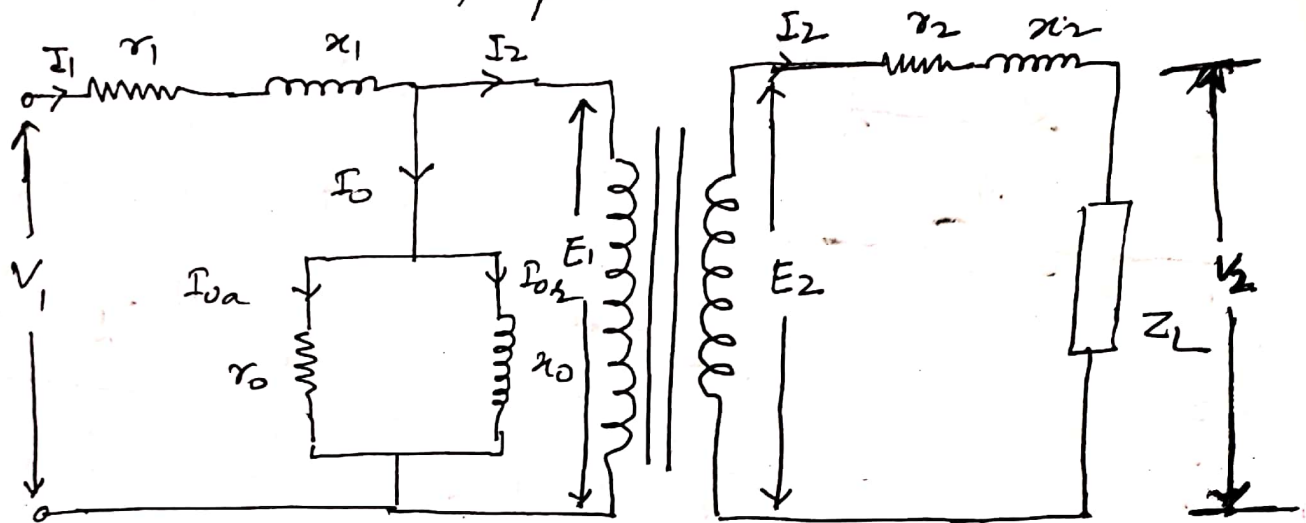


(Exact Equivalent circuit of π/ϕ^2)



$$\bar{I}_1 = \bar{I}_0 + \bar{I}'_2$$

$$\bar{I}_0 = \bar{I}_{0a} + \bar{I}_{0z}$$

I_{0a} = Active Component of $I_0 = I_0 \cos \phi_0$

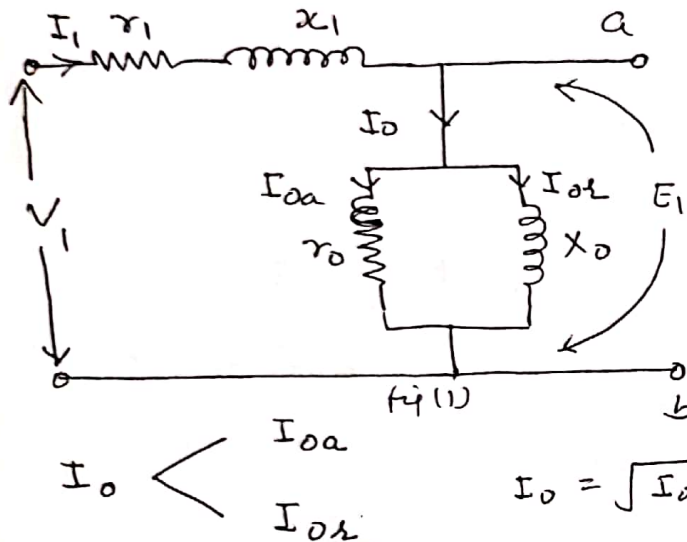
I_{0z} = Reactive Component of $I_0 = I_0 \sin \phi_0$

$$r_0 = \frac{V_1}{I_{0a}}$$

Equivalent ckt of a transformer:

Primary side Secondary side

Primary side:



r_1 = resistance of primary winding

x_1 = leakage reactance of primary winding

$$P_i = V_1 I_0 \cos \phi_0$$

I_{0a} = Active component of no load current

$$= I_0 \cos \phi_0$$

$\cos \phi_0$ = no load power factor

I_{0r} = Reactive component of no load current (magnetizing current)

$$= I_0 \sin \phi_0$$

$$P_i \text{ (Watt loss)} = I_{0a}^2 R_0$$

R_0 = fictitious resistance considered to take into account iron losses of transformer

x_0 = magnetizing reactance.

