

# Clustering Part 2

```
In [ ]: import numpy as np  
import pandas as pd  
import math  
import matplotlib.pyplot as plt  
import matplotlib
```

## DBSCAN Algorithm

DBSCAN(Density-Based Spatial Clustering of Applications with Noise) is a commonly used unsupervised clustering algorithm. DBSCAN does not need to specify the number of clusters. It can automatically detect the number of clusters based on your input data and parameters. More importantly, DBSCAN can find arbitrary shape clusters that k-means are not able to find.

## Algorithm:

- a. The algorithm proceeds by arbitrarily picking up a point in the dataset (until all points have been visited).
- b. If there are at least 'minPoint' points within a radius of ' $\epsilon$ ' to the point then we consider all these points to be part of the same cluster.
- c. The clusters are then expanded by recursively repeating the neighborhood calculation for each neighboring point

### A. Generate "N" spherical training data points.

General form for points on a circle:

$$(x, y) = (r\cos\theta, r\sin\theta)$$

For n such points,  $i^{\text{th}}$  point would have  $\theta = \frac{2\pi i}{n}$ ,  $i = 0, 1, 2, 3, \dots, n-1$

```
In [ ]: ## write your code here  
def circularData(radius, noise, n):  
    dataset = []  
    for i in range(n):  
        x = math.cos(2 * math.pi * i / n) * radius + np.random.normal(0, noise)  
        y = math.sin(2 * math.pi * i / n) * radius + np.random.normal(0, noise)  
        dataset.append((x, y))  
    return dataset  
  
def uniformNoise(bound, n):  
    dataset = []  
    for i in range(n):  
        x = np.random.uniform(-bound, bound)
```

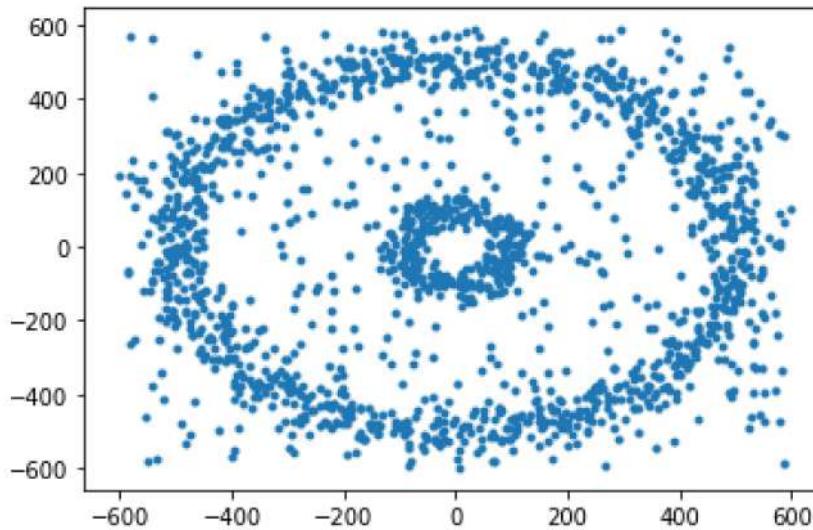
```

y = np.random.uniform(-bound, bound)
dataset.append((x, y))
return dataset

outerC = circularData(500, 35, 1000)
innerC = circularData(100, 25, 250)
noisyPts = uniformNoise(600, 400)

plt.figure()
data = pd.DataFrame(outerC)
data = data.append(innerC)
data = data.append(noisyPts)
plt.plot(data[0], data[1], '.')
plt.show()

```



## B. Perform DBSCAN Algorithm on the above generated data to obtain clusters

```

In [ ]: ## Write your code here

eps = 30
minpts = 5
x = np.array(data)

def getNeighbours(x, i, eps):
    neighbours = []
    for j in range(0, len(x)):
        # If d<eps , add it to the neighbors list.
        if np.linalg.norm(x[i] - x[j]) < eps:
            neighbours.append(j)
    return neighbours

def populateCluster(x, seed, labels, cluster_id, neighbours, eps, minpts):
    labels[seed] = cluster_id
    i = 0
    while i < len(neighbours):                      # sort of Breadth-First-Search
        j = neighbours[i]
        if labels[j] == -1:                           # was earlier labelled as noise
            labels[j] = cluster_id
        elif labels[j] == 0:                          # undiscovered point
            labels[j] = cluster_id
            jNeighbours = getNeighbours(x, j, eps)
            if len(jNeighbours) >= minpts: # else it's a border point!
                neighbours = neighbours + jNeighbours
        i = i + 1

def MyDBSCAN(eps, minpts, x):

```

```

labels = np.zeros((x.shape[0]))
# print(labels)
cluster_id = 0
for i in range(len(x)):
    if labels[i] != 0:
        continue
    neighbours = getNeighbours(x, i, eps)
    if len(neighbours) < minpts:
        labels[i] = -1           # noise:-1, not considered:0
    else:
        cluster_id = cluster_id + 1
        populateCluster(x, i, labels, cluster_id, neighbours, eps, minpts)
return labels

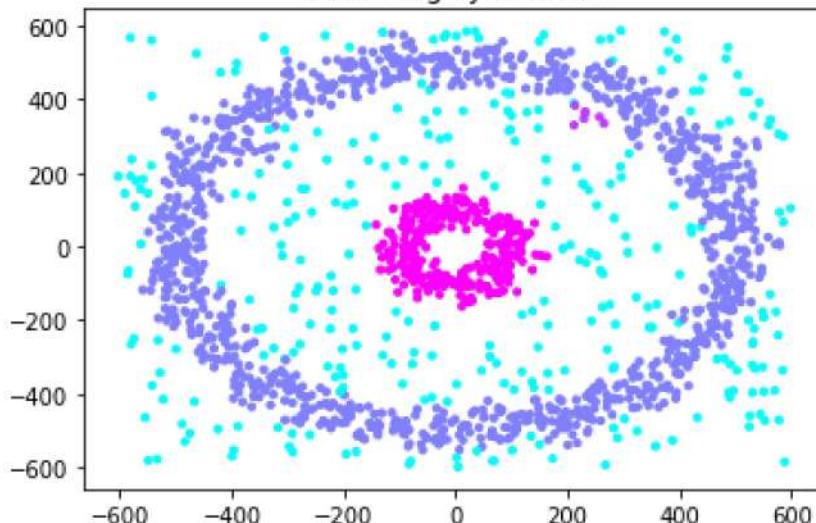
# function call
pred_labels = MyDBSCAN(eps, minpts, x)

print(pred_labels)
plt.figure()
plt.scatter(x[:,0], x[:,1], c=pred_labels, s=10, cmap = 'cool');
plt.title("Clustering by DBSCAN")
plt.show()

```

[ 1. 1. 1. ... -1. 3. -1.]

Clustering by DBSCAN



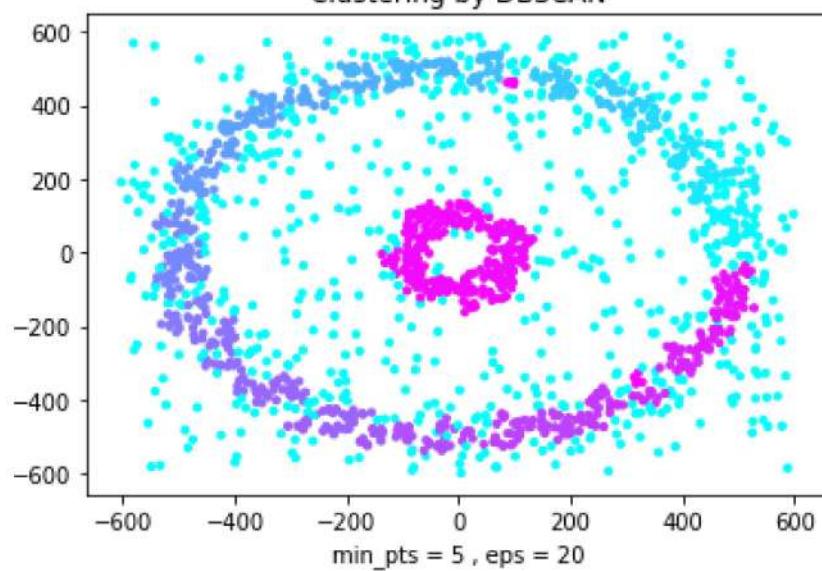
### C. Experiment by varying the number of min points and epsilon radius and plot your observations

```

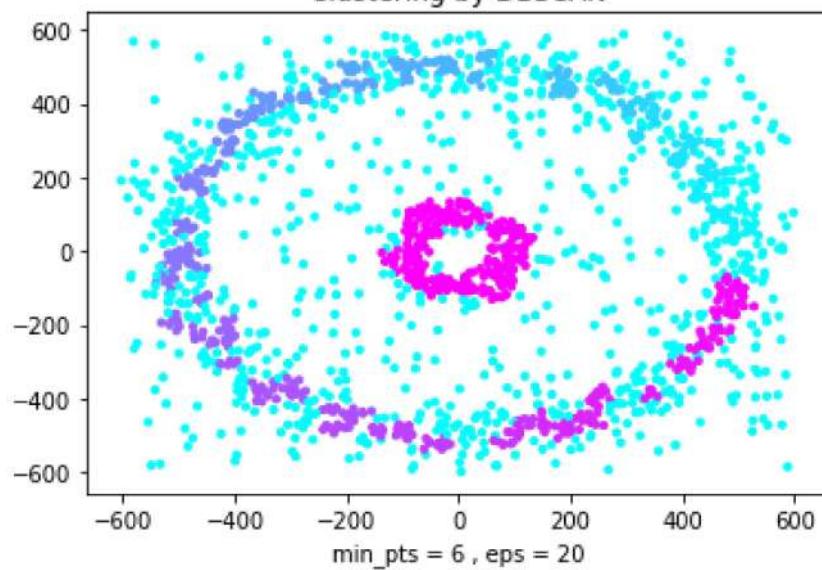
In [ ]: ## write your code here
min_pts = [5, 6, 7, 8, 9]
eps_rad = [20, 25, 30, 35, 40]
for i in eps_rad:
    for j in min_pts:
        plabels = MyDBSCAN(i, j, x)
        plt.figure()
        plt.scatter(x[:,0], x[:,1], c=plabels, s=10, cmap = 'cool');
        plt.title("Clustering by DBSCAN")
        plt.xlabel("min_pts = "+str(j)+" , eps = "+str(i))
        plt.show()

