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Weekly Journal Club

An introduction to inertial navigation

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Journal Club

Journal club介绍与自动驾驶中定位方案相关的论文， 主要关注的方向有：SLAM算法、点云数据的处理和压缩、 特征地图、传感器数据处理和融合、 GNSS信号处理等。我们一直关注领域前沿技术， 选取得到广泛认可的、或者是在我们的实际使用中结果比较好的论文， 与大家分享，共同学习成长。

每周五 北京时间12点

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传统的惯性传感器一般用来捕捉飞机、轮船、航天设备等的姿态。但是现今随着其制造工艺的提高，产生了尺寸更小，重量更轻的微电子系统惯性传感器。这种制造工艺的提高大大扩展了惯性传感器的适用范围。比如，人体动作捕捉，无人机等领域。本周介绍的文章基于捷连式Xsens Mtx惯性导航系统。通过测量和数值模拟，介绍了MEMS系统的基本原理和误差特性。通过分析发现白噪声是导致测量出现漂移的主要原因。

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Outlines

1. Introduction to inertial navigation system and algorithm
2. Analysis of error sources
3. a method of error analsys Allan Variance

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Intertial Navigation

A self-contained navigation technique in which measurements provided by accelerometers and gyroscopes are used to track the position and orientation of an object relative to a known starting point, orientation and velocity. Inertial measurement units (IMUs) typically contain three orthogonal rate-gyroscopes and three orthogonal accelerometers, measuring angular velocity and linear acceleration respectively.

Two categories IMU

Stable Platform Systems---->keeping the platform aligned with the global frame .

Strapdown Systems-------->Output quantities measured in the body frame rather than the global frame.

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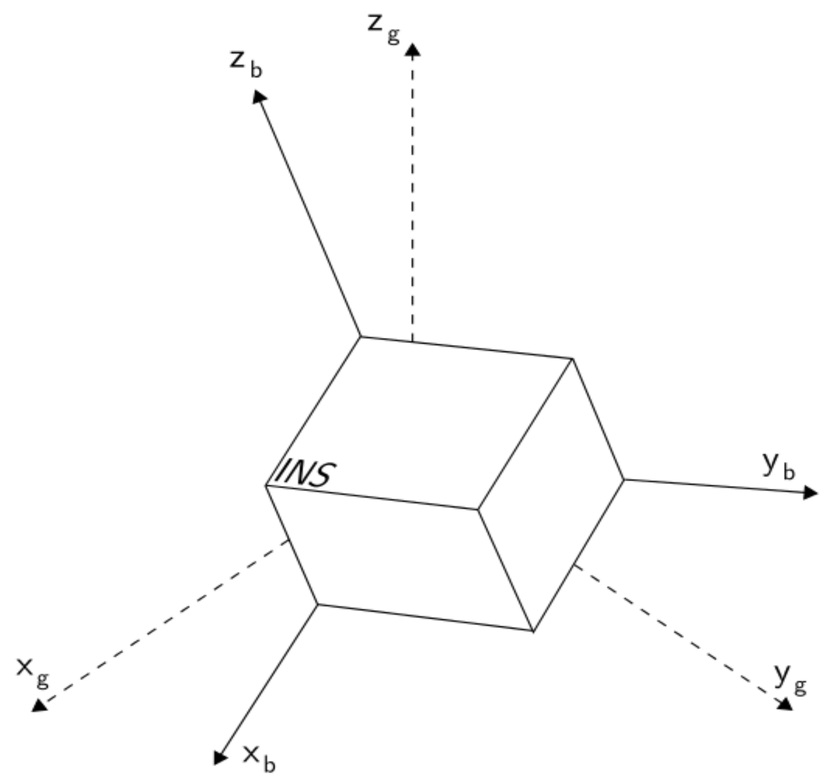

Figure 1: The body and global frames of reference.

Global frame: navigation system frame
Body frame: IMU system frame

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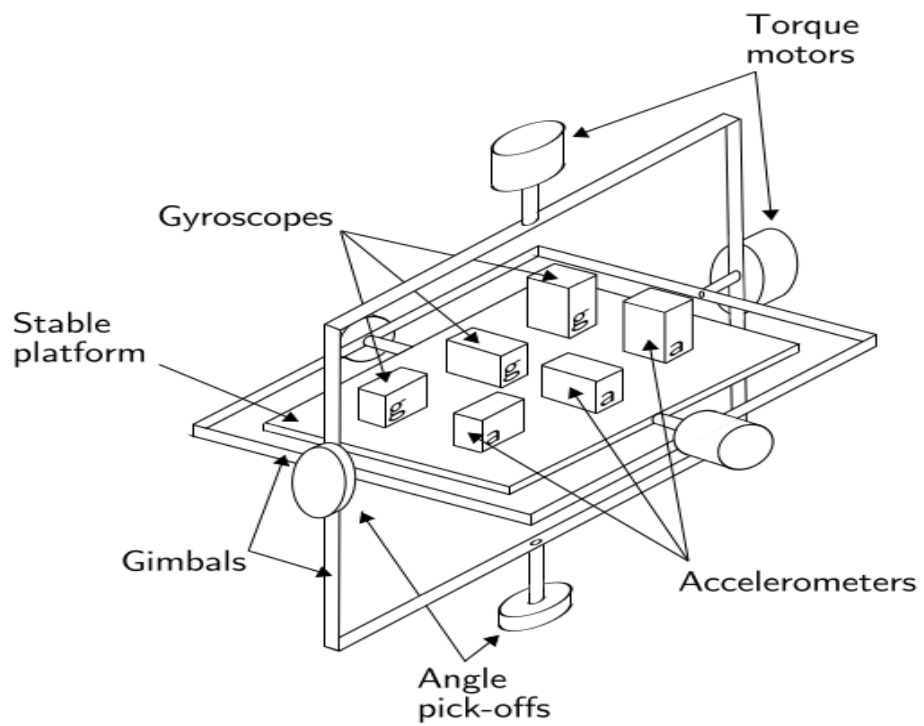
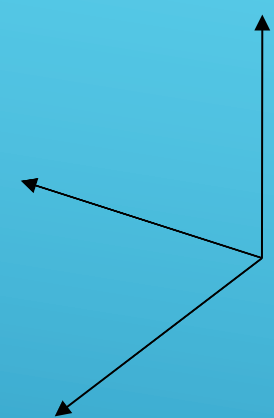


