrt{\overline{u_j^2}}C_{ik} \pm \sqrt{\overline{u_i^2}}C_{jk}\right]\right]}$$



$$C_{\beta_1} = 1.3; C_{\beta_7} = 0.37$$

$$f_{\mu_2} = 1 - \exp\left(-\frac{y^+}{26.5}\right)$$

Final equations: low *Re* *k*-*ε* type model for channel flow

Momentum:
$$\frac{d}{dy}\left[\eta_s\frac{dU}{dy}+\overline{\tau}_{p,xy}-\rho\overline{uv}\right]-\frac{d\overline{p}}{dx}=0$$

$$\overline{\tau}_{xy,p}=\frac{\eta_p}{\lambda}f\left(C_{kk}\right)C_{xy}$$

$$f\left(C_{kk}\right)C_{xy}=\lambda C_{yy}\frac{dU}{dy}+\lambda NLT_{xy}$$

$$f\left(C_{kk}\right)C_{yy}=\lambda NLT_{yy}+1$$

$$f\left(C_{kk}\right)C_{xx}=2\lambda C_{xy}\frac{dU}{dy}+\lambda NLT_{xx}+1$$

$$f\left(C_{kk}\right)C_{zz}=\lambda NLT_{zz}+1$$

$$f\left(C_{kk}\right)=\frac{L^2-3}{L^2-\left(C_{xx}+C_{yy}+C_{zz}\right)}$$

Reynolds stress:
$$-\rho\overline{uv}=\rho\nu_T\frac{dU}{dy}\quad\text{with}\quad\nu_T=C_\mu f_\mu\frac{k^2}{\tilde{\varepsilon}^N}$$

k and ε transport equations: modified Nagano & Hishida

$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho\nu_T}{\sigma_k}\right)\frac{dk}{dy}\right]+P_k-\rho\tilde{\varepsilon}^N-\rho D+\eta_p\frac{d}{dy}\left[\frac{f\left(C_{mm}\right)}{\lambda}\frac{CU_{nny}}{2}\right]-\eta_p\frac{f\left(C_{mm}\right)}{\lambda}\frac{NLT_{nn}}{2}$$

$$\varepsilon^N=\tilde{\varepsilon}^N+D^N \qquad\longleftrightarrow\qquad D^N=2\eta_s\left(\frac{d\sqrt{k}}{dy}\right)^2$$

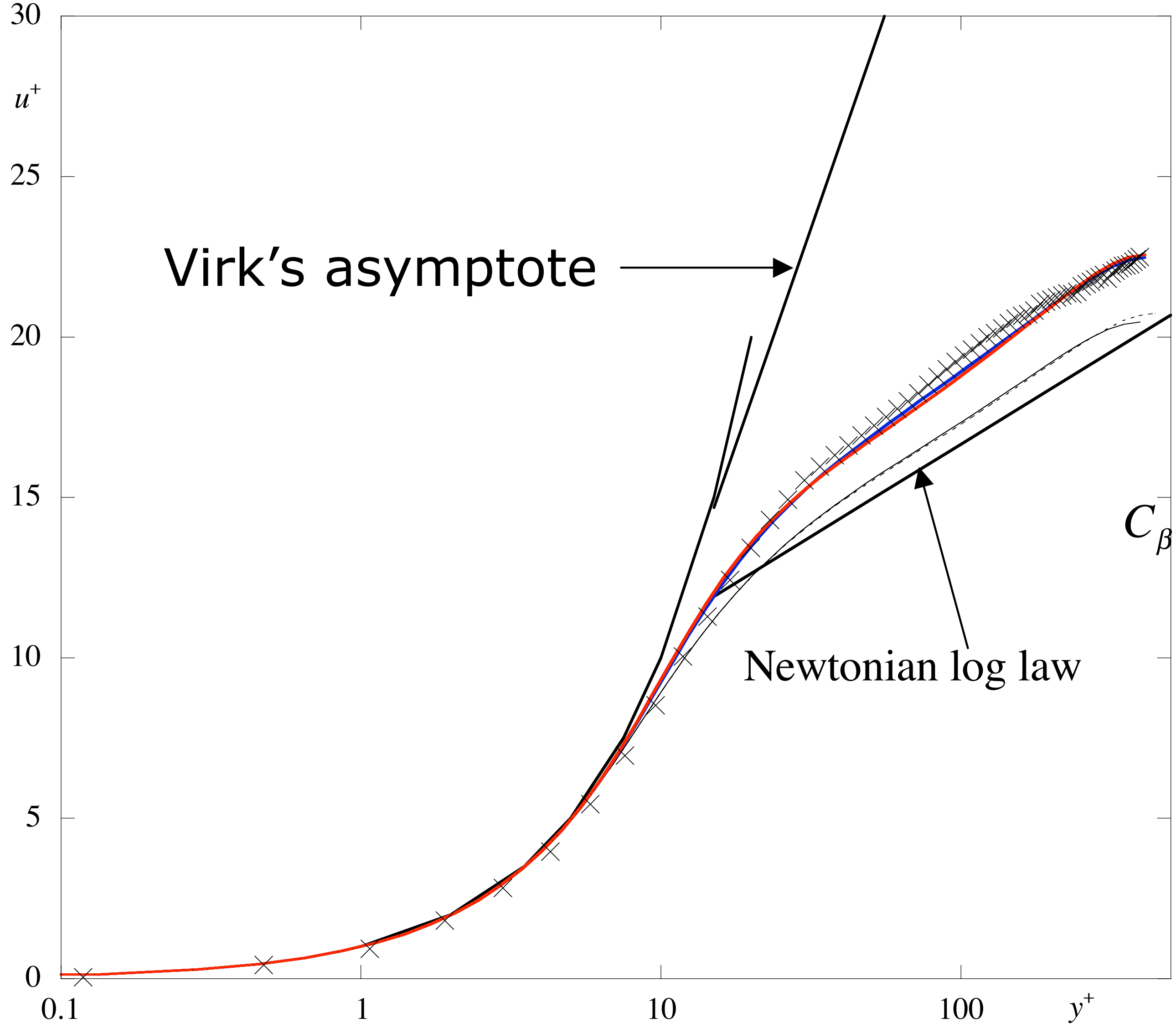
$$0=\frac{d}{dy}\left[\left(\eta_s+\frac{\rho\nu_T}{\sigma_\varepsilon}\right)\frac{d\tilde{\varepsilon}^N}{dy}\right]+\rho f_1C_{\varepsilon_1}\frac{\tilde{\varepsilon}^N}{k}\frac{P_k}{\rho}-\rho f_2C_{\varepsilon_2}\frac{{\varepsilon^N}^2}{k}+\rho E+E_{\tau_p}$$

$$E=\frac{\eta_s}{\rho}\nu_T\left(1-f_\mu\right)\left(\frac{d^2U}{dy^2}\right)^2$$

$$\boxed{f_1=1 \qquad f_2=1-0.3\exp\left(-R_T^2\right) \qquad f_\mu=\left[1-\exp\left(\frac{-y^+}{26.5}\right)\right]^2}$$

based on Newtonian model of Nagano & Hishida (1984)