$$V_1 = I_1 (r_1 + jx_1) + E_1 \quad \text{--- (1)}$$

$$R_0 = \frac{V_1}{I_{0a}}$$

$$x_0 = \frac{V_1}{I_{0r}}$$

Notes →

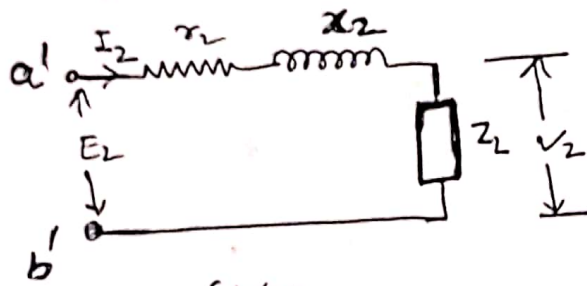
* I_{0a} & I_{0r} are voltage dependent, therefore parameters R_0 & x_0 are connected in parallel.

r_1 & x_1 are current dependent, therefore

these parameters are connected in series

* The parameters of magnetizing branch I_0 , I_{0a} , I_{0r} , R_0 , x_0 are dependent upon primary voltage applied V_1 . These parameters are independent of the load current at t/f. Therefore the magnetizing branch is shown in || to applied voltage V_1 .

Secondary side:



$Z_L = \text{load impedance}$

Fig (2)

$$E_2 = V_2 + I_2(r_2 + jx_2) \quad \text{--- (2)}$$

$$V_2 = I_2 Z_L$$

Joining of primary & secondary equivalent ccts:

In order to find a single equivalent ckt of transformer, equivalent ccts of Fig (1) & (2) should be joined together. They can only be done when voltage b/w a, b & a', b' are equal.

In order to join the two equivalent ccts, ~~the actual secondary ckt is replaced by an~~ equivalent secondary ckt such that the values E_2' , I_2' , r_2' , x_2' , V_2' and Z_L' are called referred values to primary side.

The new secondary equivalent ckt is

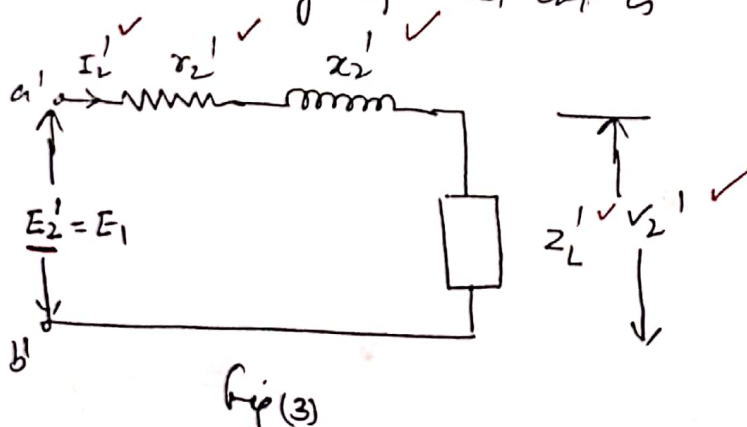
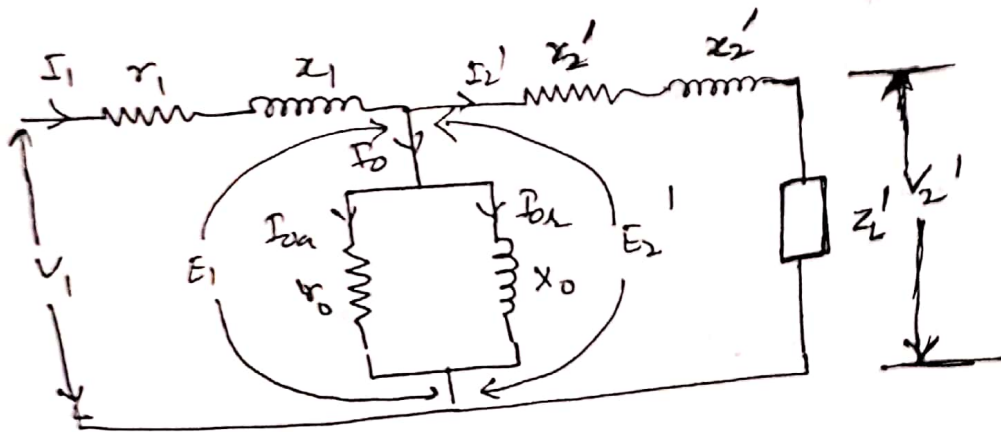


Fig (3)

$$E_2' = V_2' + I_2'(r_2' + jx_2')$$

$$V_2' = I_2' Z_L'$$

the voltage b/w a & b of fig (1) & a' & b' of fig (2) are equal.
Therefore equivalent ckt can be joined together.