```

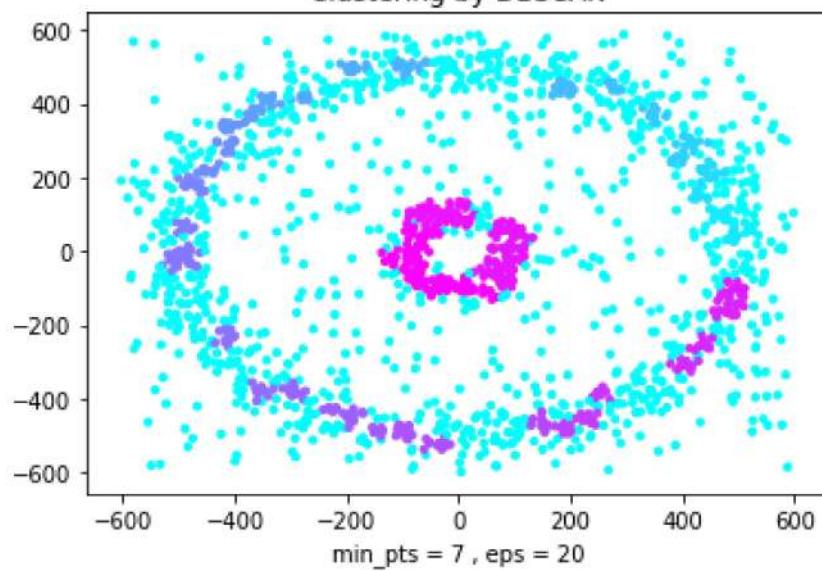
Clustering by DBSCAN



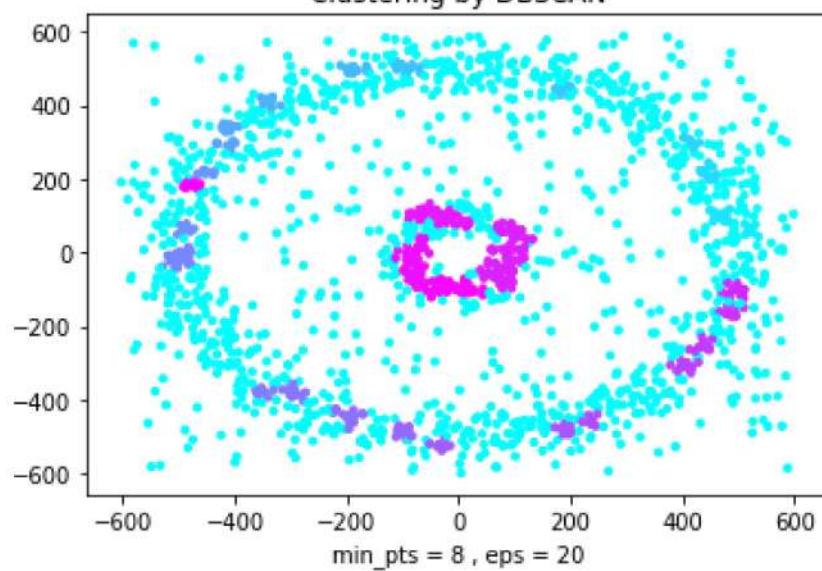
Clustering by DBSCAN



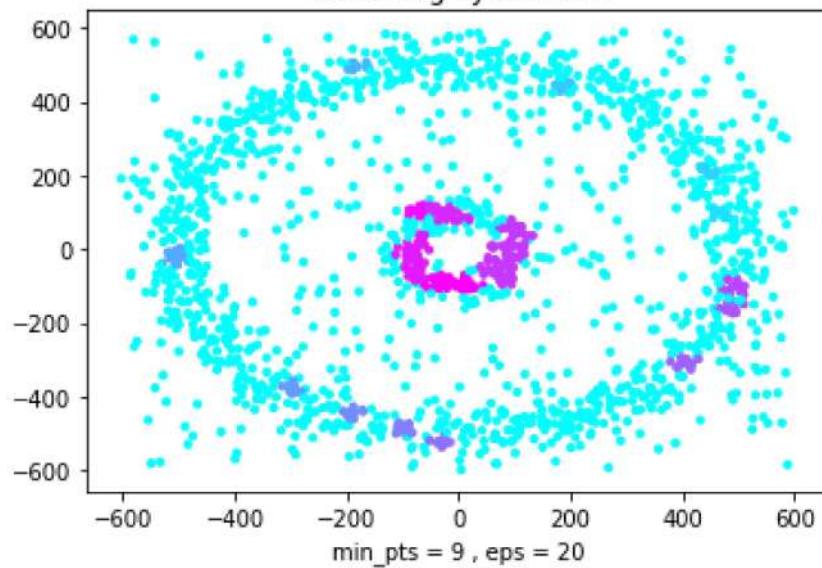
Clustering by DBSCAN



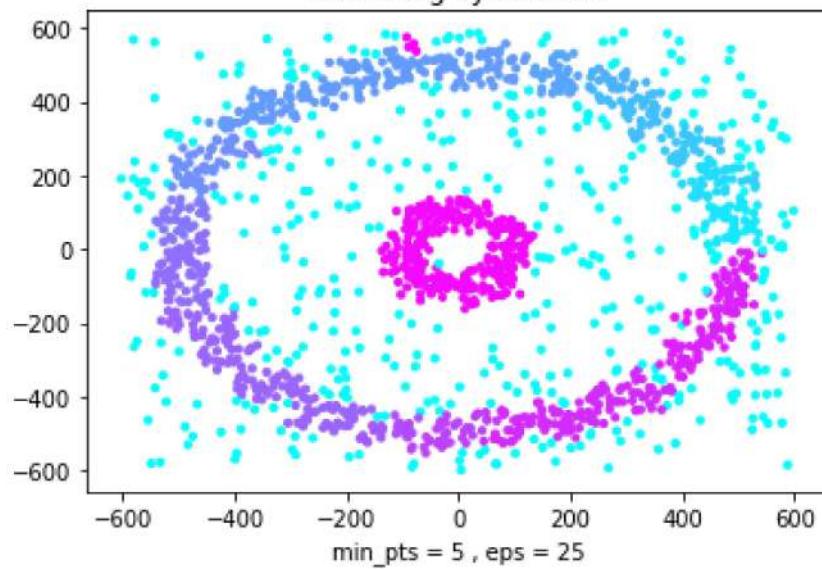
Clustering by DBSCAN



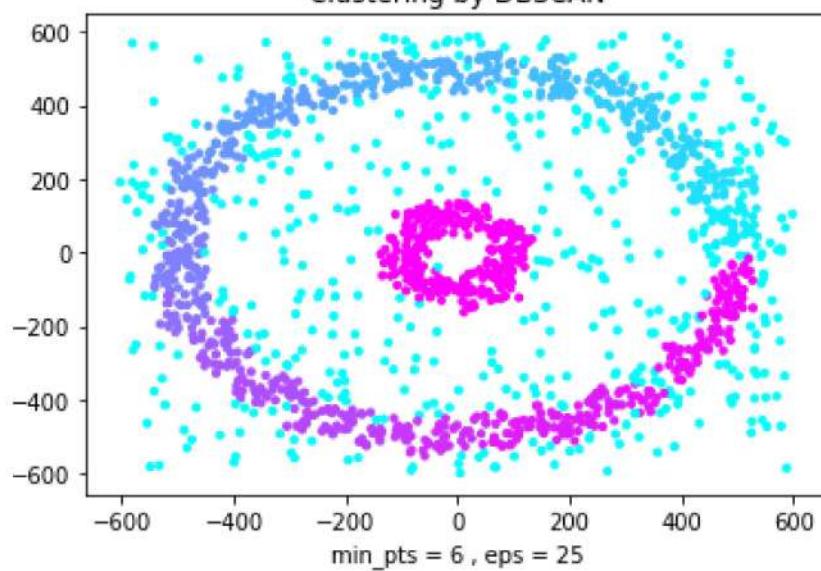
Clustering by DBSCAN



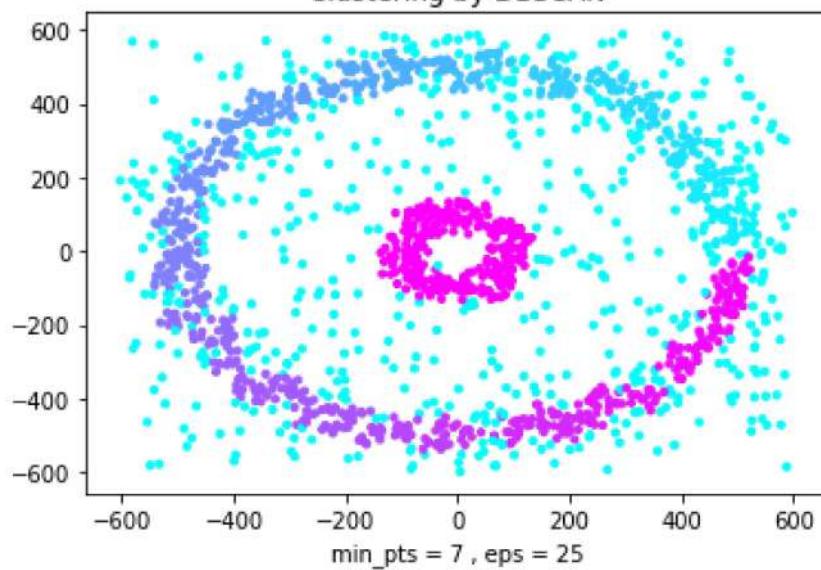
Clustering by DBSCAN



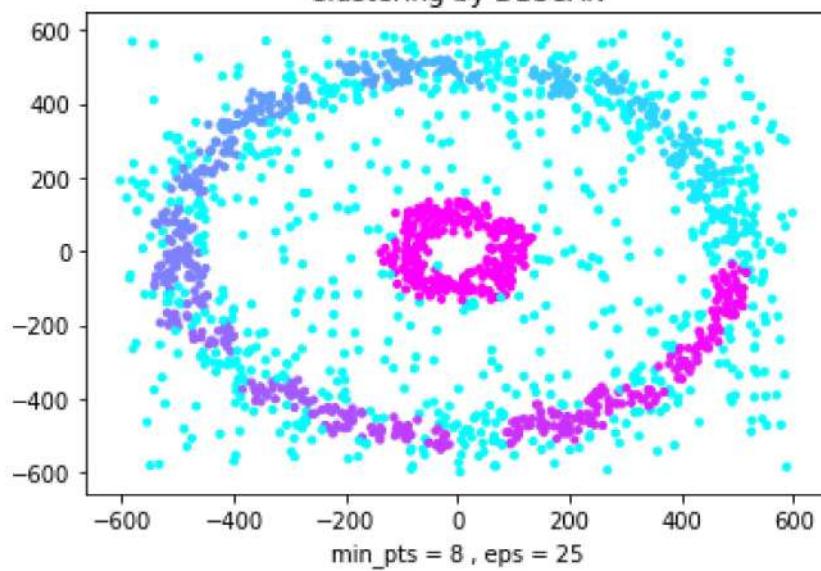
Clustering by DBSCAN



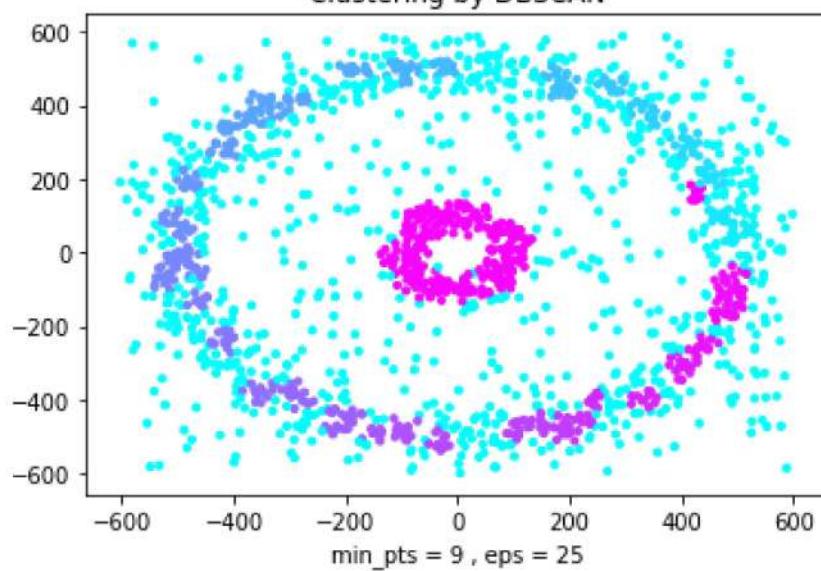
Clustering by DBSCAN



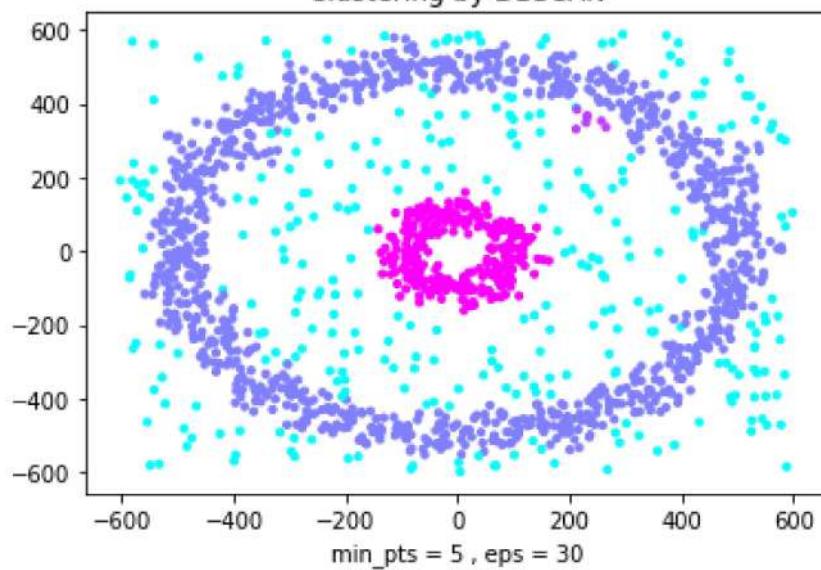
Clustering by DBSCAN



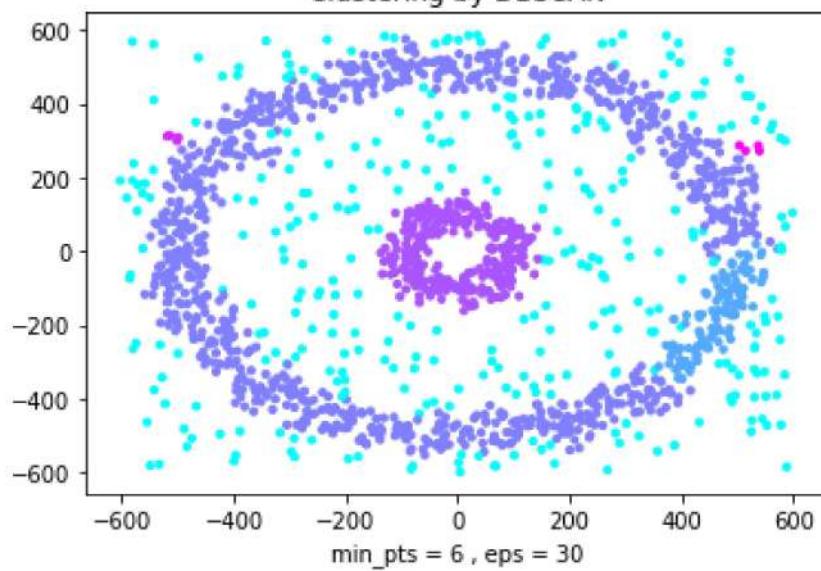
Clustering by DBSCAN



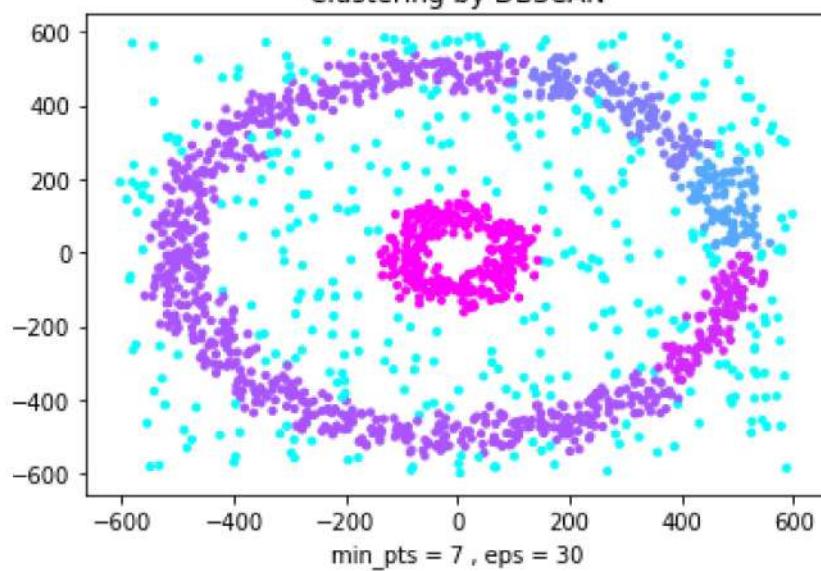
Clustering by DBSCAN



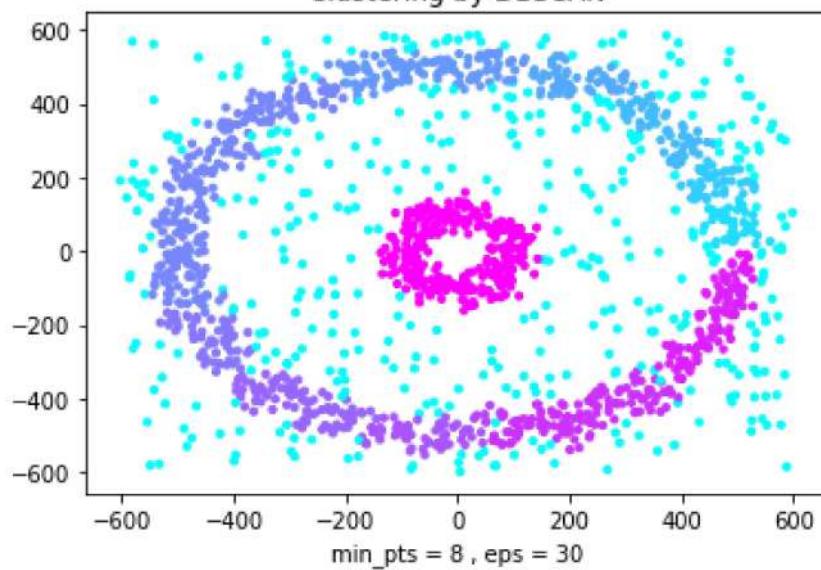
Clustering by DBSCAN



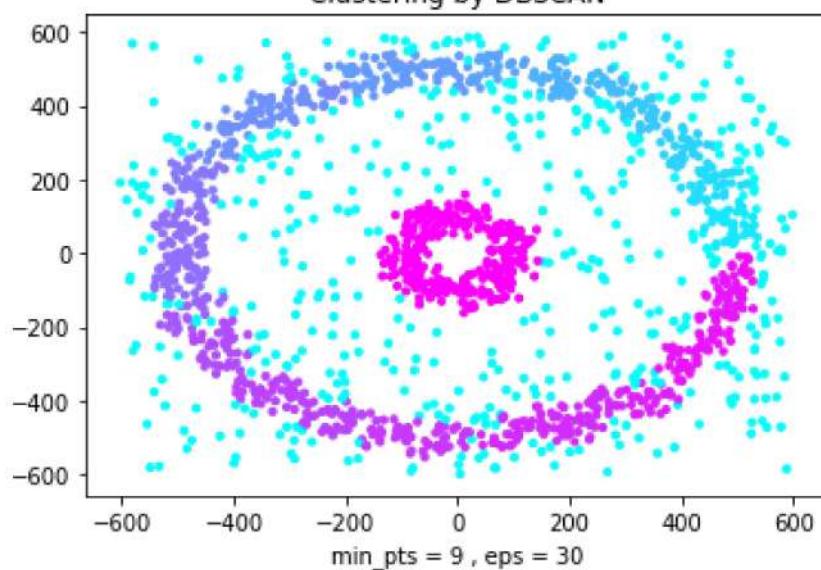
Clustering by DBSCAN



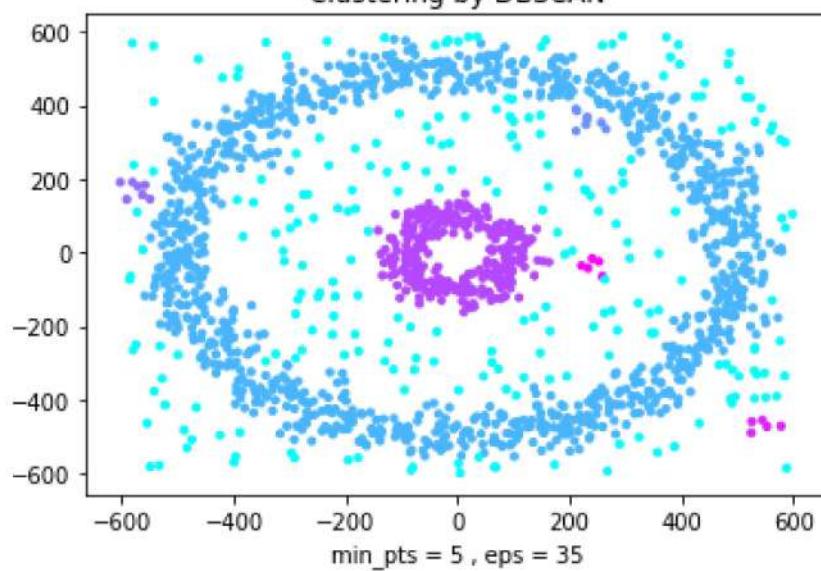
Clustering by DBSCAN



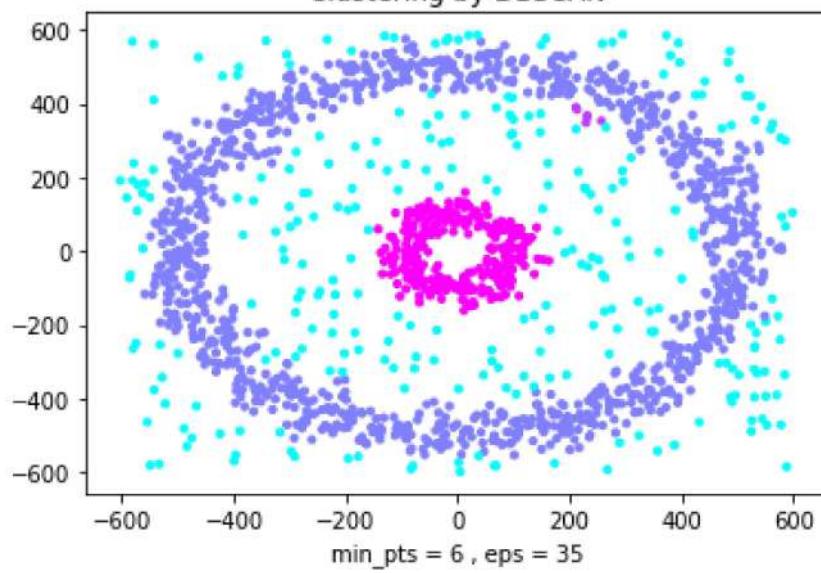
Clustering by DBSCAN



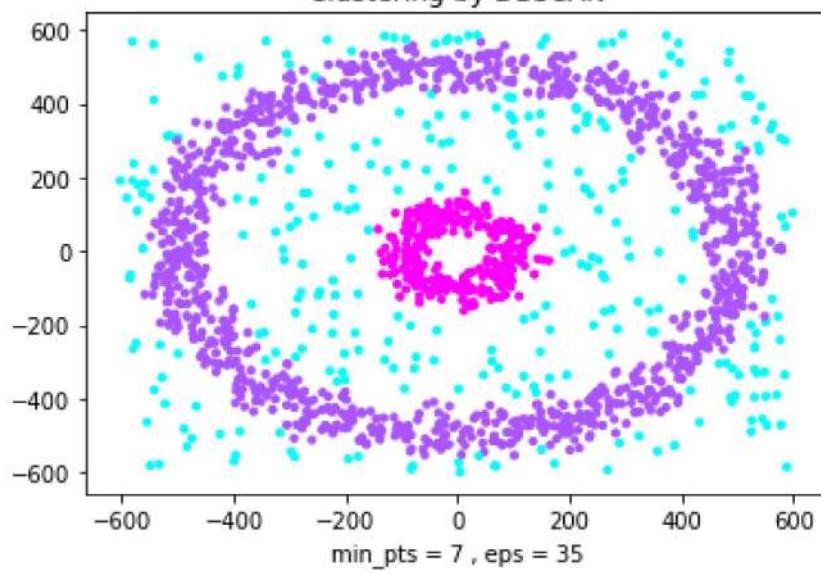
Clustering by DBSCAN



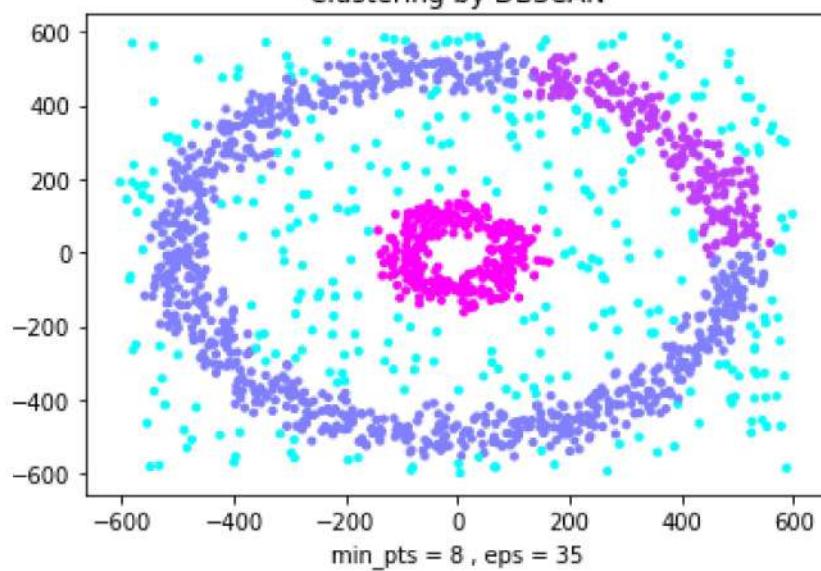
Clustering by DBSCAN



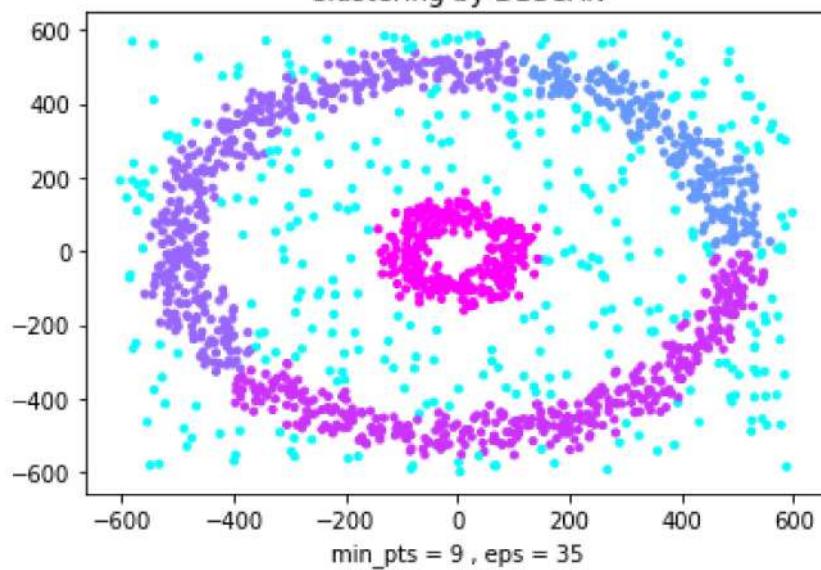
Clustering by DBSCAN



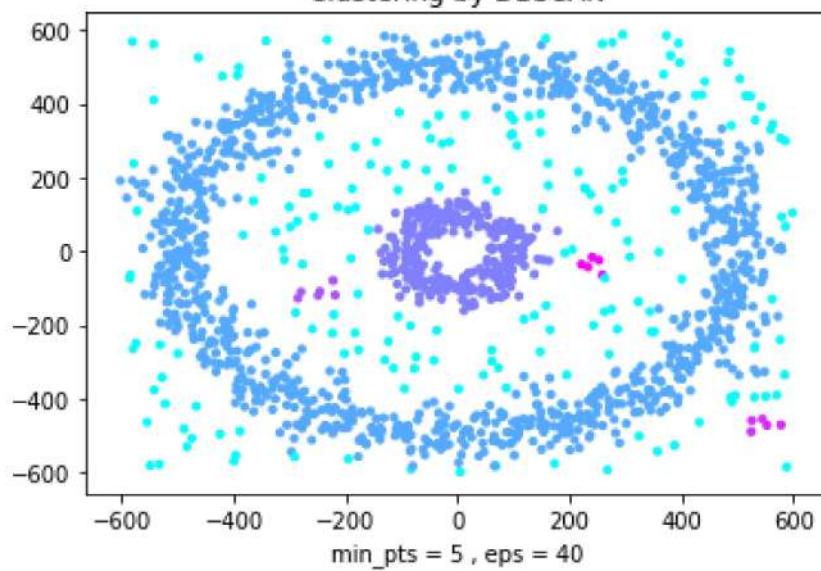
Clustering by DBSCAN



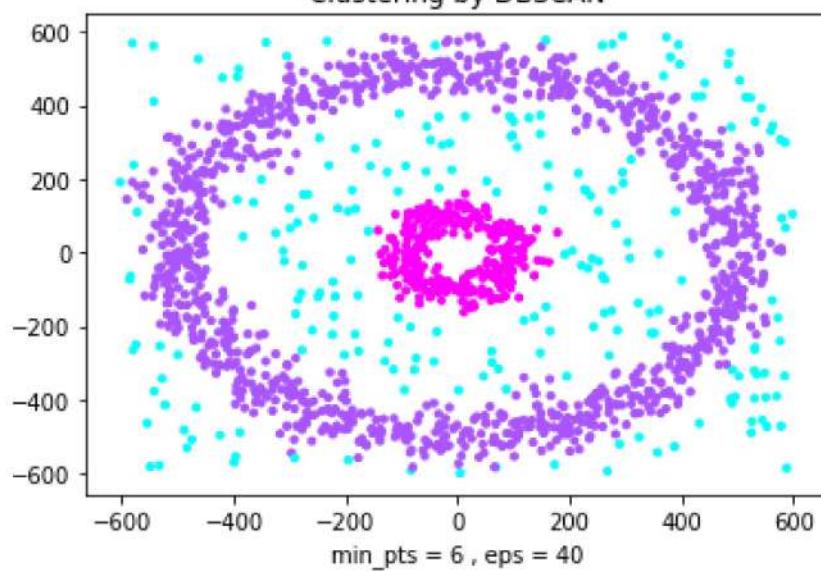
Clustering by DBSCAN



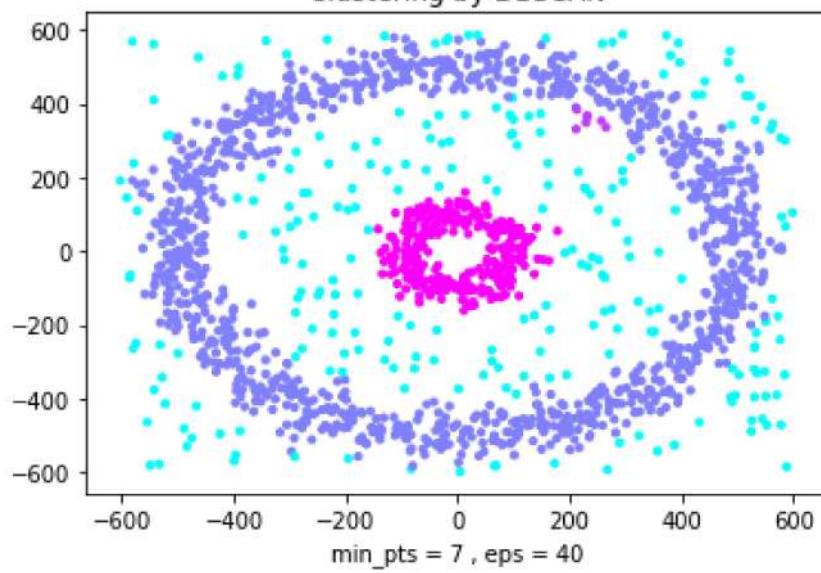
Clustering by DBSCAN



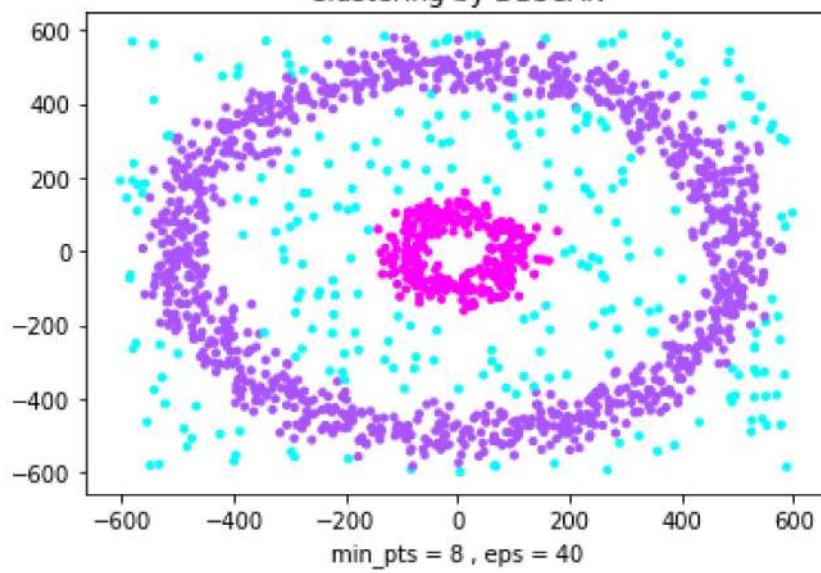
Clustering by DBSCAN

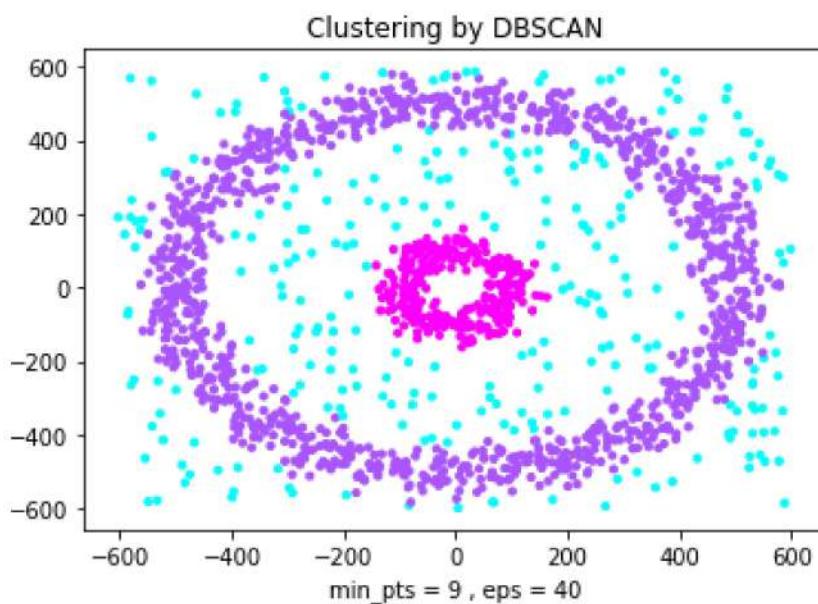


Clustering by DBSCAN



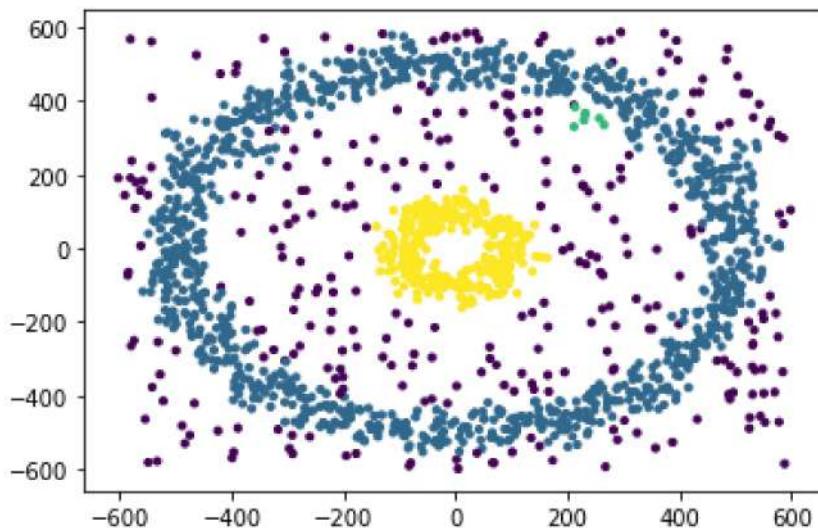
Clustering by DBSCAN





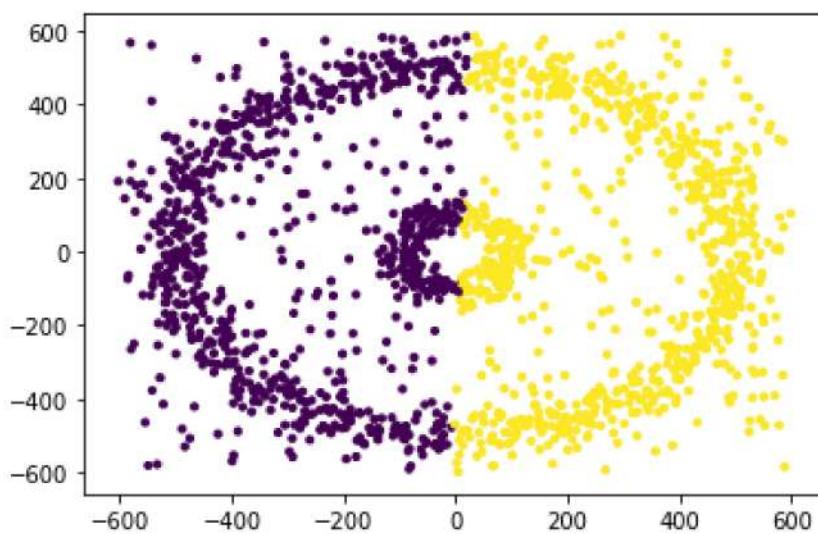
**D. Compare your model with the built in DBSCAN in Sci-kit Learn. Also compare you results with GMM and the K-means Algorithm**

```
In [ ]: from sklearn.cluster import DBSCAN
## write your code here
dBSACN = DBSCAN(eps=30, min_samples=5).fit(x)
labels = dBSACN.labels_
plt.figure()
plt.scatter(x[:, 0], x[:, 1], s=10, c=labels)
plt.show()
```



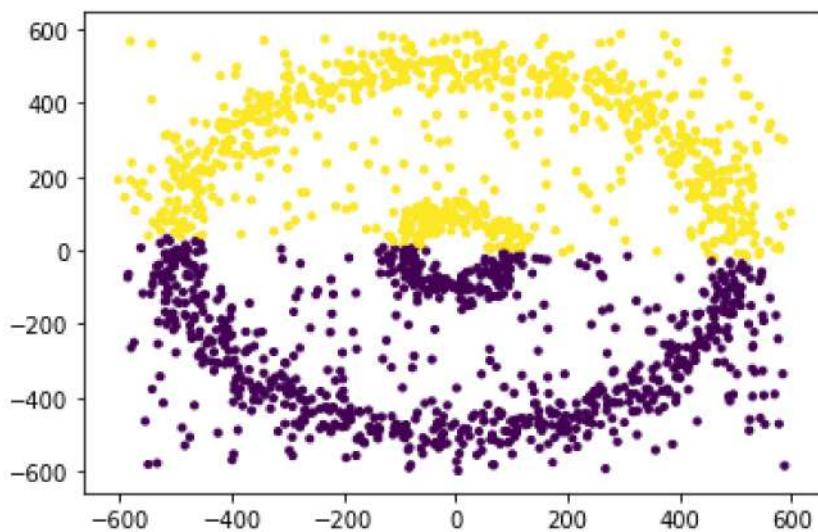
```
In [ ]: #####
from sklearn.mixture import GaussianMixture
## write your code here
gaussMix = GaussianMixture(n_components=2, random_state=0).fit(x)
labels = gaussMix.predict(x)
plt.scatter(x[:, 0], x[:, 1], s=10, c=labels)
```

Out[ ]: <matplotlib.collections.PathCollection at 0x206797b48e0>



```
In [ ]: #####  
from sklearn.cluster import KMeans  
## write your code here  
kMeans = KMeans(n_clusters=2, random_state=0).fit(x)  
y = kMeans.fit_predict(x)  
labels = kMeans.predict(x)  
plt.scatter(x[:,0], x[:,1], s=10, c=labels)
```

Out[ ]: <matplotlib.collections.PathCollection at 0x20605f449a0>



