Алешко Альберт АС-21-05 Вариант 1

Лабораторная работа №4

Регрессионный анализ

In [1]:
```
import pandas as pd
import numpy as np
from matplotlib import pyplot as plt
import seaborn as sns
import statsmodels.formula.api as smf
from itertools import chain
from scipy.stats import spearmanr, f

plt.style.use('Solarize_Light2') # Функция для задания стиля графикам
```

По данным предыдущей работы провести регрессионный анализ:

выбрать вид регрессионной зависимости (возможно преобразование данных для выбора линейной модели);

определить модельные коэффициенты;

проверить модель на значимость;

провести проверку модели на адекватность (см. дополнительные данные на отдельной вкладке).

In [2]:
```
names = [f'var{i + 1}' for i in range(5)]

df_train = pd.read_excel('data2.xlsx', sheet_name='2-1',
                   header=None, names=names)
df_train.head(6)
```

Out[2]:

	var1	var2	var3	var4	var5
0	13.576533	8.444634	42.737	7.574	2.369
1	9.988142	13.552947	23.261	1.660	6.072
2	8.190783	14.466266	13.439	-5.422	3.504
3	8.412009	10.531638	15.174	-5.483	4.532
4	10.021344	8.826599	24.573	3.247	7.447
5	6.843471	7.619135	13.896	-5.045	7.473

In [3]:
```
df_test = pd.read_excel('data2.xlsx', sheet_name='1вар-адекв',
                   header=None, names=names)
df_test.head(6)
```

Out[3]:

	var1	var2	var3	var4	var5
0	8	10.746	19.059	-1.541	6.924
1	8	10.220	16.557	-4.043	5.923
2	8	8.615	18.565	-2.035	6.726
3	8	4.791	18.115	-2.485	6.546
4	8	2.831	19.900	-0.700	7.260
5	9	5.483	21.785	-1.115	5.225

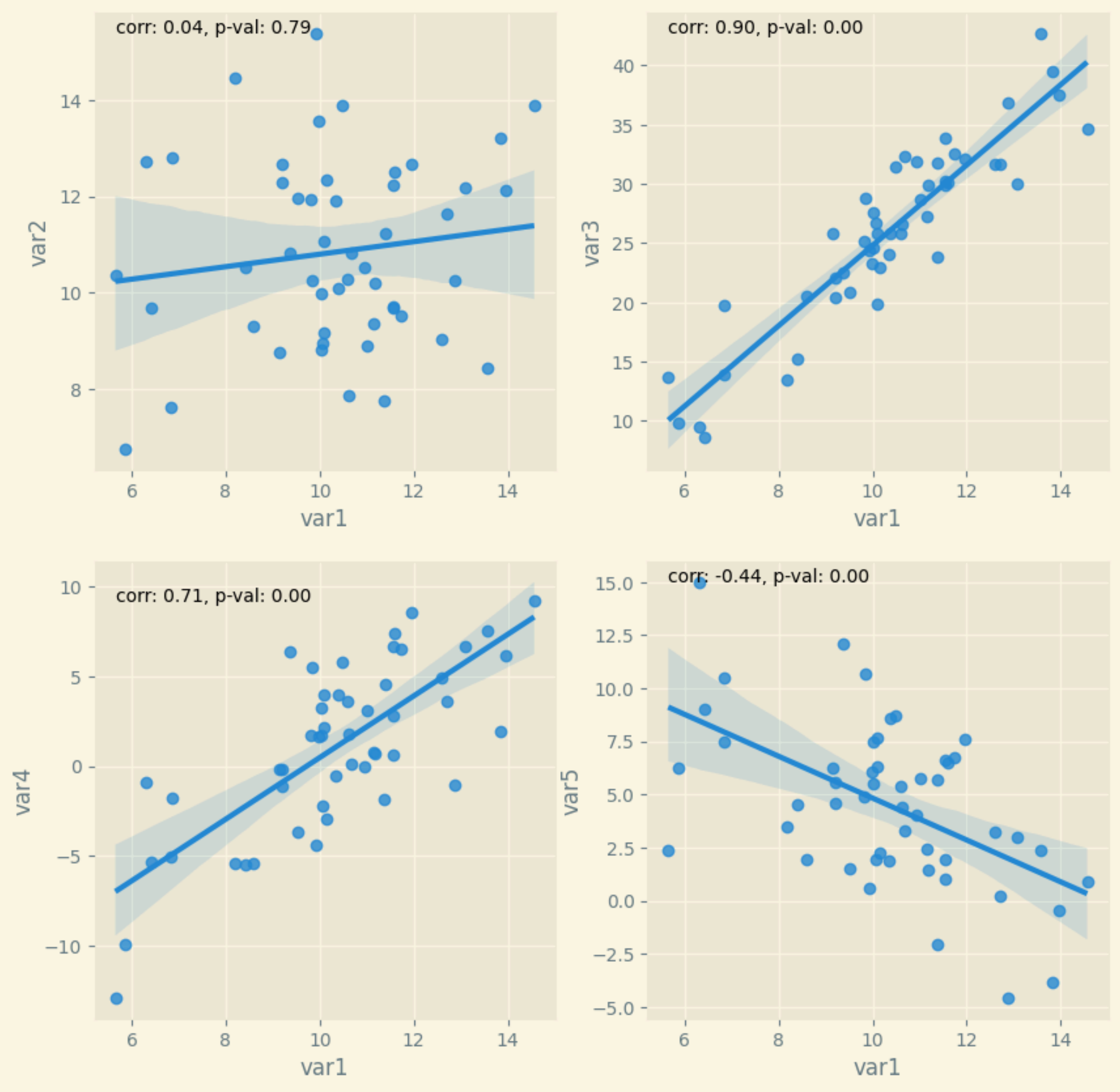
In [4]:
```
independent_var = 'var1'

y_train = df_train[[var for var in df_train.columns if var != independent_var]]
x_train = df_train[independent_var]
y_test = df_test[y_train.columns]
x_test = df_test[independent_var]
x_train.shape, y_train.shape, x_test.shape, y_test.shape
```

