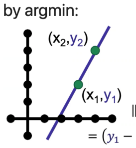
2d Transform

- Method: finding correspondences
- transformations: translation, rotation, aspect, affine (change shape), perspective
  - p' = T(p) = Mp
  - when p is in R^2 -> 2x2 M
- uniform scaling: same scalar for all components
- non uniform -> new aspect ratio
- sheer
  - $x' = x + sh_x * y$
  - $y' = sh_y * x + y$
- mirror about y
  - $x' = -x$
  - $y' = y$
- parellel lines remain parallel
- feature based alignment
- eigenvectors are orthonormal
  - symmetric A eigenvector w largest λ max $\frac{x^TAx}{x^Tx}$ unit vectors max num
- non symmetric -> sVD: U∑VT^
  - u rotation eigenvectors of AA^t
  - σ sqrt of A^tA λ's

$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix}=\begin{bmatrix}a&b&c\\d&e&f\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\w\end{bmatrix}$$

- solving least squares invert A find v (m,b) point -> minimizing error
  - by argmin:



Start with two points (x_i, y_i)

$$y = Av$$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}x_1&1\\x_2&1\end{bmatrix}\begin{bmatrix}m\\b\end{bmatrix}$$

$$\|y-Av\|^2=\left\|\begin{bmatrix}y_1\\y_2\end{bmatrix}-\begin{bmatrix}mx_1+b\\mx_2+b\end{bmatrix}\right\|^2 = (y_1-(mx_1+b))^2+(y_2-(mx_2+b))^2$$

  - satisfying:

Solution satisfies $(A^TA)v^* = A^Ty$

or

$$v^* = (A^TA)^{-1}A^Ty$$

  - when A is square, rank < rows or rank < cols
  - homo ls -> orthogonal to vectors argmin(||Av||^2) smallest ev from AtA, smallest singular v of A
  - for 2 variables need at least 2 eq

ransac - reduce noise

- not true matches -> outlier (minimising lq creates error)
  - look for inliers

For N times

1. select random seed group s points
   1. more points = more robust
2. compute transformation for seed group
3. find inliers to this transformation
   1. point whose d is < t
4. if large then recomputer estimate M on all of inliers
   1. d > inliers accept and refit using all inliers

$$\begin{bmatrix}x'\\y'\\1\end{bmatrix}=\begin{bmatrix}1&0&t_x\\0&1&t_y\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$

Translate

$$\begin{bmatrix}x'\\y'\\1\end{bmatrix}=\begin{bmatrix}s_x&0&0\\0&s_y&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$

Scale

$$\begin{bmatrix}x'\\y'\\1\end{bmatrix}=\begin{bmatrix}\cos\Theta&-\sin\Theta&0\\\sin\Theta&\cos\Theta&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$

Rotate

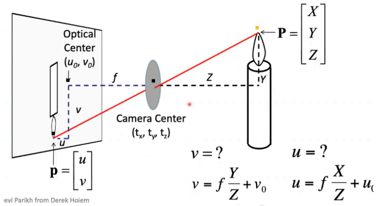
$$\begin{bmatrix}x'\\y'\\1\end{bmatrix}=\begin{bmatrix}1&sh_x&0\\sh_y&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$

Shear

- image formation
  - every point on a tree blends its colors all across the film
  - -> barrier known as aperture (only one makes it through) flipped
  - f = focal length, c = center of camera
    - f distance from pinhole and img
  - lines all to vanishing point
- facts about projection
  - line in 3d -> line in 3d
  - parallel 3d -> loses, but converge to centr point
  - distant objects are smaller
- increase aperture (when too dark)

photometric: type, direction, intensity of light, surfaces reflectance property

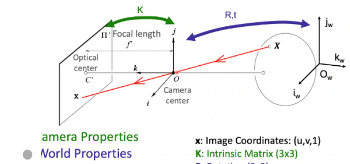
optical: focal length, fov, aperture, shutter speed



