Spagetti data model

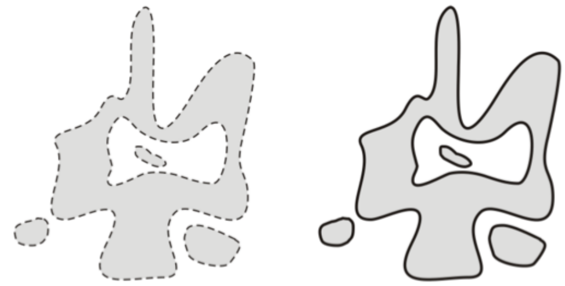
- Polygon with n vertices has $2n$ possible representations
- Polygon and polyline structure is identical
- No guarantee that a polygon/polyline is simple/convex
- Regions may have overlapping/containment relations

Topology: qualitative properties

- **Study of properties of form invariant under continuous deformation**

Topological invariants

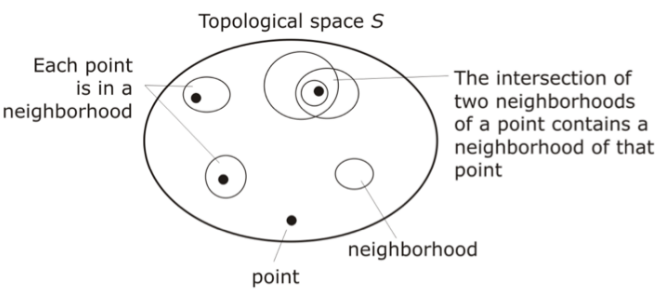
- **Homeomorphism = topological isomorphism = continuous mapping preserving topological properties**
- **Topologically invariant properties: properties preserved by continuous (topological) transformations**



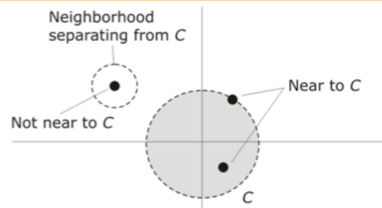
Open set

Closed set

Point-set topology



Open and Closed Sets



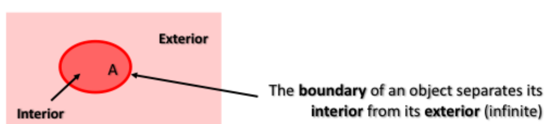
- Let S be a topological space and X be a subset of points of S. An individual point x is near to X if every neighborhood of x contains some point of X.
- X is *open* if every point of X can be surrounded by a neighborhood that is entirely within X.
- X is *closed* if it contains all its near points.

Formalisation of topological relationships

- **To query topological spatial relationships efficiently, a formalism has to be found;**
- **Such a formalism should:**
  - Be efficiently computable and enable reasoning
  - Succinct and expressive
  - Result in equivalent relationships for topologically isomorphic geometries

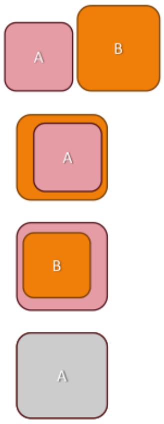
9IM

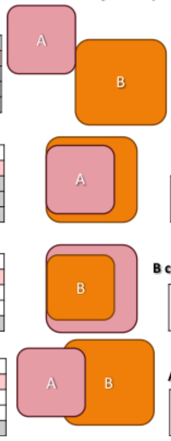
- **Egenhofer et al. worked on a formal model of topological spatial relations in the late 1980s and early 1990s**
- **The 9-intersection model describes binary topological relationships between in terms of *interiors, boundaries and exteriors* for two spatial entities in two dimensions**



Named spatial predicates in 9IM

0 = FALSE, 1 = TRUE (not dimensionality, here)





A disjoint B

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	0	0	1
	Boundary	0	0	1
	Exterior	1	1	1

A meets B

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	0	0	1
	Boundary	0	1	1
	Exterior	1	1	1

A inside B

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	1	1	1
	Boundary	0	0	1
	Exterior	0	0	1

B covers A

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	1	1	1
	Boundary	0	1	1
	Exterior	0	0	1

A contains B

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	1	0	0
	Boundary	1	0	0
	Exterior	1	1	1

B covered by A

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	1	0	0
	Boundary	1	1	0
	Exterior	1	1	1