## Fuzzy C-Means Based clustering

1. Randomly initialize the centroids and clusters K, and compute the probability that each data point  $x_i$  is a member of a given cluster  $k$ ,  $P(\text{point } x_i \text{ has label } k | x_i, k)$ .
2. Iteration: Recalculate the centroids of the clusters as the weighted centroid given the probabilities of membership of all data points  $x_i$ :

$$\mu_k(n+1) = \frac{\sum_{x_i \in k} x_i * P(\mu_k | x_i)^b}{\sum_{x_i \in k} P(\mu_k | x_i)^b}$$

1. Implement it on the data for which Kmeans was implemented.

```
In [ ]: import random
```

```

import numpy as np
import math
import matplotlib.pyplot as plt

d1 = np.random.multivariate_normal((0.5, 0.0), np.identity(2), 100)
d2 = np.random.multivariate_normal((5.0, 5.0), np.identity(2), 100)
d3 = np.random.multivariate_normal((5.0, 1.0), np.identity(2), 100)
d4 = np.random.multivariate_normal((10.0, 1.5), np.identity(2), 100)
zeroarr = np.zeros(100)
onearr = np.ones(100)
twoarr = 2*onearr
threearr = 3*onearr
# concatenate all this to form an unlabelled dataset
real_labels = np.concatenate((zeroarr, onearr, twoarr, threearr))
data = np.concatenate((d1,d2,d3,d4))

class FuzzyC:
    # func for euclidean distance
    def dist(self, a, b):
        d = (a[1] - b[1])**2 + (a[0] - b[0])**2
        return d

    # initialization parameters
    def __init__(self, c, data):
        self.data = data
        self.b = 2           # fuzzification parameter
        self.n = len(data)
        self.c = c
        self.p=len(data[0])
        self.max_iter=100

    # initialize the membership matrix
    def initialize_membership_matrix(self, n, c):
        mem_mat = list()
        # generate c random numbers in [0,1), sum them and divide (w.r.t. each point
        for i in range(n):
            random_list = [random.random() for x in range(c)]
            summation = sum(random_list)
            for i in range(len(random_list)):
                random_list[i] = random_list[i]/summation
            mem_mat.append(random_list)
        return mem_mat

    # update centroids
    # (can also be used for cluster initialization, since we let mem matrix be rand
    def update_centroid(self, mem_mat):
        data = self.data
        centroids = {}
        for j in range(self.c):                                # consider jth cluster
            temp=[]
            for k in range(self.p):                            # w.r.t each feature
                num = 0
                for i in range(self.n):                      # sum(x * p^b)
                    num = num + (mem_mat[i][j]**self.b)*(data[i][k])
                den = 0