Exact Equivalent ckt of transformer.

① Referring secondary values to primary side:

VA → Same $E_1 I_1 = E_2 I_2$ ① (volt-ampere should be same)
VA

$$\frac{I_2'}{I_2} = \frac{E_2}{E_1}$$

$$I_2' = \frac{E_2}{E_1} I_2$$

$$I_2' = \frac{1}{k} I_2$$

$$E_2 = E_1$$

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} = k$$

② Load-volt ampere should be same.

Load VA → Same $V_2' I_2' = V_2 I_2 \Rightarrow V_2' = \frac{I_2}{I_2'} V_2$

$$V_2' = \frac{V_2}{I_2'} I_2$$

$$= \frac{V_2}{k} \quad \checkmark$$

$$V_2' = k V_2$$

$$V_2' = \frac{I_2}{I_1} V_2$$

$$\frac{I_2}{I_1} = a = k \quad \checkmark$$

$$\frac{I_2}{I_2'} = a$$

$$I_1 \sim I_2'$$

$$\frac{I_1}{I_2} = k$$

③ Copper loss should be same.

(P_{cu}) → Same $I_2'^2 r_2' = I_2^2 r_2$

$$r_2' = \frac{I_2^2}{I_2'^2} r_2 = \frac{I_2^2}{I_1^2} k^2 r_2$$

$$r_2' = k^2 r_2 \quad \checkmark$$

$$r_2' = \left(\frac{I_2}{I_1}\right)^2 r_2$$

$$r_2' = \left(\frac{I_2}{I_1}\right)^2 r_2$$

④ Per unit leakage reactance ^{ref} should be same.

$$\frac{I_2' X_2'}{E_2'} = \frac{I_2 X_2}{E_2}$$

$$X_2' = \frac{I_2 X_2}{E_2} \cdot \frac{E_2'}{I_2'}$$

$$= \frac{I_2 X_2}{E_2} \cdot \frac{E_2'}{I_2} \cdot k \quad E_2' = E_1$$

$$X_2' = k^2 X_2$$

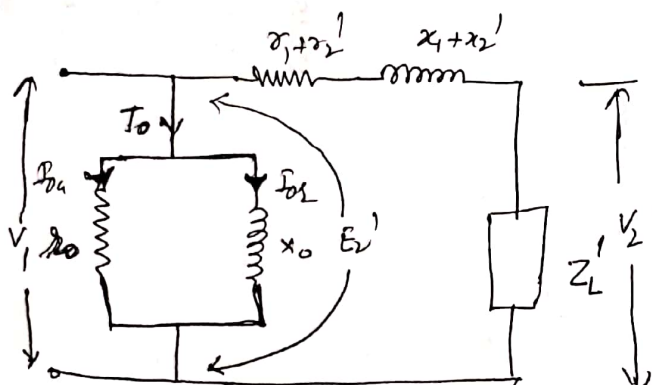
$$= \frac{E_2'}{E_2} \cdot \frac{I_2}{I_1}$$

$$= \frac{E_1}{E_2} \cdot \frac{I_2}{I_1} \cdot E_2$$

$$= \frac{1}{k} \cdot \frac{1}{k} E_2$$

$$X_2' = \frac{1}{k^2} X_2$$

In order to simplify the calculations, approximate equivalent ckt:



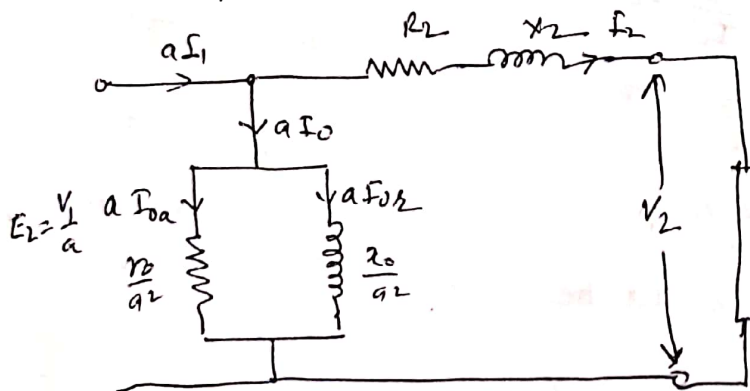
$$R_{01} = r_1 + r_2' \quad r_2' = k^2 r_2$$

$$R_{01} = r_1 + k^2 r_2$$

$$X_{01} = x_1 + x_2' \quad x_2' = k^2 x_2$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

Approximate equivalent circuit referred to the secondary side



$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{I_1}{I_2} = \frac{1}{a} \quad \text{or } k$$

$$R_{02} = r_2 + \frac{r_1}{a^2}$$

$$X_{02} = x_2 + \frac{x_1}{a^2}$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