A equals B

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	1	0	0
	Boundary	0	1	0
	Exterior	0	0	1

A overlaps B

		Entity A		
		Interior	Boundary	Exterior
Entity B	Interior	1	1	1
	Boundary	1	1	1
	Exterior	1	1	1

Masks for 9-IM

$$\mathrm{bin}(\mathrm{DE9IM}(a,b)) = \mathrm{9IM}(a,b) = \begin{bmatrix} a^o \cap b^o \neq \emptyset & a^o \cap \partial b \neq \emptyset & a^o \cap b^e \neq \emptyset \\ \partial a \cap b^o \neq \emptyset & \partial a \cap \partial b \neq \emptyset & \partial a \cap b^e \neq \emptyset \\ a^e \cap b^o \neq \emptyset & a^e \cap \partial b \neq \emptyset & a^e \cap b^e \neq \emptyset \end{bmatrix}$$

Alternative notation:

$\neg\varnothing$

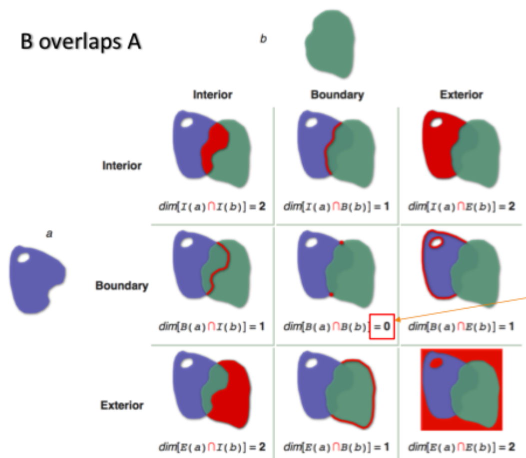
Equals	$\begin{bmatrix} T & * & F \\ * & * & F \\ F & F & * \end{bmatrix}$			
	T*F**FFF*			
Disjoint	$\begin{bmatrix} F & F & * \\ F & F & * \\ * & * & * \end{bmatrix}$			
	$\boxed{\text{FF*FF****}}$			
Touches (meets)	$\begin{bmatrix} F & T & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} F & * & * \\ T & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} F & * & * \\ * & T & * \\ * & * & * \end{bmatrix}$	
	FT*******	F**T*****	F***T****	
Contains	$\begin{bmatrix} T & * & * \\ * & * & * \\ F & F & * \end{bmatrix}$			
	T*****FF*			
Covers	$\begin{bmatrix} T & * & * \\ * & * & * \\ F & F & * \end{bmatrix}$	$\begin{bmatrix} * & T & * \\ * & * & * \\ F & F & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ T & * & * \\ F & F & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & T & * \\ F & F & * \end{bmatrix}$
	T*****FF*	*T****FF*	***T**FF*	****T*FF*

<< Mask string code:
T – TRUE,
F – FALSE,
* - don't care
Or dimensions:
0 – point
1 - line
2 - polygon
* - don't care

```
ST_Relate(geomA, geomB, intersectionMatrixPattern);
```

Dimensionally extended 9-IM

B overlaps A



Note: 9IM (and DE9IM) can be applied to geometries of different dimensionalities (e.g., 1D with 2D: line crosses polygon)

0, but not NULL
This is a dimensionality!

Clementini; Di Felice; van Oosterom, (1993). "A small set of formal topological relationships suitable for end-user interaction". In Abel; Ooi. Advances in Spatial Databases: Third International Symposium, SSD '93 Singapore. Lecture Notes in Computer Science. 692/1993. Springer. pp. 277–295

Topological queries

- Spatial join – two ways to write:
  - `SELECT s.state, r.name FROM SPATIAL.US_STATES AS s, SPATIAL.US_RIVERS as r WHERE ST_WITHIN(r.geom,s.geom)`
  - `SELECT * FROM SPATIAL.US_STATES as s JOIN SPATIAL.US_RIVERS as r ON ST_WITHIN(r.geom,s.geom)`
- Return intersection geometry:
  - `SELECT ST_AsText(ST_Intersection('POINT(0 0)'::geometry, 'LINESTRING ( 0 0, 0 2 )'::geometry));`
- Using Masks
  - **Find mask:** `SELECT ST_Relate(ST_GeometryFromText('POINT(1 2)'), ST_Buffer(ST_GeometryFromText('POINT(1 2)'),2));`
  - **Or test:** `SELECT ST_Relate(ST_GeometryFromText('POINT(1 2)'), ST_Buffer(ST_GeometryFromText('POINT(1 2)'),2), '*FF*FF212');`
  - Using binary predicates (based on envelopes only)
  - http://postgis.net/docs/manual-2.2/reference.html#Operators
  - Faster evaluation, approximate results! (see Lecture on indexes)