                for i in range(self.n):                      # get sum(p^b)
                    den = den + mem_mat[i][j]**self.b
                val = num/den
                temp.append(val)
            centroids[j] = temp
        return centroids

```

```

# update membership matrix
def update_membership_matrix(self, mem_mat, centroids):
    ratio = float(2/(self.b-1))
    data = self.data
    for i in range(self.n):
        distances = list()
        for j in range(self.c):
            distances.append(self.dist(data[i], centroids[j]))
        for j in range(self.c):
            den = sum([math.pow(float(distances[j]/distances[q]), ratio) for q
                      in range(self.c)])
            mem_mat[i][j] = float(1/den)
    return mem_mat

# Labelling based on final clustering
def find_cluster(self, mem_mat):
    clusters=list()
    for i in range(self.n):
        max_val, idx = max((val, idx) for (idx, val) in enumerate(mem_mat[i]))
        clusters.append(idx)
    return clusters

# check if membership matrix isn't changing much
def check(self, prev_mat, mem_mat):
    diff=0
    for i in range(self.n):
        for j in range(self.c):
            diff+=prev_mat[i][j]-mem_mat[i][j]
    if(diff<0.01):
        return True
    return False

# main thing here!!!!!
def fuzzy_c_mean(self):
    mem_mat=self.initialize_membership_matrix(self.n, self.c)
    for i in range(self.max_iter):
        centroids=self.update_centroid(mem_mat)
        prev_mat=mem_mat
        mem_mat=self.update_membership_matrix(mem_mat, centroids)
        cluster=self.find_cluster(mem_mat)
        if i<=10:
            self.showGraphs(cluster, centroids, i)
        if(self.check(prev_mat, mem_mat))<0.01:
            print(i)
            break
    return cluster, centroids

def showGraphs(self, clusters, centroids, i):
    print("Iteration ",i)
    plt.figure()
    plt.scatter(self.data[:,0], self.data[:,1], c=clusters, s=12, cmap="cool")
    x = []
    y = []
    for i in range(len(centroids)):
        cent = centroids[i]
        x.append(cent[0])
        y.append(cent[1])
    plt.scatter(x, y, marker='x', color='r', s=50*np.ones(len(centroids)))
    plt.show()

fcm = FuzzyC(4, data)

```

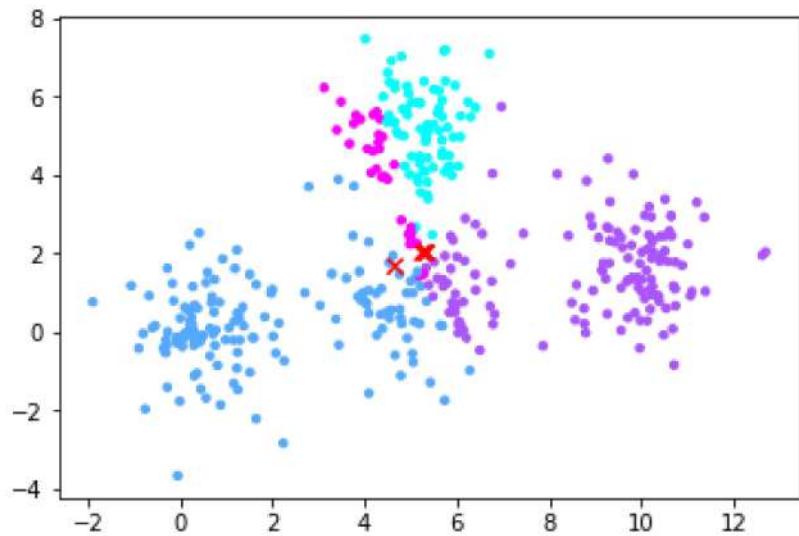
```

clusters, centroids = fcm.fuzzy_c_mean()
x = []
y = []
for i in range(len(centroids)):
    cent = centroids[i]
    x.append(cent[0])
    y.append(cent[1])

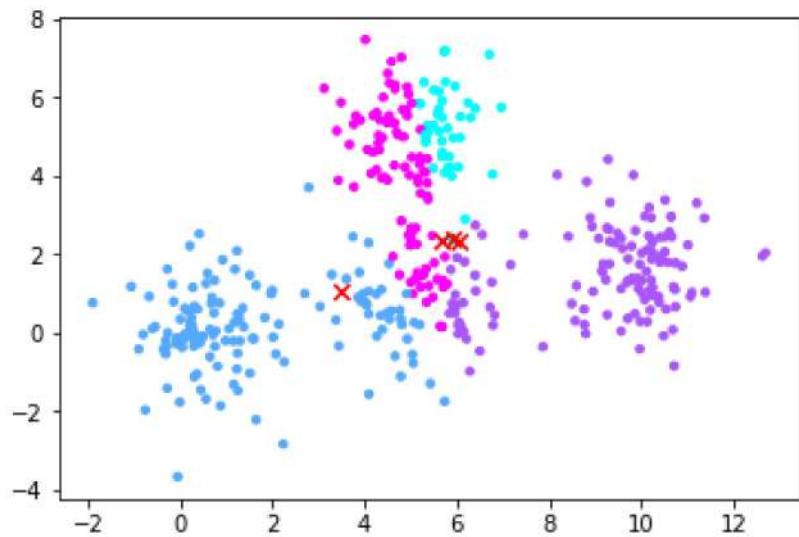
print("Final Clustering:")
plt.figure()
plt.scatter(data[:,0], data[:,1], c=clusters, s=12, cmap="cool")
plt.scatter(x, y, marker='x', color='r', s=50*np.ones(len(centroids)))
plt.show()