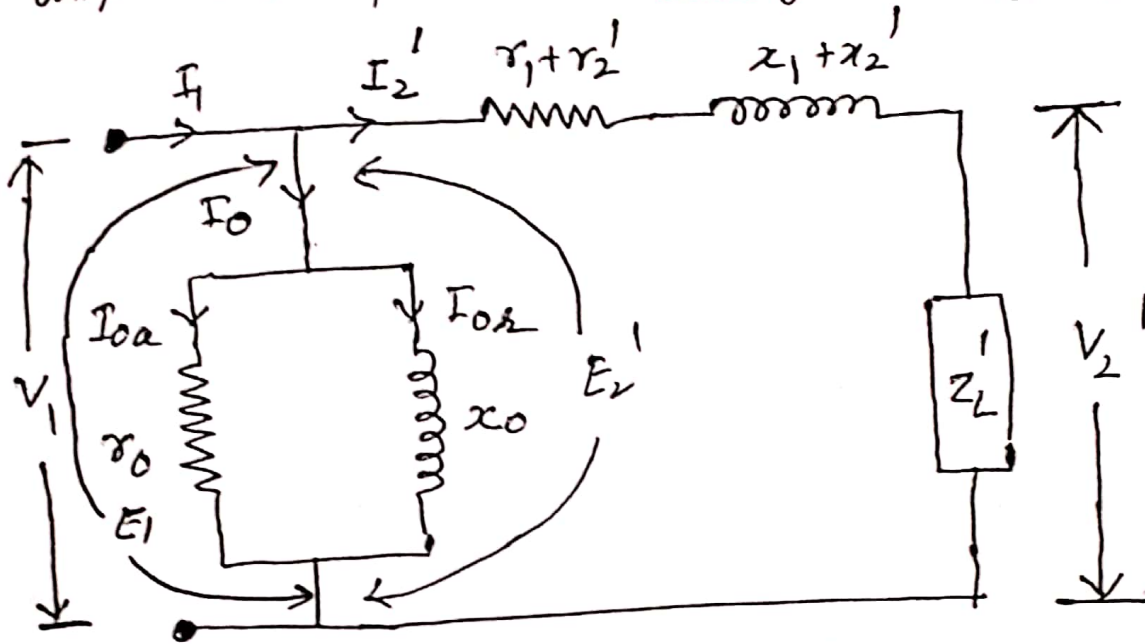
$$R_2 = r_2 + \frac{r_1}{a^2} \quad X_2 = x_2 + \frac{x_1}{a^2}$$

$$Z_L = R_2 + jX_2$$

$$I_2 = \frac{V_2}{Z_L} = \frac{E_2}{Z_2 + Z_L}$$

$$\frac{V_1}{a} = E_2 = V_2 + I_2 Z_2$$

* Approximate equivalent ckt of transformers when
 as parameters referred to primary side. $\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} = k$



$$R_{eq1} = r_1 + r_2' \quad ; \quad r_2' = k^2 r_2$$

$$X_{eq1} = x_1 + x_2' \quad ; \quad x_2' = k^2 x_2$$

$$Z_{eq1} = \sqrt{R_{eq1}^2 + X_{eq1}^2}$$

$$I_2' = \frac{I_2}{k}$$

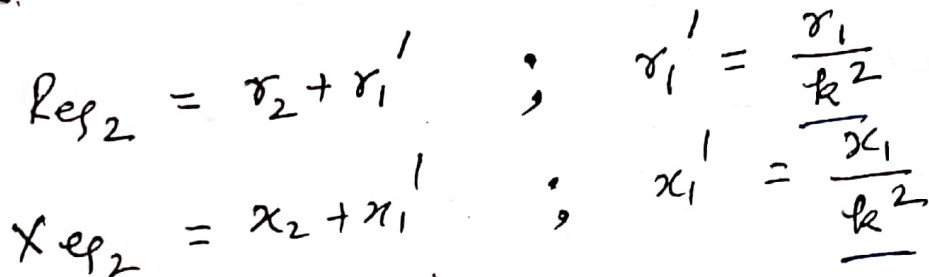
$$I \propto \frac{1}{V} \rightarrow k \quad \checkmark$$

$$V_2' = k V_2 \text{ at (f.s.d.)}$$

$$V_2' = I_2' Z_L'$$

or Equivalent resistance
 $R_{eq1} =$ Total resistance as referred to primary side
or Equivalent leakage reactance
 $X_{eq1} =$ Total leakage reactance as referred to primary side

* In order to simplify calculations.

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} = k$$


$$k_{e2} = \frac{\text{Total resistance as referred to secondary side}}{\text{Total leakage reactance as referred to secondary side}}$$

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