Алешко Альберт АС-21-05
Лабораторная работа №5

Задание:

1. Классификация данных
   1.1. Дискриминантный анализ.
      1.1.1. Построить классификационное правило, используя обучающую выборку (p = 8; k = 2).
      1.1.2. Провести оценку качества построенного правила.
      1.1.3. Провести классификацию 10 новых объектов.
   1.2. Кластерный анализ (p = 8)
      1.2.1. Провести классификацию, используя различные метрики и алгоритмы кластерного анализа.
      1.2.2. Сравнить результаты кластеризации с классификацией полученной другими методами.
      1.2.3. Определить характеристики случайных величин, представляющих полученные классы. Построить регрессионную модель для каждого кластера.
2. Снижение размерности. Метод главных компонент (p = 8; p’ = 2)
   2.1. Найти выражение для двух главных компонент Z1 , Z2.
   2.2. Определить основные характеристики компонент.
   2.3. Провести визуальную классификацию данных, спроектированных в пространство первых двух главных компонент.
   2.4. Какую часть дисперсии объясняют эти компоненты.
   2.5. Сколько компонент необходимо для объяснения 90% дисперсии (p’-?).

In [463…
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
import seaborn as sns
from sklearn.linear_model import LogisticRegression
from scipy.cluster.vq import whiten, kmeans, vq
from sklearn.metrics import silhouette_score
from scipy.cluster.hierarchy import dendrogram, linkage, fcluster
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
from bioinfokit.visuz import cluster
```

In [464…
```
def get_info(df):
    columns = ['count', 'nunique', 'mean',  'range',  'std', 'varience', 'min', '25%', '50%', '75%', 'max', 'skew', 'kurt', 'sem', 'sum']
    info = df.describe().T
    info['nunique'] = df.nunique()
    info['range'] = df.max(0) - df.min(0)
    info['varience'] = df.var()
    info['skew'] = df.skew()
    info['kurt'] = df.kurtosis()
    info['sem'] = df.sem()
    info['sum'] = df.sum()
    mode = df.mode(axis=0).T
    mode_names = [f'mode{i + 1}' for i in range(len(mode.T))]
    mode.columns = [f'mode{i + 1}' for i in range(len(mode.T))]
    columns = columns + mode_names[:3]
    info = pd.concat([info, mode], axis=1)
    return info[columns]
```

Считывание данных

In [465…
```
column_names = [f'var{i + 1}' for i in range(8)]

df = pd.read_excel('ms-data3.xlsx', '3', header=None,
                   names=column_names + ['None'] + ['cls'])
df = df.drop('None', axis=1)
df_train = df.iloc[1:21]
df_train['index'] = range(20)
df_train = df_train.set_index('index')
df_train['cls'] = df_train['cls'].values - 1
```

```
C:\Users\aerty\AppData\Local\Temp\ipykernel_10964\1572387737.py:7: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  df_train['index'] = range(20)
```

In [466…
```
df_train
```

Out[466…

index	var1	var2	var3	var4	var5	var6	var7	var8	cls
0	13.829	13.054	2.072	9.316	-75.047	0.728	-5.127	0.520	0
1	13.595	14.198	1.949	4.541	23.833	-0.557	8.205	2.712	0
2	15.199	12.208	2.295	6.440	-7.980	3.087	4.551	1.538	0
3	3.449	2.002	2.870	7.656	-36.830	0.339	-2.888	1.273	0
4	9.297	1.674	7.481	6.199	-2.556	4.813	4.066	0.943	0
5	13.285	17.957	1.687	4.993	5.851	-4.215	5.515	1.177	0
6	6.74	-4.922	-0.852	2.361	1.984	5.337	1.861	0.683	0
7	7.929	7.038	2.152	9.282	-72.814	0.480	-6.536	1.354	0
8	4.929	-0.397	-14.155	4.212	-1.863	1.782	-0.630	1.036	0
9	7.679	3.864	3.188	1.683	4.001	1.989	3.312	-0.156	0
10	-3.056	11.077	9.879	10.818	-8.632	6.164	3.904	9.663	1
11	10.219	37.730	19.992	14.205	-27.490	15.293	9.600	12.009	1
12	1.624	26.560	14.205	13.141	-25.556	10.936	5.693	12.907	1
13	4.915	33.325	16.151	10.585	-29.385	11.883	6.524	11.986	1
14	5.884	38.263	15.818	6.457	-52.694	10.727	3.340	11.680	1
15	-5.536	19.209	3.969	7.454	-19.132	7.280	1.929	10.077	1
16	-1.544	36.043	9.125	16.400	66.582	10.520	18.435	11.779	1
17	-5.346	22.670	4.675	12.285	-60.253	8.561	-2.276	11.423	1
18	-0.776	29.698	10.563	11.567	-89.128	9.874	-3.765	10.781	1
19	-1.843	26.677	9.597	7.489	-32.518	9.479	2.632	9.366	1

In [467…