Out[4]:
```
((50,), (50, 4), (35,), (35, 4))
```

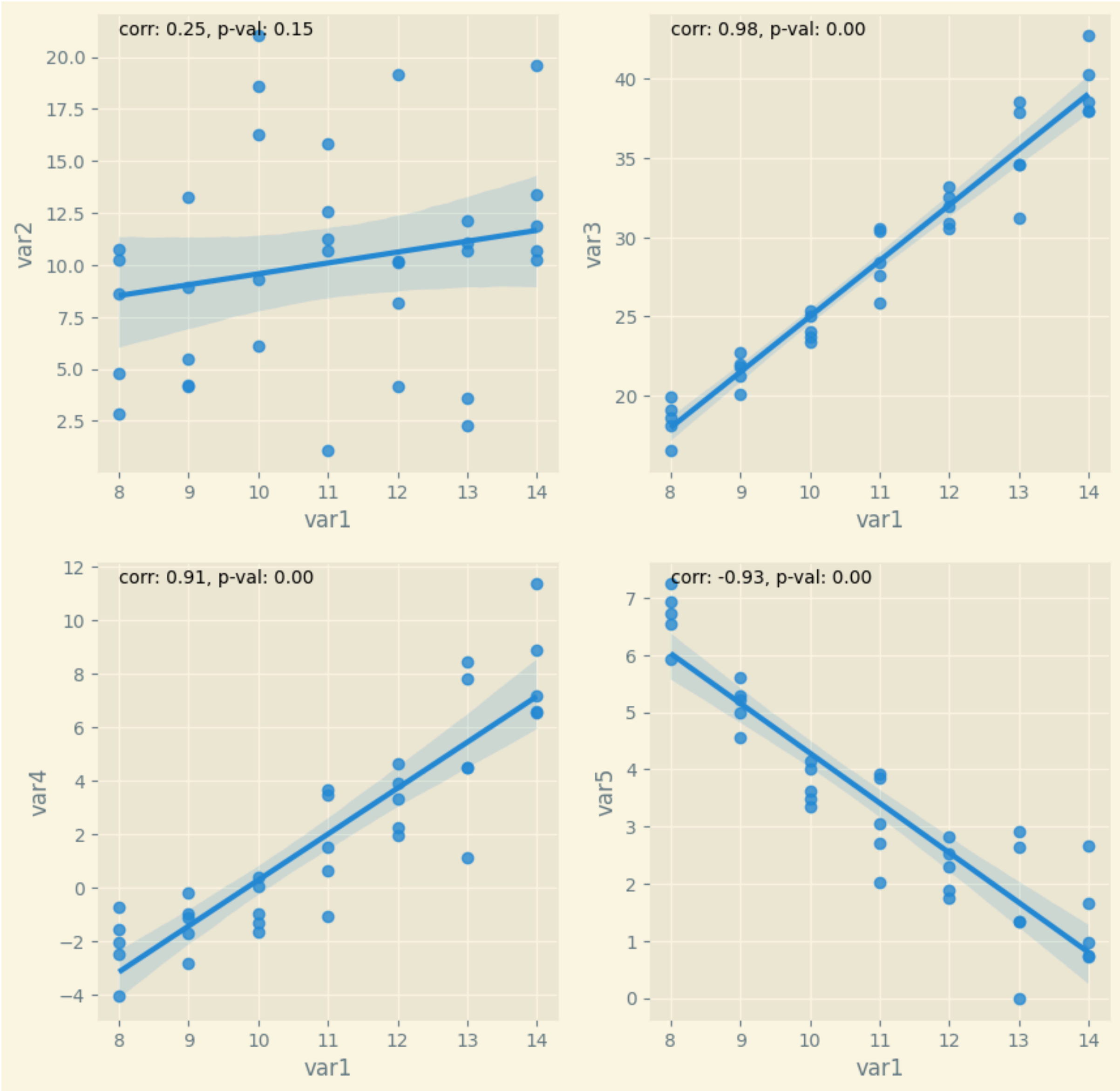
In [5]:
```
def plot_data(x, y):
    fig, axs = plt.subplots(y.shape[1]//2, y.shape[1]//2, figsize=(10, 10))

    axs = list(chain.from_iterable(axs))
```

```python
    for name, ax in zip(y_train.columns, axs):
        res = spearmanr(x, y[name])
        sns.regplot(x=x, y=y[name], ax=ax)
        ax.set_ylabel(name)
        ax.set_xlabel(independent_var)
        ax.text(x.min(), y[name].max(),
                f'corr: {res[0]:.2f}, p-val: {res[1]:.2f}')
```