$v = ?$ $u = ?$

$$v = f\frac{Y}{Z} + v_0 \qquad u = f\frac{X}{Z} + u_0$$

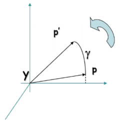
$$K[Rt]X$$



- extrinsic ass: no rotation, camera at 000 4th col
- intrinsic ass: opticalcenter000,unit aspe,!skew
- rotations:

Rotation around the coordinate axes, counter-clockwise:

$$R = R_x(\alpha)R_y(\beta)R_z(\gamma)$$



$$R_x(\alpha)=\begin{bmatrix}1&0&0\\0&\cos\alpha&-\sin\alpha\\0&\sin\alpha&\cos\alpha\end{bmatrix}$$

$$R_y(\beta)=\begin{bmatrix}\cos\beta&0&\sin\beta\\0&1&0\\-\sin\beta&0&\cos\beta\end{bmatrix}$$

$$R_z(\gamma)=\begin{bmatrix}\cos\gamma&-\sin\gamma&0\\\sin\gamma&\cos\gamma&0\\0&0&1\end{bmatrix}$$

5 6

$$w\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}\alpha&s&u_0\\0&\beta&v_0\\0&0&1\end{bmatrix}\begin{bmatrix}r_{11}&r_{12}&r_{13}&t_x\\r_{21}&r_{22}&r_{23}&t_y\\r_{31}&r_{32}&r_{33}&t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Stereo: given 2 cameras/correspondence

- find: world coordinate for known point

Epipolar Geometry

baseline connects both origins

- epioles where baseline intersects w img planes
- epiploar lines btw point and e -> p' (must be from same light ray)
  - rays given by ep line
- only translation -> ep lines horizontal, e ∞
- forward motion -> ep lines out from optical cent

Calibrated case

- know intrinsic & extrinsic -> set coord to cam 1
  - $M_1 = K[I,0]$, $M_2 = K'[R,t]$, M produce p
- $K^{-1}p$ producing normailized coordinate p^ with K as identity matrix (canonical view)
- coplanar: Rp^, t, p^ -> $\hat{p}'^T(t \times R\hat{p}) = 0$

$$\hat{p}'^T\begin{bmatrix}0&-t_3&t_2\\t_3&0&-t_1\\-t_2&t_1&0\end{bmatrix}R\hat{p}=0$$

  - $x^TEy = 0$ E is t_x: essential matrix
    - bc of p^ being normalized
    - $\hat{p}'^TE\hat{p} = 0$ E = $[t_x]R$
  - Ep^ gives eq for ep line for o'
  - E^Tp^ gives ep in o
  - epipoles in nullspace of E: $E^T\vec{e}' = 0$ and $E\vec{e} = 0$
- Set: $F = K'^{-T}EK^{-1}$ Then: $p'^TFp = 0$ Fp F^tp' are ep lines w p, p'

Each point gives an equation:

$$[uu', uv', u, vu', vv', v, u', v', 1] \cdot [f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33}] = 0$$

Stack equations to yield U:

$$U = \begin{bmatrix}u_iu_i' & u_iv_i' & u_i & v_iu_i' & v_iv_i' & v_i & u_i' & v_i' & 1\end{bmatrix}$$

How to solve for f (F unrolled)?:

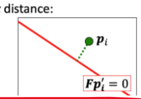
$$\arg\min_{\|f\|=1} \|Uf\|_2^2$$ → Eigenvector of U^TU with smallest eigenvalue