```
df_test = df.iloc[22:, :-1]
df_test['index'] = range(10)
df_test = df_test.set_index('index')
df_test
```

Out[467…

index	var1	var2	var3	var4	var5	var6	var7	var8
0	7.067112	14.126164	1.398492	12.248382	15.720863	-3.387396	6.533666	-0.731764
1	5.404137	21.548591	22.137885	8.843795	-5.157064	16.663980	9.734781	10.677927
2	1.821365	27.883897	14.096621	11.050607	-24.370467	10.914315	5.736915	11.610423
3	4.459267	3.676102	2.256440	2.327041	-1.380254	0.237134	1.676466	0.455382
4	3.838606	33.584304	14.948935	8.628067	-33.720832	11.061429	5.259817	11.713002
5	6.16695	2.178221	4.204207	4.956740	-4.611220	1.670255	2.196810	1.973321
6	2.394137	27.222858	15.018086	11.618131	-21.099159	11.423781	6.463490	10.750459
7	9.099304	2.970610	4.483372	2.367415	2.432537	3.786637	3.769759	0.949605
8	4.976684	26.629139	18.619270	8.900759	-9.059375	14.223080	8.929193	11.923076
9	10.732772	5.193872	3.283666	5.332875	-2.416008	4.036576	3.898864	0.856805

Дискриминантный анализ

In [468…

```
df_train1 = df_train.loc[df_train['cls'] == 0][column_names]
df_train2 = df_train.loc[df_train['cls'] == 1][column_names]

n1, n2 = len(df_train1), len(df_train2)
mean1, mean2 = df_train1.mean(), df_train2.mean()
cov1, cov2 = df_train1.cov(), df_train2.cov()
unestablished_estimates = np.linalg.inv((n1*cov1 + n2*cov2) / (n1 + n2 - 2))
unestablished_estimates
coeffs = unestablished_estimates @ (mean1 - mean2)
mean = (np.mean(df_train1.values @ coeffs) + np.mean(df_train2.values @ coeffs)) / 2
```

Провести оценку качества построенного правила.

In [469…

```
classes = (df_train[column_names] @ coeffs - mean) < 0
(classes == df_train['cls']).mean() * 100
```

Out[469…

```
100.0
```

In [470…

```
clf = LogisticRegression(random_state=0, solver='liblinear').fit(df_train[column_names], df_train['cls'].astype(int))
y_test = clf.predict(df_test[column_names])
classes = (df_test[column_names] @ coeffs - mean) < 0
(classes == y_test).mean() * 100
```

Out[470…

```
100.0
```

Провести классификацию 10 новых объектов.

In [471…

```
classes.astype(int)
```

Out[471…

```
index
0    0
1    1
2    1
3    0
4    1
5    0
6    1
7    0
8    1
9    0
dtype: int32
```

Кластерный анализ (p = 8)

In [472…

```
for n_clust in range(2, 10):
    data = whiten(df_train[column_names].values.astype(float))
    centroids, mean_value = kmeans(data, n_clust)
```

```
    clusters, distances = vq(data, centroids)
    print(f'{n_clust = }, {silhouette_score(data, clusters) = }')
```