Predictions 1: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900



X DNS

Black: Newtonian

——— $\eta = \eta_s + \eta_p$

— — — $\eta = \eta_{wall}$

- - - - - - - $\eta = \eta_{wall}$; same $\dot{Q}$; $\mathrm{Re}_{\tau_0} = 443$

FENE-P simulations

$$C_{\beta 1} = 1.3; C_{\beta 7} = 0.37; C_{\alpha 14} = 1.5 \times 10^{-4}$$

$$C_{\varepsilon^V} = 1.076$$

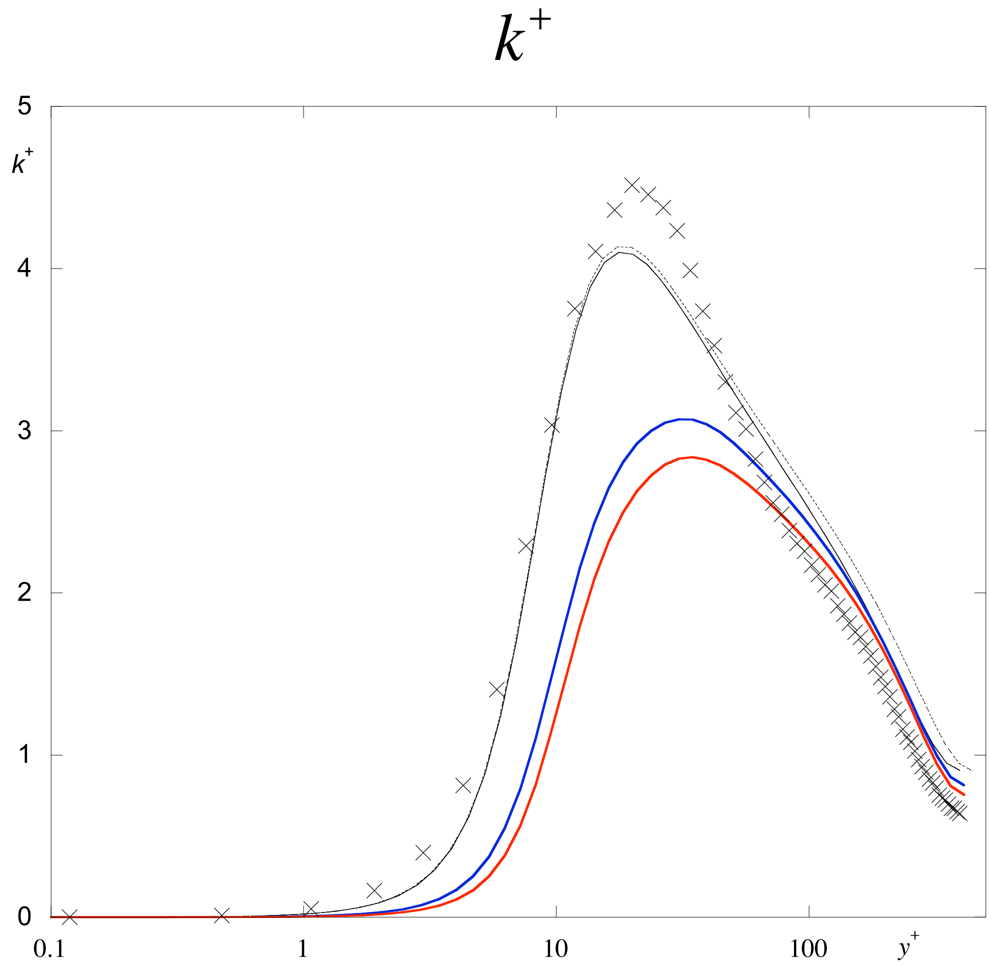
With ε^p

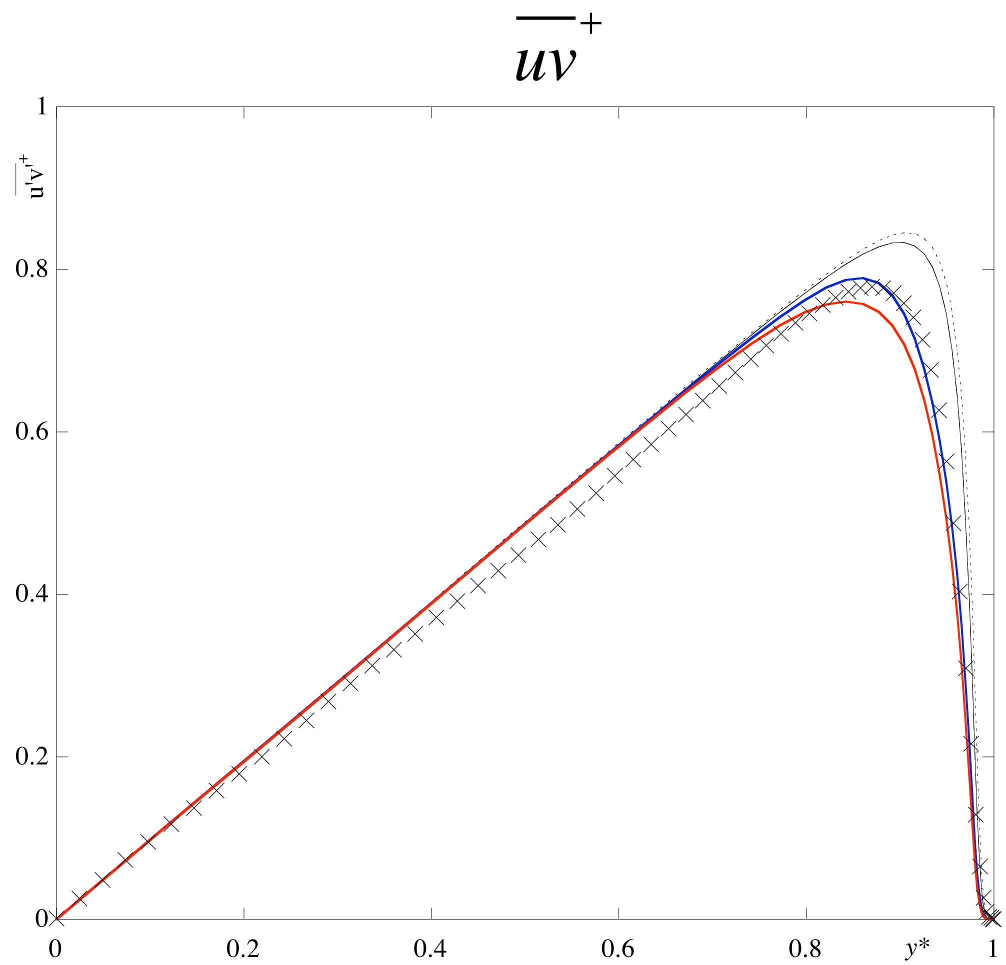
——— $C_{E3} = 1.93 \times 10^{-4}$

Without ε^p

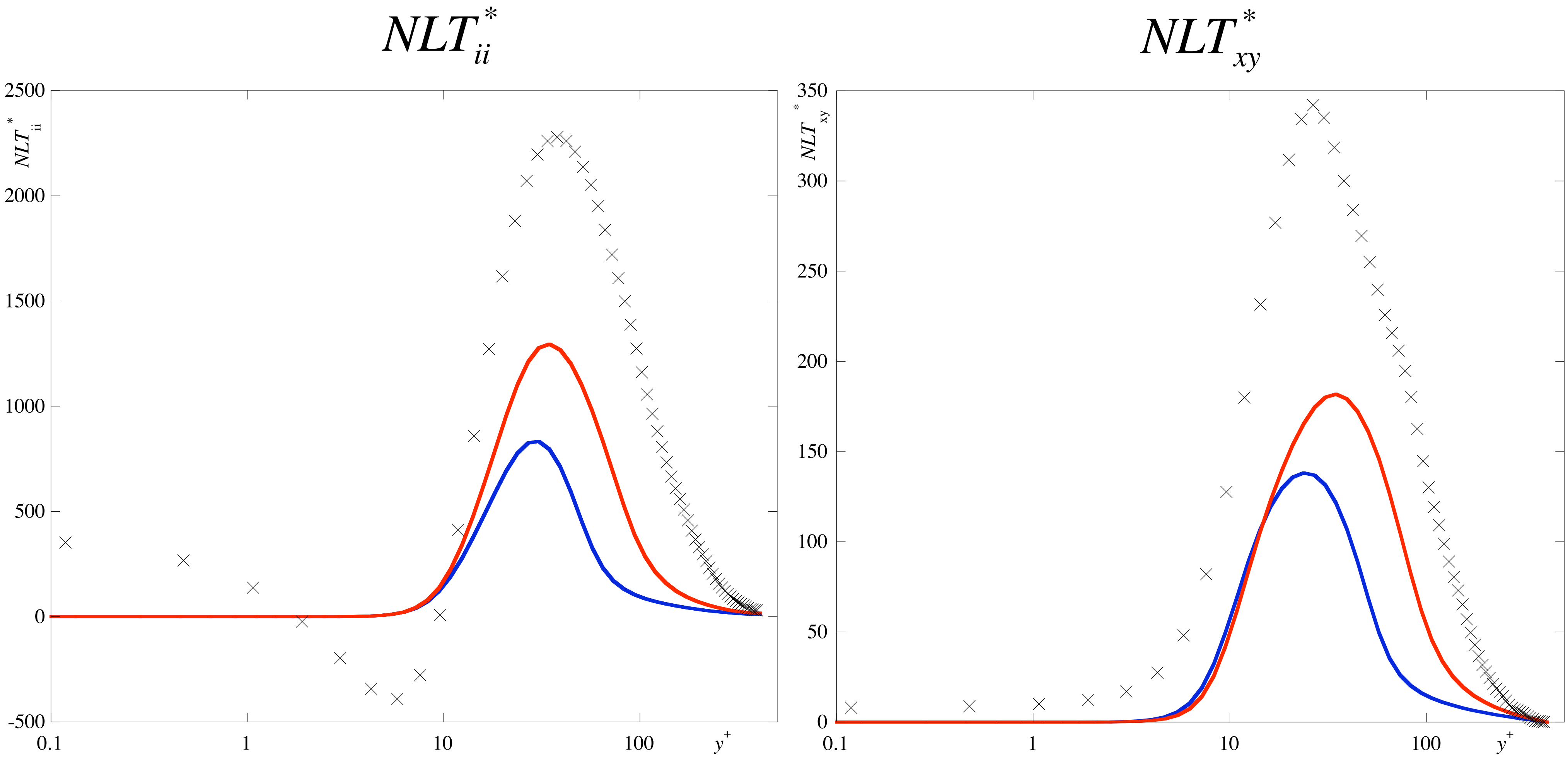