Figure 2: A stable platform IMU.



Body
frame

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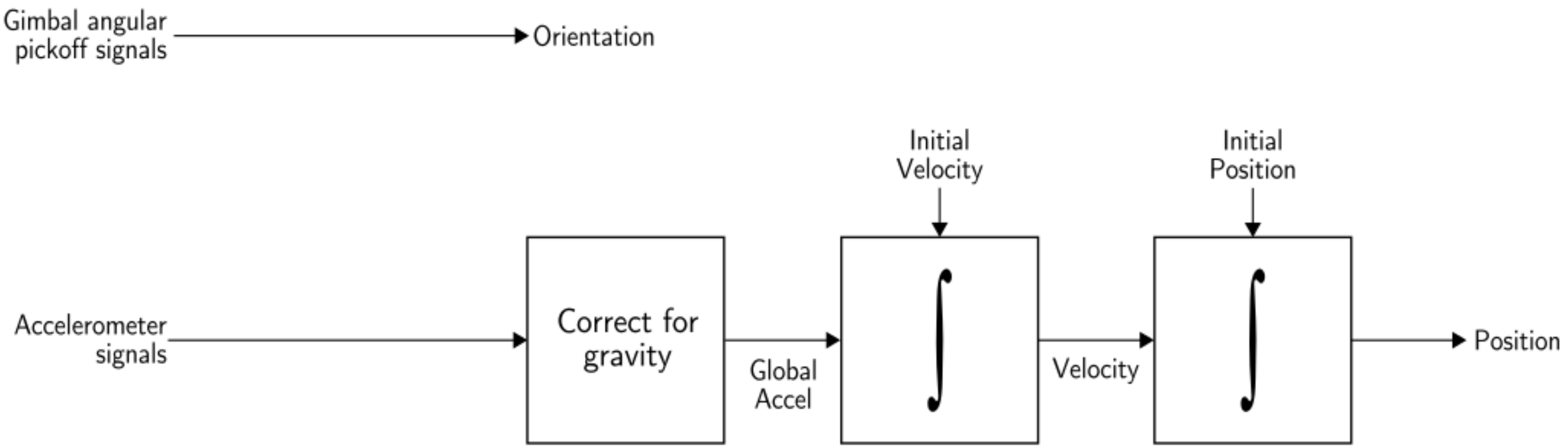

Figure 3: Stable platform inertial navigation algorithm.

It is necessary to subtract acceleration due to gravity from the vertical channel before performing the integration

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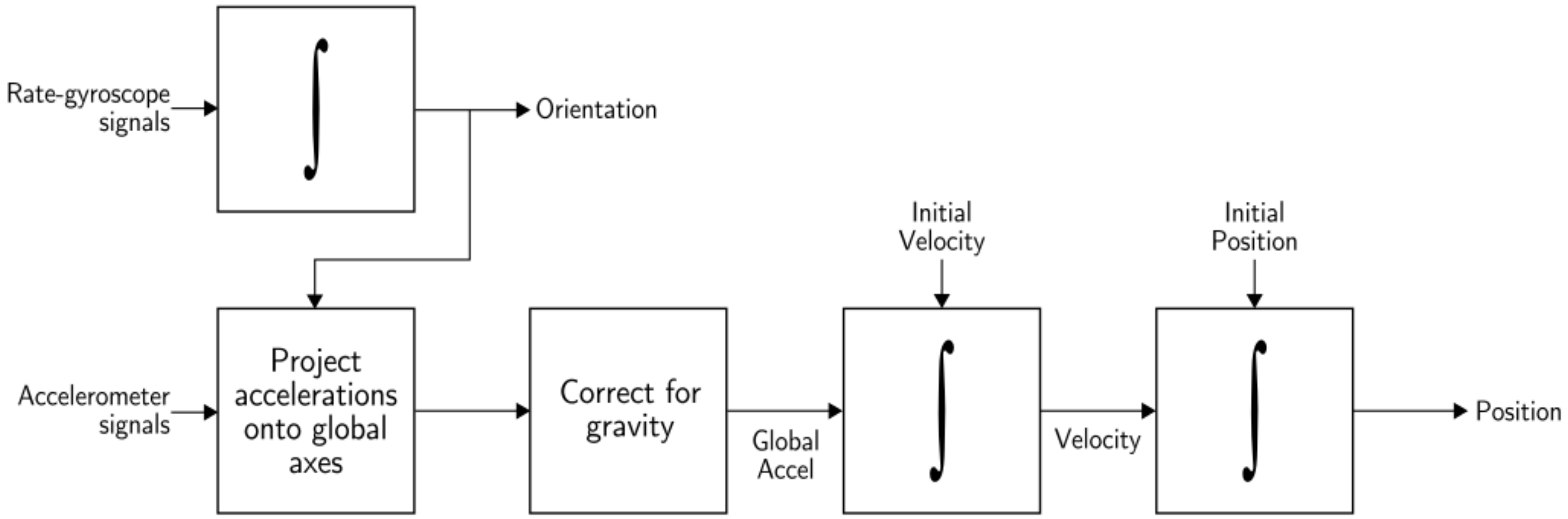