```

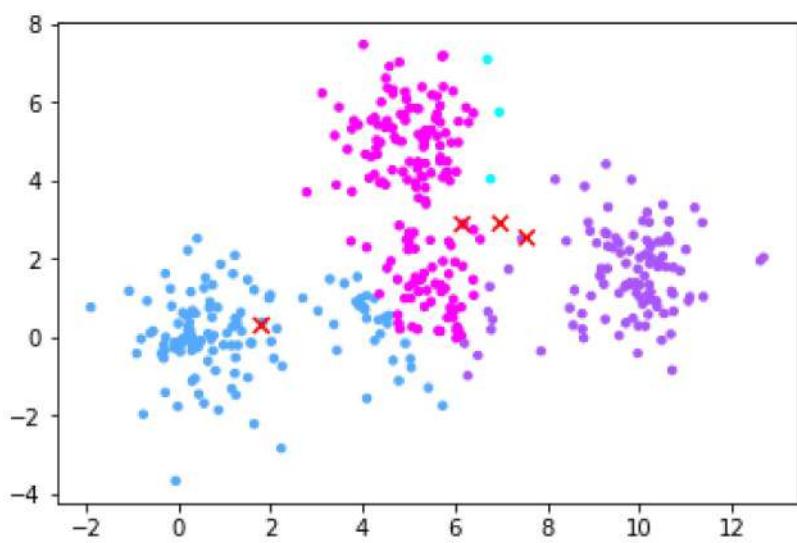
Iteration 0



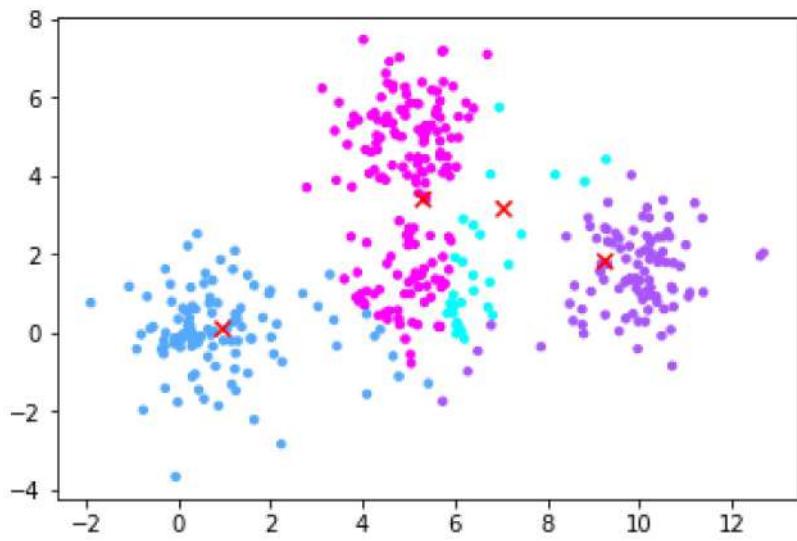
Iteration 1



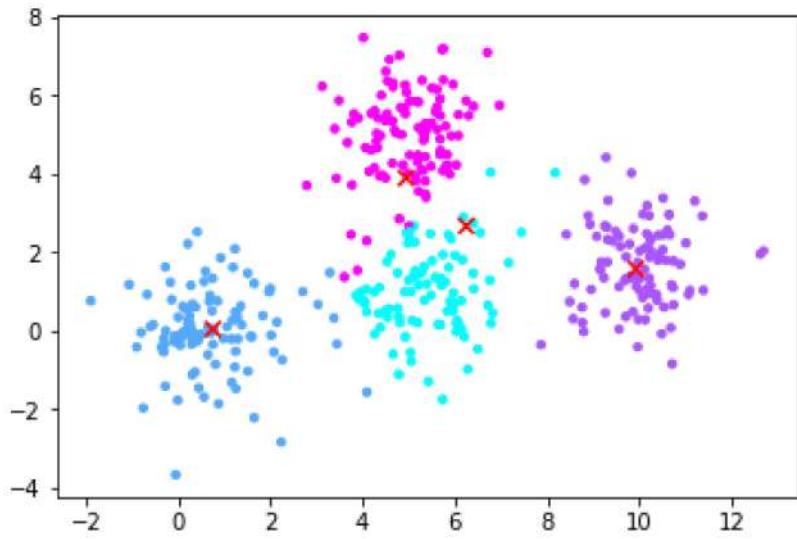
Iteration 2



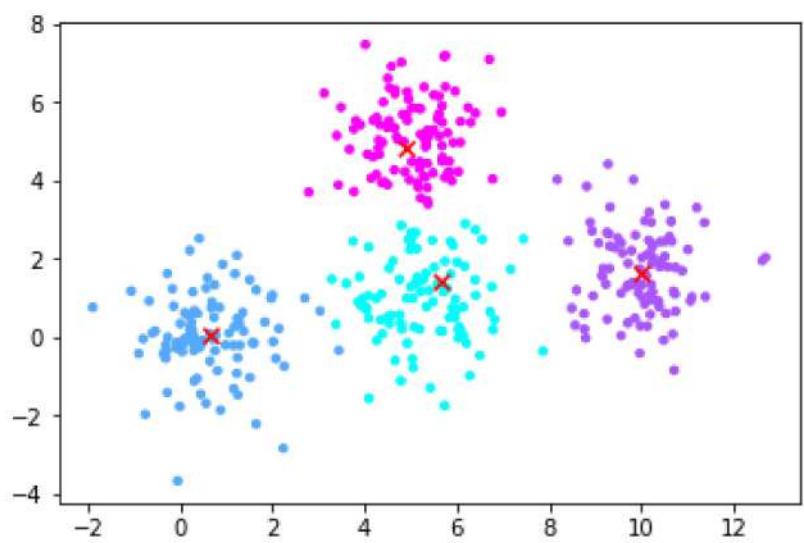
Iteration 3



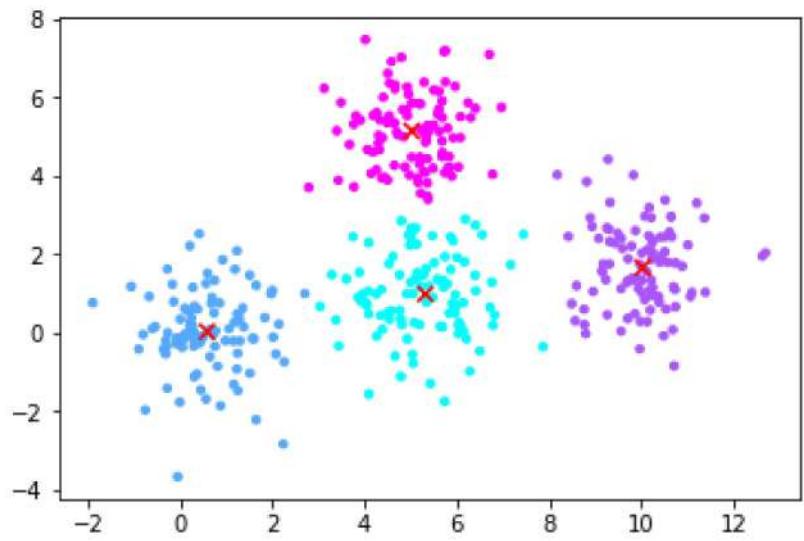
Iteration 4



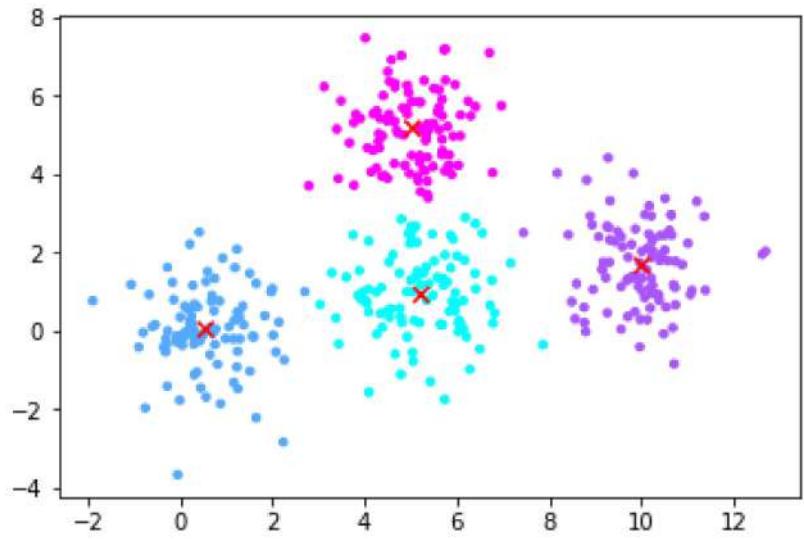
Iteration 5



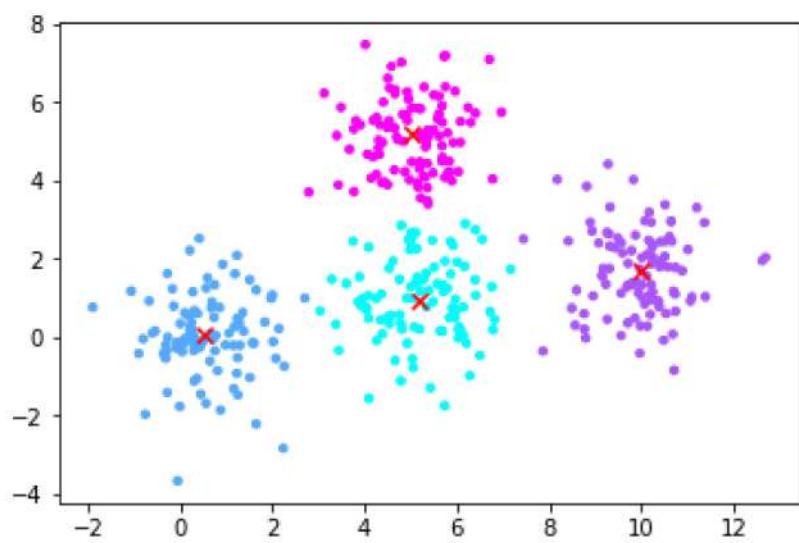
Iteration 6



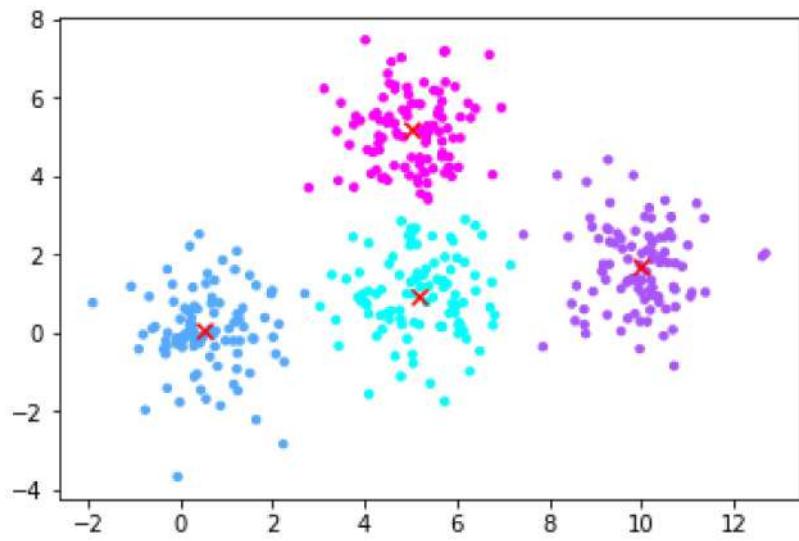
Iteration 7



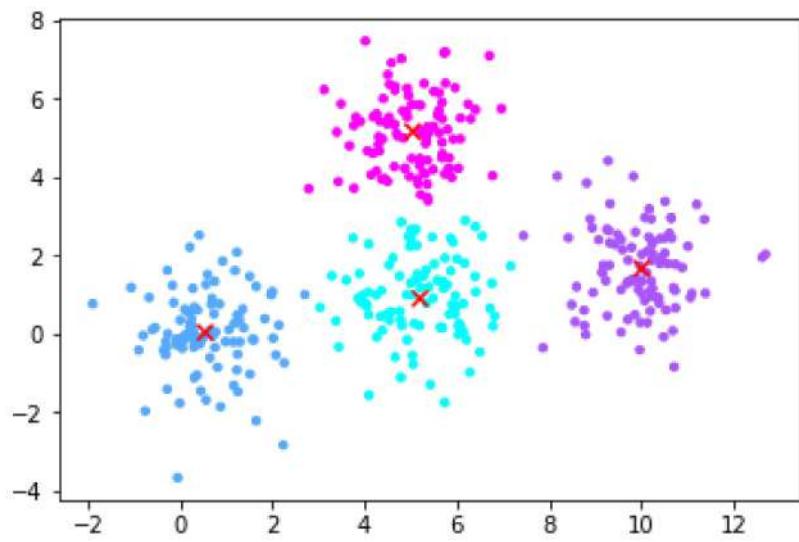
Iteration 8



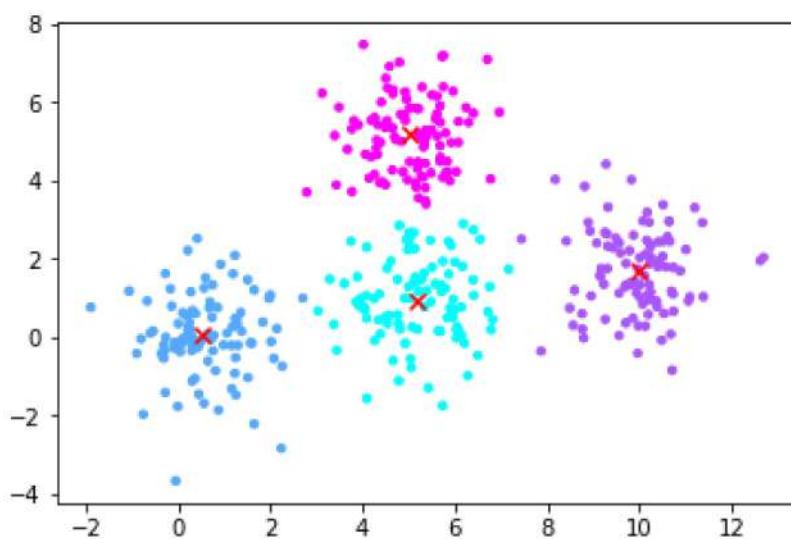
Iteration 9



Iteration 10



Final Clustering:



# Hierarchical Clustering

Hierarchical clustering is an unsupervised clustering technique which groups together the unlabelled data of similar characteristics.

There are two types of hierarchical clustering:

- Agglomerative Clustering
- Divisive Clustering

## Agglomerative Clustering:

In this type of hierarchical clustering all data set are considered as individual cluster and at every iterations clusters with similar characteristics are merged to give bigger clusters. This is repeated until one single cluster is reached. It is also called bottom-top approach.

## Agglomerative Clustering:

Lets start with some dummy example :

$X = [x_1, x_2, \dots, x_5]$ , with

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}, x_4 = \begin{bmatrix} 6 \\ 5 \end{bmatrix}, x_5 = \begin{bmatrix} 6.5 \\ 6 \end{bmatrix}$$

## Steps to perform Agglomerative Clustering:

1. Compute Distance matrix ( $N \times N$  matrix, where  $N$  number of vectors present in the dataset):  $D(a, b) = \|x_a - x_b\|_2$
2. Replace the diagonal elements with  $\inf$  and find the index of the minimum element present in the distance matrix (suppose we get the location  $(l, k)$ ).
3. Replace  $x_{min(l,k)} = .5 \times [x_l + x_m]$  and delete  $x_{max(l,m)}$  vector from  $X$ (i.e now  $(N = N - 1)$ ),

**repeat from step 1 again untill all the vectors combined to a single cluster.**

```
In [ ]: from cmath import inf

def Euclidean_Dist(x,y):
    return ((x[0]-y[0])**2+(x[1]-y[1])**2)**0.5

def Dist_mat(X):
    ## write your code here
    dist_mat=np.empty(shape=(X.shape[0],X.shape[0]))
    for i in range(X.shape[0]):
        for j in range(X.shape[0]):
            dist_mat[i][j]=Euclidean_Dist(X[i,:],X[j,:])
    for i in range(X.shape[0]):
        dist_mat[i][i]=inf
    return dist_mat

def combine(X,cluster):
    dist_mat=Dist_mat(X=X)
    points=np.where(dist_mat == np.amin(dist_mat))[0]
    minx=points[0]
    miny=points[1]
    if(minx>miny):
        minx,miny=miny,minx
    cluster.append([minx,miny])
    X[minx]=0.5*(X[minx]+X[miny])
    X=np.delete(X,miny,0)
    return X,cluster
```

```
In [ ]: %pip install plotly
```

Requirement already satisfied: plotly in c:\users\bsidd\appdata\local\programs\python\python39\lib\site-packages (5.10.0)  
Note: you may need to restart the kernel to use updated packages.

Requirement already satisfied: tenacity>=6.2.0 in c:\users\bsidd\appdata\local\programs\python\python39\lib\site-packages (from plotly) (8.1.0)

WARNING: You are using pip version 22.0.4; however, version 22.2.2 is available.  
You should consider upgrading via the 'c:\Users\bsidd\AppData\Local\Programs\Python\Python39\python.exe -m pip install --upgrade pip' command.

```
In [ ]: X=np.array([[1,1],[2,1],[5,4],[6,5],[6.5,6]])
import plotly.figure_factory as ff
## write your code here
cluster=[]
print(X)
while X.shape!=(1,2):
    X1=X.transpose()
    lab=np.linspace(1,X1.shape[1],X1.shape[1])
    fig = ff.create_dendrogram(X1.T, labels=lab)
    fig.update_layout(width=800, height=300)
    fig.show()
    X,cluster = combine(X,cluster)
    print("Combined Points",cluster[-1]+np.ones((1,2)))

    print('\nMean of clusters after every iteration: \n\n',X)
    print('\ncluster combination order: \n\n',cluster+np.ones((len(cluster),2)))

## validate from inbuilt Dendogram
```

```
[[1. 1. ]
 [2. 1. ]
 [5. 4. ]
 [6. 5. ]
 [6.5 6. ]]
```

Combined Points [[1. 2.]]

Mean of clusters after every iteration:

```
[[1.5 1. ]
 [5. 4. ]
 [6. 5. ]
 [6.5 6. ]]
```

Combined Points [[3. 4.]]

Mean of clusters after every iteration:

```
[[1.5 1. ]
 [5. 4. ]
 [6.25 5.5 ]]
```

Combined Points [[2. 3.]]

Mean of clusters after every iteration:

```
[[1.5 1. ]
 [5.625 4.75 ]]
```

Combined Points [[1. 2.]]