——— $C_{E3} = 2.86 \times 10^{-4}$

Predictions 2: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

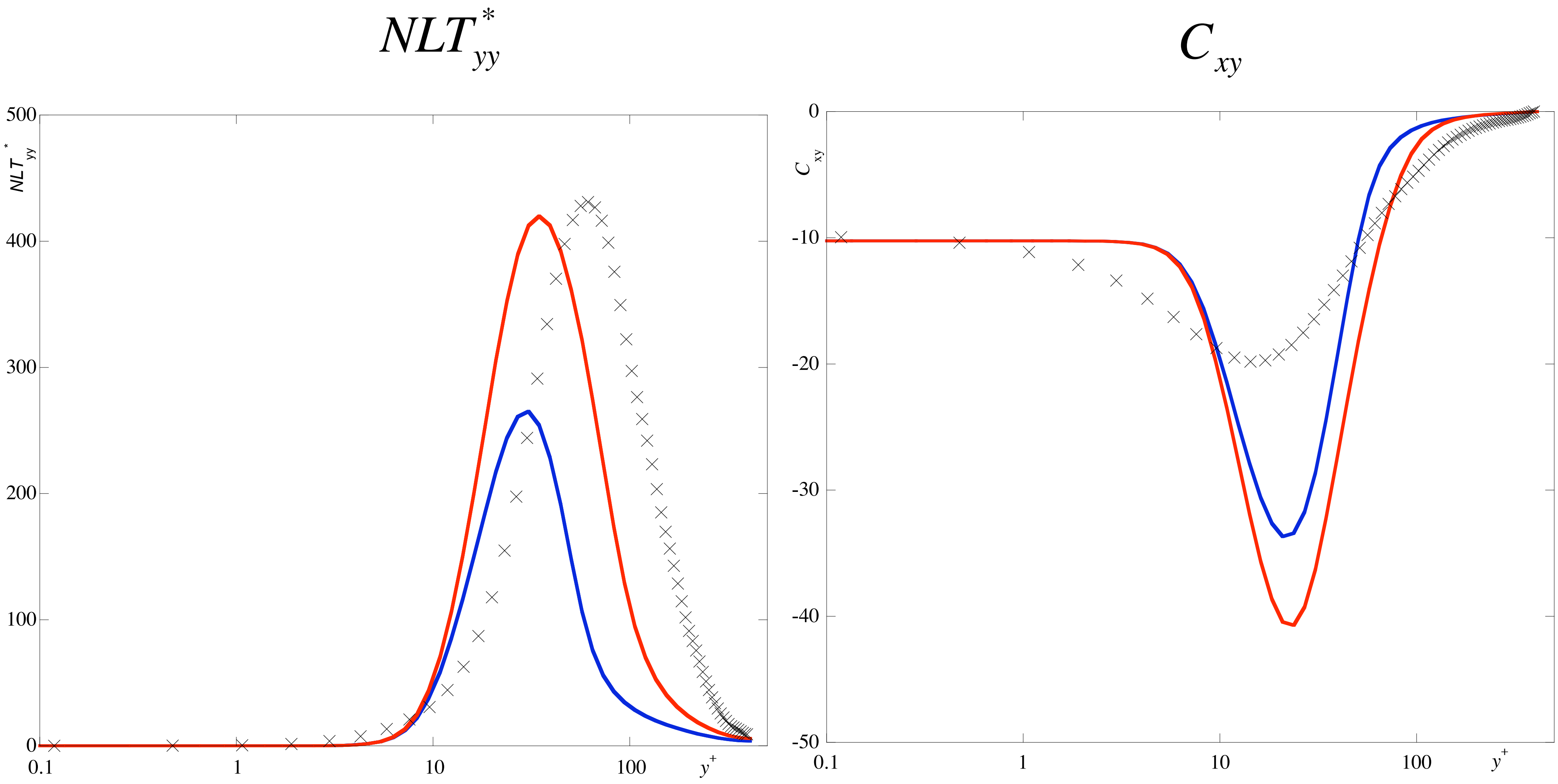




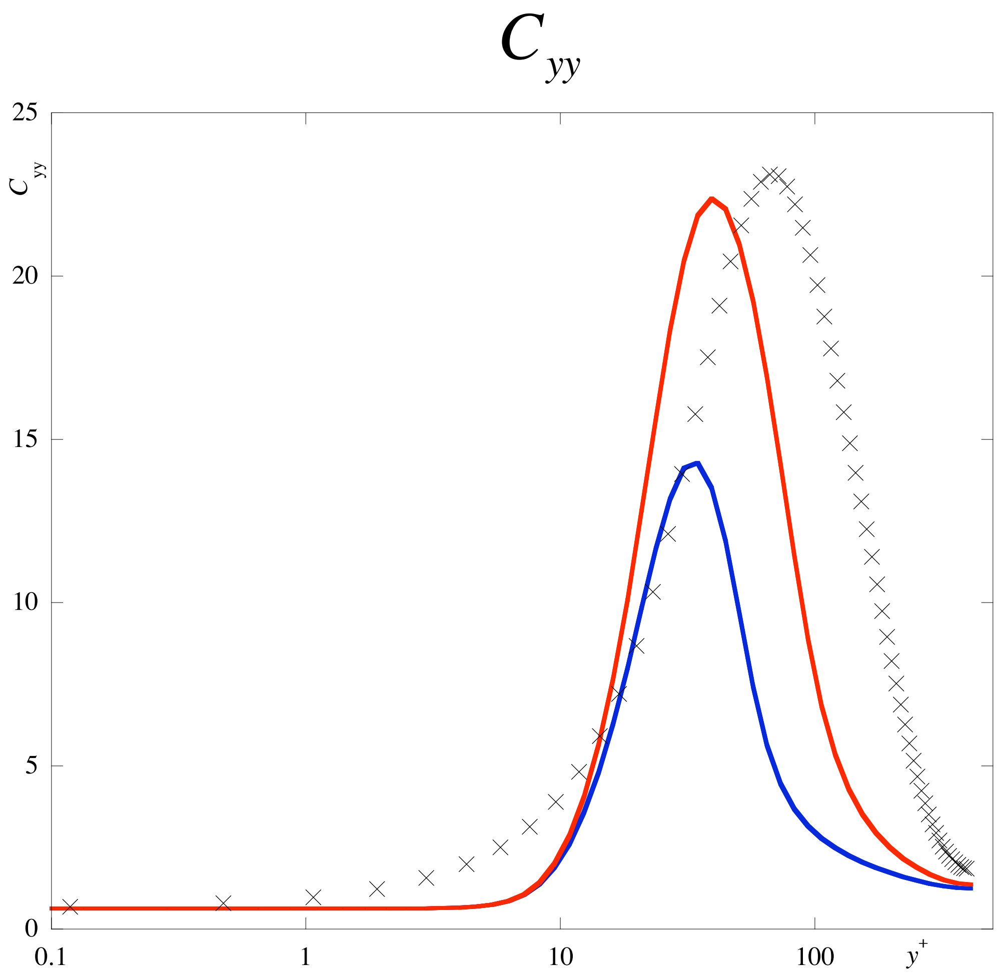
Predictions 3: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