In [6]:
```python
plot_data(x_train, y_train)
```



In [7]:
```python
plot_data(x_test, y_test)
```



var1 - var2 - независимые

var1 - var3 - линейная зависимость

var1 - var4 - экспоненциальная зависимость (предположительно)

var1 - var5 - обратная экспоненциальная зависимость (предположительно)

С целью преобразования экспоненциальная зависимости в линейную, выполним преобразование: $x' = log(1 + x - min)$

где min = min(x), max = max(x)

In [8]:
```
# var4_min = y_train['var4'].min()
var5_min = y_train['var5'].min()
# y_train.loc[:, 'var4'] = np.power(y_train['var4'] - var4_min, 0.95).values
y_train.loc[:, 'var5'] = np.log(1 + np.exp(y_train['var5']) - np.exp(y_train['var5']).min()).values
# y_test.loc[:, 'var4'] = np.power(y_test['var4'] - y_test['var4'].min(), 0.95).values
y_test.loc[:, 'var5'] = np.log(1 + np.exp(y_test['var5'] - y_test['var5'].min()) - np.exp(y_test['var5'] - y_test['var5'].
```

В итоге, после проведённых испытаний, получилось следующее

первые 2 прогноза остались

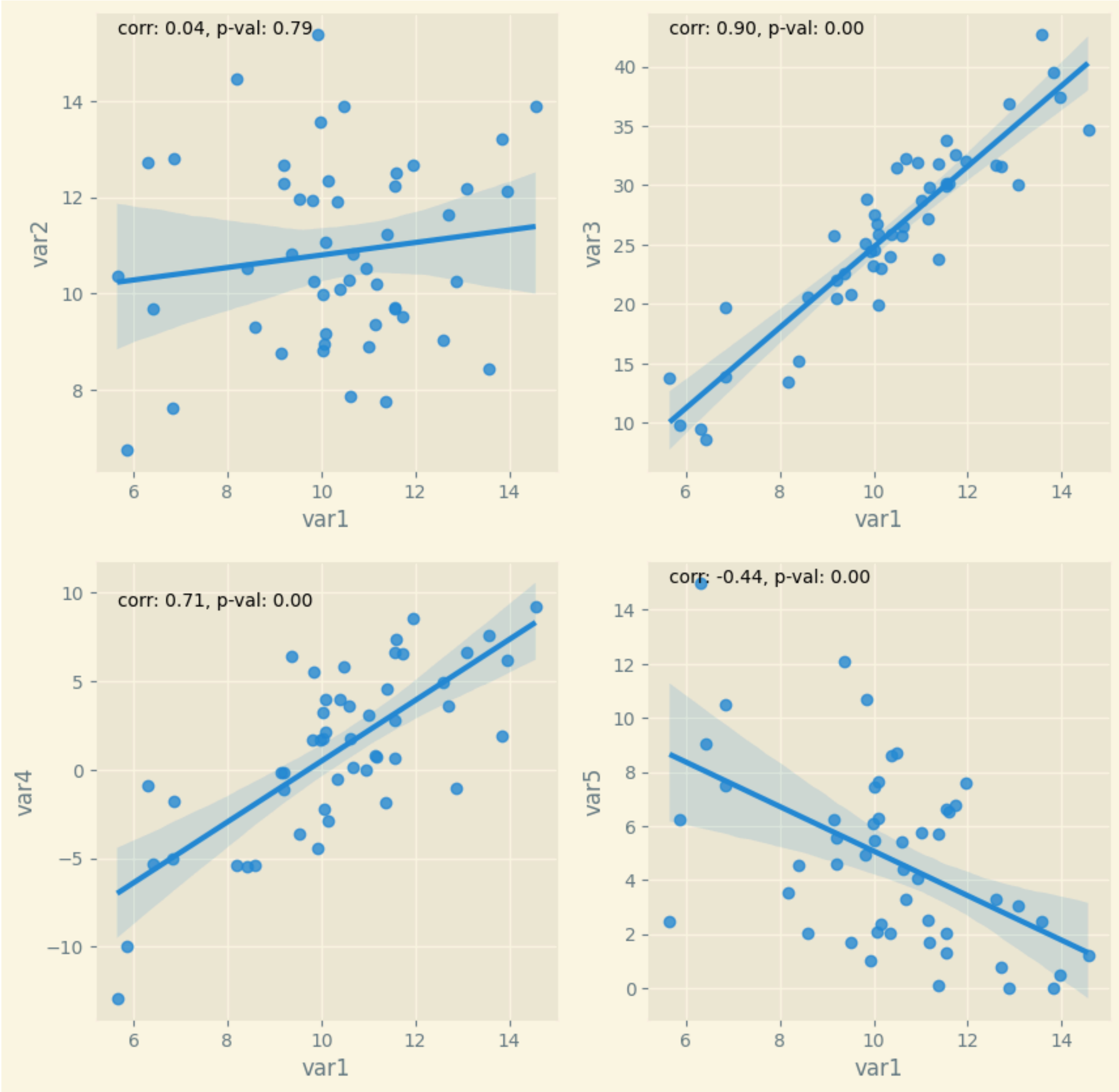
var1 - var4 - линейная зависимость

var1 - var5 - сложная зависимость

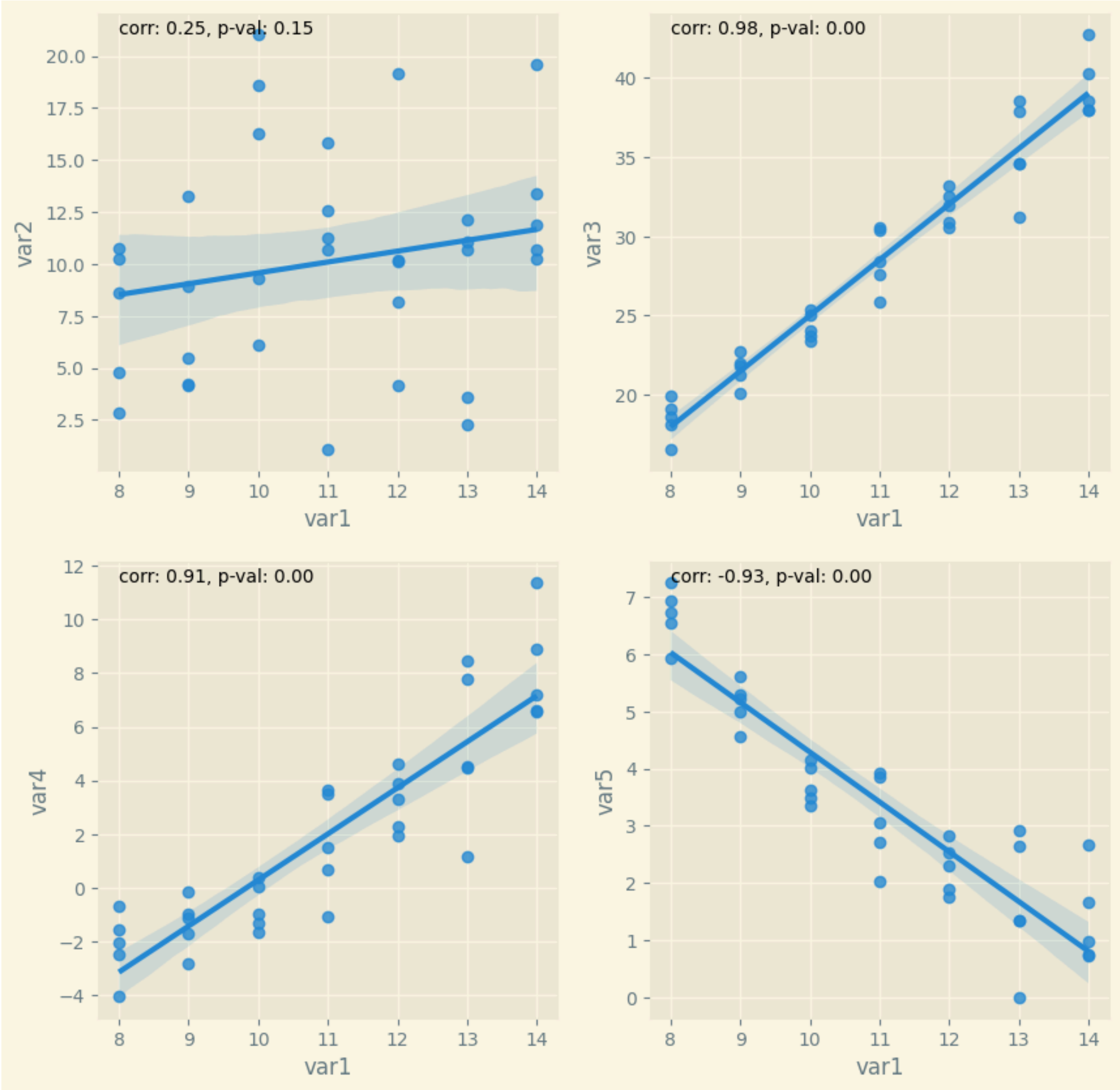
С целью преобразования зависимости в линейную, я выполнил преобразование: $x' = log(1 + e^{(x-min)} - min_e)$

где $min = min(x)$,
$min_e = min(exp(x))$.

In [9]:
```
plot_data(x_train, y_train)
```



In [10]:
```
plot_data(x_test, y_test)
```



In [11]:
```
def get_means(x_test, y_test):
    means = dict()
    for x in x_test.unique():
        means[x] = np.mean(y_test[x_test == x])

    def f(x):
        nonlocal means, x_test
        return means[x], (x_test == x).sum()
    return f
```

In [12]:
```
def is_adequately(linear_model, x_test, y_test):
    n = 30
    m = 6
    f_means = get_means(x_test, y_test)
    x_test_unique = x_test.unique()
    preds = linear_model.predict(pd.Series(x_test_unique, name='var1'))
    Q2 = 0
    for x, pred in zip(x_test_unique, preds):