Figure 4: Strapdown inertial navigation algorithm.

To keep track of orientation the signals from the rate gyroscopes are ‘integrated’.

These benefits are achieved at the cost of increased computational complexity. As the cost of computation has decreased strapdown systems have become the dominant type of INS.

Gyroscope
Mechanical,Optical,MEMS

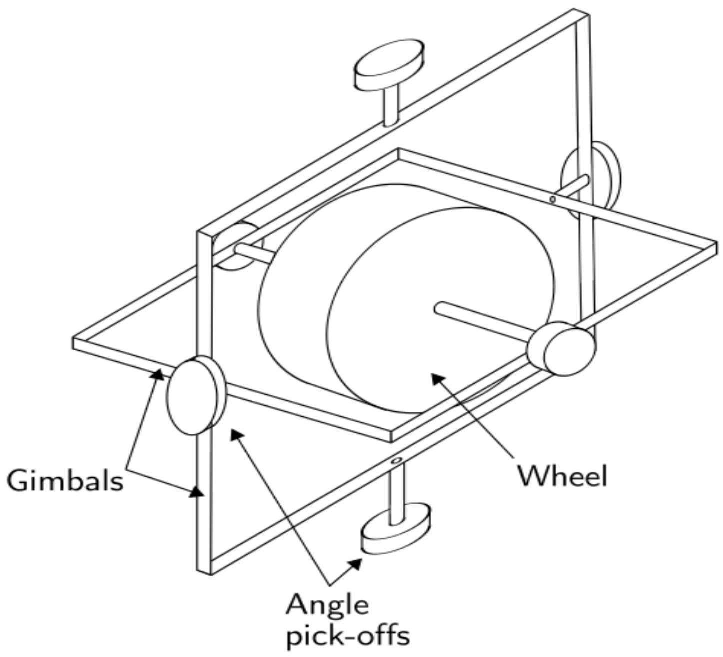

Figure 5: A conventional mechanical gyroscope (source: [1]).

conservation of angular
momentum

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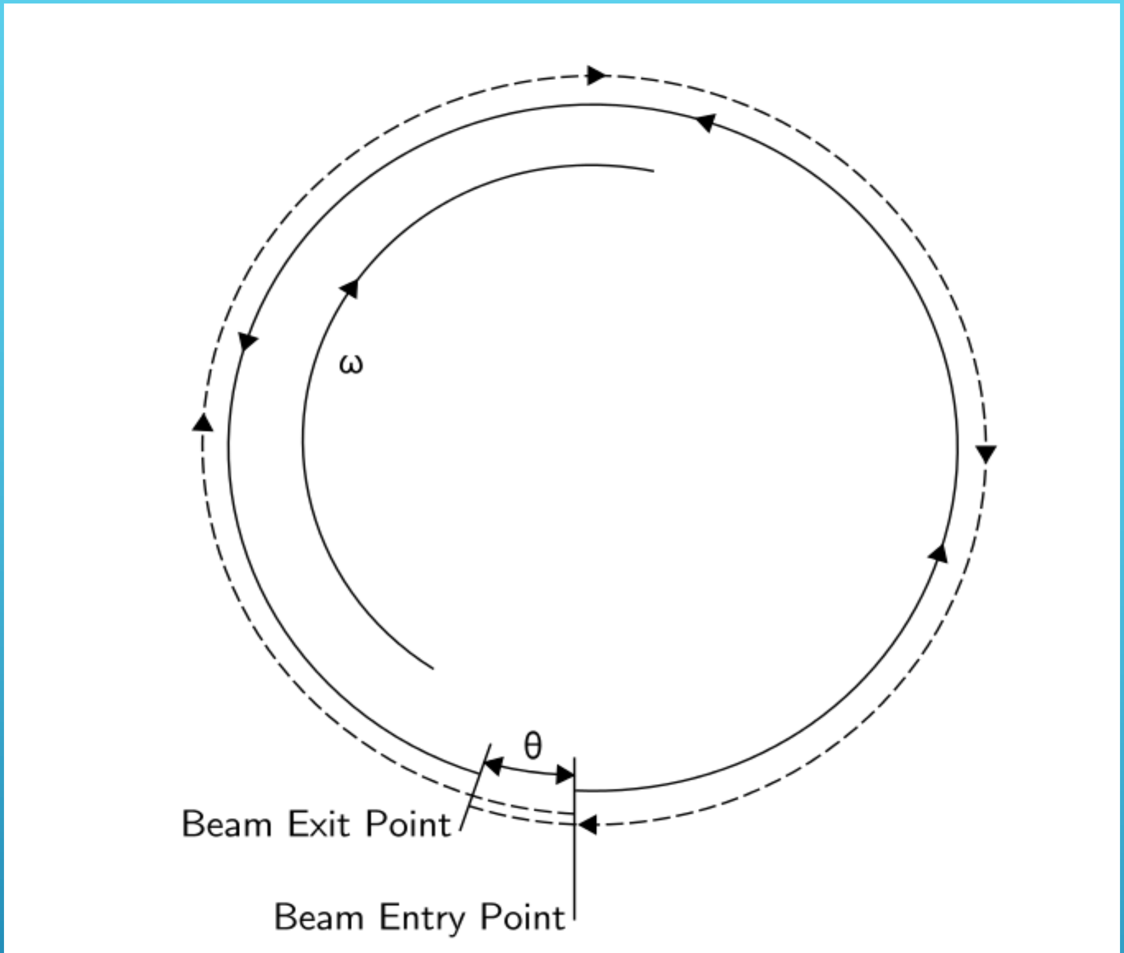