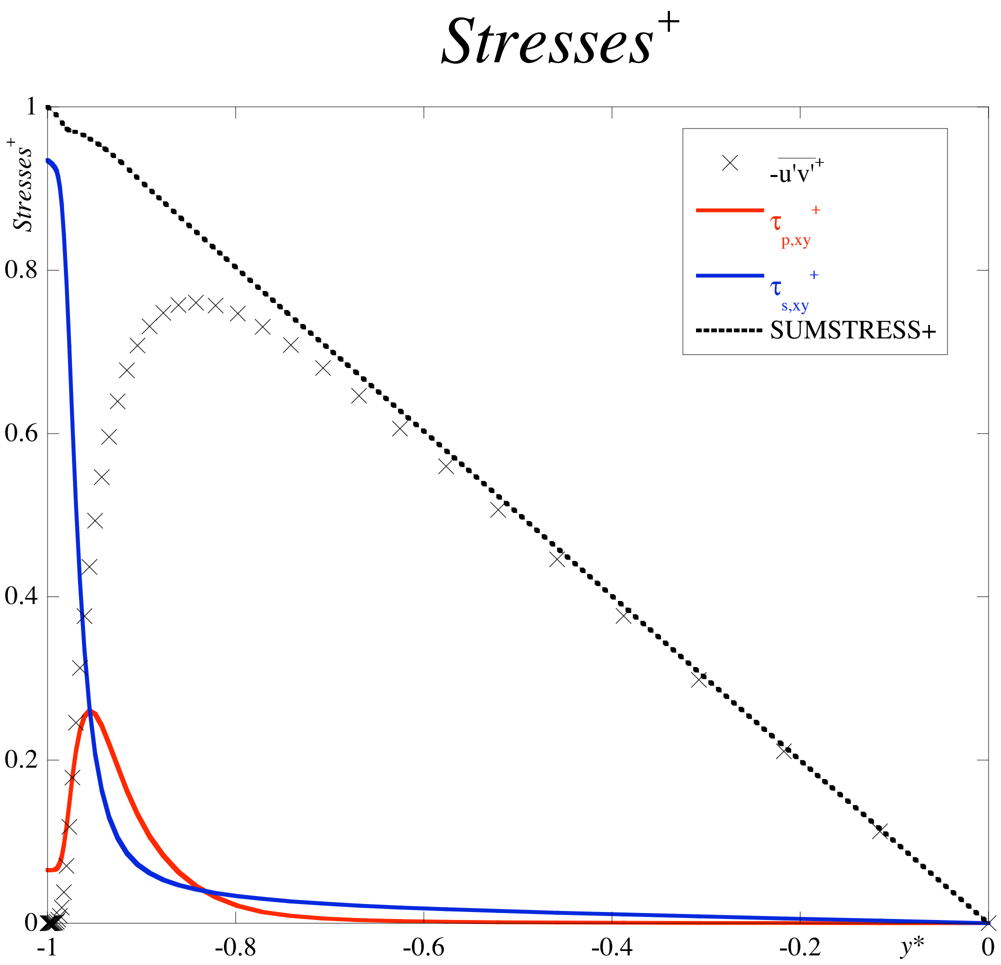


Predictions 4: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900

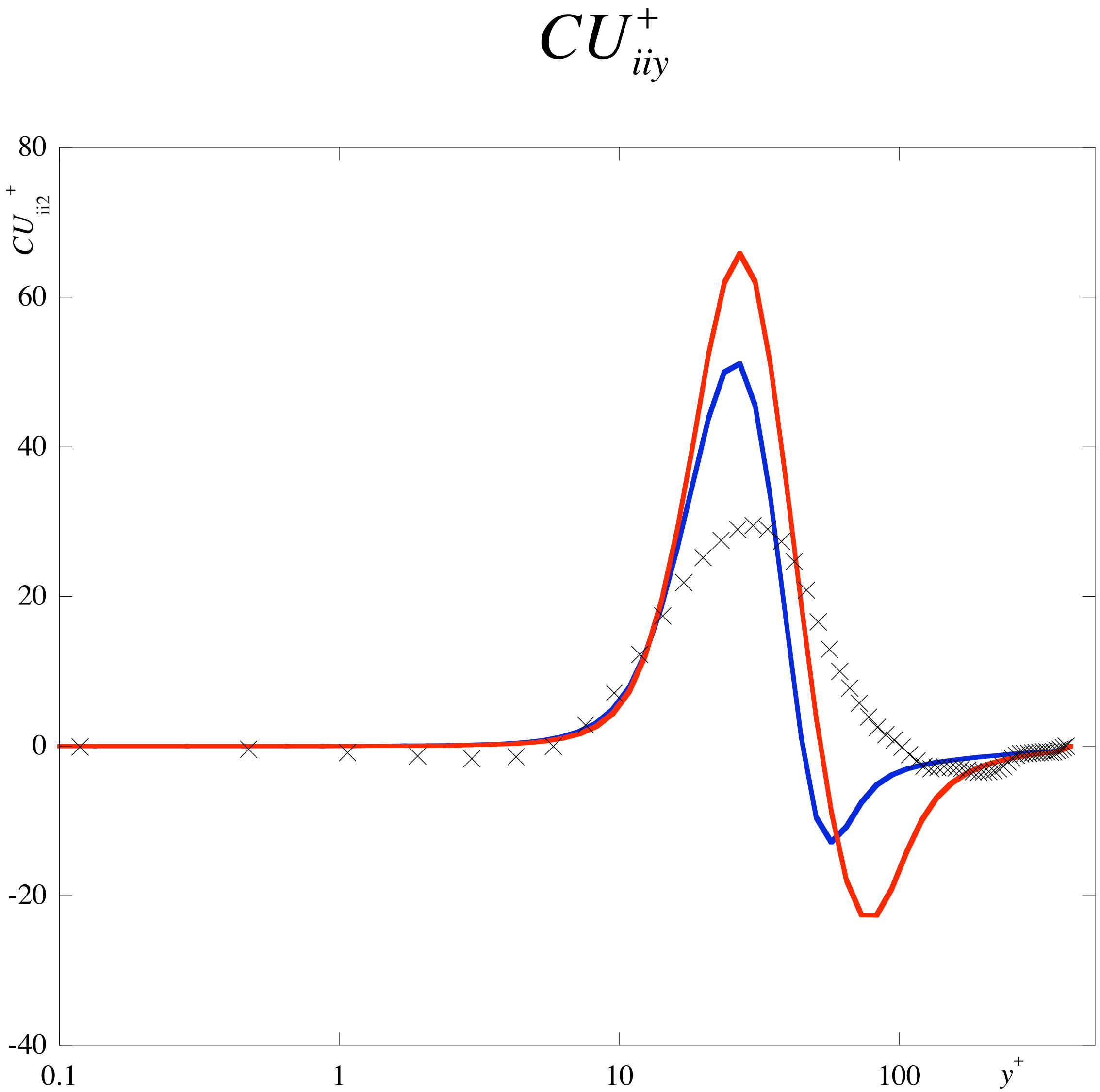


Predictions 5: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900





Predictions 6: $Re_{\tau 0}$= 395; $We_{\tau 0}$= 25; β=0.9, L^2=900



Conclusions

- Developed simplified k-ε model: code and closures are working
- Viscoelastic stress power well modeled by NLT_{ij}
- Viscoelastic turbulent transport (CU_{ijk}) is not that relevant at 18%
- NLT_{ij} is also required for C_{ij}

-Closure for NLT_{ij} has deficiencies and needs significant improvement

- Excessive dissipation of turbulence
- Need to model viscoelastic turbulence production close to wall
- Isotropic turbulence does not allow a good model
- Need to consider anisotropic turbulence: anisotr. k-ε and RSM

- Closure for CU_{ijk} is fair but also needs improvement: small impact

Conclusions: Models for other constitutive equations

Constitutive equations can be rewritten as a function of the conformation tensor

$$\tau_{ij} = 2\eta_s S_{ij} + \frac{\eta_p}{\lambda}\left[f_1\left(C_{kk}, L, \ldots\right) C_{ij} - f_2\left(C_{kk}, L, \ldots\right)\delta_{ij}\right]$$

Different functions are used

Turbulence models can be modifications of turbulence models developed for FENE-P

Summary

- **Relevance: some facts about drag reduction**
- **Governing equations for FENE-P in RANS form**
- **Reynolds decomposition of FENE-P equations**

- **Closure needs and analysis of DNS case (LDR)**

- **A simplified closure with *a priori* testing of DNS data**
- **Some concerns regarding limiting cases**