```
n_clust = 2, silhouette_score(data, clusters) = 0.41690851141777685
n_clust = 3, silhouette_score(data, clusters) = 0.34993259277881567
n_clust = 4, silhouette_score(data, clusters) = 0.3151807648849406
n_clust = 5, silhouette_score(data, clusters) = 0.3267105257729622
n_clust = 6, silhouette_score(data, clusters) = 0.3110498224700978
n_clust = 7, silhouette_score(data, clusters) = 0.3187604541806587
n_clust = 8, silhouette_score(data, clusters) = 0.26079103202342324
n_clust = 9, silhouette_score(data, clusters) = 0.29252784100360046
```

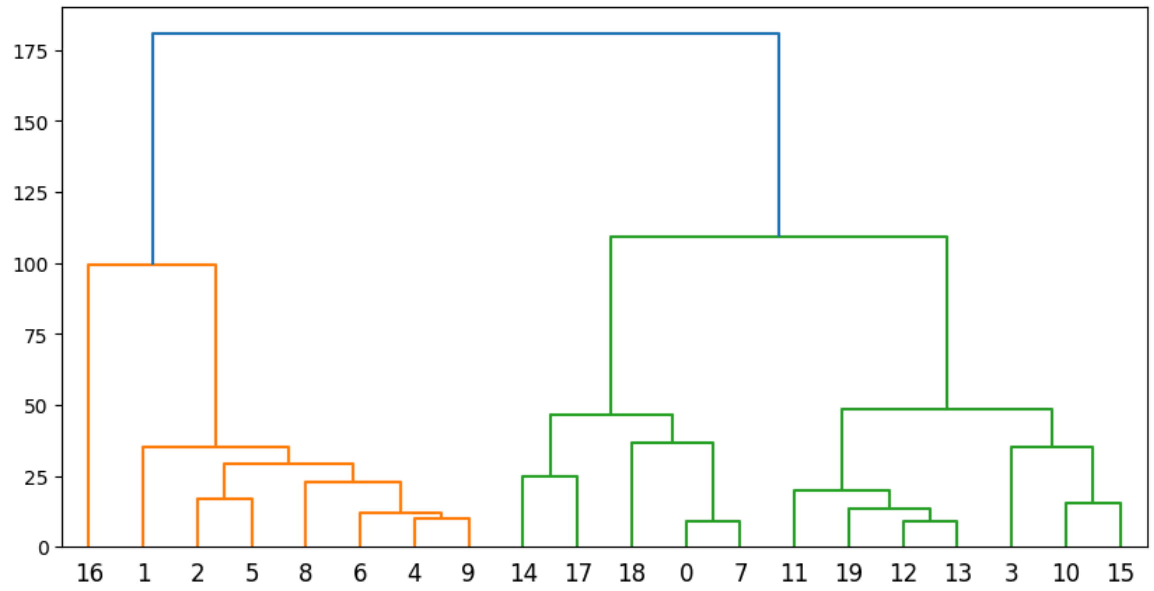
In [473…
```
data = whiten(df_train[column_names].values.astype(float))
centroids, mean_value = kmeans(data, 2)
100 - (vq(data, centroids)[0::-1] == df_train['cls']).mean() * 100
```

Out[473… 100.0

In [474…
```
data = whiten(df_test[column_names].values.astype(float))
centroids, mean_value = kmeans(data, 2)
(vq(data, centroids)[0::-1] == y_test).mean() * 100
```

Out[474… 100.0

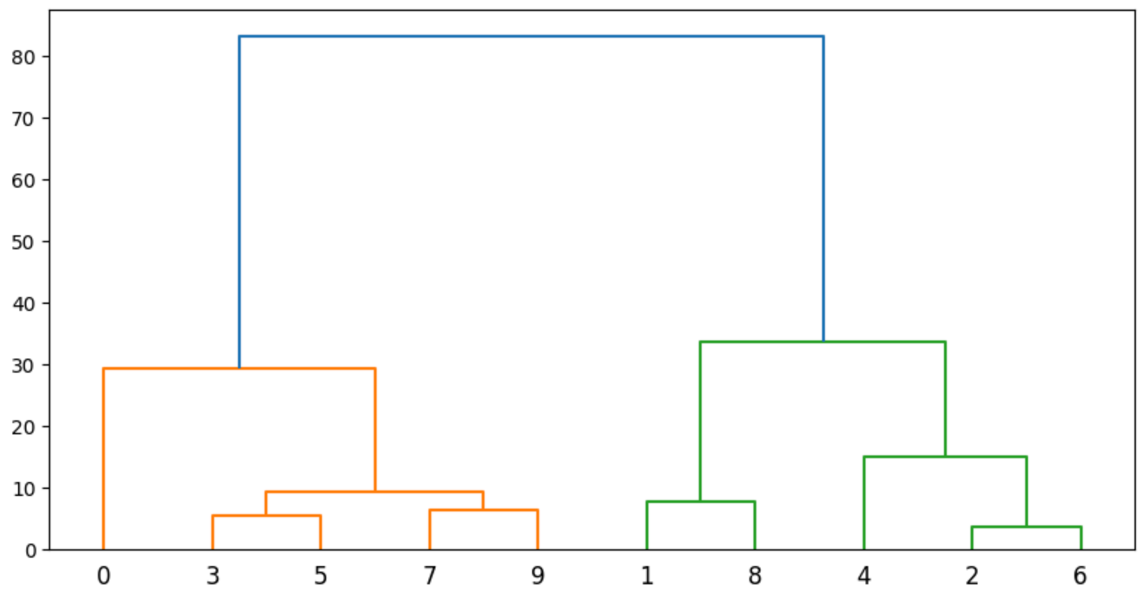
In [475…
```
Z = linkage(df_train[column_names], method='ward')
fig = plt.figure(figsize=(10, 5))
dn = dendrogram(Z)
```



In [476…
```