        mean, n_i = f_means(x)
        Q2 += n_i * (mean - pred) ** 2

    Q3 = 0
    mean = y_test_part.mean()
    for y in y_test_part:
        Q3 += (mean - y) ** 2

    t = Q2 * (n - m) / (Q3 * (m - 2))
    tcr = f.ppf(0.95, m-2, n-m)

    print(f'{t = :.2f}, {tcr = :.2f}')

    return t < tcr
```

In [13]:
```
target_var = 'var2'
y_test_part = y_test[target_var]
```

```python
model_linear_ols = smf.ols(formula=f'{target_var} ~ var1', data=df_train)
linear_model = model_linear_ols.fit()
linear_model.summary()
```

Out[13]:

OLS Regression Results

Dep. Variable:	var2	**R-squared:**	0.019
Model:	OLS	**Adj. R-squared:**	-0.001
Method:	Least Squares	**F-statistic:**	0.9459
Date:	Sun, 19 May 2024	**Prob (F-statistic):**	0.336
Time:	14:07:27	**Log-Likelihood:**	-102.82
No. Observations:	50	**AIC:**	209.6
Df Residuals:	48	**BIC:**	213.5
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	9.5047	1.406	6.762	0.000	6.679	12.331
var1	0.1297	0.133	0.973	0.336	-0.138	0.398