The Sagnac effect. The dashed line is the path taken by the beam travelling in the direction of rotation. The solid line is the beam travelling against the rotation. θ is the angle through which the gyro turns whilst the beams are in flight

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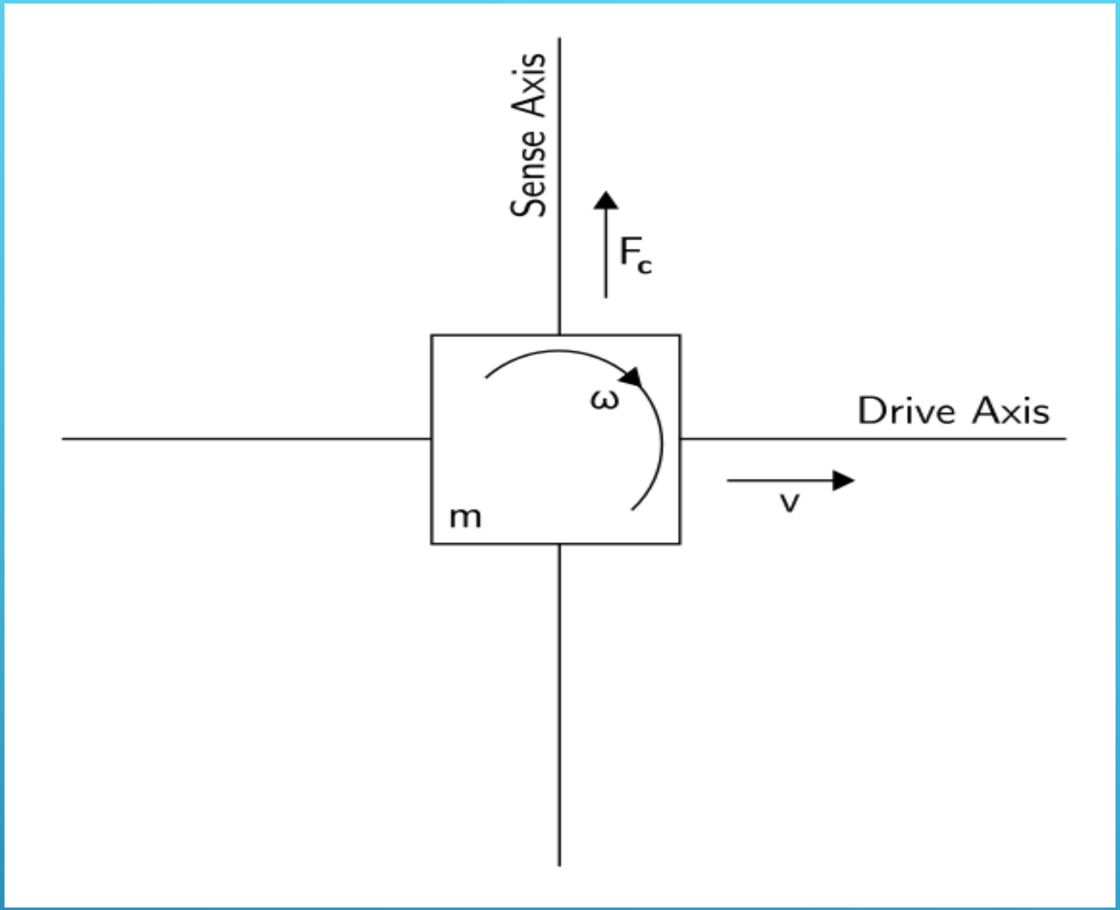

MEMS gyroscopes make use of the Coriolis effect, which states that in a frame of
reference rotating at angular velocity ω, a mass m moving with velocity v experiences a force F c = −2m(ω × v)

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MEMS Gyro Error Characteristics

Constant Bias

The bias of a rate gyro is the average output from the gyroscope when it is not undergoing any rotation in ◦ /h.A constant bias error of c, when integrated,causes an angular error which grows linearly with time θ(t) = c · t.