clusters = fcluster(Z, 120, criterion="distance") - 1
(clusters == df_train['cls']).mean() * 100
```

Out[476… 80.0

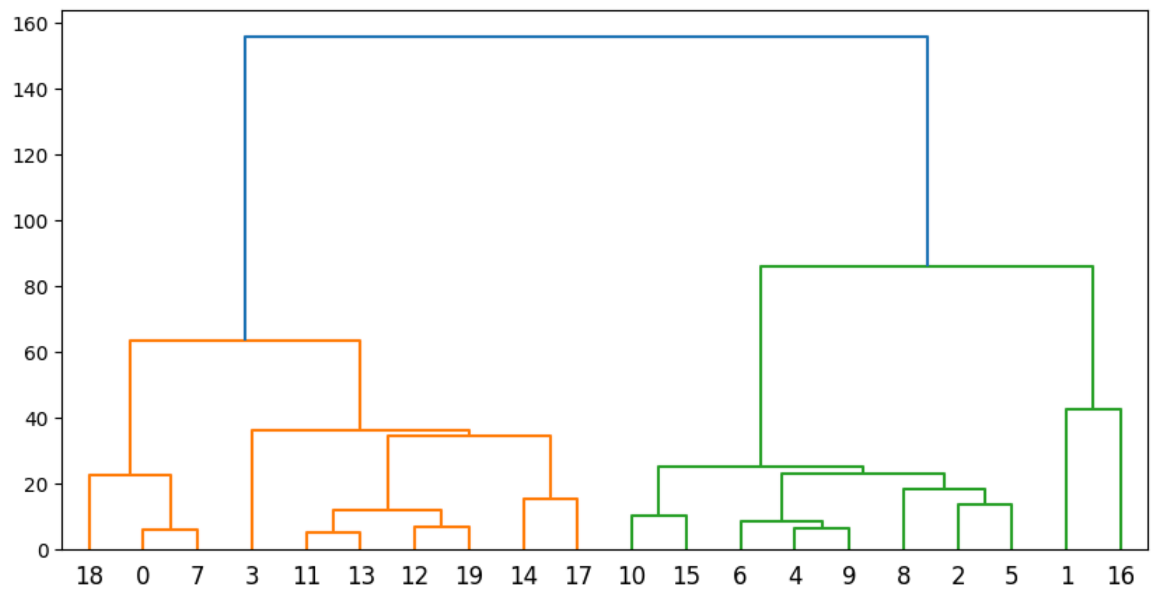
In [477…
```
Z = linkage(df_test[column_names], method='ward')
fig = plt.figure(figsize=(10, 5))
dn = dendrogram(Z)
```



In [478…
```
clusters = fcluster(Z, 60, criterion="distance") - 1
(clusters == y_test).mean() * 100
```

Out[478… 100.0

In [479…
```
Z = linkage(df_train[column_names], method='complete', metric='chebyshev')
fig = plt.figure(figsize=(10, 5))
dn = dendrogram(Z)
```



In [480…
```
clusters = fcluster(Z, 120, criterion="distance") - 1
100 - (clusters == df_train['cls']).mean() * 100
```

Out[480…
```
70.0
```

Определить характеристики случайных величин, представляющих полученные классы. Построить регрессионную модель для каждого кластера.

In [481…
```
get_info(df_train[column_names][clusters == 0])
```

Out[481…

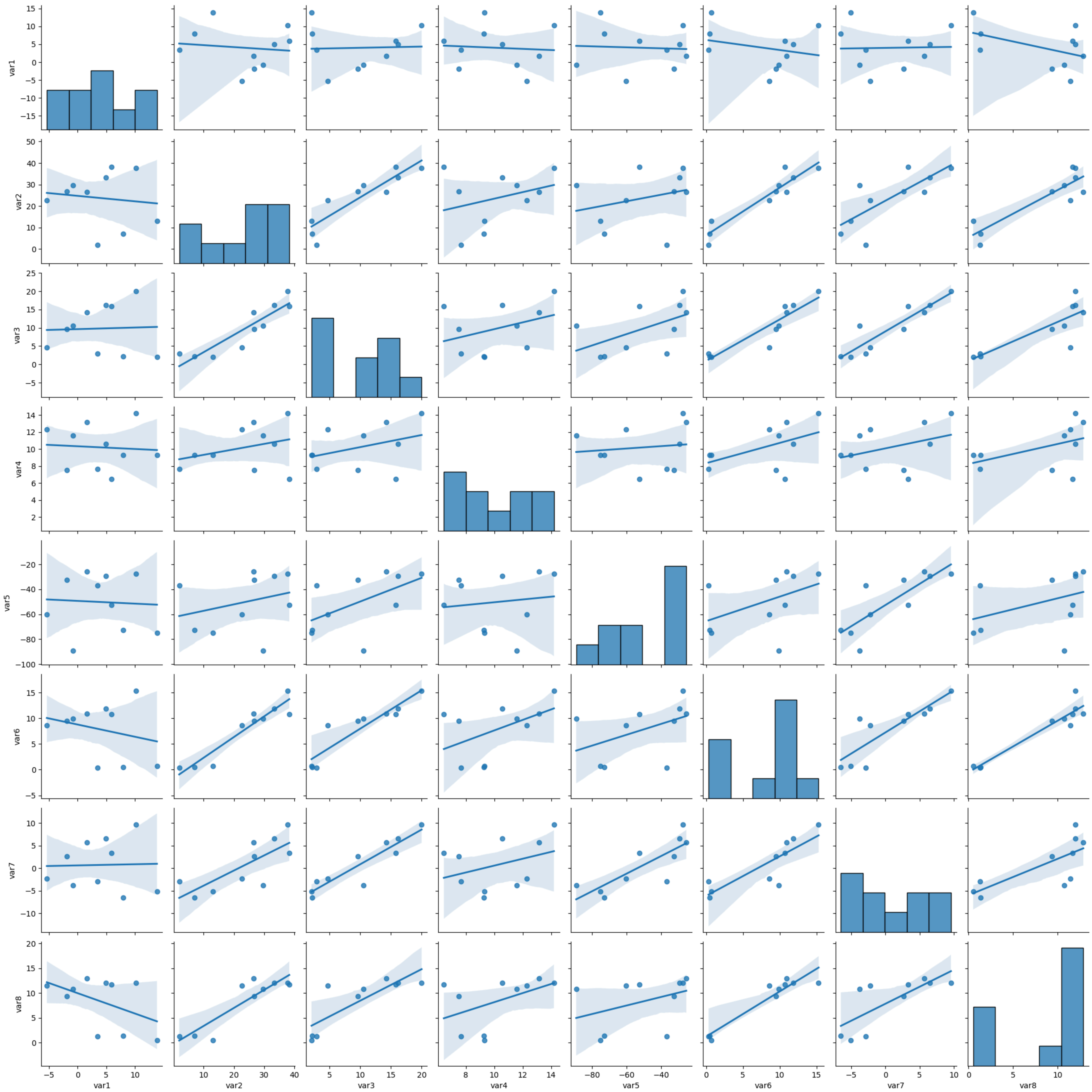
	count	nunique	mean	range	std	varience	min	25%	50%	75%	max	skew	kurt	sem	sum	mode1	mode2	mode3
var2	10.0	10.0	23.7017	36.261	12.549477	157.489362	2.002	15.45800	26.6185	32.41825	38.263	-0.635882	-0.772091	3.968493	237.017	2.002	7.038	13.054