Omnibus:	1.183	**Durbin-Watson:**	2.083
Prob(Omnibus):	0.554	**Jarque-Bera (JB):**	1.146
Skew:	0.235	**Prob(JB):**	0.564
Kurtosis:	2.426	**Cond. No.**	54.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [14]:
```python
is_adequately(linear_model, x_test, y_test_part)
```

```
t = 1.64, tcr = 2.78
```

Out[14]: True

In [15]:
```python
target_var = 'var3'
y_test_part = y_test[target_var]

model_linear_ols = smf.ols(formula=f'{target_var} ~ var1', data=df_train)
linear_model = model_linear_ols.fit()
linear_model.summary()
```

Out[15]: OLS Regression Results

Dep. Variable:	var3	**R-squared:**	0.844
Model:	OLS	**Adj. R-squared:**	0.840
Method:	Least Squares	**F-statistic:**	258.8
Date:	Sun, 19 May 2024	**Prob (F-statistic):**	5.76e-21
Time:	14:07:27	**Log-Likelihood:**	-125.74
No. Observations:	50	**AIC:**	255.5
Df Residuals:	48	**BIC:**	259.3
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-9.1250	2.223	-4.105	0.000	-13.595	-4.656
var1	3.3935	0.211	16.086	0.000	2.969	3.818

Omnibus:	0.422	**Durbin-Watson:**	1.907
Prob(Omnibus):	0.810	**Jarque-Bera (JB):**	0.567
Skew:	-0.008	**Prob(JB):**	0.753
Kurtosis:	2.478	**Cond. No.**	54.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [16]:
```
is_adequately(linear_model, x_test, y_test_part)
```

```
t = 0.03, tcr = 2.78
```

Out[16]: True

In [17]:
```
target_var = 'var4'
y_test_part = y_test[target_var]

model_linear_ols = smf.ols(formula=f'{target_var} ~ var1', data=df_train)
linear_model = model_linear_ols.fit()
linear_model.summary()
```

Out[17]: OLS Regression Results

Dep. Variable:	var4	**R-squared:**	0.560
Model:	OLS	**Adj. R-squared:**	0.551
Method:	Least Squares	**F-statistic:**	61.17
Date:	Sun, 19 May 2024	**Prob (F-statistic):**	4.11e-10
Time:	14:07:27	**Log-Likelihood:**	-127.77
No. Observations:	50	**AIC:**	259.5
Df Residuals:	48	**BIC:**	263.4
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-16.7060	2.315	-7.215	0.000	-21.361	-12.051
var1	1.7185	0.220	7.821	0.000	1.277	2.160

Omnibus:	0.920	**Durbin-Watson:**	2.333
Prob(Omnibus):	0.631	**Jarque-Bera (JB):**	0.841
Skew:	-0.015	**Prob(JB):**	0.657
Kurtosis:	2.366	**Cond. No.**	54.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [18]:
```
is_adequately(linear_model, x_test, y_test_part)
```

```
t = 0.21, tcr = 2.78
```

Out[18]: True

In [19]:
```
target_var = 'var5'
y_test_part = y_test[target_var]

model_linear_ols = smf.ols(formula=f'{target_var} ~ var1', data=df_train)
linear_model = model_linear_ols.fit()
linear_model.summary()
```

Out[19]: OLS Regression Results

Dep. Variable:	var5	**R-squared:**	0.285
Model:	OLS	**Adj. R-squared:**	0.270
Method:	Least Squares	**F-statistic:**	19.14
Date:	Sun, 19 May 2024	**Prob (F-statistic):**	6.53e-05
Time:	14:07:27	**Log-Likelihood:**	-129.05
No. Observations:	50	**AIC:**	262.1
Df Residuals:	48	**BIC:**	265.9
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	14.6881	2.375	6.184	0.000	9.912	19.464
var1	-0.9861	0.225	-4.375	0.000	-1.439	-0.533

Omnibus:	0.209	**Durbin-Watson:**	2.308
Prob(Omnibus):	0.901	**Jarque-Bera (JB):**	0.411
Skew:	-0.003	**Prob(JB):**	0.814
Kurtosis:	2.556	**Cond. No.**	54.7

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [20]:
```
is_adequately(linear_model, x_test, y_test_part)
```

```
t = 0.67, tcr = 2.78
```

Out[20]: True