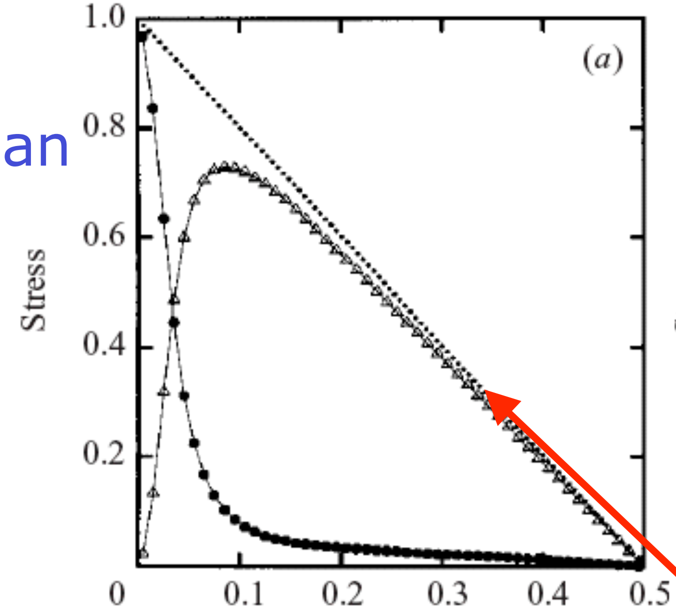
- **Some preliminary results**

- **Conclusions**

Direct evidence of Reynolds stress deficit

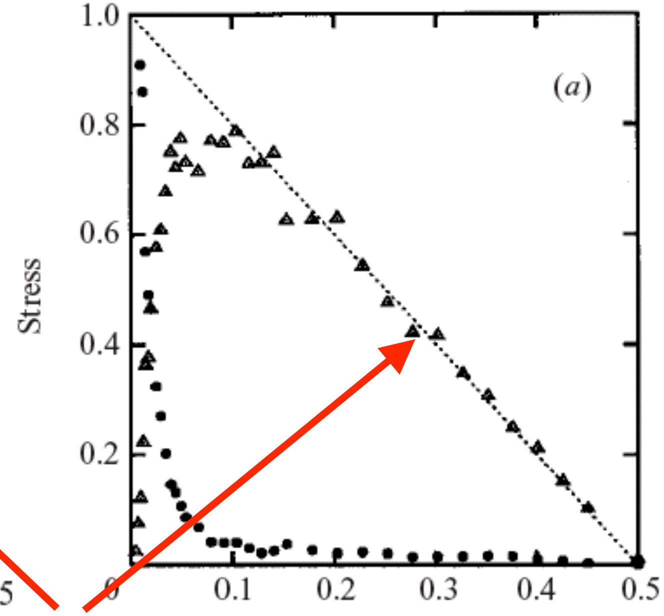
DNS

Newtonian



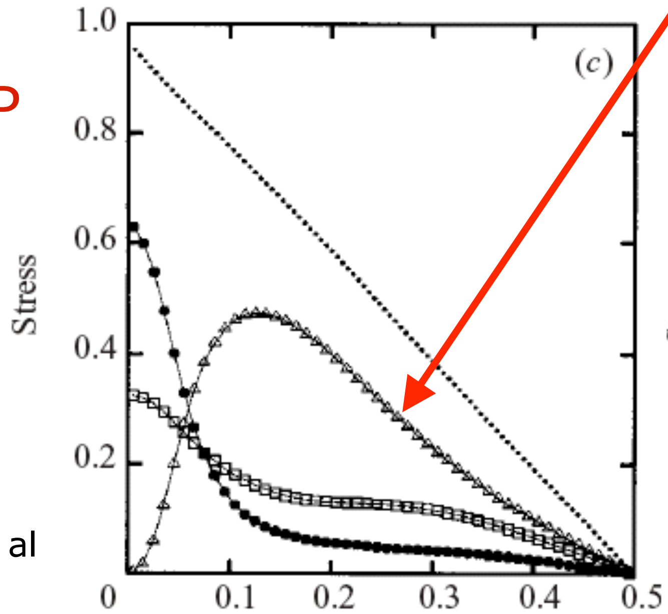
Experimental

Newtonian



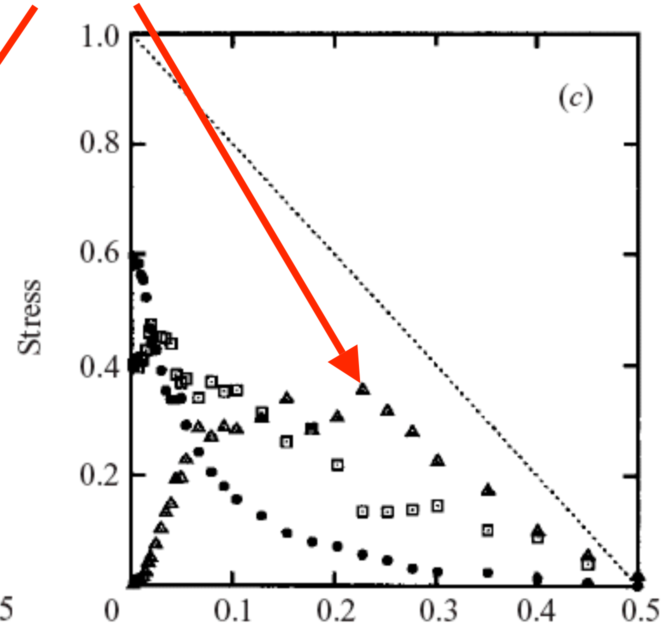
Reynolds stresses

FENE-P



Ptasinski et al
JFM (2003)

PAA solutions



Ptasinski et al
Flow, Turb. & Comb.
(2001)

FENE-P model

$$\boxed{\tau_{ij} = 2\eta_s S_{ij} + \frac{\eta_p}{\lambda}\left[f(C_{kk})C_{ij} - f(L)\delta_{ij}\right]}$$

Molecular conformation

$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

$$\boxed{f(C_{kk})C_{ij} + \lambda \overset{\nabla}{C}_{ij} = \delta_{ij}}$$

with

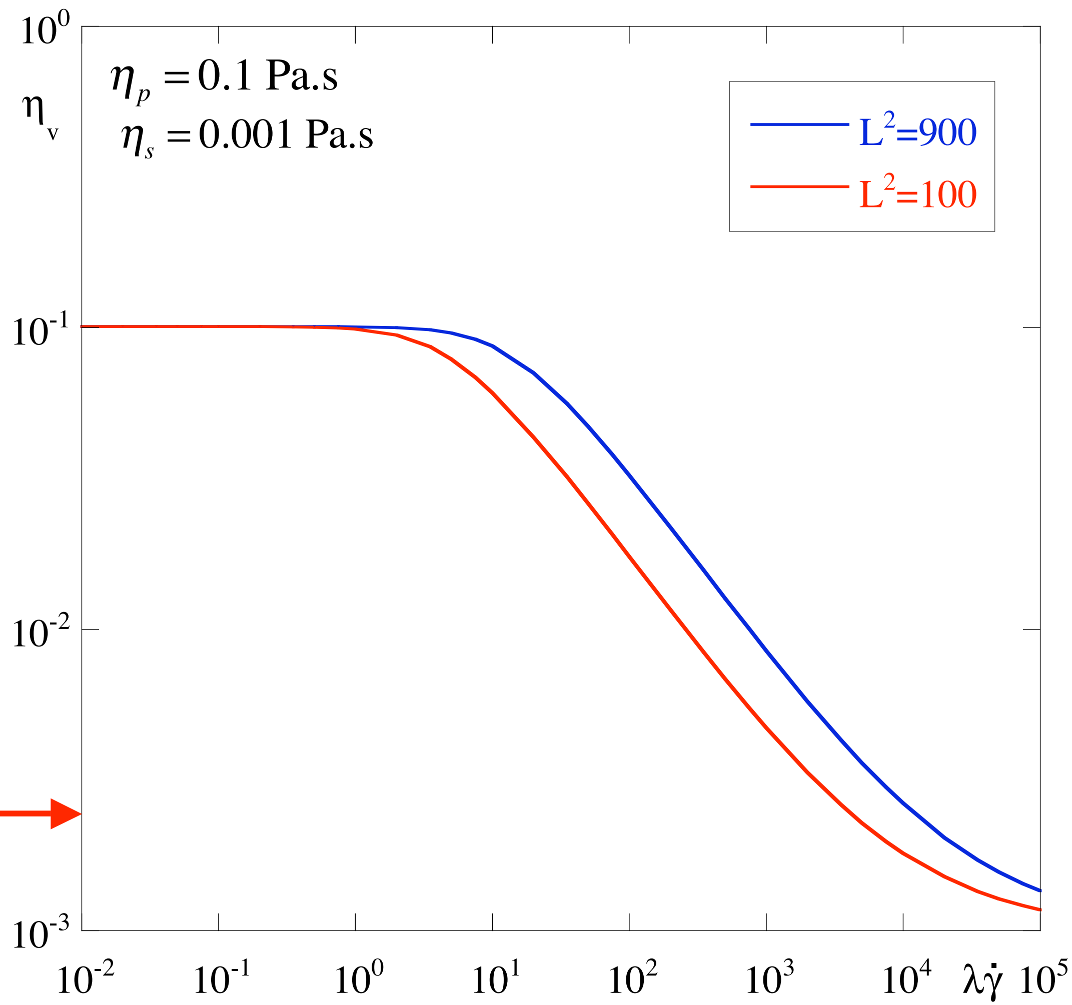
$$f(C_{kk}) = \frac{L^2}{L^2 - C_{kk}}$$