var3	10.0	10.0	9.8095	17.92	6.607734	43.662145	2.072	3.32125	10.0800	15.41475	19.992	0.111321	-1.582339	2.089549	98.095	2.072	2.152	2.87
var4	10.0	10.0	10.1983	7.748	2.592924	6.723256	6.457	8.06250	9.9505	12.10550	14.205	0.09058	-1.210059	0.819955	101.983	6.457	7.489	7.656
var5	10.0	10.0	-50.1715	63.572	23.092027	533.241711	-89.128	-69.67375	-44.7620	-30.16825	-25.556	-0.493144	-1.340762	7.30234	-501.715	-89.128	-75.047	-72.814
var6	10.0	10.0	7.8300	14.954	5.355895	28.685612	0.339	2.68625	9.6765	10.88375	15.293	-0.565641	-1.110841	1.693683	78.3	0.339	0.48	0.728
var7	10.0	10.0	0.7197	16.136	5.545143	30.748615	-6.536	-3.54575	0.1780	5.10475	9.600	0.242707	-1.425936	1.753528	7.197	-6.536	-5.127	-3.765
var8	10.0	10.0	8.3299	12.387	5.111882	26.131342	0.520	3.35700	11.1020	11.90950	12.907	-0.925029	-1.263231	1.616519	83.299	0.52	1.273	1.354
var1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	-5.346	-1.843	-0.776

In [482…
```
get_info(df_train[column_names][clusters == 1])
```

Out[482…

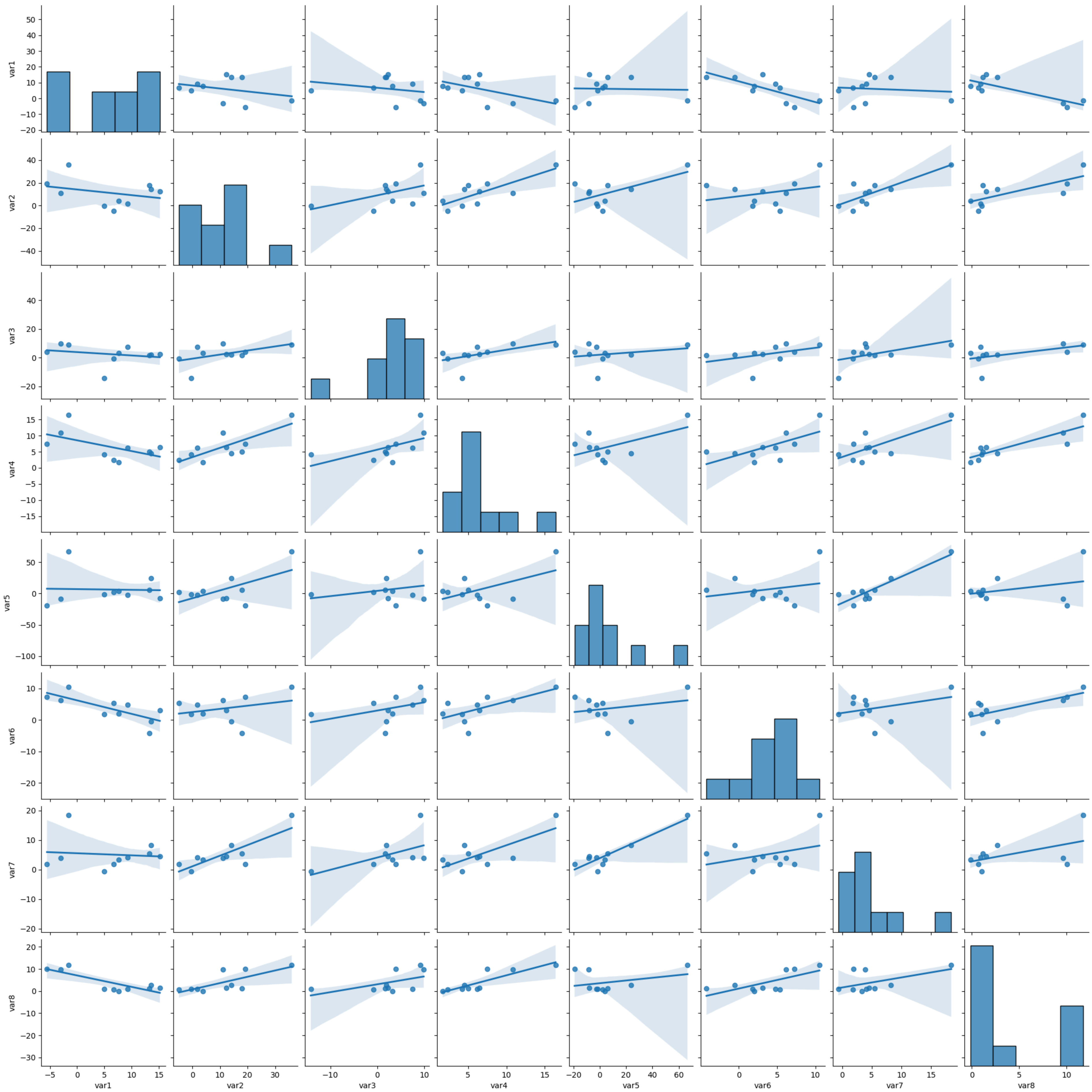
	count	nunique	mean	range	std	varience	min	25%	50%	75%	max	skew	kurt	sem	sum	mode1	mode2	mode3