Maintaining Topological Relationships

- **Certain entity types may have to comply to specified spatial relationships:**
  - Ship cannot float on land;
  - Tree has to stand in a parcel (not in a lake)
  - Car has to be on the road…
- **We can maintain such relationships using spatial triggers (a trigger is an automatically executed piece of code within a database):**
  - On insert, update;
  - On delete,…

Spatial trigger

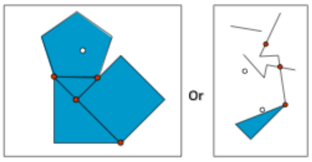
- **Make sure a point is within a polygon**

Persistent topology

- We may want to persist a certain topological relationship between geometries in a database:
  - For (topo)logical integrity assurance (parcels not overlapping, streets connected at junctions…)
  - To increase speed and efficiency of processing (many comparisons need not be re-computed)
  - To reduce storage load (no duplicate storage of shared edges,…)

Topological data model

- Explicitly stores topological relationships of 0-nD geometries
- Topological elements:
  - **Nodes** [point, edge]
  - **Arcs** (edge, if orientation is not important) [node-start, node-end,right-face, left-face]
  - **Faces** (= polygon) [{arcs}]
- Distinguishes:
  - Start and End nodes (for connectivity)
  - Left and Right faces (for adjacency)



Planar partitions of space

- Polygons completely covering space;
- No overlaps, no gaps;
- Some mathematical properties of *finite*, *connected*, *planar* graphs of f faces (n-nodes, e-edges, f-faces):
  - Euler's characteristic:

$$n-e+f=2$$

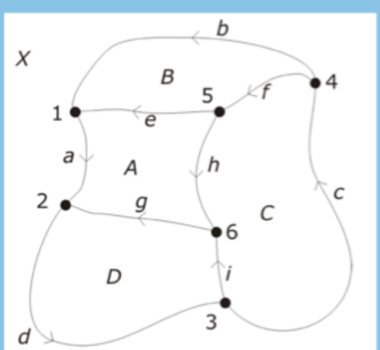
  - Average node *degree* (number of edges incident with a node) <6
  - Note: we will revisit graphs in the lecture on networks

Node – Arc – Area Topology

- Each directed arc has exactly one start and one end node.
- Each node must be the start node or end node (maybe both) of at least one directed arc.
- Each area is bounded by one or more directed arcs.
- Directed arcs may intersect only at their end nodes.
- Each directed arc has exactly one area on its right and one area on its left.
- Each area must be the left area or right area (maybe both) of at least one directed arc.

NAA Planar decomposition

- Node-Arc-Area Topology



Arcs (Edges)

Arc	S_node	E_node	L_Face	R_Face
a	1	2	A	X
b	4	1	B	X
c	3	4	C	X
d	2	3	D	X
e	5	1	A	B
f	4	5	C	B
g	6	2	D	A
h	5	6	C	A
i	3	6	D	C

Nodes: 6
Arcs: 9
Faces: 5

N-A+F= 6-9+5 = 2

NAA Conceptual model

ARC(ARC_ID, BEGIN_NODE, END_NODE, LEFT_AREA, RIGHT_AREA)

POLYGON(AREA_ID, ARC_ID, SEQUENCE_NO)

POLYLINE(ARC_ID, POINT_ID, SEQUENCE_NO)

POINT(POINT_ID, X_COORD,Y_COORD)

NODE(NODE_ID, POINT_ID)

ArcGIS: AAT(FNODE, TNODE, LPOLY, RPOLY, LENGTH, Adm-ID)

43

What to do with topology

- Create topology for cleaning data;
- Create topology before simplification of boundaries of planar decompositions;
- Create topology if neighbourhoods matter (e.g., cadaster)