Thermo-Mechanical White Noise

1.thermo-mechanical noise which fluctuates at a rate much greater than the sampling rate of the sensor

2.zero-mean uncorrelated random variables,each random variable is identically distributed and has a finite variance σ 2

N i be the i th random variable in the white noise sequence

E(N i) = E(N) = 0; Var(N i) = Var(N) = σ 2;Cov(N i , N j) = 0 for all i = j

integrate the white noise signal ϙ(t) over a timespan t = n · δt

$$\int_0^t \epsilon(\tau)\, d\tau = \delta t \sum_{i=1}^{n} N_i$$

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Using the standard formulae E(aX + bY) = aE(X) + bE(y) and Var(aX + bY)
=
a 2 Var(X) + b 2 Var(Y) + 2abCov(X, Y)

$$\begin{aligned} \mathrm{E}\left(\int_0^t \epsilon(\tau)\, d\tau\right) &= \delta t \cdot n \cdot \mathrm{E}(N) = 0 \\ \mathrm{Var}\left(\int_0^t \epsilon(\tau)\, d\tau\right) &= \delta t^2 \cdot n \cdot \mathrm{Var}(N) = \delta t \cdot t \cdot \sigma^2. \end{aligned}$$

$$\sigma_\theta(t) = \sigma \cdot \sqrt{\delta t \cdot t}$$

$$\mathrm{ARW} = \sigma_\theta(1)$$

$$\begin{aligned} \mathrm{ARW}\ (^\circ/\sqrt{\mathrm{h}}) &= \frac{1}{60} \cdot \sqrt{\mathrm{PSD}\ ((^\circ/\mathrm{h})^2/\mathrm{Hz})} \\ \mathrm{ARW}\ (^\circ/\sqrt{\mathrm{h}}) &= \frac{1}{60} \cdot \mathrm{FFT}\ (^\circ/\mathrm{h}/\sqrt{\mathrm{Hz}}) \end{aligned}$$

An example

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	GG1320AN (Laser Gyro)	GG5300 (MEMS 3xGyro)
Size	88 mm × 88 mm × 45 mm	50 mm × 50 mm × 30 mm
Weight	454 g	136 g
Start-Up Time	< 4 s	< 1 s
Power	15 Vdc, 1.6 watts nominal 5 Vdc, 0.375 watts nominal	5 Vdc, $<$ 800 mA
Operating Temperature Range	-54 °C to 85 °C	-45 °C to 85 °C
Angular Random Walk	$0.0035°/\sqrt{\text{h}}$	$0.2°/\sqrt{\text{h}}$
Bias Stability	$0.0035°/\text{h}$	$< 70°/\text{h}$

Table 1: Specifications for the Honeywell GG1320AN and GG5300 gyroscopes.

Flicker Noise / Bias Stability

BS (◦ /h) = 1σ standard deviation

$$\text{BRW } (°/\sqrt{\text{h}}) = \frac{\text{BS } (°/\text{h})}{\sqrt{t \text{ (h)}}}$$

Temperature

Effects

Calibration Errors

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Accelerometer
Mechanical,Solid State,MEMS Accelerometers

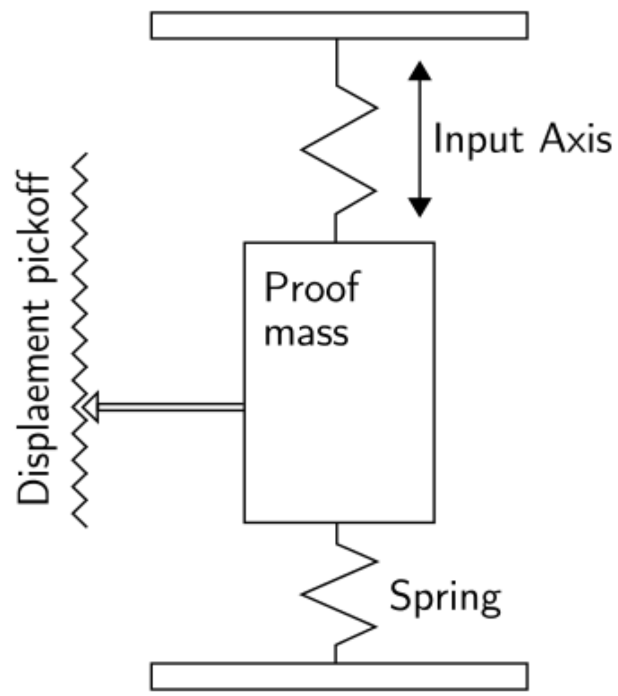

Figure 8: A mechanical accelerometer (source: [1]).

