2d Transform

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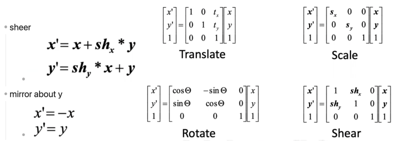

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

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  - u rotation eigenvectors of AA^t
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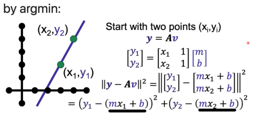
  - by argmin:



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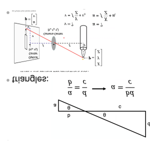
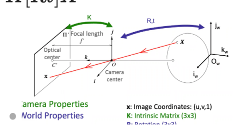
$K[Rt]X$





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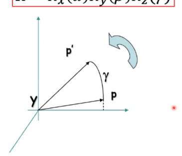
Rotation around the coordinate axes, counter-clockwise:

$\boxed{R = R_x(\alpha) R_y(\beta) R_z(\gamma)}$



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5 6

$$w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Stereo: given 2 cameras/correspondence

- find: world coordinate for known point

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baseline connects both origins

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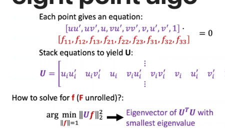
eight point algo



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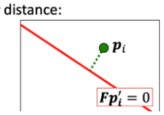
Minimizing via $U^T U$ minimizes sum of squared *algebraic* distances between points p_i and epipolar lines Fp'_i (or points p'_i and epipolar lines $F^T p_i$):

$$\sum_i (p_i^T F p'_i)^2$$

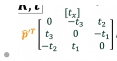
May want to minimize *geometric* distance:

$$\sum_i d(p_i, Fp'_i)^2 + d(p'_i, F^T p_i)^2$$



Calibrated case

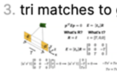
- know intrinsic & extrinsic -> set coord to cam 1
  - $M_1 = K[I, 0], M_2 = K'[R, t]$, M produce p
- $K^{-1}p$ producing normailized coordinate p^ with K as identity matrix (canonical view)
- coplanar: Rp^, t, p^ -> $\hat{p}^T (t \times R\hat{p}) = 0$



  - $x^T E y = 0$ E is t_x: essential matrix
    - bc of p^ being normalized
    - $\hat{p}'^T E \hat{p} = 0$ E = $[t_x]R$
  - Ep^ gives eq for ep line for o'
  - E^Tp^ gives ep in o
  - epipoles in nullspace of E: $E^T \hat{e}' = 0$ and $E\hat{e} = 0$
- Set: $F = K^{-T} E K'^{-1}$ Then: $p^T F p' = 0$ Fp F^tp' are ep lines w p, p'

- F is estimate known as weak calibration
- E = $E = K'^T F K$
- from E calc relative rotation/translation
- scene point z direction
- disparity: x - x', inversely proportional to depth For each pixel

1. find coreresponding ep line in right
2. search along and find best match (SSD)
3. tri matches to get depth

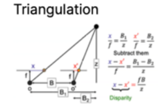


y points are the same

Stereo image rectificiation

- create virtual planes that only differ by translation
- → pixel motion is horiz → 2 homo for each img
- distance min, similarity max
  - sum(l - r)^2 vs

Triangulation



z = B*f/(z-x')

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- sheer

$$x' = x + sh_x * y$$
$$y' = sh_y * x + y$$

- mirror about y

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

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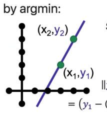

Start with two points (x_i, y_i)

$$y = Av$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix}$$

$$\|y - Av\|^2 = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} mx_1 + b \\ mx_2 + b \end{bmatrix} \right\|^2 = (y_1 - (\underline{mx_1 + b}))^2 + (y_2 - (\underline{mx_2 + b}))^2$$

  - satisfying:

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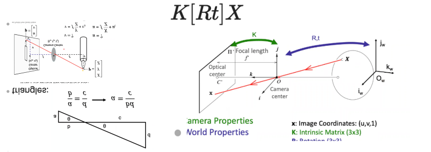

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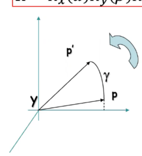

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Each point gives an equation:

$$[uu', uv', u, vu', vv', v, u', v', 1] \cdot [f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33}] = 0$$

Stack equations to yield **U**:

$$U = \begin{bmatrix} & & & & \vdots & & & & \\ u_i u_i' & u_i v_i' & u_i & v_i u_i' & v_i v_i' & v_i & u_i' & v_i' & 1 \\ & & & & \vdots & & & & \end{bmatrix}$$

How to solve for **f** (**F** unrolled)?:

$$\arg\min_{\|f\|=1} \|Uf\|_2^2 \rightarrow$$ Eigenvector of $U^T U$ with smallest eigenvalue

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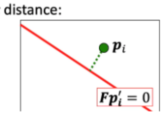
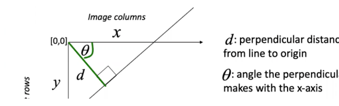


Process:

- y=mx + b -> point based on parameters m and b
- point in x,y -> b = -xm + y (line in hough space)
- intersection of lines in hough space = line that passes through two coordinates in image space -> quantize and bin each grid spot for votes
- polar representation:



$$x\cos(\theta) + y\sin(\theta) = d$$

- point in img -> sinusoid segment in hough

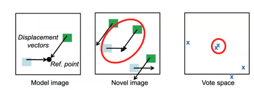
Algorithm

1. $H[d, (\theta)] = 0$$
2. for each edge point I[x,y]
   1. for theta = $[\theta_{min} to \theta_{max}]$
   2. theta = graident at x,y
   3. $x\cos(\theta) + y\sin(\theta) = d$
   4. $H[d, (\theta)] += 1$
3. find (d, theta) where H[] is max
4. detect line given by d

Tuning

- same d and theta can't vote for multiple lines
- still see peaks in noise voting space -> gradients & orientation
  - or give more votes to stronger edges
  - or smapling of d, theta to give more or less res

- now cirlces instead of lines intersecting -> intersecting circles centered at points of hough space -> on edges of circle in img
- unknown radius = cone in hough space
  - can use gradient angle of img space -> hough space line ray



- offline procedure: at each boundary point find r = a - p_{i} a is potential center, r vote
  - dispaclement vectors in table indexed by gradient orientation
  - look for overlaps in r