Mean of clusters after every iteration:

```
[[3.5625 2.875 ]]
```

cluster combination order:

```
[[1. 2.]
 [3. 4.]
 [2. 3.]
 [1. 2.]]
```

WARNING: You are using pip version 22.0.4; however, version 22.2.2 is available.  
You should consider upgrading via the 'c:\Users\bsidd\AppData\Local\Programs\Python\Python39\python.exe -m pip install --upgrade pip' command.

Collecting plotly

```
  Downloading plotly-5.10.0-py2.py3-none-any.whl (15.2 MB)
----- 15.2/15.2 MB 4.5 MB/s eta 0:00:00
```

Collecting tenacity>=6.2.0

```
  Downloading tenacity-8.1.0-py3-none-any.whl (23 kB)
```

Installing collected packages: tenacity, plotly

Successfully installed plotly-5.10.0 tenacity-8.1.0

# Clustering Algorithms on MNIST Digit dataset

Perform Kmeans and gmm clustering on MNIST dataset

1. Load MNIST data from the given images and labels

2. Consider any 2 classes

```
In [ ]: %pip install idx2numpy

Collecting idx2numpy
  Downloading idx2numpy-1.2.3.tar.gz (6.8 kB)
    Preparing metadata (setup.py): started
      Preparing metadata (setup.py): finished with status 'done'
Requirement already satisfied: numpy in c:\users\bsidd\appdata\local\programs\python\python39\lib\site-packages (from idx2numpy) (1.21.2)
Requirement already satisfied: six in c:\users\bsidd\appdata\local\programs\python\python39\lib\site-packages (from idx2numpy) (1.16.0)
Using legacy 'setup.py install' for idx2numpy, since package 'wheel' is not installed.
Installing collected packages: idx2numpy
  Running setup.py install for idx2numpy: started
    Running setup.py install for idx2numpy: finished with status 'done'
Successfully installed idx2numpy-1.2.3
Note: you may need to restart the kernel to use updated packages.

WARNING: You are using pip version 22.0.4; however, version 22.2.2 is available.
You should consider upgrading via the 'c:\Users\bsidd\AppData\Local\Programs\Python\Python39\python.exe -m pip install --upgrade pip' command.
```

```
In [ ]: import idx2numpy
# from keras.utils import np_utils
img_path = "t10k-images-idx3-ubyte" ## write your code here
label_path = "t10k-labels-idx1-ubyte" ## write your code here

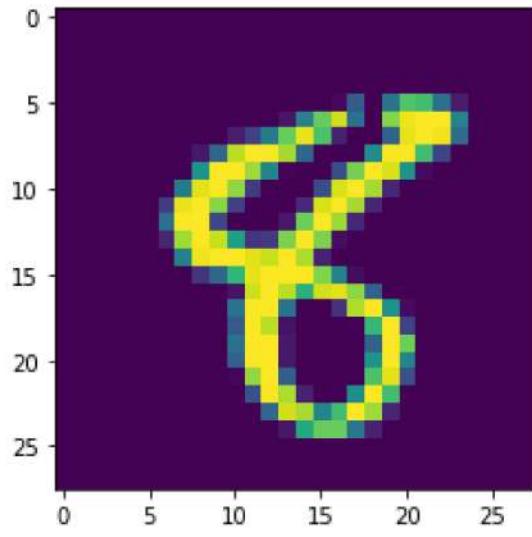
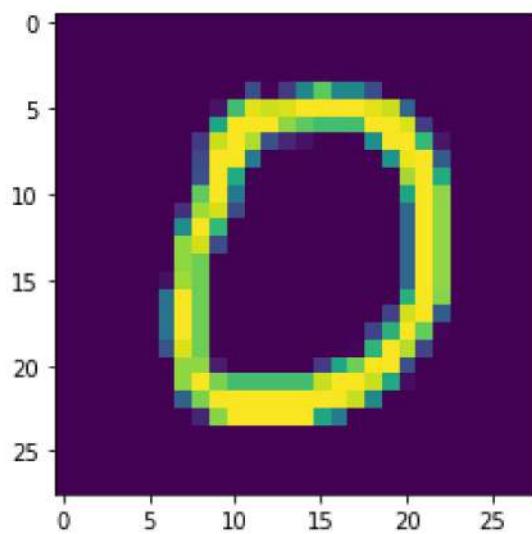
Images = idx2numpy.convert_from_file(img_path)
labels = idx2numpy.convert_from_file(label_path)

## write your code here
id_0 = np.where(labels==0)
id0 = id_0[0]
Im_0 = Images[id0,:,:]
label_0=labels[id0]
plt.figure()
plt.imshow(Im_0[1,:,:])

id_8=np.where(labels==8)
id8=id_8[0]
Im_8=Images[id8,:,:]
label_8=labels[id8]
plt.figure()
plt.imshow(Im_8[1,:,:])

data=np.concatenate((Im_0, Im_8))
data=np.reshape(data,(data.shape[0],data.shape[1]*data.shape[2]))
y=np.concatenate((label_0, label_8))
print(data.shape)
```

(1954, 784)



Use the K-means clustering algorithm from the last lab to form the clusters

```
In [ ]: ## Write your code here
from sklearn.cluster import KMeans
kmeans = KMeans(n_clusters=2, n_init=100, max_iter=100, verbose=1)
kmeans.fit(data)

cents = kmeans.cluster_centers_
im = np.reshape(cents[0,:], (28,28))
plt.figure()
plt.imshow(im)

im = np.reshape(cents[1,:], (28,28))
plt.figure()
plt.imshow(im)
```

Initialization complete  
Iteration 0, inertia 10569376621.0  
Iteration 1, inertia 6027558060.99791  
Iteration 2, inertia 5948722623.144613  
Iteration 3, inertia 5946248218.593399  
Iteration 4, inertia 5946185599.385244  
Converged at iteration 4: strict convergence.  
Initialization complete  
Iteration 0, inertia 11984286835.0  
Iteration 1, inertia 6823503613.92376  
Iteration 2, inertia 6649751605.540089  
Iteration 3, inertia 6290397494.620843  
Iteration 4, inertia 5990593254.258571  
Iteration 5, inertia 5955393779.2508135  
Iteration 6, inertia 5948976094.530472  
Iteration 7, inertia 5947108351.10419  
Iteration 8, inertia 5946423579.1520195  
Iteration 9, inertia 5946256719.009511  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10722176791.0  
Iteration 1, inertia 6097476521.307847  
Iteration 2, inertia 6002977891.074748  
Iteration 3, inertia 5968607097.054088  
Iteration 4, inertia 5955291322.269452  
Iteration 5, inertia 5949077797.836747  
Iteration 6, inertia 5947265844.410739  
Iteration 7, inertia 5946598710.140918  
Iteration 8, inertia 5946291825.920058  
Iteration 9, inertia 5946193609.350716  
Iteration 10, inertia 5946185599.385245  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10115717620.0  
Iteration 1, inertia 5956317984.9538555  
Iteration 2, inertia 5946590689.183146  
Iteration 3, inertia 5946185599.385244  
Converged at iteration 3: strict convergence.  
Initialization complete  
Iteration 0, inertia 11015004344.0  
Iteration 1, inertia 6045114284.792912  
Iteration 2, inertia 5984972087.56122  
Iteration 3, inertia 5960827186.477382  
Iteration 4, inertia 5951903000.074925  
Iteration 5, inertia 5947768679.273354  
Iteration 6, inertia 5946758585.869441  
Iteration 7, inertia 5946331861.227085  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 12162943376.0  
Iteration 1, inertia 6390036806.580162  
Iteration 2, inertia 6025609289.508827  
Iteration 3, inertia 5948753351.979566  
Iteration 4, inertia 5946393570.793601  
Iteration 5, inertia 5946193508.036913  
Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 10374585372.0

Iteration 1, inertia 5985143924.109375  
Iteration 2, inertia 5947723067.063828  
Iteration 3, inertia 5946582959.193299  
Iteration 4, inertia 5946419653.38891  
Iteration 5, inertia 5946344403.565366  
Iteration 6, inertia 5946268777.538702  
Iteration 7, inertia 5946207359.724583  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11141968432.0  
Iteration 1, inertia 5993984303.846148  
Iteration 2, inertia 5952213263.494222  
Iteration 3, inertia 5947080249.165954  
Iteration 4, inertia 5946423579.1520195  
Iteration 5, inertia 5946256719.009511  
Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 10830213554.0  
Iteration 1, inertia 6064327505.756428  
Iteration 2, inertia 5951567503.421175  
Iteration 3, inertia 5947396938.500655  
Iteration 4, inertia 5946611992.986639  
Iteration 5, inertia 5946502268.271992  
Iteration 6, inertia 5946392068.1635895  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11901043794.0  
Iteration 1, inertia 6154778313.528939  
Iteration 2, inertia 5997534535.650227  
Iteration 3, inertia 5963793701.4774275  
Iteration 4, inertia 5953219387.590105  
Iteration 5, inertia 5948568064.163213  
Iteration 6, inertia 5946868948.924838  
Iteration 7, inertia 5946351256.172853  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385246  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11257317451.0  
Iteration 1, inertia 6266008009.135151  
Iteration 2, inertia 6045026787.081464  
Iteration 3, inertia 5953084638.600369  
Iteration 4, inertia 5947196729.9658575  
Iteration 5, inertia 5946518148.018414  
Iteration 6, inertia 5946369610.190655  
Iteration 7, inertia 5946268777.538702  
Iteration 8, inertia 5946207359.724581  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10946378266.0  
Iteration 1, inertia 6092035204.933018  
Iteration 2, inertia 5958997736.074331  
Iteration 3, inertia 5948252590.127128  
Iteration 4, inertia 5947083692.400879  
Iteration 5, inertia 5946625645.947666  
Iteration 6, inertia 5946474702.761869

Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.45052  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385245  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11657132496.0  
Iteration 1, inertia 6550742767.400741  
Iteration 2, inertia 6431383510.341793  
Iteration 3, inertia 6277414755.740333  
Iteration 4, inertia 6095034477.909592  
Iteration 5, inertia 5983286353.604475  
Iteration 6, inertia 5950729175.371016  
Iteration 7, inertia 5947135351.642101  
Iteration 8, inertia 5946423579.1520195  
Iteration 9, inertia 5946256719.009511  
Iteration 10, inertia 5946185599.385245  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10079374277.0  
Iteration 1, inertia 5955762475.601915  
Iteration 2, inertia 5946741966.518094  
Iteration 3, inertia 5946369258.084686  
Iteration 4, inertia 5946224172.898862  
Iteration 5, inertia 5946185599.385245  
Converged at iteration 5: strict convergence.  
Initialization complete  
Iteration 0, inertia 10856870778.0  
Iteration 1, inertia 5984635492.215052  
Iteration 2, inertia 5949019984.789828  
Iteration 3, inertia 5947188647.311945  
Iteration 4, inertia 5946625645.947666  
Iteration 5, inertia 5946474702.761869  
Iteration 6, inertia 5946366132.679892  
Iteration 7, inertia 5946295921.45052  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385245  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10721788954.0  
Iteration 1, inertia 6552021297.3167925  
Iteration 2, inertia 6487199502.0063095  
Iteration 3, inertia 6397149282.817568  
Iteration 4, inertia 6225910335.098821  
Iteration 5, inertia 6032675195.924904  
Iteration 6, inertia 5955659195.364047  
Iteration 7, inertia 5947060928.467709  
Iteration 8, inertia 5946384283.931921  
Iteration 9, inertia 5946206130.517454  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10864517422.0  
Iteration 1, inertia 6009835268.6407  
Iteration 2, inertia 5951371702.412695  
Iteration 3, inertia 5947860647.719056  
Iteration 4, inertia 5946808275.991884  
Iteration 5, inertia 5946568492.25512  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.45052  
Iteration 9, inertia 5946207359.724583

Iteration 10, inertia 5946185599.385245  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 9909908896.0  
Iteration 1, inertia 5948366998.414898  
Iteration 2, inertia 5946473661.776793  
Iteration 3, inertia 5946344403.565365  
Iteration 4, inertia 5946268777.538702  
Iteration 5, inertia 5946207359.724583  
Iteration 6, inertia 5946185599.385246  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 11665302408.0  
Iteration 1, inertia 6023766429.658562  
Iteration 2, inertia 5960442074.962462  
Iteration 3, inertia 5950804847.451724  
Iteration 4, inertia 5947615614.623979  
Iteration 5, inertia 5946715011.478968  
Iteration 6, inertia 5946331861.227085  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385246  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 12932559682.0  
Iteration 1, inertia 6293958068.179632  
Iteration 2, inertia 6001491692.141198  
Iteration 3, inertia 5959423489.526887  
Iteration 4, inertia 5949920677.658673  
Iteration 5, inertia 5947408627.285728  
Iteration 6, inertia 5946598710.140918  
Iteration 7, inertia 5946291825.920058  
Iteration 8, inertia 5946193609.350716  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10862666504.0  
Iteration 1, inertia 6083940534.027683  
Iteration 2, inertia 5957756202.609805  
Iteration 3, inertia 5947553264.309313  
Iteration 4, inertia 5946625936.537294  
Iteration 5, inertia 5946502268.271992  
Iteration 6, inertia 5946392068.1635895  
Iteration 7, inertia 5946295921.45052  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385246  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11572816186.0  
Iteration 1, inertia 6134509263.494941  
Iteration 2, inertia 5957509077.994882  
Iteration 3, inertia 5946951566.282738  
Iteration 4, inertia 5946192992.975294  
Iteration 5, inertia 5946185599.385244  
Converged at iteration 5: strict convergence.  
Initialization complete  
Iteration 0, inertia 10904448842.0  
Iteration 1, inertia 6026441501.672522  
Iteration 2, inertia 5948740871.240011  
Iteration 3, inertia 5946235127.455228  
Iteration 4, inertia 5946185599.385244  
Converged at iteration 4: strict convergence.  
Initialization complete

Iteration 0, inertia 10060607870.0  
Iteration 1, inertia 6004151096.822786  
Iteration 2, inertia 5949037492.367149  
Iteration 3, inertia 5946880037.553509  
Iteration 4, inertia 5946481777.3466425  
Iteration 5, inertia 5946369610.190655  
Iteration 6, inertia 5946268777.538702  
Iteration 7, inertia 5946207359.724583  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 12530362496.0  
Iteration 1, inertia 6260936981.818479  
Iteration 2, inertia 5997015956.536039  
Iteration 3, inertia 5950769733.297568  
Iteration 4, inertia 5946832998.055262  
Iteration 5, inertia 5946384283.931921  
Iteration 6, inertia 5946206130.517454  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 12042977410.0  
Iteration 1, inertia 6205278848.505245  
Iteration 2, inertia 6004853872.287383  
Iteration 3, inertia 5961857764.258535  
Iteration 4, inertia 5951281217.60515  
Iteration 5, inertia 5947669684.573002  
Iteration 6, inertia 5946758585.869441  
Iteration 7, inertia 5946331861.227086  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10207505784.0  
Iteration 1, inertia 5954812594.202124  
Iteration 2, inertia 5947779654.281613  
Iteration 3, inertia 5946634121.839708  
Iteration 4, inertia 5946256719.009511  
Iteration 5, inertia 5946185599.385244  
Converged at iteration 5: strict convergence.  
Initialization complete  
Iteration 0, inertia 12586495225.0  
Iteration 1, inertia 6516168049.072311  
Iteration 2, inertia 5985136965.28231  
Iteration 3, inertia 5949371361.879644  
Iteration 4, inertia 5947188647.311945  
Iteration 5, inertia 5946625645.947666  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.45052  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11992390680.0  
Iteration 1, inertia 6090990455.30163  
Iteration 2, inertia 5951014025.783516  
Iteration 3, inertia 5947756972.745643  
Iteration 4, inertia 5946766281.909214  
Iteration 5, inertia 5946568492.255119  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892