Minimizing via U^TU minimizes sum of squared *algebraic* distances between points p_i and epipolar lines Fp'_i (or points p'_i and epipolar lines F^Tp_i):

$$\sum_i (p_i^T F p_i')^2$$

May want to minimize *geometric* distance:

$$\sum_i d(p_i, Fp'_i)^2 + d(p'_i, F^Tp_i)^2$$



- old*
  - linear independence
  - span: all linear combiantions of set of vectors
  - Ax output is constrained to span of cols of A
  - full rank: one-to-one mapping -> invertible
- Symmetric $A = X^TX$
- rotation matrix: all row/col have l2 norm, lin ind, transpose is inverse, det = 1, eigenvlaue = 1
- eigensystems: lagest eigenvector = vector with largest value

Approximating Laplacian

$$L = \sigma^2\left(G_{xx}(x,y,\sigma) + G_{yy}(x,y,\sigma)\right)$$

(Laplacian)

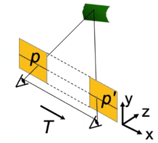
$$DoG = G(x,y,k\sigma) - G(x,y,\sigma)$$

(Difference of Gaussians)

- F is estimate known as weak calibration
- E = $E = K'^TFK$
- from E calc relative rotation/translation
- scene point z direction
- disparity: x - x', inversely proportional to depth

For each pixel

1. find coreresponding ep line in right
2. search along and find best match (SSD)
3. tri matches to get depth



$p'^TEp = 0$ $E = [t_x]R$

What's R? $R = I$

What's t? $t = [T, 0, 0]$

$$E = [t_x]R = \begin{bmatrix}0&0&0\\0&0&-T\\0&T&0\end{bmatrix}$$

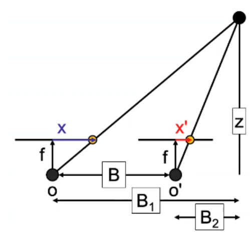
$$[u' \; v' \; 1]\begin{bmatrix}0&0&0\\0&0&-T\\0&T&0\end{bmatrix}\begin{bmatrix}u\\v\\1\end{bmatrix} = 0 \rightarrow [u' \; v' \; 1]\begin{bmatrix}0\\-T\\Tv\end{bmatrix} = 0 \rightarrow -Tv' + Tv = 0 \quad Tv = Tv'$$

y points are the same

Stereo image rectificiation

- create virtual planes that only differ by translation
- -> pixel motion is horiz -> 2 homo for each img
- distance min, similarity max
  - sum(l - r)^2 vs

Triangulation



$$\frac{x}{f} = \frac{B_1}{z} \quad \frac{x'}{f} = \frac{B_2}{z}$$

Subtract them

$$\frac{x - x'}{f} = \frac{B_1 - B_2}{z}$$

$$x - x' = \frac{fB}{z}$$

Disparity

$$z = Bf/(z-x')$$

- associate pateches with object of interest
- frames of a single shot, faces, background vs what is moving

Gestalt: symmetry, similarity, common fate (coherent motion), proximity, subjective contors (why shape exists)

- whole is other than sum of its parts

segmentation: separate into coherent objects

- top down: pixels same object -> group
- bot up: pixels look similar -> group
- superpixels: similar looking pixels

i. want to find centers
- minimize SSD between all points in cluster

ii. methods
- k means: randomly initilize k clusters -> for each point find closest c -> given points avg for c -> if c changed iterate again (without random)
  - gives local min (converges reasonably in time), sensitive to start/#initial clusters(must cluster all points(outliers suck), detects spherical clusters
- Feature space
  - intensity+position or color
- normalized cuts: have similar appearance forming parts of an object
  - build graph node for every pixel -> every edge has weight which is similarity (affinity)

Line fitting

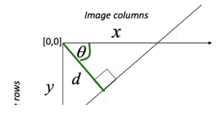
- why fit lines: many objects characterized by presence of straight lines
- problem: extra edge points clutter it, missing lines, noise in edge points

Voting

- all local features vote for all models compatible with it (look at parameters with lots of votes) - noise will make votes but will be as outlier
  - for every edge point
- look for lines that get many votes

Process:

- y=mx + b -> point basd on parameters m and b
- point in x,y -> b = -xm + y (line in hough space)
- intersection of lines in hough space = line that passes through two coordinates in image space -> quantize and bin each grid spot for votes
- polar representation:



d: perpendicular distance from line to origin

θ: angle the perpendicular makes with the x-axis

$$x\cos(\theta) + y\sin(\theta) = d$$

- point in img -> sinusoid segment in hough

Algorithm

1. $H[d, (\theta)] = 0\$$
2. for each edge point I[x,y]
   1. for theta = $[\theta_{min} to \theta_{max}]$
   2. theta = graident at x,y
   3. $x\cos(\theta) + y\sin(\theta) = d$
   4. $H[d, (\theta)] += 1$
3. find (d, theta) where H[] is max
4. detect line given by d

- same d and theta can't vote for multiple lines
- still see peaks in noise voting space -> gradients & orientation
  - or give more votes to stronger edges
  - or smapling of d, theta to give more or less res

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- now circles instead of lines intersecting -> intersecting circles centered at points of hough space -> on edges of circle in img
- unknown radius = cone in hough space
  - can use gradient angle of img space -> hough space line ray
- for circle:

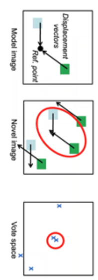
```
For every edge pixel (x,y) :
  For each possible radius value r:
    For each possible gradient direction θ:
      // or use estimated gradient at (x,y)
      a = x – r cos(θ) // column
      b = y – r sin(θ) // row
      H[a,b,r] += 1
    end
  end
```

- minimize irrelevant tokens first
- choose a good grid/ discretization
  - too fine<---->too course
  - voting for nearby (smoothing in acc array)
- use direction of edge to reduce para by 1

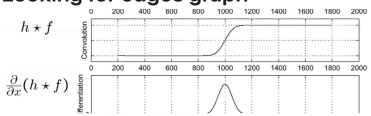
Pros: all points are processed ind, don't need all points on line, noise points unlikely to contribute big to bin, detect multiple instances of a model in single pass

Cons: more complex shapes for each parameter, non-target shapes offset, quantizations of grid need to bet specified

- offline procedure: at each boundary point find r = a - b_{i} a is potential center, r vote
  - dispaclement vectors in table indexed by gradient orientation
  - look for overlaps in r
- not rotation invariant



Looking for edges graph

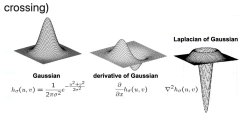


Gaussian derivative (speed things up):

- don't have to differentiate whole thing -diff gausssian * f

- x high -> low
- y low -> high

Take laplacian of gaussian (2nd derivative - zero crossing)



- more sigma smooths it out over wider -> less edges in more detailed areas

1. smoothing
2. edge enhancement

1&2 can be done through derivative of gaussian

Canny edge detection

3. edge localization (threshold) -> binary image
    1. pixel less than t set to 0
    2. others to 1
- non max suppression edge width-> single pixel
    - specific location by gradient direction
- low and high threshold (high starts low finishes edge curves)

Derivatives

- smoothing *negative signs* used to get high response in regions of high contrast
- sum to 0 -> no response in constant regions
- high abs at points of high contrast

edge strength = gradient magnitude

- choose min seam

$$\mathbf{M}(i,j) = Energy(i,j) + \min(\mathbf{M}(i-1,j-1), \mathbf{M}(i-1,j), \mathbf{M}(i-1,j+1))$$

Harris Corner Detection

Need:

- repeatability: despite geometric transformations - correspondence
- Saliency: small distinctive subset
- compactness/opt: fewer features than pixels
- locality: small area of image (no cluter)

How:

- eigenvalues give us basis vector
    - eigenvalue >> other -> edge
- rotation invariant: by eigen to find new basis rotation
- scale invariant: no -> yes
    - choosing window size: expand radius -> get most activation for each pixel

1. cornerness: M matrix
2. threshhold: keep high
3. local filter for most cornerlike
    1. non-max suppression