var2	10.0	10.0	11.0911	40.965	11.906335	141.760805	-4.922	2.22150	11.6425	17.01725	36.043	0.78915	0.961164	3.765114	110.911	-4.922	-0.397	1.674
var3	10.0	10.0	2.4566	24.034	6.793744	46.154958	-14.155	1.75250	2.7415	6.60300	9.879	-1.676318	4.053816	2.14837	24.566	-14.155	-0.852	1.687
var4	10.0	10.0	6.5101	14.717	4.337650	18.815212	1.683	4.29425	5.5960	7.20050	16.400	1.431067	2.318794	1.371686	65.101	1.683	2.361	4.212
var5	10.0	10.0	6.2088	85.714	24.000209	576.010028	-19.132	-6.62400	0.0605	5.38850	66.582	2.027887	4.800254	7.589532	62.088	-19.132	-8.632	-7.98
var6	10.0	10.0	3.6200	14.735	4.176733	17.445102	-4.215	1.83375	3.9500	5.95725	10.520	-0.312198	0.38688	1.320799	36.2	-4.215	-0.557	1.782
var7	10.0	10.0	5.1148	19.065	5.238856	27.445607	-0.630	2.27475	3.9850	5.27400	18.435	2.055175	5.190472	1.656672	51.148	-0.63	1.861	1.929
var8	10.0	10.0	3.9452	11.935	4.613716	21.286377	-0.156	0.96625	1.3575	7.92525	11.779	0.998173	-1.019426	1.458985	39.452	-0.156	0.683	0.943
var1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	-5.536	-3.056	-1.544

In [483…
```
g = sns.pairplot(df_train[column_names][clusters == 0].astype(float), kind="reg")
```



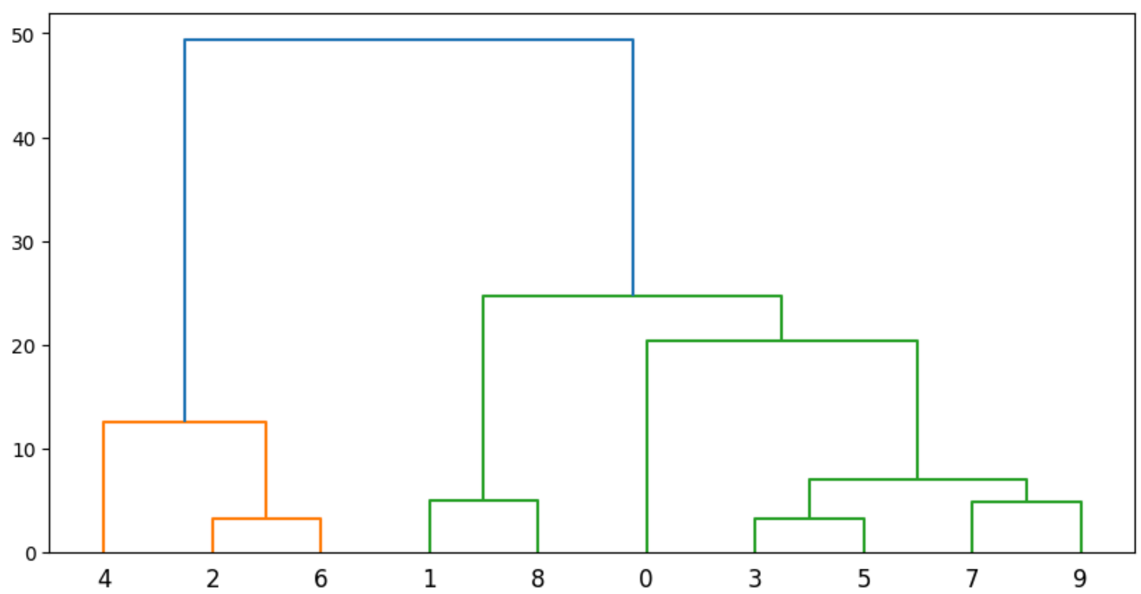
In [484…