Iteration 8, inertia 5946295921.4505205  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11543839892.0  
Iteration 1, inertia 6024068162.013538  
Iteration 2, inertia 5955358828.610191  
Iteration 3, inertia 5947284726.95615  
Iteration 4, inertia 5946504872.691041  
Iteration 5, inertia 5946291825.920057  
Iteration 6, inertia 5946193609.350716  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 10471352076.0  
Iteration 1, inertia 6131870226.921689  
Iteration 2, inertia 5969359201.428627  
Iteration 3, inertia 5949238649.226777  
Iteration 4, inertia 5947239020.83382  
Iteration 5, inertia 5946651556.219193  
Iteration 6, inertia 5946502268.271992  
Iteration 7, inertia 5946392068.1635895  
Iteration 8, inertia 5946295921.45052  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 12372138019.0  
Iteration 1, inertia 6260242962.64969  
Iteration 2, inertia 5951272146.753332  
Iteration 3, inertia 5947168795.679857  
Iteration 4, inertia 5946573945.050664  
Iteration 5, inertia 5946472194.272189  
Iteration 6, inertia 5946366132.679892  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10764839983.0  
Iteration 1, inertia 6115761383.486792  
Iteration 2, inertia 5993289908.022993  
Iteration 3, inertia 5962499997.107114  
Iteration 4, inertia 5952841416.160238  
Iteration 5, inertia 5948525713.293636  
Iteration 6, inertia 5946868948.924838  
Iteration 7, inertia 5946351256.172853  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385245  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10979659900.0  
Iteration 1, inertia 5985859229.061696  
Iteration 2, inertia 5953255084.011823  
Iteration 3, inertia 5948147723.490656  
Iteration 4, inertia 5946824660.984224  
Iteration 5, inertia 5946351256.172853  
Iteration 6, inertia 5946206130.517454  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete

Iteration 0, inertia 12000795313.0  
Iteration 1, inertia 6273487021.269246  
Iteration 2, inertia 5986239533.049507  
Iteration 3, inertia 5948737618.547138  
Iteration 4, inertia 5946492234.434719  
Iteration 5, inertia 5946256719.009511  
Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 11133595263.0  
Iteration 1, inertia 6548090550.769834  
Iteration 2, inertia 6434420383.569464  
Iteration 3, inertia 6172882150.752765  
Iteration 4, inertia 5975011389.026878  
Iteration 5, inertia 5950118655.45491  
Iteration 6, inertia 5946823130.473685  
Iteration 7, inertia 5946256719.009511  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11011853818.0  
Iteration 1, inertia 6052656606.295334  
Iteration 2, inertia 5949691400.453703  
Iteration 3, inertia 5946931954.549732  
Iteration 4, inertia 5946592654.820652  
Iteration 5, inertia 5946474702.761869  
Iteration 6, inertia 5946366132.679892  
Iteration 7, inertia 5946295921.45052  
Iteration 8, inertia 5946207359.724582  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 13482397190.0  
Iteration 1, inertia 6597656250.639009  
Iteration 2, inertia 6080859384.970302  
Iteration 3, inertia 5949479096.347073  
Iteration 4, inertia 5946952115.469556  
Iteration 5, inertia 5946592654.820652  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.4505205  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11500599626.0  
Iteration 1, inertia 6268331441.366072  
Iteration 2, inertia 5985642552.902413  
Iteration 3, inertia 5954623741.569504  
Iteration 4, inertia 5948976094.530472  
Iteration 5, inertia 5947108351.10419  
Iteration 6, inertia 5946423579.15202  
Iteration 7, inertia 5946256719.009511  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11235363962.0  
Iteration 1, inertia 5994338436.759376  
Iteration 2, inertia 5950907320.731824  
Iteration 3, inertia 5947002160.946369  
Iteration 4, inertia 5946423579.15202  
Iteration 5, inertia 5946256719.009511

Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 11325459600.0  
Iteration 1, inertia 6096896037.14918  
Iteration 2, inertia 5974938731.183369  
Iteration 3, inertia 5952986056.28975  
Iteration 4, inertia 5948496842.986118  
Iteration 5, inertia 5946803428.99884  
Iteration 6, inertia 5946331861.227085  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 10709264148.0  
Iteration 1, inertia 6319852032.884639  
Iteration 2, inertia 5976541787.833776  
Iteration 3, inertia 5948327613.666501  
Iteration 4, inertia 5946609422.30431  
Iteration 5, inertia 5946501318.116011  
Iteration 6, inertia 5946389187.952923  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10307169278.0  
Iteration 1, inertia 6059590998.920707  
Iteration 2, inertia 5973784457.574446  
Iteration 3, inertia 5951161725.566212  
Iteration 4, inertia 5947557979.014988  
Iteration 5, inertia 5946650938.437969  
Iteration 6, inertia 5946291825.920057  
Iteration 7, inertia 5946193609.350716  
Iteration 8, inertia 5946185599.385245  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11850179198.0  
Iteration 1, inertia 5997457678.5759325  
Iteration 2, inertia 5953227520.829029  
Iteration 3, inertia 5947987407.629555  
Iteration 4, inertia 5946758585.869441  
Iteration 5, inertia 5946331861.227085  
Iteration 6, inertia 5946206130.517454  
Iteration 7, inertia 5946185599.385245  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 11425808290.0  
Iteration 1, inertia 6228655596.946611  
Iteration 2, inertia 6007585488.3437  
Iteration 3, inertia 5949322023.053587  
Iteration 4, inertia 5946320018.15456  
Iteration 5, inertia 5946192576.515545  
Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 11592078462.0  
Iteration 1, inertia 6050219871.308249  
Iteration 2, inertia 5950891449.367537  
Iteration 3, inertia 5947617683.213665  
Iteration 4, inertia 5946766281.909214  
Iteration 5, inertia 5946568492.25512

Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.4505205  
Iteration 9, inertia 5946207359.724582  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11674527010.0  
Iteration 1, inertia 6175686813.866407  
Iteration 2, inertia 5958074764.600211  
Iteration 3, inertia 5948311862.376384  
Iteration 4, inertia 5946915493.191978  
Iteration 5, inertia 5946625645.947666  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.4505205  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 12174465002.0  
Iteration 1, inertia 5952779525.908057  
Iteration 2, inertia 5946622226.024435  
Iteration 3, inertia 5946331861.227085  
Iteration 4, inertia 5946206130.517454  
Iteration 5, inertia 5946185599.385244  
Converged at iteration 5: strict convergence.  
Initialization complete  
Iteration 0, inertia 10859829593.0  
Iteration 1, inertia 6137584748.815824  
Iteration 2, inertia 6021867657.697381  
Iteration 3, inertia 5974927466.858312  
Iteration 4, inertia 5957145014.974785  
Iteration 5, inertia 5949809991.235873  
Iteration 6, inertia 5947416885.314002  
Iteration 7, inertia 5946598710.140918  
Iteration 8, inertia 5946291825.920058  
Iteration 9, inertia 5946193609.350716  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10874655466.0  
Iteration 1, inertia 5999469479.984511  
Iteration 2, inertia 5965351626.374612  
Iteration 3, inertia 5953467647.736193  
Iteration 4, inertia 5948296177.143916  
Iteration 5, inertia 5946868948.924838  
Iteration 6, inertia 5946351256.172853  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11371524891.0  
Iteration 1, inertia 6013946282.406817  
Iteration 2, inertia 5960169963.316824  
Iteration 3, inertia 5951812419.595436  
Iteration 4, inertia 5947768679.273354  
Iteration 5, inertia 5946758585.869441  
Iteration 6, inertia 5946331861.227086  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.

Initialization complete  
Iteration 0, inertia 11856140868.0  
Iteration 1, inertia 6496189253.589516  
Iteration 2, inertia 6281291957.85269  
Iteration 3, inertia 6045293096.032471  
Iteration 4, inertia 5951917477.906582  
Iteration 5, inertia 5947007914.272045  
Iteration 6, inertia 5946481777.3466425  
Iteration 7, inertia 5946369610.190655  
Iteration 8, inertia 5946268777.538702  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11109355081.0  
Iteration 1, inertia 6139908337.385467  
Iteration 2, inertia 5977967328.0322  
Iteration 3, inertia 5947757601.1742  
Iteration 4, inertia 5946502226.722412  
Iteration 5, inertia 5946369610.190655  
Iteration 6, inertia 5946268777.538702  
Iteration 7, inertia 5946207359.724583  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 10522438272.0  
Iteration 1, inertia 6022326072.335923  
Iteration 2, inertia 5953561587.096657  
Iteration 3, inertia 5948083857.0082  
Iteration 4, inertia 5946857262.254427  
Iteration 5, inertia 5946568492.25512  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.4505205  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11877781192.0  
Iteration 1, inertia 6002699748.243535  
Iteration 2, inertia 5962619380.847886  
Iteration 3, inertia 5953079383.033131  
Iteration 4, inertia 5948568064.163212  
Iteration 5, inertia 5946868948.924838  
Iteration 6, inertia 5946351256.172853  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 10372616378.0  
Iteration 1, inertia 5973079104.681058  
Iteration 2, inertia 5950528005.536718  
Iteration 3, inertia 5947449122.795771  
Iteration 4, inertia 5946650938.437969  
Iteration 5, inertia 5946291825.920058  
Iteration 6, inertia 5946193609.350716  
Iteration 7, inertia 5946185599.385245  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 11672010318.0  
Iteration 1, inertia 6180593026.999825  
Iteration 2, inertia 5965399230.809962

Iteration 3, inertia 5947712762.505862  
Iteration 4, inertia 5946318604.954279  
Iteration 5, inertia 5946185599.385244  
Converged at iteration 5: strict convergence.  
Initialization complete  
Iteration 0, inertia 11356267446.0  
Iteration 1, inertia 6129591837.598859  
Iteration 2, inertia 5960648913.491602  
Iteration 3, inertia 5948600090.677914  
Iteration 4, inertia 5947161313.562705  
Iteration 5, inertia 5946625645.947666  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.45052  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10377639160.0  
Iteration 1, inertia 5976895687.198111  
Iteration 2, inertia 5951113775.201029  
Iteration 3, inertia 5947702023.002999  
Iteration 4, inertia 5946758585.869441  
Iteration 5, inertia 5946331861.227086  
Iteration 6, inertia 5946206130.517454  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 11784956179.0  
Iteration 1, inertia 6392895782.708006  
Iteration 2, inertia 6191126641.866333  
Iteration 3, inertia 6001350088.378908  
Iteration 4, inertia 5950480707.359138  
Iteration 5, inertia 5946559079.690792  
Iteration 6, inertia 5946310831.400655  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 13413108856.0  
Iteration 1, inertia 6582289283.613349  
Iteration 2, inertia 6328720393.9771  
Iteration 3, inertia 6072905963.539662  
Iteration 4, inertia 5955330162.852629  
Iteration 5, inertia 5947429668.024757  
Iteration 6, inertia 5946582959.193299  
Iteration 7, inertia 5946419653.38891  
Iteration 8, inertia 5946344403.565366  
Iteration 9, inertia 5946268777.538702  
Iteration 10, inertia 5946207359.724583  
Iteration 11, inertia 5946185599.385244  
Converged at iteration 11: strict convergence.  
Initialization complete  
Iteration 0, inertia 12731633278.0  
Iteration 1, inertia 6422898825.480052  
Iteration 2, inertia 6017974326.173583  
Iteration 3, inertia 5972144842.387388  
Iteration 4, inertia 5956037613.798416  
Iteration 5, inertia 5949293274.193438  
Iteration 6, inertia 5947333956.037614  
Iteration 7, inertia 5946598710.140919  
Iteration 8, inertia 5946291825.920057

Iteration 9, inertia 5946193609.350716  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 10855426436.0  
Iteration 1, inertia 6038786301.642751  
Iteration 2, inertia 5984530007.019499  
Iteration 3, inertia 5960791164.495849  
Iteration 4, inertia 5951903000.074925  
Iteration 5, inertia 5947768679.273354  
Iteration 6, inertia 5946758585.869441  
Iteration 7, inertia 5946331861.227085  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385245  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 12222380400.0  
Iteration 1, inertia 6379830583.734245  
Iteration 2, inertia 6146235973.324803  
Iteration 3, inertia 6016811439.948845  
Iteration 4, inertia 5969727876.673958  
Iteration 5, inertia 5955291419.566433  
Iteration 6, inertia 5948989880.147812  
Iteration 7, inertia 5947185497.878584  
Iteration 8, inertia 5946475375.774194  
Iteration 9, inertia 5946256719.009511  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11830272956.0  
Iteration 1, inertia 5995683635.268729  
Iteration 2, inertia 5957063105.763138  
Iteration 3, inertia 5949729351.827102  
Iteration 4, inertia 5947416885.314002  
Iteration 5, inertia 5946598710.140919  
Iteration 6, inertia 5946291825.920057  
Iteration 7, inertia 5946193609.350716  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11772064340.0  
Iteration 1, inertia 6002133760.392621  
Iteration 2, inertia 5949652406.900532  
Iteration 3, inertia 5946419376.561036  
Iteration 4, inertia 5946227381.154176  
Iteration 5, inertia 5946185599.385244  
Converged at iteration 5: strict convergence.  
Initialization complete  
Iteration 0, inertia 10879978582.0  
Iteration 1, inertia 6032326149.84403  
Iteration 2, inertia 5965801267.7616825  
Iteration 3, inertia 5951496930.354331  
Iteration 4, inertia 5947837735.234199  
Iteration 5, inertia 5946758585.869441  
Iteration 6, inertia 5946331861.227086  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 10493876701.0  
Iteration 1, inertia 6056068696.881008  
Iteration 2, inertia 5953761179.543274

Iteration 3, inertia 5946989432.659873  
Iteration 4, inertia 5946321497.698899  
Iteration 5, inertia 5946248899.678625  
Iteration 6, inertia 5946207359.724583  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 10595786306.0  
Iteration 1, inertia 5978544274.60758  
Iteration 2, inertia 5956960922.61988  
Iteration 3, inertia 5949437598.518428  
Iteration 4, inertia 5947333956.037614  
Iteration 5, inertia 5946598710.140918  
Iteration 6, inertia 5946291825.920057  
Iteration 7, inertia 5946193609.350716  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11134435984.0  
Iteration 1, inertia 5996543919.134007  
Iteration 2, inertia 5963538235.008598  
Iteration 3, inertia 5952773575.029356  
Iteration 4, inertia 5948188739.500652  
Iteration 5, inertia 5946868948.924838  
Iteration 6, inertia 5946351256.172853  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11233232130.0  
Iteration 1, inertia 5968314473.619905  
Iteration 2, inertia 5951978133.250246  
Iteration 3, inertia 5947962659.807809  
Iteration 4, inertia 5946760583.618365  
Iteration 5, inertia 5946331861.227085  
Iteration 6, inertia 5946206130.517454  
Iteration 7, inertia 5946185599.385245  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 10753943826.0  
Iteration 1, inertia 6012160658.163033  
Iteration 2, inertia 5947872828.238397  
Iteration 3, inertia 5946636387.949522  
Iteration 4, inertia 5946502268.271992  
Iteration 5, inertia 5946392068.1635895  
Iteration 6, inertia 5946295921.4505205  
Iteration 7, inertia 5946207359.724583  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11359238288.0  
Iteration 1, inertia 6374950430.319717  
Iteration 2, inertia 6095657895.632221  
Iteration 3, inertia 5964753663.18974  
Iteration 4, inertia 5948292774.749086  
Iteration 5, inertia 5946501609.817481  
Iteration 6, inertia 5946291825.920058  
Iteration 7, inertia 5946193609.350716  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11661598748.0