Formulas:
R = det(M) - alphatrace(m)^2
R small: flat, R > 0: corner,

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Blob destection

- edge = ripple, (2nd of guass) -> blob is is where ripples dip lower
    - mag of laplacian will provide scale of blob - smaller laplacian dip wants smaller blob -> (characteristic scale)

trying to do multiple scales

- simple descriptors: raw pixel vectorized (highly sensitive to noise/shifintg) -> SIFT
- Histograms to bin pixels sub-patches accoridng to thier graident orientation (don't vectorize)
    - mag: old determines weight of histo
    - angle: tan
- compute histograms for different patches

Oreint patch by max weight and make this the 0

- less bins -> less orientations = less info
- more bins -> too much info

partially invariant to

- illumination changes
- camera viewpoint
- clutter

$$M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$$

- find patches that have most similar (lowest SSD)
    - **(Xi – Xj)2 + (Yi – Yj)2** for every distinct pair **(i, j)**
- robustness: distance to best match/distance to second best match
    - =1 is ambiguous
    - lowest: first match looks good
- hypothesize tranformation -> apply trans and see if more match

Clustering algorithms

i. want to find centers

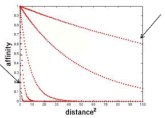
- minimize SSD between all points in cluster

ii. methods

- k means: randomly initilize k clusters -> for each point find closet c -> given points avg for c -> if c changed iterate again (without random)
    - gives local min (converges reasonably in time), sensitive to start/#initial clusters/must cluster all points(outliers suck), detects spherical clusters
- Feature space
    - intensity+position or color
- normalzied cuts: have similar appearance forming parts of an object
    - build graph node for every pixel -> every edge has weight which is similarity (affinity)

$$aff(x,y) = \exp\left\{-\left(\frac{1}{2\sigma_d^2}\right)\|x-y\|^2\right\}$$

Small sigma: group only nearby points



Large sigma: group distant points

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

- exp(-1/(2sigma^2) * ||xi - xj||^2)
- delete links that cross btw segments
    - cut low affinities
    - min cut (summing up all the weights taht you cut) - removal makes graph disconnected
- generalized eigenvalue problem
- pro: does not require model fo data distribution, flexible choice of affinities
- cons: comp high, dense, preference fore balanced partitions (equal weights - or else could favor really small clutsters ex: small object on big background)

Sampling

- sample the 2d space on grid -> quantize each sample (int)
    - one value per pixel
- sample across R, G, B (can be avged together for one image)

Filter (denoise, resize, extract texture, edges, detect patterns)

- enhance image - denoise
    - raw pixel of same image won't be same
- Salt and pepper: white pixels - guassian noise sample from guassian N(u, sigma) more var = more noise

Reducing noise: average of neighbors - expect neighbors to be similar [1, 1, 1, 1, 1]/5 (uniform) [1, 4, 5, 4, 1]/16

H = 1/9 [matrix of 1's]

Correlation filtering: G[i,j] = 1/(2k+1)^2 $\sum_{u=-k}^{k}(\sum_{v=-k}^{k} F[i+u+, j+v])$ G = HxF H is mask produces flipped -> convolution then apply cross (symmetric will output same)

- full = any part of g touches f, same = same size as f, valid = doesn't fall of edge
- boundary (clip filter black, wrap around, copy edge, reflect across edge)
- window size doesn't imapct gaussian -but variance does directly (filter size ~ 6σ) choose window size from this
- remove hf; low pass filter

Runtime: O(n^2m^2) -> 1d G * 1d G = 2d G

O(N) (by seperability outer product)
prop: convolution is linear, can shift

Non-linear filters: median (comp heavy)

low freq: smoothing

high freq: og + (og - smooth = details)

$$Filter_{ij} \propto \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$Filter_{ij} \propto \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right)$$

Prewitt

$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Roberts

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$