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MEMSThe first class consists of mechanical accelerometers manufactured using MEMS techniques.
The second class consists of devices which measure the change in frequency of a vibrating element caused by a change of tension, as in SAW accelerometers

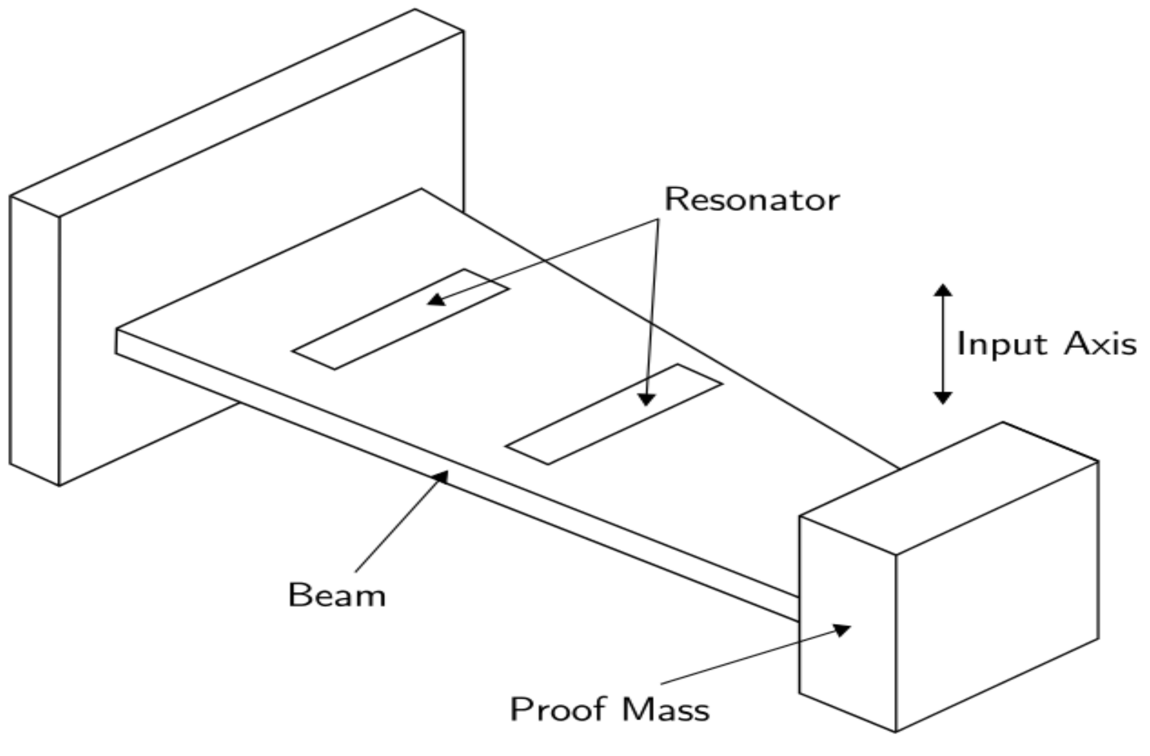

Figure 9: A surface acoustic wave accelerometer (source: [1]).

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MEMS Accelerometer Error Characteristics
Constant Bias

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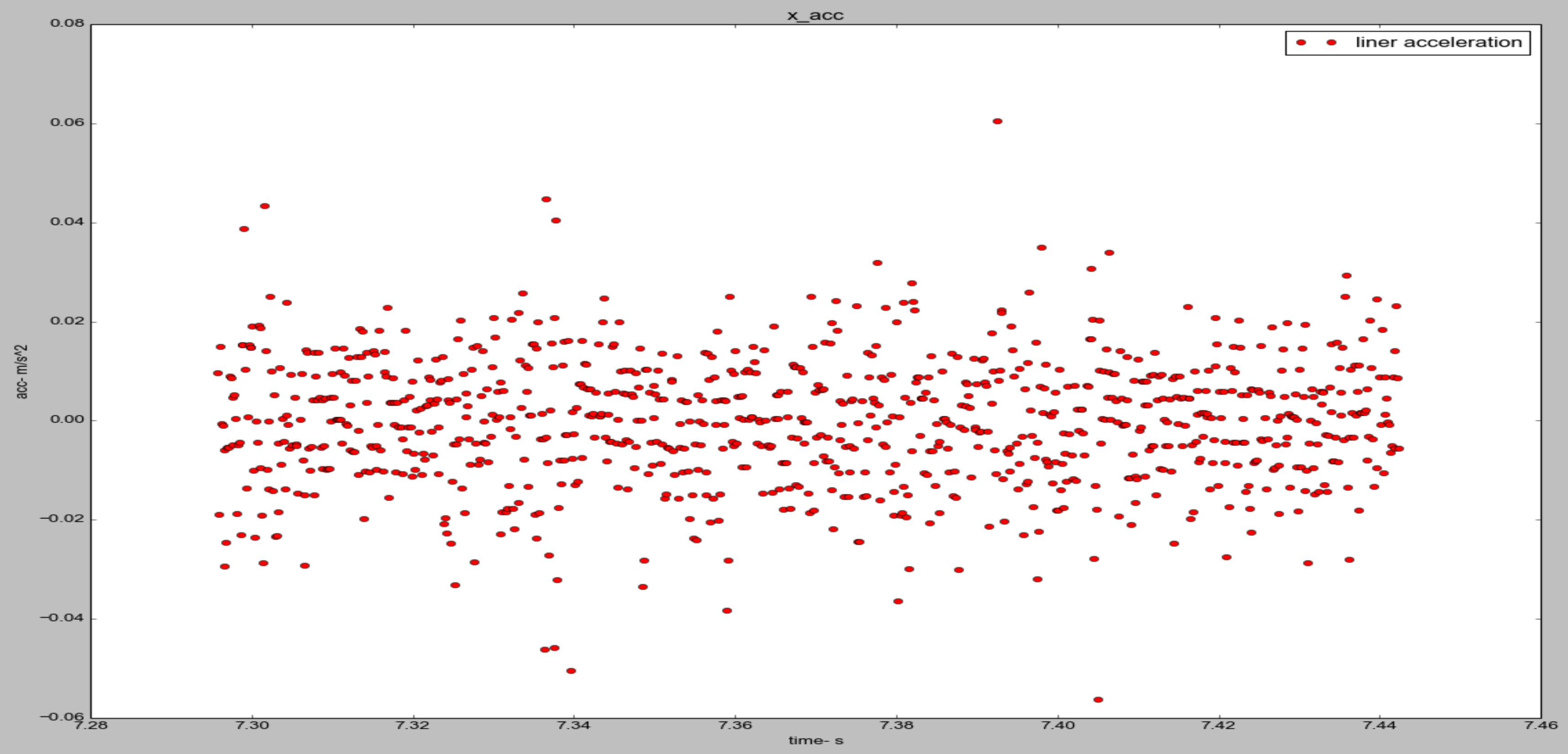
$$s(t) = \epsilon \cdot \frac{t^2}{2}$$