Iteration 1, inertia 6591657396.895796  
Iteration 2, inertia 6271963552.69635  
Iteration 3, inertia 6072670753.695386  
Iteration 4, inertia 5971558759.125284  
Iteration 5, inertia 5947264894.505356  
Iteration 6, inertia 5946384283.931921  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385245  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11452478251.0  
Iteration 1, inertia 6135387146.310196  
Iteration 2, inertia 5966042811.0212  
Iteration 3, inertia 5948412815.0949955  
Iteration 4, inertia 5947170871.589587  
Iteration 5, inertia 5946651556.219193  
Iteration 6, inertia 5946502268.271993  
Iteration 7, inertia 5946392068.1635895  
Iteration 8, inertia 5946295921.4505205  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385246  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11792891814.0  
Iteration 1, inertia 6315206342.723371  
Iteration 2, inertia 6097505235.095007  
Iteration 3, inertia 6005159598.087448  
Iteration 4, inertia 5968749466.251608  
Iteration 5, inertia 5955291322.269451  
Iteration 6, inertia 5949077797.836747  
Iteration 7, inertia 5947265844.410739  
Iteration 8, inertia 5946598710.140919  
Iteration 9, inertia 5946291825.920058  
Iteration 10, inertia 5946193609.350716  
Iteration 11, inertia 5946185599.385244  
Converged at iteration 11: strict convergence.  
Initialization complete  
Iteration 0, inertia 10778678601.0  
Iteration 1, inertia 6022614096.131845  
Iteration 2, inertia 5950560871.704993  
Iteration 3, inertia 5946934782.118774  
Iteration 4, inertia 5946481777.3466425  
Iteration 5, inertia 5946369610.190655  
Iteration 6, inertia 5946268777.538702  
Iteration 7, inertia 5946207359.724583  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 10191189996.0  
Iteration 1, inertia 6035043231.38682  
Iteration 2, inertia 5956288860.299123  
Iteration 3, inertia 5947040745.120452  
Iteration 4, inertia 5946384283.931921  
Iteration 5, inertia 5946206130.517454  
Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 10835330535.0  
Iteration 1, inertia 5995702731.849979  
Iteration 2, inertia 5948459034.844713  
Iteration 3, inertia 5946968193.094673  
Iteration 4, inertia 5946561116.380888

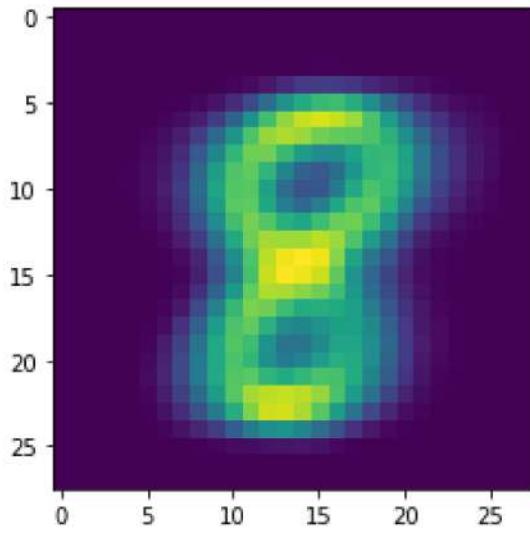
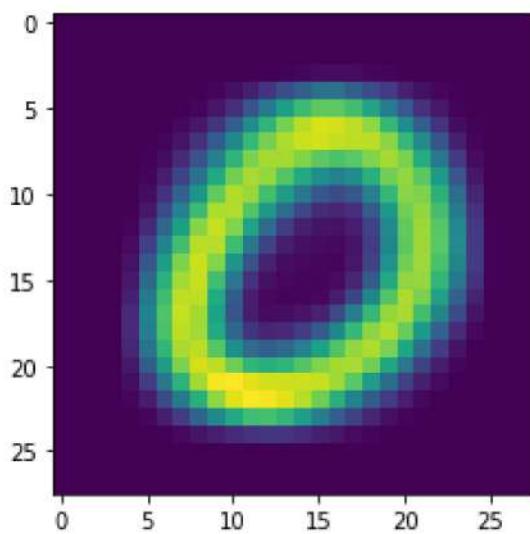
Iteration 5, inertia 5946458240.598179  
Iteration 6, inertia 5946366132.679892  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 12065781640.0  
Iteration 1, inertia 6424308061.430551  
Iteration 2, inertia 6176225482.500801  
Iteration 3, inertia 5985182849.52048  
Iteration 4, inertia 5948916603.182275  
Iteration 5, inertia 5946521757.056422  
Iteration 6, inertia 5946209600.946692  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 12514646252.0  
Iteration 1, inertia 5984855302.895817  
Iteration 2, inertia 5948150785.071257  
Iteration 3, inertia 5946708323.96322  
Iteration 4, inertia 5946331861.227085  
Iteration 5, inertia 5946206130.517454  
Iteration 6, inertia 5946185599.385246  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 10645017329.0  
Iteration 1, inertia 5973770500.398857  
Iteration 2, inertia 5952528863.3443365  
Iteration 3, inertia 5948079343.86223  
Iteration 4, inertia 5946803428.99884  
Iteration 5, inertia 5946331861.227085  
Iteration 6, inertia 5946206130.517454  
Iteration 7, inertia 5946185599.385244  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 10896838223.0  
Iteration 1, inertia 6282560164.002617  
Iteration 2, inertia 6039130509.227372  
Iteration 3, inertia 5954158183.813697  
Iteration 4, inertia 5947865162.437876  
Iteration 5, inertia 5946857262.254429  
Iteration 6, inertia 5946568492.25512  
Iteration 7, inertia 5946474702.761869  
Iteration 8, inertia 5946366132.679892  
Iteration 9, inertia 5946295921.450521  
Iteration 10, inertia 5946207359.724583  
Iteration 11, inertia 5946185599.385244  
Converged at iteration 11: strict convergence.  
Initialization complete  
Iteration 0, inertia 9816464313.0  
Iteration 1, inertia 5950723687.383961  
Iteration 2, inertia 5946692316.5493555  
Iteration 3, inertia 5946501318.116011  
Iteration 4, inertia 5946389187.952923  
Iteration 5, inertia 5946295921.45052  
Iteration 6, inertia 5946207359.724583  
Iteration 7, inertia 5946185599.385246  
Converged at iteration 7: strict convergence.  
Initialization complete  
Iteration 0, inertia 10641725380.0  
Iteration 1, inertia 6045862403.363358

Iteration 2, inertia 5975534405.117009  
Iteration 3, inertia 5954020973.838659  
Iteration 4, inertia 5948690412.646926  
Iteration 5, inertia 5946931223.698053  
Iteration 6, inertia 5946351256.172853  
Iteration 7, inertia 5946206130.517454  
Iteration 8, inertia 5946185599.385244  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 11325904552.0  
Iteration 1, inertia 6113083129.576727  
Iteration 2, inertia 6006087970.446456  
Iteration 3, inertia 5962607215.714321  
Iteration 4, inertia 5951194962.855941  
Iteration 5, inertia 5947669684.573002  
Iteration 6, inertia 5946758585.869441  
Iteration 7, inertia 5946331861.227085  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 12312944296.0  
Iteration 1, inertia 6010142312.653195  
Iteration 2, inertia 5953136666.927179  
Iteration 3, inertia 5946770416.758584  
Iteration 4, inertia 5946384283.931921  
Iteration 5, inertia 5946206130.517454  
Iteration 6, inertia 5946185599.385244  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 10596421905.0  
Iteration 1, inertia 5955580428.122269  
Iteration 2, inertia 5947732526.9659  
Iteration 3, inertia 5946732059.730334  
Iteration 4, inertia 5946552591.978764  
Iteration 5, inertia 5946454819.509368  
Iteration 6, inertia 5946366132.679892  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11911708573.0  
Iteration 1, inertia 6066347057.203327  
Iteration 2, inertia 5953416511.993177  
Iteration 3, inertia 5947649946.852065  
Iteration 4, inertia 5946666995.8142395  
Iteration 5, inertia 5946502268.271992  
Iteration 6, inertia 5946392068.1635895  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385245  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11941564400.0  
Iteration 1, inertia 5981156980.349543  
Iteration 2, inertia 5955471974.202061  
Iteration 3, inertia 5949175563.832167  
Iteration 4, inertia 5947333956.037614  
Iteration 5, inertia 5946598710.140919  
Iteration 6, inertia 5946291825.920057  
Iteration 7, inertia 5946193609.350716

Iteration 8, inertia 5946185599.385245  
Converged at iteration 8: strict convergence.  
Initialization complete  
Iteration 0, inertia 12207746837.0  
Iteration 1, inertia 6079970097.079408  
Iteration 2, inertia 5954393164.363008  
Iteration 3, inertia 5947937328.091304  
Iteration 4, inertia 5946857262.254427  
Iteration 5, inertia 5946568492.25512  
Iteration 6, inertia 5946474702.761869  
Iteration 7, inertia 5946366132.679892  
Iteration 8, inertia 5946295921.45052  
Iteration 9, inertia 5946207359.724583  
Iteration 10, inertia 5946185599.385244  
Converged at iteration 10: strict convergence.  
Initialization complete  
Iteration 0, inertia 11383505986.0  
Iteration 1, inertia 6139879378.911595  
Iteration 2, inertia 6021493514.636542  
Iteration 3, inertia 5968082338.467373  
Iteration 4, inertia 5953106816.142641  
Iteration 5, inertia 5948104276.698193  
Iteration 6, inertia 5946824660.984224  
Iteration 7, inertia 5946351256.172853  
Iteration 8, inertia 5946206130.517454  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11788417070.0  
Iteration 1, inertia 6046337455.612163  
Iteration 2, inertia 5974083654.763396  
Iteration 3, inertia 5956598586.826509  
Iteration 4, inertia 5949331028.379788  
Iteration 5, inertia 5947333956.037614  
Iteration 6, inertia 5946598710.140919  
Iteration 7, inertia 5946291825.920057  
Iteration 8, inertia 5946193609.350716  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 10981022764.0  
Iteration 1, inertia 6051199687.756385  
Iteration 2, inertia 5952363966.434395  
Iteration 3, inertia 5947042269.140477  
Iteration 4, inertia 5946302196.209322  
Iteration 5, inertia 5946207359.724583  
Iteration 6, inertia 5946185599.385246  
Converged at iteration 6: strict convergence.  
Initialization complete  
Iteration 0, inertia 10744033900.0  
Iteration 1, inertia 5952962767.447474  
Iteration 2, inertia 5947588195.067987  
Iteration 3, inertia 5946732059.730334  
Iteration 4, inertia 5946552591.978764  
Iteration 5, inertia 5946454819.509368  
Iteration 6, inertia 5946366132.679892  
Iteration 7, inertia 5946295921.4505205  
Iteration 8, inertia 5946207359.724583  
Iteration 9, inertia 5946185599.385244  
Converged at iteration 9: strict convergence.  
Initialization complete  
Iteration 0, inertia 11928060530.0

```
Iteration 1, inertia 6064271793.324489
Iteration 2, inertia 5965547916.897862
Iteration 3, inertia 5948945546.284864
Iteration 4, inertia 5946599019.810866
Iteration 5, inertia 5946291825.920057
Iteration 6, inertia 5946193609.350716
Iteration 7, inertia 5946185599.385244
Converged at iteration 7: strict convergence.
Initialization complete
Iteration 0, inertia 10628618600.0
Iteration 1, inertia 5965292025.609367
Iteration 2, inertia 5953322722.938694
Iteration 3, inertia 5948296177.143916
Iteration 4, inertia 5946868948.924838
Iteration 5, inertia 5946351256.172853
Iteration 6, inertia 5946206130.517454
Iteration 7, inertia 5946185599.385246
Converged at iteration 7: strict convergence.
Initialization complete
Iteration 0, inertia 11360580040.0
Iteration 1, inertia 6159377347.462243
Iteration 2, inertia 6028523726.990302
Iteration 3, inertia 5973583076.675297
Iteration 4, inertia 5955300718.868937
Iteration 5, inertia 5949043691.257874
Iteration 6, inertia 5947266047.162573
Iteration 7, inertia 5946559502.414055
Iteration 8, inertia 5946256719.009511
Iteration 9, inertia 5946185599.385246
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 11906485472.0
Iteration 1, inertia 6365662127.358145
Iteration 2, inertia 6143854191.542116
Iteration 3, inertia 6016907424.105479
Iteration 4, inertia 5969662244.772013
Iteration 5, inertia 5955291419.566433
Iteration 6, inertia 5948989880.147812
Iteration 7, inertia 5947185497.878584
Iteration 8, inertia 5946475375.774194
Iteration 9, inertia 5946256719.009511
Iteration 10, inertia 5946185599.385244
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 12511199778.0
Iteration 1, inertia 5983531944.924501
Iteration 2, inertia 5947683407.425686
Iteration 3, inertia 5946502226.722412
Iteration 4, inertia 5946369610.190655
Iteration 5, inertia 5946268777.538702
Iteration 6, inertia 5946207359.724583
Iteration 7, inertia 5946185599.385244
Converged at iteration 7: strict convergence.
```

Out[ ]: <matplotlib.image.AxesImage at 0x1a662fa36a0>



```
In [ ]: from sklearn import metrics

pred_lab = kmeans.predict(data)
# print(pred_lab)
print('performance=' ,metrics.homogeneity_score(pred_lab, y))

performance= 0.7387783429447129
```

Use the GMM clustering algorithm from the last lab to form the clusters

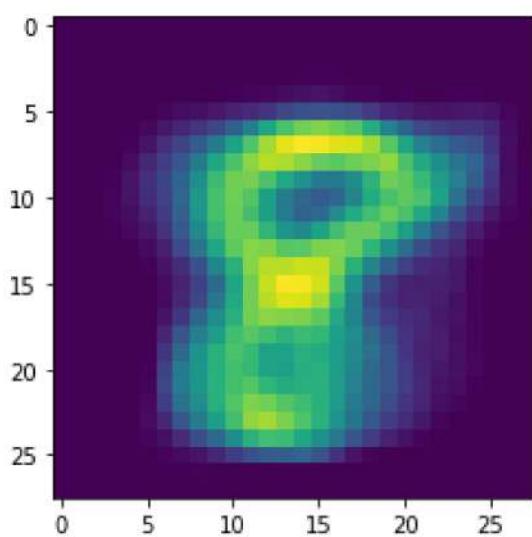
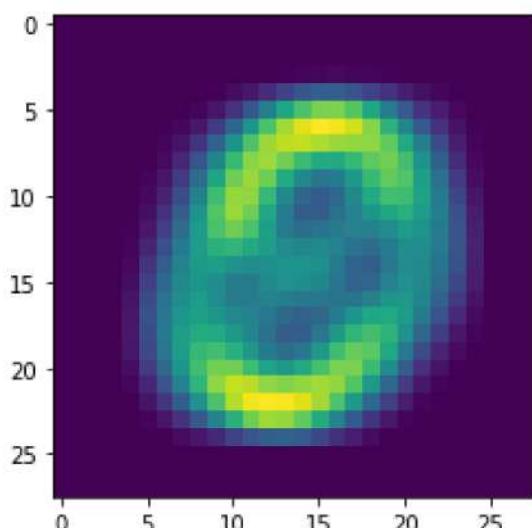
```
In [ ]: ## Write your code here
from sklearn.mixture import GaussianMixture
gauss = GaussianMixture(n_components=2, init_params='kmeans', covariance_type='diag')
gauss.fit(data)
gauss.get_params(deep=True)

means = gauss.means_
vars = gauss.covariances_
weights = gauss.weights_

im=np.reshape(means[0,:],(28,28))
plt.figure()
plt.imshow(im)

im=np.reshape(means[1,:],(28,28))
plt.figure()
plt.imshow(im)
```

```
Initialization 0
Iteration 10
Initialization converged: True
Out[ ]: <matplotlib.image.AxesImage at 0x1a663085c10>
```



```
In [ ]: pred_lab = gauss.predict(data)
print(pred_lab)
print('performance=' ,metrics.homogeneity_score(pred_lab, y))
```

```
[0 0 0 ... 0 0 0]
performance= 0.1213186660952875
```