$$f(L) = \frac{L^2}{L^2 - 3}$$

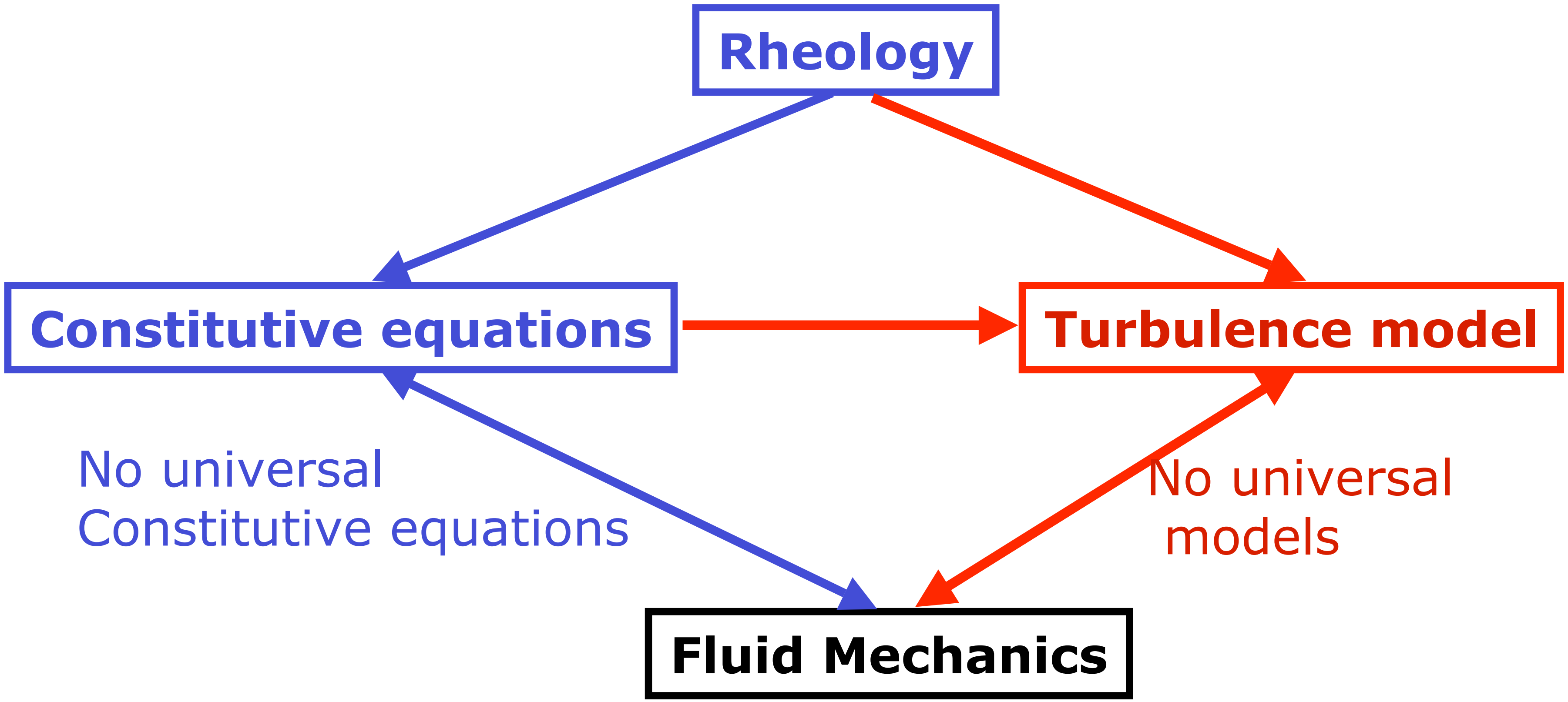
Couette flow

$$\eta(\dot{\gamma}) = \eta_p C_{22}(\dot{\gamma}) + \eta_s$$

$C_{22}(\dot{\gamma})$: analytical solution



Important relationships



Extensive DNS: **FENE-P model** → Basis for turbulence model

Other constitutive models → **Extensions of this turbulence model**

Function *f*()

$$f\left(C_{kk}\right)=\frac{L^2}{L^2-C_{kk}} \quad \text{e} \quad f\left(L\right)=\frac{L^2}{L^2-3}$$

Most used in laminar flow

$$f\left(C_{kk}\right)=\frac{L^2}{L^2-C_{kk}} \quad \text{e} \quad f\left(L\right)=1$$

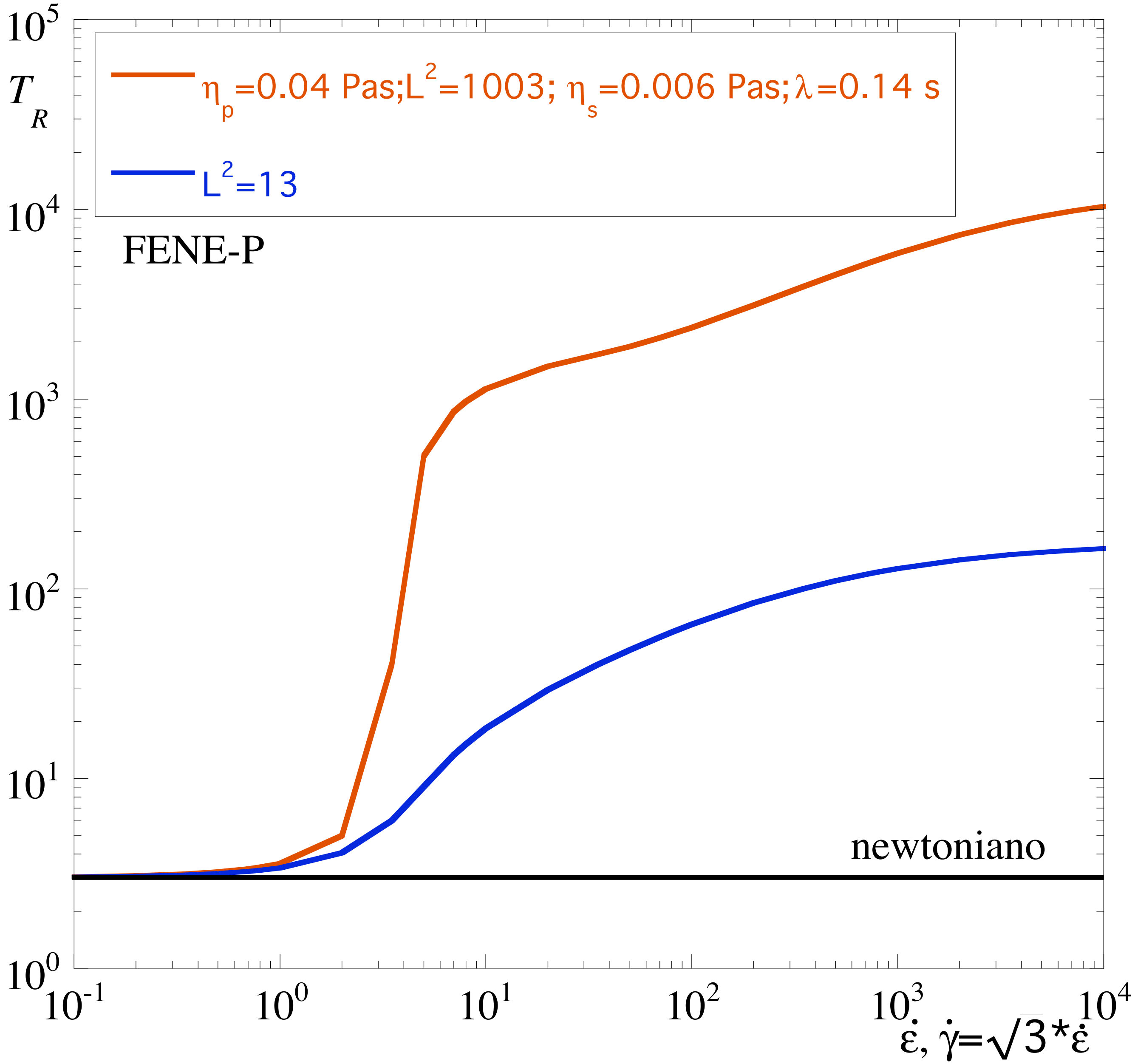
Most used in DNS
(Beris et al)

$$f\left(C_{kk}\right)=\frac{L^2-3}{L^2-C_{kk}} \quad \text{e} \quad f\left(L\right)=1$$

Sureshkumar et al

Generally speaking: $f\left(C_{kk}\right)$ and $f\left(L\right)$

Trouton ratio: FENE-P model



Transport equation for the Reynolds stresses 2

$$P_{ij} = -\rho\left(\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}\right)$$ Production of Reynolds stresses

$$Q_{ij} = -\frac{\partial}{\partial x_k}\left(\rho\overline{u_i u_j u_k}\right)$$ “Turbulent” diffusion

$$\Pi_{ij} = -\left(\overline{u_i\frac{\partial p'}{\partial x_j}} + \overline{u_j\frac{\partial p'}{\partial x_i}}\right)$$ Pressure fluctuations: redistribution (pressure-strain) and turbulent transport

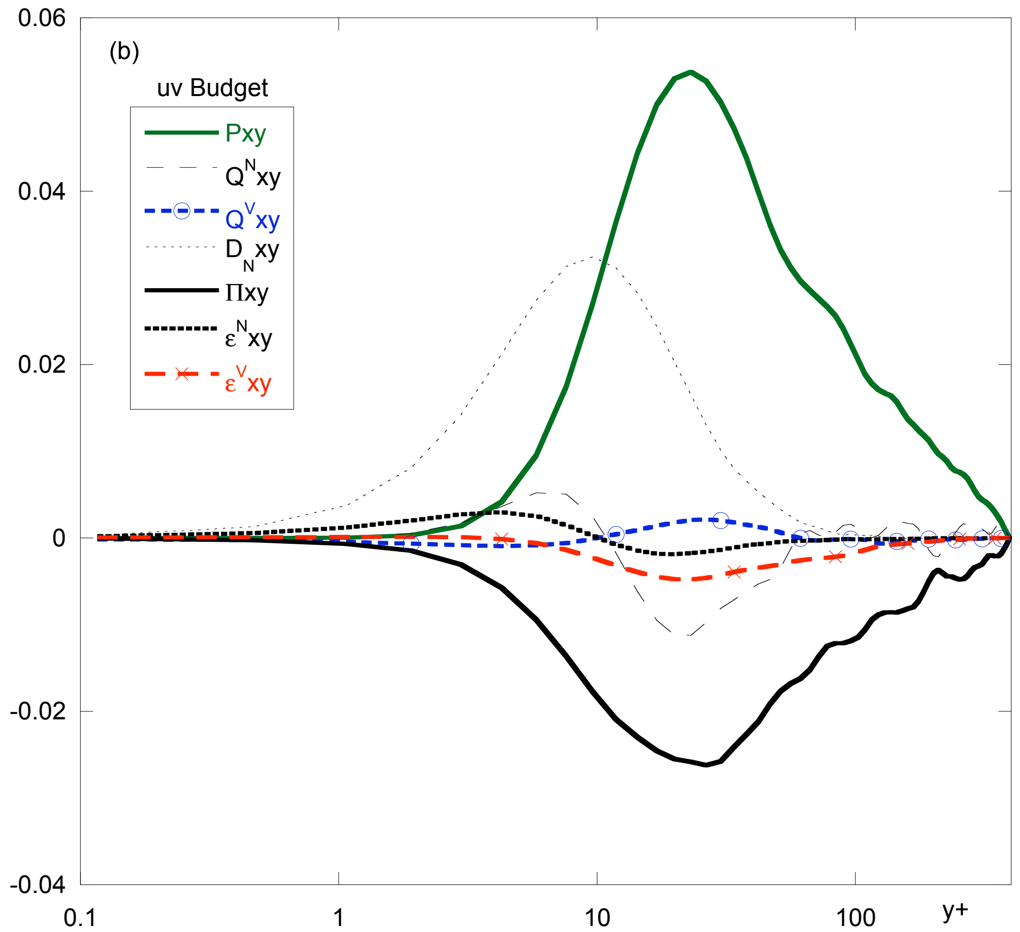
$$\overline{p'\frac{\partial u_i}{\partial x_j}} + \overline{p'\frac{\partial u_j}{\partial x_i}} \qquad -\frac{\partial\overline{p'u_i}}{\partial x_j} - \frac{\partial\overline{p'u_j}}{\partial x_i}$$

$$D_{ij}^N = \eta_s\frac{\partial^2\overline{u_i u_j}}{\partial x_k\partial x_k}$$ Molecular diffusion by solvent

$$\varepsilon_{ij}^N = 2\nu_s\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}$$ Viscous dissipation by solvent → Transport eq. of ε

Solvent viscosity

Budget of Reynolds stress 2



- Viscoelastic turbulent transport is not so important

Model for Cu_{ijk} 1

Same key ideas, but simpler implementation

1) Write down exact equation for $CU_{ijk} = \overline{f\left(\hat{C}_{mm}\right)\left(u_i c_{kj} + u_j c_{ik}\right)}$

2) Identify new and existing terms

3) Apply previous simplifying assumptions

4) Found: new terms have a kin existing term: $\overline{u_i u_m \frac{\partial c_{kj}}{\partial x_m}}; \overline{u_i u_m} \frac{\partial C_{kj}}{\partial x_m}$

5) Perform the de-correlation

$$\overline{u_i u_m \frac{\partial c_{kj}}{\partial x_m}} + \overline{u_i u_m} \frac{\partial C_{kj}}{\partial x_m} \approx Coef \times \overline{u_i u_m} \frac{\partial C_{kj}}{\partial x_m}$$

Alternative model: Corrections for limiting behaviour

$$\lambda \to 0 \longrightarrow \tau_{ij} = \underbrace{2\eta_s S_{ij}}_{\tau_s} + \underbrace{2\eta_p S_{ij}}_{\tau_p}$$

Predictions must be identical to

$$\tau_{ij} = 2\left(\eta_s + \eta_p\right)_{Newtonian} S_{ij}$$

In the absence of "correct" models

$$\varepsilon^V \approx \frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\frac{NLT_{nn}}{2} \to \varepsilon^V \approx \varepsilon^p + \frac{\eta_p}{\rho\lambda} f\left(C_{mm}\right)\frac{NLT_{nn}}{2}$$

(The trouble is double role of NLT_{ij})

Newtonian-like dissipation due to shear-thinning polymer

$$Q^V \approx \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[f\left(C_{mm}\right)\frac{CU_{iik}}{2}\right]$$

with

$$\frac{f\left(C_{mm}\right)CU_{nny}}{2\lambda} = \frac{\mu_p}{\eta_p}\frac{\partial^2 k}{\partial x_k \partial x_k} + \ldots$$

or

$$Q^V \approx \mu_p \frac{\partial^2 k}{\partial x_i \partial x_i} + \frac{\eta_p}{\lambda}\frac{\partial}{\partial x_k}\left[f\left(C_{mm}\right)\frac{CU_{iik}}{2}\right]$$

k and ε equations: including alternative model

$$\tilde{\varepsilon}^{N} = \frac{\eta_s}{\eta_s + \mu_p}\tilde{\varepsilon}^{Np} \qquad \tilde{\varepsilon}^{p} = \frac{\mu_p}{\eta_s + \mu_p}\tilde{\varepsilon}^{Np}$$

$$0 = \frac{d}{dy}\left[\left(\eta_s + \mu_p + \frac{\rho \nu_T}{\sigma_k}\right)\frac{dk}{dy}\right] + P_k - \rho\tilde{\varepsilon}^{Np} - \rho D + \eta_p \frac{d}{dy}\left[\frac{f(C_{mm})}{\lambda}\frac{CU_{nny}}{2}\right] - \eta_p \frac{f(C_{mm})}{\lambda}\frac{NLT_{nn}}{2}$$

$$\varepsilon^{Np} = \tilde{\varepsilon}^{Np} + D^{Np} \qquad D^{Np} = D^{N} + D^{p} = 2(\eta_s + \mu_p)\left(\frac{d\sqrt{k}}{dy}\right)^2$$

$$0 = \frac{d}{dy}\left[\left(\eta_s + \mu_p + \frac{\rho \nu_T}{\sigma_\varepsilon}\right)\frac{d\tilde{\varepsilon}^{Np}}{dy}\right] + \rho f_1 C_{\varepsilon_1}\frac{\tilde{\varepsilon}^{Np}}{k}\frac{P_k}{\rho} - \rho f_2 C_{\varepsilon_2}\frac{{\varepsilon^{Np}}^2}{k} + \rho E + E_{\tau_p}$$

$$E = \frac{\eta_s + \mu_p}{\rho}\nu_T\left(1 - f_\mu\right)\left(\frac{d^2U}{dy^2}\right)^2$$

$$f_1 = 1 \qquad f_2 = 1 - 0.3\exp\left(-R_T^2\right) \qquad f_\mu = \left[1 - \exp\left(\frac{-y^+}{26.5}\right)\right]^2$$