```
g = sns.pairplot(df_train[column_names][clusters == 1].astype(float), kind="reg")
```



In [485…

```
Z = linkage(df_test[column_names], method='complete', metric='chebyshev')
fig = plt.figure(figsize=(10, 5))
dn = dendrogram(Z)
```



In [486…

```
clusters = fcluster(Z, 40, criterion="distance") - 1
100 - (clusters == y_test).mean() * 100
```

Out[486…

```
80.0
```

Снижение размерности. Метод главных компонент (p = 8; p′ = 2)

In [487…

```
stand_scaler = StandardScaler()
stand_scaler = stand_scaler.fit(df_train[column_names])
df_train_norm = stand_scaler.transform(df_train[column_names])
```

```
pca = PCA(n_components=2)
pca.fit(df_train_norm)
df_train2c = pd.DataFrame(
    np.column_stack((pca.transform(df_train_norm), df_train['cls'])
              ), columns=['var1', 'var2', 'cls'])
df_train2c
```

Out[487…

	var1	var2	cls
0	-1.788326	-1.658465	0
1	-1.762518	1.957775	0
2	-1.571388	0.892515	0
3	-1.782689	-0.973243	0
4	-1.346723	0.72403	0
5	-2.129626	1.313295	0
6	-2.363677	0.525615	0
7	-1.74808	-1.995316	0
8	-3.049508	0.090103	0
9	-2.276803	0.835273	0
10	0.945462	0.01448	1
11	3.400855	0.666237	1
12	2.517775	-0.028337	1
13	2.489414	0.172011	1
14	1.96083	-0.533896	1
15	0.718986	-0.434227	1
16	3.224902	3.187631	1
17	1.470179	-1.85029	1
18	1.799389	-2.421098	1
19	1.291547	-0.48409	1

In [488…

```
pca.components_.T
```

Out[488…

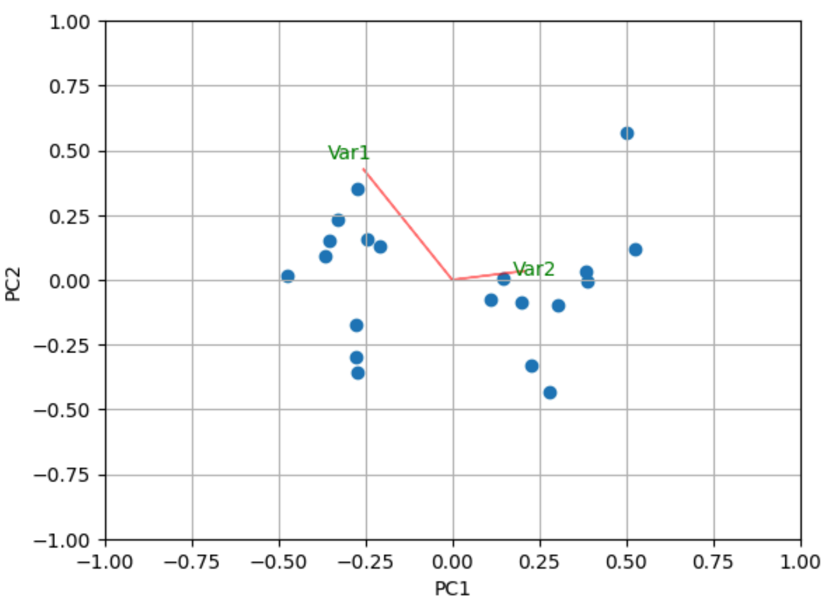
```
array([[-0.25417969,  0.20518878],
       [ 0.42473553,  0.03368089],
       [ 0.40039205, -0.00075756],
       [ 0.3862074 , -0.07458999],
       [-0.04785591,  0.71031922],
       [ 0.43131966, -0.0326741 ],
       [ 0.21255472,  0.66506489],
       [ 0.46045208, -0.05715437]])
```

In [489…

```
def myplot(score,coeff,labels=None):
    xs = score[:,0]
    ys = score[:,1]
    n = coeff.shape[0]
    scalex = 1.0/(xs.max() - xs.min())
    scaley = 1.0/(ys.max() - ys.min())
    plt.scatter(xs * scalex,ys * scaley)
    for i in range(n):
        plt.arrow(0, 0, coeff[i,0], coeff[i,1],color = 'r',alpha = 0.5)
        if labels is None:
            plt.text(coeff[i,0]* 1.15, coeff[i,1] * 1.15, "Var"+str(i+1), color = 'g', ha = 'center', va = 'center')
        else:
            plt.text(coeff[i,0]* 1.15, coeff[i,1] * 1.15, labels[i], color = 'g', ha = 'center', va = 'center')
    plt.xlim(-1,1)
    plt.ylim(-1,1)
    plt.xlabel("PC{}".format(1))
    plt.ylabel("PC{}".format(2))
    plt.grid()
```

In [490…