Thermo-Mechanical White Noise / Velocity Random Walk

E(N i) = E(N) = 0,Var(N i) = Var(N) = σ 2,t = n · δt;

$$\int_0^t \int_0^t \epsilon(\tau)\, d\tau d\tau = \delta t \sum_{i=1}^{n} \delta t \sum_{j=1}^{i} N_j$$

$$= \delta t^2 \sum_{i=1}^{n} (n-i+1) N_i$$

$$\mathrm{E}\left(\int_0^t \int_0^t \epsilon(\tau)\, d\tau d\tau\right) = \delta t^2 \sum_{i=1}^{n} (n-i+1)\mathrm{E}(N_i)$$

$$= 0$$

$$\mathrm{Var}\left(\int_0^t \int_0^t \epsilon(\tau)\, d\tau d\tau\right) = \delta t^4 \sum_{i=1}^{n} (n-i+1)^2 \mathrm{Var}(N_i)$$

$$= \frac{\delta t^4 n(n+1)(2n+1)}{6} \mathrm{Var}(N)$$

$$\approx \frac{1}{3} \cdot \delta t \cdot t^3 \cdot \sigma^2$$

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$$\sigma_s(t) \approx \sigma \cdot t^{3/2} \cdot \sqrt{\frac{\delta t}{3}}$$

PRW

Flicker Noise / Bias Stability
random walk in velocity whose uncertainty grows proportionally to
t 3/2 , and a third order random walk in position which grows proportionally to t 5/2

Temperature Effects
Calibration Errors

Signal Noise
Analysis
Allan Variance

1. Take a long sequence of data and divide it into bins of length t. There must be enough data for at least 9 bins (otherwise the results obtained begin to lose their significance).

2. Average the data in each bin to obtain a list of averages $(a(t)_1, a(t)_2, ..., a(t)_n)$, where n is the number of bins.

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3. The Allan Variance is then given by

$$\text{AVAR}(t) = \frac{1}{2 \cdot (n-1)} \sum_i (a(t)_{i+1} - a(t)_i)^2$$

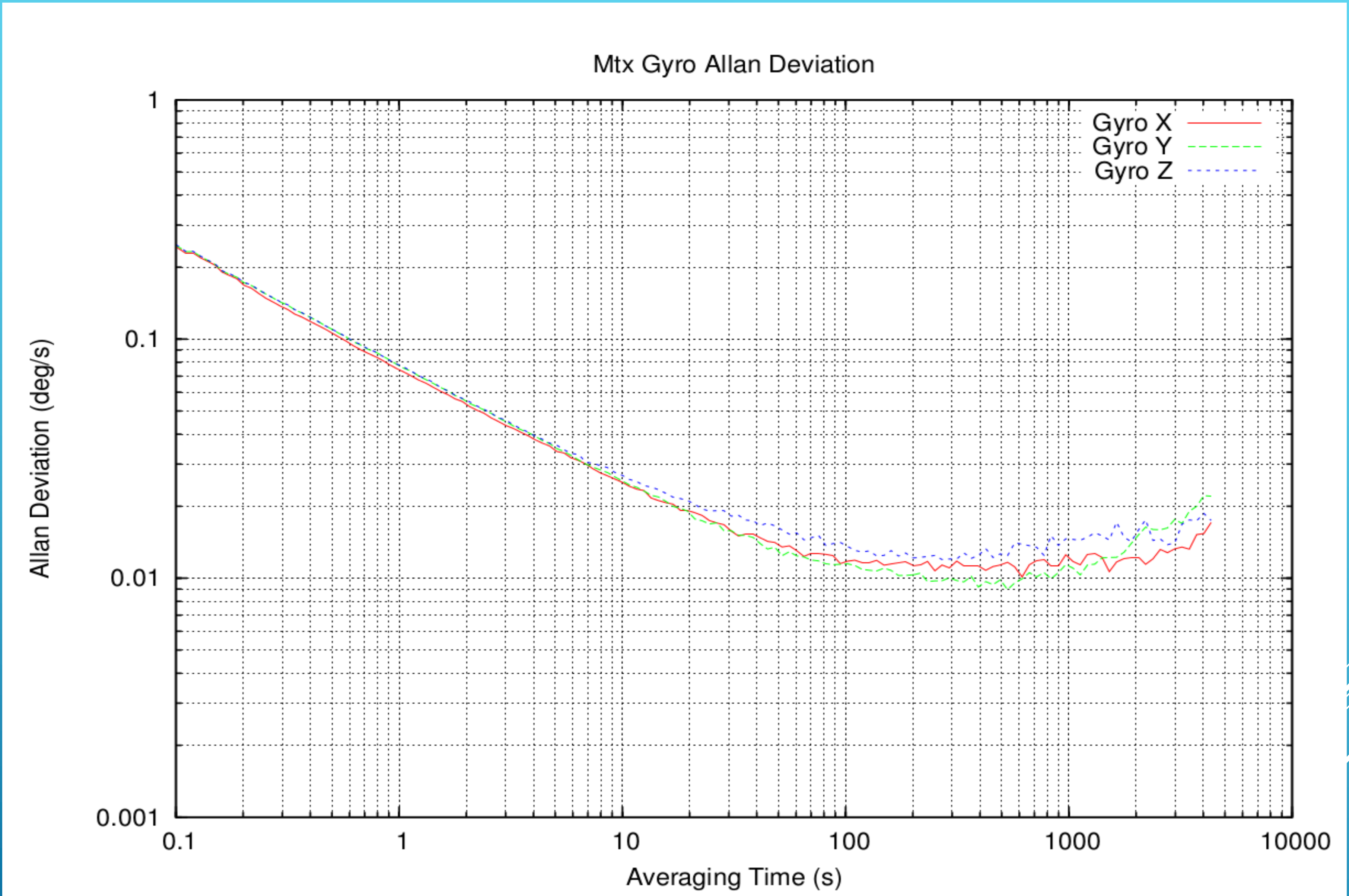
To determine the characteristics of the underlying noise processes, Allan Deviation

$$\text{AD}(t) = \sqrt{\text{AVAR}(t)}$$

The Allan Variance technique was applied to a 12 hour log of raw output data gathered from a stationary Mtx device with a 100 Hz sampling rateThe random walk measurement for this noise (ARW for a rate-gyro, VRW for an accelerometer) is obtained by fitting a straight line through the slope and reading its value at t = 1. Bias Instability appears on the plot as a flat region around the minimum. The numerical value is the minimum value on the Allan Deviation curve.

	Bias Instability	Angle Random Walk
X Axis	$0.010°/\text{s} = 36°/\text{h}$ (at 620 s)	$0.075°/\sqrt{\text{s}} = 4.6°/\sqrt{\text{h}}$
Y Axis	$0.009°/\text{s} = 32°/\text{h}$ (at 530 s)	$0.078°/\sqrt{\text{s}} = 4.8°/\sqrt{\text{h}}$
Z Axis	$0.012°/\text{s} = 43°/\text{h}$ (at 270 s)	$0.079°/\sqrt{\text{s}} = 4.8°/\sqrt{\text{h}}$

Table 4: Gyroscope Noise Measurements.



Mtx Gyro Allan Deviation
Allan Deviation (deg/s)
Averaging Time (s)
Gyro X
Gyro Y
Gyro Z
1
0.1
0.01
0.001
0.1
1
10
100
1000
10000

Error Type	Description	Result of Integration
Bias	A constant bias ϵ	A steadily growing angular error $$\theta(t) = \epsilon \cdot t$$
White Noise	White noise with some standard deviation σ	An angle random walk, whose standard deviation $$\sigma_\theta(t) = \sigma \cdot \sqrt{\delta t \cdot t}$$ grows with the square root of time
Temperature Effects	Temperature dependent residual bias	Any residual bias is integrated into the orientation, causing an orientation error which grows linearly with time
Calibration	Deterministic errors in scale factors, alignments and gyro linearities	Orientation drift proportional to the rate and duration of motion
Bias Instability	Bias fluctuations, usually modelled as a bias random walk	A second-order random walk

Table 2: A Summary of Gyro Error Sources.

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Error Type	Description	Result of Double Integration
Bias	A constant bias ϵ in the accelerometer's output signal	A quadratically growing position error $$s(t) = \epsilon \cdot \frac{t^2}{2}$$
White Noise	White noise with some standard deviation σ	A second-order random walk. The standard deviation of the position error grows as $$\sigma_s(t) = \sigma \cdot t^{3/2} \cdot \sqrt{\frac{\delta t}{3}}$$
Temperature Effects	Temperature dependent residual bias	Any residual bias causes an error in position which grows quadratically with time
Calibration	Deterministic errors in scale factors, alignments and accelerometer linearities	Position drift proportional to the squared rate and duration of acceleration
Bias Instability	Bias fluctuations, usually modelled as a bias random walk	A third-order random walk in position

Table 3: A Summary of Accelerometer Error Sources.

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Loose and Tight GNSS INS Intergrations: Comparison of Performance Assessed in Real Urban Scenarios

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