```
myplot(df_train2c[['var1', 'var2']].values, pca.components_)
```



In [491…

```
get_info(df_train2c[['var1', 'var2']].astype(float))
```

Out[491…

	count	nunique	mean	range	std	varience	min	25%	50%	75%	max	skew	kurt	sem	sum	mode1	mode2	mode3
var1	20.0	20	2.220446e-17	6.450363	2.155190	4.644846	-3.049508	-1.784098	-0.313868	1.839749	3.400855	0.190587	-1.63687	0.481915	-4.440892e-16	-3.049508	-2.363677	-2.276803
var2	20.0	20	-3.330669e-17	5.608730	1.379706	1.903589	-2.421098	-0.643733	0.052292	0.751841	3.187631	0.222154	0.31128	0.308512	-6.661338e-16	-2.421098	-1.995316	-1.850290

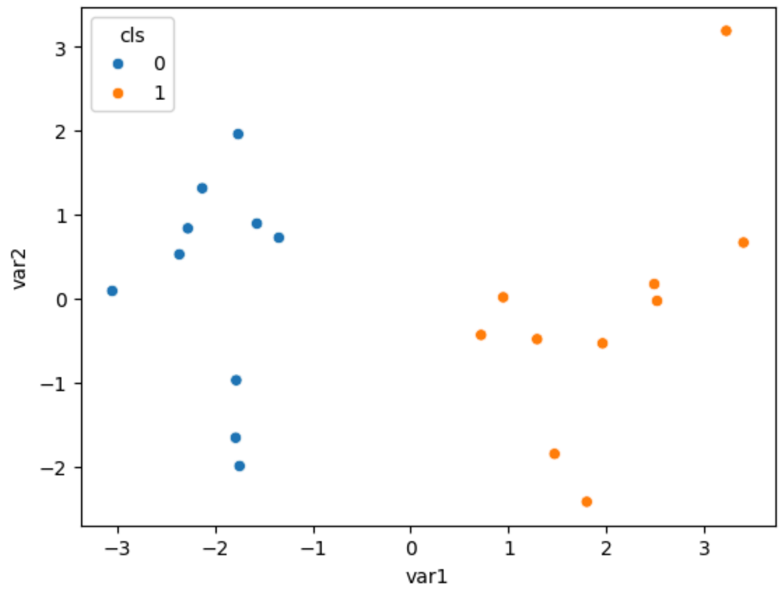
Провести визуальную классификацию данных, спроектированных в пространство первых двух главных компонент.

In [492…

```
sns.scatterplot(data=df_train2c, x='var1', y='var2', hue='cls')
```

Out[492…

```
<Axes: xlabel='var1', ylabel='var2'>
```



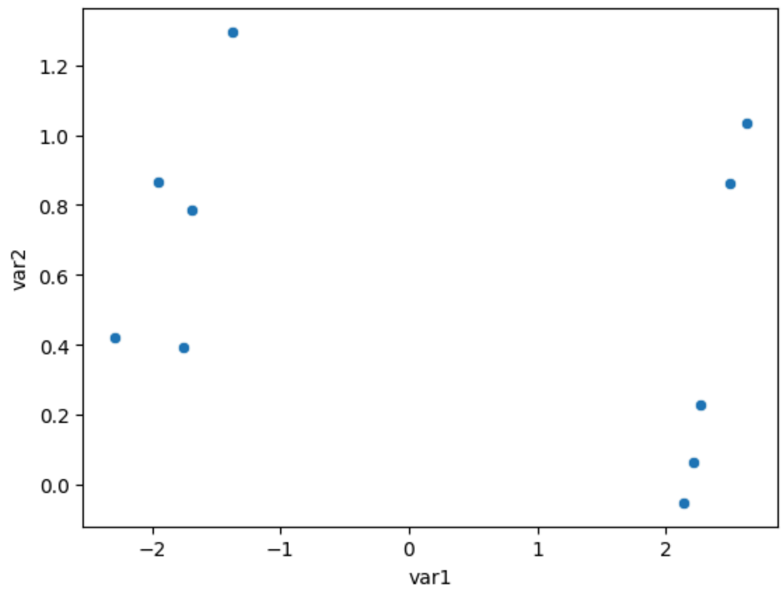
In [493…
```
df_test2c = pd.DataFrame(pca.transform(
    stand_scaler.transform(df_test[column_names])
    ), columns=['var1', 'var2'])
df_test2c
```

Out[493…

	var1	var2
0	-1.369068	1.295282
1	2.626671	1.036150
2	2.218836	0.065109
3	-2.291303	0.421593
4	2.141711	-0.052283
5	-1.754328	0.393858
6	2.271445	0.228493
7	-1.949557	0.865070
8	2.497815	0.863293
9	-1.687453	0.785310

In [494…
```
sns.scatterplot(data=df_test2c, x='var1', y='var2')
```

Out[494…
```
&lt;Axes: xlabel='var1', ylabel='var2'&gt;
```



Какую часть дисперсии объясняют эти компоненты.

In [495…
```
pca = PCA(n_components=8)
pca.fit(df_train_norm)
pca.explained_variance_ratio_[:2].sum()
```

Out[495…
```
0.7776266385028869
```

Сколько компонент необходимо для объяснения 90% дисперсии (p'-4).

In [496…
```
pca = PCA(n_components=8)
pca.fit(df_train_norm)
#print(pca.explained_variance_ratio_[:8])
for i in range(1,len(pca.explained_variance_ratio_)+1):
    print(pca.explained_variance_ratio_[:i].sum())
    if(pca.explained_variance_ratio_[:i].sum() > 0.9):
        break
```

```
0.551575456337787
0.7776266385028869
0.8939441951005733
0.9456067017407405
```

In [ ]: