

# CSC 3210

## Computer Organization and Programming

### Chapter 1 Basic Concepts

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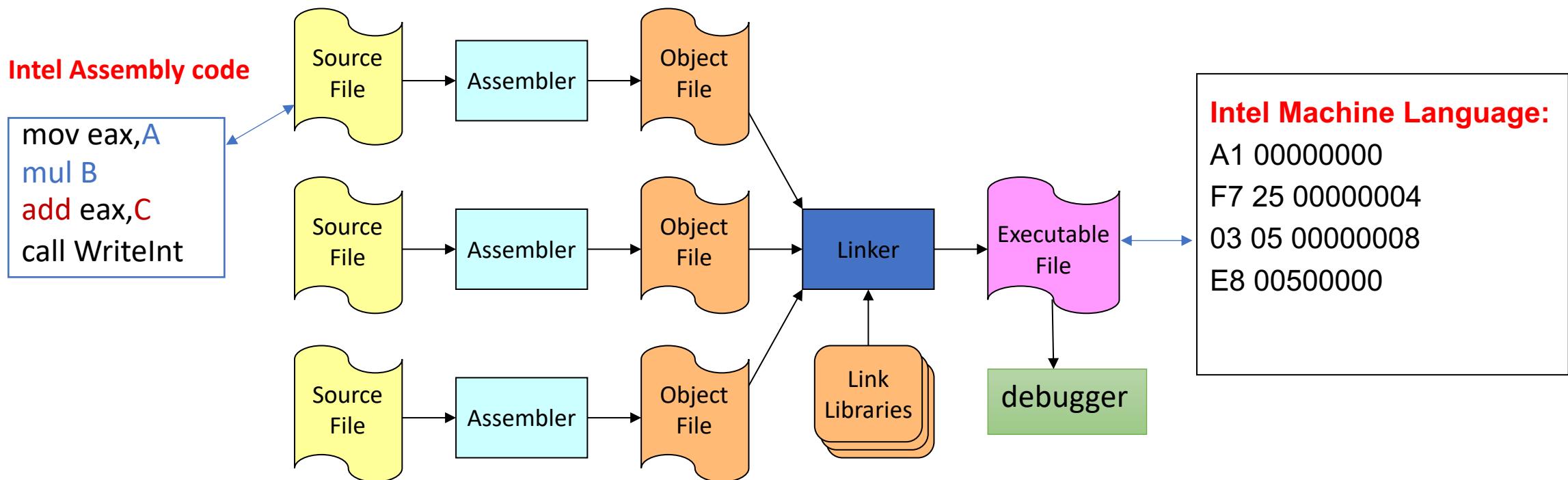
# Outline

- Welcome to Assembly Language
  - Some concerns
  - Applications
- Virtual Machine Concepts
  - Virtual Machines
  - Machine Levels
  - Translating Language
- Data Representation
- Boolean Operations

# Welcome to Assembly language!

Programming is fun!

# What is an assembler, a linker, a debugger?



# Some issues

- What **hardware/software** do I need?
  - Computer 32/64 bit
  - Windows + Microsoft Visual Studio
  - Mac + Virtualization software (Virtualbox) + Windows + Microsoft Visual Studio
- What **types of programs** will I create?
  - Mainly **32-Bit**, Some 64 bit
- What do I get with this book?
  - <http://www.asmirvine.com/>

# More issues

- How does **assembly language** (AL) relate to **machine language**?
  - One-to-one relationship
- How do **C/C++** relates to **AL**?

C/C++

```
int Y;  
int X = (Y + 4) * 3;
```

Assembly

mov eax,Y ;	- move Y to the EAX register
add eax,4 ;	- add 4 to the EAX register
mov ebx,3 ;	- move 3 to the EBX register
imul ebx ;	- multiply EAX by EBX
mov X,eax ;	- move EAX to X

- Is **AL** portable? No. Why?

# Assembly Language Applications

- Some representative types of applications:
  - Hardware device driver
  - Embedded systems & computer games
  - Business application for single platform (No)
  - Business application for multiple platforms (No)

# Think again!

Is the assembly language for x86 processors the same as those for computer systems such as the Vax, Motorola 68x00, or SPARC?

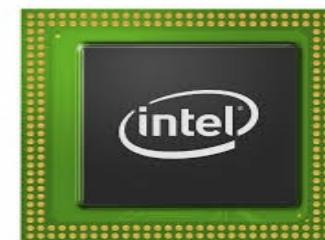
## SPARC

```
mov 5,r1  
mov 6,r2  
add r1,r2,r3
```



## Intel

```
mov eax,5  
add eax,6
```



# What's Next

- - Welcome to Assembly Language
  - Some Good Questions to Ask
  - Assembly Language Applications
- - **Virtual Machine Concept**
  - Virtual Machines
  - Specific Machine Levels
  - Translating Languages
- - Data Representation
- - Boolean Operations

# Virtual Machine Concepts

Abstraction that hides all the low level complexities

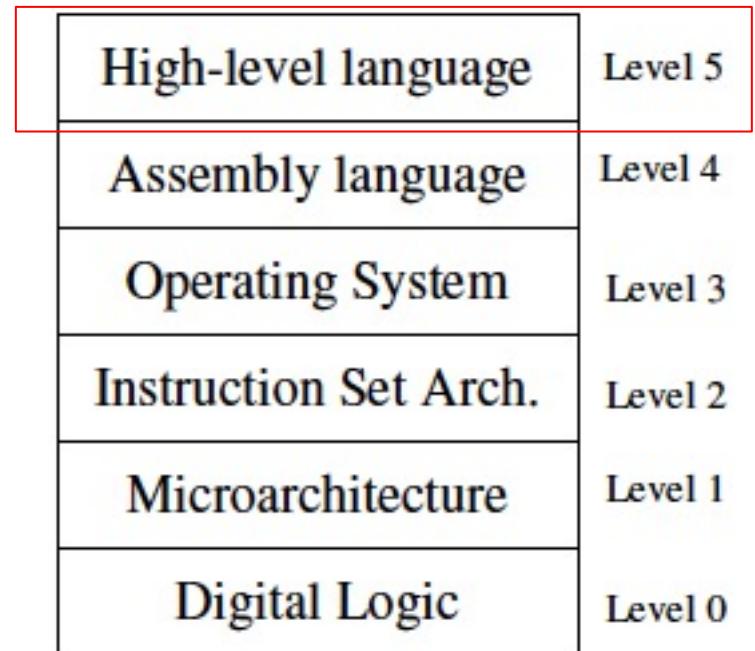
# Specific Machine Levels

- **Virtual Machine Concept**
  - Explain how a computer's hardware and software are related.
  - The main thing a computer does is executing programs
  - A computer execute programs written in its **native machine language (ML)**.
    - difficult to write programs in ML
      - What to do?
    - programs could be written in another language  
(Interpretation vs. Translation)

High-level language	Level 5
Assembly language	Level 4
Operating System	Level 3
Instruction Set Arch.	Level 2
Microarchitecture	Level 1
Digital Logic	Level 0

# Specific Machine Levels: High-Level Language

- **Level 5**
- Application-oriented languages
  - C++, Java, Visual Basic . . .
- Programs compile into assembly language (Level 4)

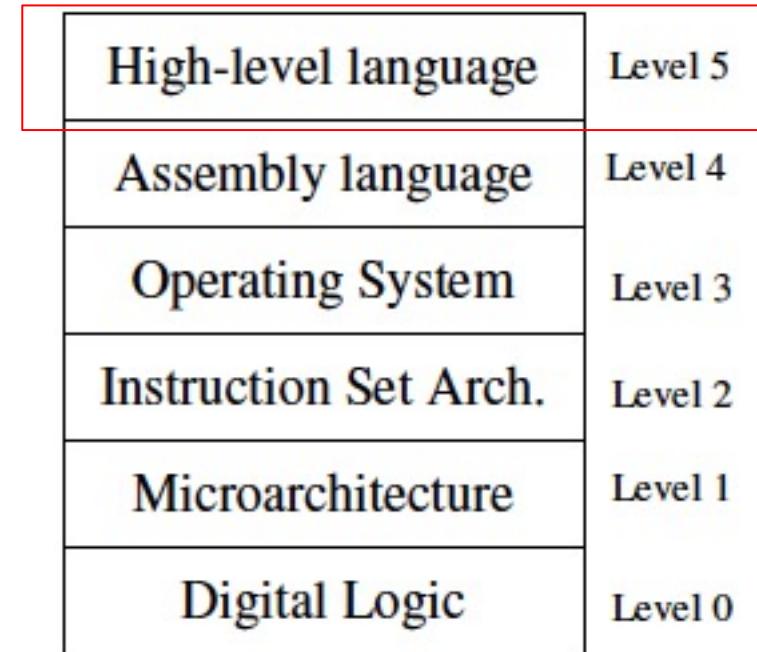
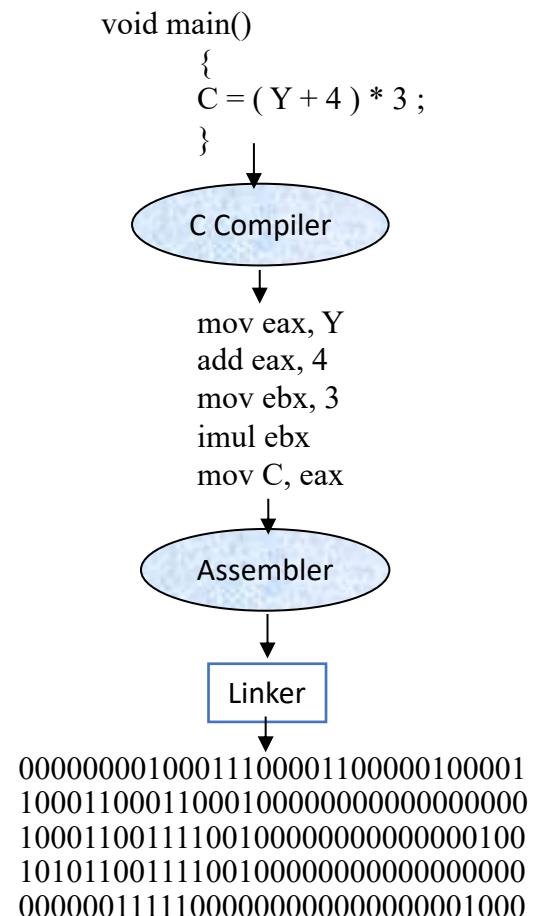


# Specific Machine Levels: High-Level Language

## *High-level language program (in C)*

## *Assembly language program*

## *Binary machine language program*



# What's Next

- - Welcome to Assembly Language
  - Some Good Questions to Ask
  - Assembly Language Applications
- - **Virtual Machine Concept**
  - Virtual Machines
  - Specific Machine Levels
  - Translating Languages
- - **Data Representation**
- - Boolean Operations

# Data Representation

Lots of math!

# Data Representation

- **Binary Numbers**
  - Translating between binary and decimal
- **Binary Addition**
- Integer Storage Sizes
- **Hexadecimal Integers**
  - Translating between decimal and hexadecimal
  - Hexadecimal subtraction
- **Signed Integers**
  - Binary subtraction
- **Character Storage**

# Data Representation: Numbering System

- Assembly language deals with **Data at the physical level**
  - so you need to examine registers and memory
- Binary and hexadecimal numbers are commonly used to describe those contents (other systems used as well)
- **Need to learn how to translate from one format to another**

Table 1-2 Binary, Octal, Decimal, and Hexadecimal Digits.

System	Base	Possible Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

Base: maximum number of symbols assigned to every digit

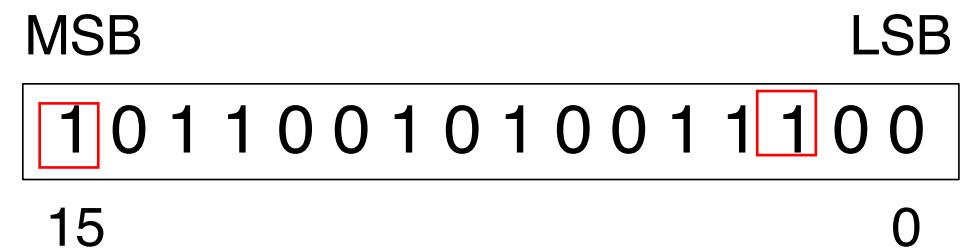


# Data Representation: Binary Numbers (**Integers**)

- Binary integers can be **signed** or **unsigned**.
  - A **signed** integer is **positive** or **negative**.
  - An **unsigned** integer is by default **positive**.
    - **Zero** is considered **positive**.

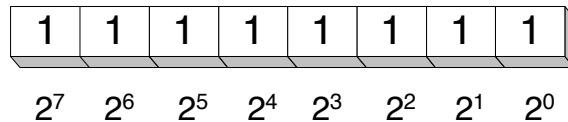
# Data Representation: Binary Numbers (**Integers**)

- Digits are 1 and 0
  - 1 = true
  - 0 = false
- **MSB** – most significant bit
- **LSB** – least significant bit
- **Bit numbering:**



# Data Representation: Binary Numbers (Integers)

- Each digit (bit) is either 1 or 0
- Each bit represents **a power of 2**:



**Table 1-3** Binary Bit Position Values.

$2^n$	Decimal Value	$2^n$	Decimal Value
$2^0$	1	$2^8$	256
$2^1$	2	$2^9$	512
$2^2$	4	$2^{10}$	1024
$2^3$	8	$2^{11}$	2048
$2^4$	16	$2^{12}$	4096
$2^5$	32	$2^{13}$	8192
$2^6$	64	$2^{14}$	16384
$2^7$	128	$2^{15}$	32768

Every binary number is a sum of powers of 2

# Data Representation: Translating Binary to Decimal

- Weighted positional notation shows how to calculate the decimal value of each binary bit:

$$dec = (D_{n-1} \times 2^{n-1}) + (D_{n-2} \times 2^{n-2}) + \dots + (D_1 \times 2^1) + (D_0 \times 2^0)$$

D = binary digit

**binary 00001001 = decimal 9:**

$$(1 \times 2^3) + (1 \times 2^0) = 9$$

# Data Representation: Translating Binary to Decimal

# Data Representation: Translating Unsigned Decimal to Binary

- Repeatedly divide the decimal integer by 2.
- Each remainder is a binary digit in the translated value:

$$37 = 100101$$

Division	Quotient	Remainder
$37 / 2$	18	1
$18 / 2$	9	0
$9 / 2$	4	1
$4 / 2$	2	0
$2 / 2$	1	0
$1 / 2$	0	1

# Data Representation: Binary Addition

- Starting with the LSB, add each pair of digits,  
**include the carry if present.**

A binary number represented as a sequence of 16 bits: 10110010100011100. The most significant bit (MSB) is at the top left, and the least significant bit (LSB) is at the bottom right. Below the number, the value 15 is aligned under the first four bits, and the value 0 is aligned under the last four bits.

	carry:	1						
0	0	0	0					
+	0	0	0					
<hr/>								
0	0	0	0					
1	0	1	1					
(4)	(7)	(11)						
position:	7	6	5	4	3	2	1	0

# Data Representation: Integer Storage Sizes

- Standard sizes (**x86**):

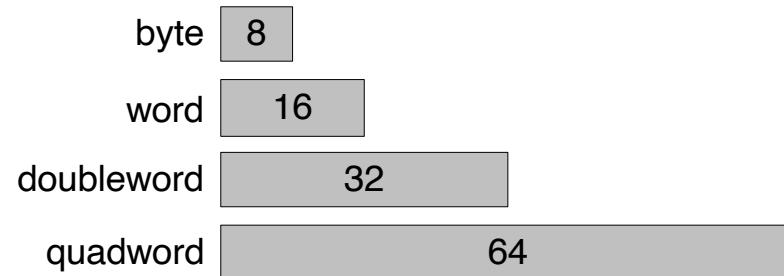


Table 1-4 Ranges and Sizes of Unsigned Integer Types.

Type	Range	Storage Size in Bits
Unsigned byte	0 to $2^8 - 1$	8
Unsigned word	0 to $2^{16} - 1$	16
Unsigned doubleword	0 to $2^{32} - 1$	32
Unsigned quadword	0 to $2^{64} - 1$	64
Unsigned double quadword	0 to $2^{128} - 1$	128

What is the largest unsigned integer that may be stored in 20 bits?



# Data Representation: **Hexadecimal** Integers

- Binary values are represented in hexadecimal (Why).
  - Large binary numbers are hard to read

**Table 1-5** Binary, Decimal, and Hexadecimal Equivalents.

Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	10	A
0011	3	3	1011	11	B
0100	4	4	1100	12	C
0101	5	5	1101	13	D
0110	6	6	1110	14	E
0111	7	7	1111	15	F

# Data Representation: Translating Binary to Hexadecimal

- Each hexadecimal digit corresponds to **4 binary bits** (Why?).
- **Example:** Translate the binary integer

00010110101001110010100

to hexadecimal:

1	6	A	7	9	4
0001	0110	1010	0111	1001	0100

# Data Representation: Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16:

$$\text{dec} = (D_3 \times 16^3) + (D_2 \times 16^2) + (D_1 \times 16^1) + (D_0 \times 16^0)$$

- Hex 1234 equals  $(1 \times 16^3) + (2 \times 16^2) + (3 \times 16^1) + (4 \times 16^0)$ , or decimal 4,660.
- Hex 3BA4 equals  $(3 \times 16^3) + (11 * 16^2) + (10 \times 16^1) + (4 \times 16^0)$ , or decimal 15,268.

# Data Representation: Converting Hexadecimal to Decimal

-

# Data Representation: Converting Decimal to Hexadecimal

Division	Quotient	Remainder
$422 / 16$	26	6
$26 / 16$	1	A
$1 / 16$	0	1

decimal 422 = 1A6 hexadecimal

# Data Representation: **Hexadecimal Addition**

- Divide the sum of two digits by the number base (16). The quotient becomes the carry value, and the remainder is the sum digit.

$$\begin{array}{r} 36 & 28 & 28 & 6A \\ 42 & 45 & 58 & 4B \\ \hline 78 & 6D & 80 & B5 \end{array}$$

↑  
21 / 16 = 1, rem 5

Important skill: Programmers frequently add and subtract the addresses of variables and instructions.

# Data Representation: **Hexadecimal Subtraction**

- When a borrow is required from the digit to the left, **add 16 (decimal) to the current digit's value:**

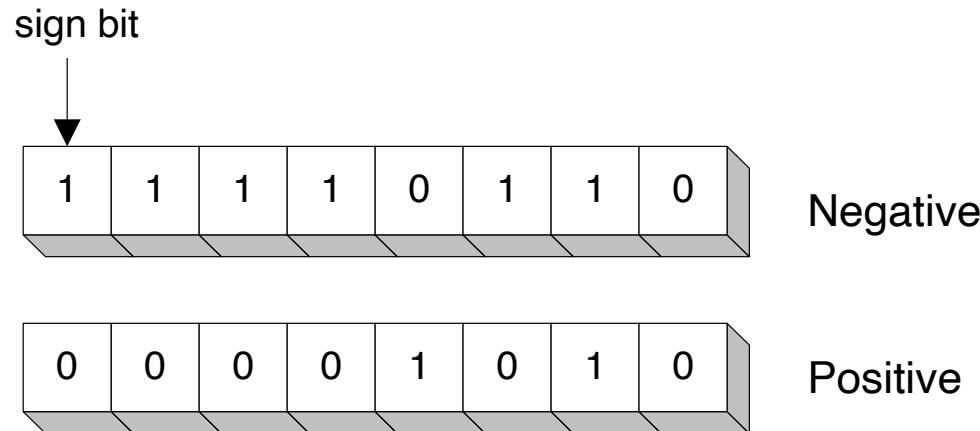
$$\begin{array}{r} 16 + 5 = 21 \\ \downarrow \\ \begin{array}{r} C6 \\ A2 \\ \hline 24 \end{array} \quad \begin{array}{r} 75 \\ 47 \\ \hline 2E \end{array} \\ -1 \end{array}$$

# Data Representation: **Signed** Integers

- Challenges -
  - Dealing with both positive and negative numbers
  - Simplify the Circuit design

# Data Representation: **Signed** Integers

- The highest bit indicates the sign. 1 = negative, 0 = positive



If the highest digit of a hexadecimal integer is > 7, the value is negative. Examples: 8A, C5, A2, 9D

# Data Representation: Forming the **Two's Complement**

- Negative integer numbers are stored in two's complement notation, (Why?)
  - Removes the need to separate digital circuits for addition and subtraction
- When subtracting  $A - B$ , processor will convert it to an addition expression

$$A + (-B)$$



# Data Representation: Forming the Two's Complement

- Negative integer numbers are stored in two's complement notation
- Represents the additive Inverse

Starting value	00000001
Step 1: reverse the bits	11111110
Step 2: add 1 to the value from Step 1	11111110 +00000001
Sum: two's complement representation	11111111

## Algorithm

- **First:** represent the negative number in binary
- **Second:** do One's complement:  
Replace every 0 with a 1,  
and replace every 1 with a 0
- **Third:** do Two's complement: add 1
- **Fourth:** represent the second number in binary
- **Fifth:** add the two numbers

Note that  $00000001 + 11111111 = 00000000$



# Data Representation: Binary Subtraction

- When subtracting  $A - B$ , convert  $B$  to **its two's complement**

**$A + (-B)$**

A: 0 0 0 0 1 1 0 0      

B: (-) 0 0 0 0 0 0 1 1

A: 0 0 0 0 1 1 0 0

2's complement of B: (+) 1 1 1 1 1 1 0 1

(where is the carry?)

0 0 0 0 1 0 0 1

# Data Representation: Binary Subtraction

- What if the result is a negative number?

Consider  $2 - 4$ :

$$2 = 0010$$

$$4 = 0100$$

$$\text{one's complement 1's} = 1011$$

$$\text{two's complement 2's} = 1100$$

$$\begin{array}{r} 0010 \\ 1100 \\ \hline 1110 \end{array} +$$

1110 must be a negative number, so let us complement it:

- convert the  
negative number  
to its positive  
equivalent

$$\begin{array}{r} 1110 \\ 1's \quad 0001 \\ 2's \quad 0010 \quad -2 \end{array}$$

# Data Representation: Learn How To Do the Following:

- Form the two's complement of a hexadecimal integer (page 16)
- Convert signed binary to decimal (page 17)
- Convert signed decimal to binary (page 17)
- Convert signed decimal to hexadecimal (page 17)
- Convert signed hexadecimal to decimal (page 17)

You must know these conversions!

# Data Representation: Ranges of **Signed Integers**

- The **highest bit is reserved for the sign**. This limits the range:

Storage Type	Range (low–high)	Powers of 2
Signed byte	-128 to +127	$-2^7$ to $(2^7 - 1)$
Signed word	-32,768 to +32,767	$-2^{15}$ to $(2^{15} - 1)$
Signed doubleword	-2,147,483,648 to 2,147,483,647	$-2^{31}$ to $(2^{31} - 1)$
Signed quadword	-9,223,372,036,854,775,808 to +9,223,372,036,854,775,807	$-2^{63}$ to $(2^{63} - 1)$

Practice: What is the largest positive value that may be stored in 20 bits?



# Data Representation: Signed Integers

- List all the **Four-bit two's complement** numbers:

- Total numbers can be stored :  $2^4 - 1 = 15$

Binary	Unsigned value	Signed value	Decimal	Bit Pattern
0000	0	0	7	0111
0001	1	1	6	0110
0010	2	2	5	0101
0011	3	3	4	0100
0100	4	4	3	0011
0101	5	5	2	0010
0110	6	6	1	0001
0111	7	7	0	0000
1000	8	-8	-1	1111
1001	9	-7	-2	1110
1010	10	-6	-3	1101
1011	11	-5	-4	1100
1100	12	-4	-5	1011
1101	13	-3	-6	1010
1110	14	-2	-7	1001
1111	15	-1	-8	1000

# Data Representation: **Character Storage**

- Computers only store binary data, **how do they represent characters?**
  - **Character sets (a mapping of characters to integers)**
    - Standard ASCII (0 – 127)
    - Extended ASCII (0 – 255)
    - ANSI (0 – 255)
    - Unicode (0 – 65,535)
- **Null-terminated String**
  - Array of characters followed by a *null byte*
- **Using the ASCII table**
  - ASCII (American Standard Code for Information Interchange) **maps each character to a specific number (decimal, hexadecimal, and bit pattern)**

# ASCII

<u><a href="#">Ads by Google</a></u>				<u><a href="#">ASCII</a></u>				<u><a href="#">Unicode Character</a></u>				<u><a href="#">Engine Codes</a></u>				<u><a href="#">ASCII How To</a></u>			
Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	
0	0 000	000	<b>NUL</b> (null)	32	20 040	&#32;	<b>Space</b>	64	40 100	&#64;	<b>Ø</b>	96	60 140	&#96;	<b>~</b>				
1	1 001	001	<b>SOH</b> (start of heading)	33	21 041	&#33;	<b>!</b>	65	41 101	&#65;	<b>A</b>	97	61 141	&#97;	<b>a</b>				
2	2 002	002	<b>STX</b> (start of text)	34	22 042	&#34;	<b>"</b>	66	42 102	&#66;	<b>B</b>	98	62 142	&#98;	<b>b</b>				
3	3 003	003	<b>ETX</b> (end of text)	35	23 043	&#35;	<b>#</b>	67	43 103	&#67;	<b>C</b>	99	63 143	&#99;	<b>c</b>				
4	4 004	004	<b>EOT</b> (end of transmission)	36	24 044	&#36;	<b>\$</b>	68	44 104	&#68;	<b>D</b>	100	64 144	&#100;	<b>d</b>				
5	5 005	005	<b>ENQ</b> (enquiry)	37	25 045	&#37;	<b>%</b>	69	45 105	&#69;	<b>E</b>	101	65 145	&#101;	<b>e</b>				
6	6 006	006	<b>ACK</b> (acknowledge)	38	26 046	&#38;	<b>&amp;</b>	70	46 106	&#70;	<b>F</b>	102	66 146	&#102;	<b>f</b>				
7	7 007	007	<b>BEL</b> (bell)	39	27 047	&#39;	<b>'</b>	71	47 107	&#71;	<b>G</b>	103	67 147	&#103;	<b>g</b>				
8	8 010	010	<b>BS</b> (backspace)	40	28 050	&#40;	<b>(</b>	72	48 110	&#72;	<b>H</b>	104	68 150	&#104;	<b>h</b>				
9	9 011	011	<b>TAB</b> (horizontal tab)	41	29 051	&#41;	<b>)</b>	73	49 111	&#73;	<b>I</b>	105	69 151	&#105;	<b>i</b>				
10	A 012	012	<b>LF</b> (NL line feed, new line)	42	2A 052	&#42;	<b>*</b>	74	4A 112	&#74;	<b>J</b>	106	6A 152	&#106;	<b>j</b>				
11	B 013	013	<b>VT</b> (vertical tab)	43	2B 053	&#43;	<b>+</b>	75	4B 113	&#75;	<b>K</b>	107	6B 153	&#107;	<b>k</b>				
12	C 014	014	<b>FF</b> (NP form feed, new page)	44	2C 054	&#44;	<b>,</b>	76	4C 114	&#76;	<b>L</b>	108	6C 154	&#108;	<b>l</b>				
13	D 015	015	<b>CR</b> (carriage return)	45	2D 055	&#45;	<b>-</b>	77	4D 115	&#77;	<b>M</b>	109	6D 155	&#109;	<b>m</b>				
14	E 016	016	<b>SO</b> (shift out)	46	2E 056	&#46;	<b>.</b>	78	4E 116	&#78;	<b>N</b>	110	6E 156	&#110;	<b>n</b>				
15	F 017	017	<b>SI</b> (shift in)	47	2F 057	&#47;	<b>/</b>	79	4F 117	&#79;	<b>O</b>	111	6F 157	&#111;	<b>o</b>				
16	10 020	020	<b>DLE</b> (data link escape)	48	30 060	&#48;	<b>Ø</b>	80	50 120	&#80;	<b>P</b>	112	70 160	&#112;	<b>p</b>				
17	11 021	021	<b>DC1</b> (device control 1)	49	31 061	&#49;	<b>1</b>	81	51 121	&#81;	<b>Q</b>	113	71 161	&#113;	<b>q</b>				
18	12 022	022	<b>DC2</b> (device control 2)	50	32 062	&#50;	<b>2</b>	82	52 122	&#82;	<b>R</b>	114	72 162	&#114;	<b>r</b>				
19	13 023	023	<b>DC3</b> (device control 3)	51	33 063	&#51;	<b>3</b>	83	53 123	&#83;	<b>S</b>	115	73 163	&#115;	<b>s</b>				
20	14 024	024	<b>DC4</b> (device control 4)	52	34 064	&#52;	<b>4</b>	84	54 124	&#84;	<b>T</b>	116	74 164	&#116;	<b>t</b>				
21	15 025	025	<b>NAK</b> (negative acknowledge)	53	35 065	&#53;	<b>5</b>	85	55 125	&#85;	<b>U</b>	117	75 165	&#117;	<b>u</b>				
22	16 026	026	<b>SYN</b> (synchronous idle)	54	36 066	&#54;	<b>6</b>	86	56 126	&#86;	<b>V</b>	118	76 166	&#118;	<b>v</b>				
23	17 027	027	<b>ETB</b> (end of trans. block)	55	37 067	&#55;	<b>7</b>	87	57 127	&#87;	<b>W</b>	119	77 167	&#119;	<b>w</b>				
24	18 030	030	<b>CAN</b> (cancel)	56	38 070	&#56;	<b>8</b>	88	58 130	&#88;	<b>X</b>	120	78 170	&#120;	<b>x</b>				
25	19 031	031	<b>EM</b> (end of medium)	57	39 071	&#57;	<b>9</b>	89	59 131	&#89;	<b>Y</b>	121	79 171	&#121;	<b>y</b>				
26	1A 032	032	<b>SUB</b> (substitute)	58	3A 072	&#58;	<b>:</b>	90	5A 132	&#90;	<b>Z</b>	122	7A 172	&#122;	<b>z</b>				
27	1B 033	033	<b>ESC</b> (escape)	59	3B 073	&#59;	<b>:</b>	91	5B 133	&#91;	<b>[</b>	123	7B 173	&#123;	<b>{</b>				
28	1C 034	034	<b>FS</b> (file separator)	60	3C 074	&#60;	<b>&lt;</b>	92	5C 134	&#92;	<b>\</b>	124	7C 174	&#124;	<b> </b>				
29	1D 035	035	<b>GS</b> (group separator)	61	3D 075	&#61;	<b>=</b>	93	5D 135	&#93;	<b>]</b>	125	7D 175	&#125;	<b>}</b>				
30	1E 036	036	<b>RS</b> (record separator)	62	3E 076	&#62;	<b>&gt;</b>	94	5E 136	&#94;	<b>^</b>	126	7E 176	&#126;	<b>~</b>				
31	1F 037	037	<b>US</b> (unit separator)	63	3F 077	&#63;	<b>?</b>	95	5F 137	&#95;	<b>_</b>	127	7F 177	&#127;	<b>DEL</b>				

Source: [www.LookupTables.com](http://www.LookupTables.com)

# Data Representation: Numeric Data Representation

- **Pure binary**
  - can be calculated directly
- **ASCII binary**
  - string of digits: "01100001" , character "a"
- **ASCII decimal**
  - string of digits: "97" a
- **ASCII hexadecimal**
  - string of digits: "61" a

# What's Next

- - **Welcome to Assembly Language**
  - Some Good Questions to Ask
  - Assembly Language Applications
- - **Virtual Machine Concept**
  - Virtual Machines
  - Specific Machine Levels
  - Translating Languages
- - **Data Representation**
- - **Boolean Operations**

# Boolean Operations

The world of logic

# Boolean Operations

- NOT
- AND
- OR
- Operator Precedence
- Truth Tables

XOR  $\Rightarrow$

inputs are different

X	X	XOR
0	0	0
0	1	1
1	0	1
1	1	0

# Boolean Operations: Boolean Algebra

- Based on symbolic logic, designed by George Boole
- Boolean expressions created from:
  - NOT, AND, OR

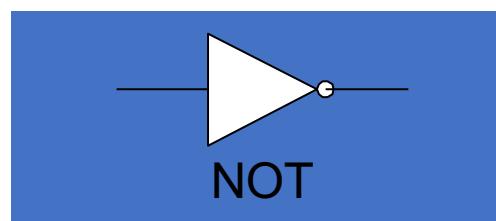
Expression	Description
$\neg X$	NOT X
$X \wedge Y$	X AND Y
$X \vee Y$	X OR Y
$\neg X \vee Y$	( NOT X ) OR Y
$\neg(X \wedge Y)$	NOT ( X AND Y )
$X \wedge \neg Y$	X AND ( NOT Y )

# Boolean Operations: NOT

- Inverts (reverses) a boolean value
- Truth table for Boolean NOT operator:

X	$\neg X$
F	T
T	F

Digital gate diagram for NOT:



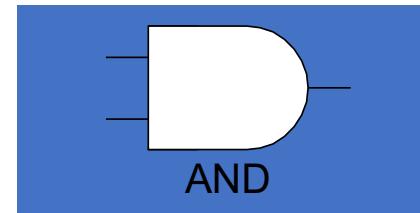
- The NOT operator is a **UNARY** operator (i.e. it acts on one variable or expressions)

# Boolean Operations: AND

- Truth table for Boolean AND operator:

X	Y	$X \wedge Y$
F	F	F
F	T	F
T	F	F
T	T	T

Digital gate diagram for AND:



- AND operator is a BINARY operator (i.e. it acts on two variables or expressions – “binary” in this context does not mean “base-2”)

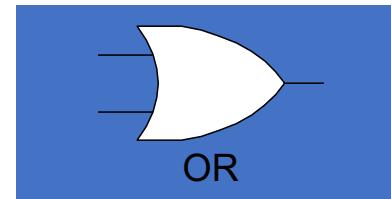
# Attendance!

# Boolean Operations: OR

- Truth table for Boolean OR operator:

X	Y	$X \vee Y$
F	F	F
F	T	T
T	F	T
T	T	T

Digital gate diagram for OR:



- The **OR** operator is a **BINARY** operator (i.e. it acts on two variables or expressions)

# Boolean Operations: Operator Precedence

- Examples showing the **order of operations**:

Expression	Order of Operations
$\neg X \vee Y$	NOT, then OR
$\neg(X \vee Y)$	OR, then NOT
$X \vee(Y \wedge Z)$	AND, then OR

# Boolean Operations: Truth Tables

- A **Boolean function** has one or more Boolean inputs, and returns a single Boolean output.
- A **truth table** shows all the inputs and outputs of a Boolean function

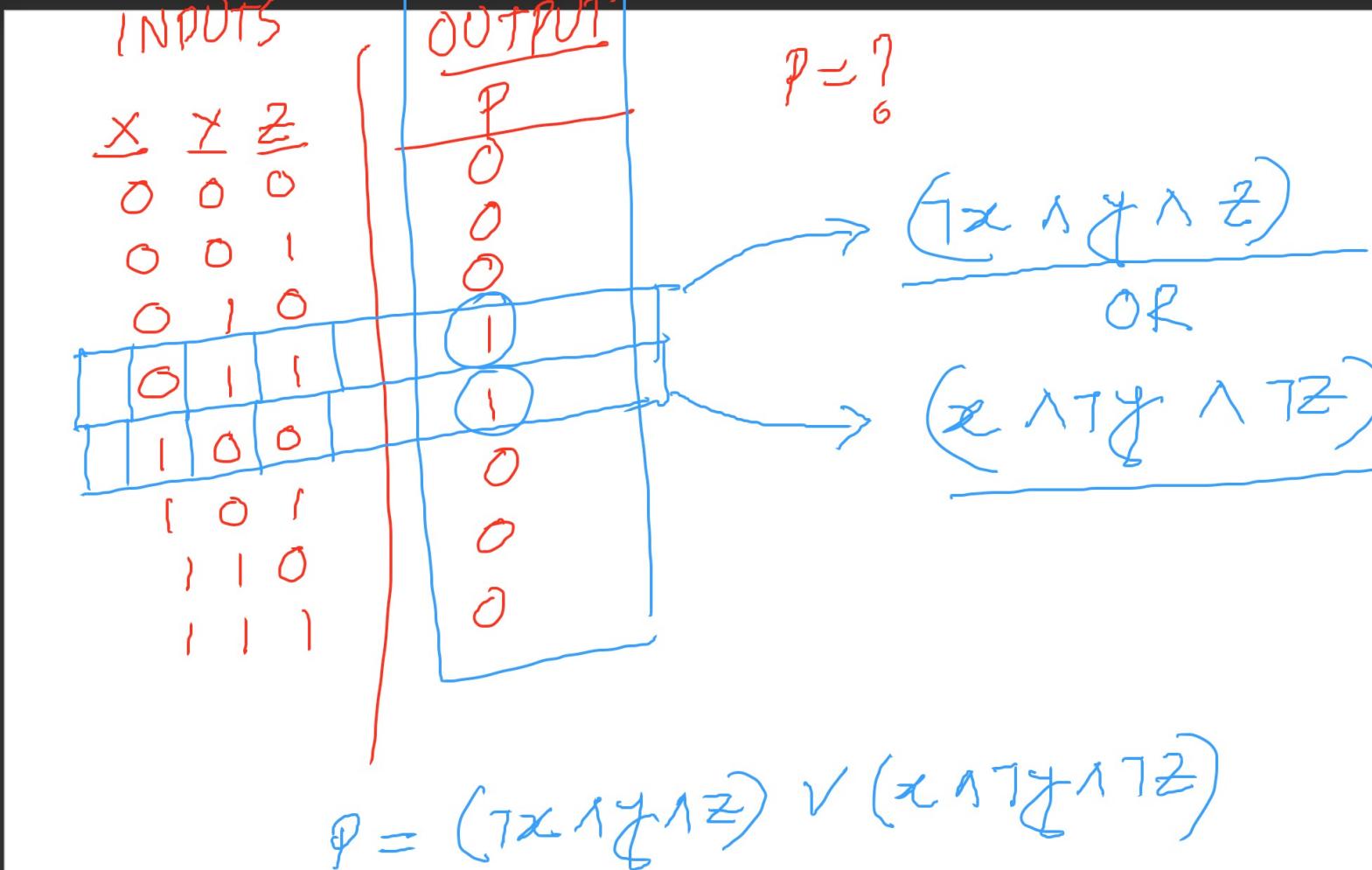
Example:  $\neg X \vee Y$

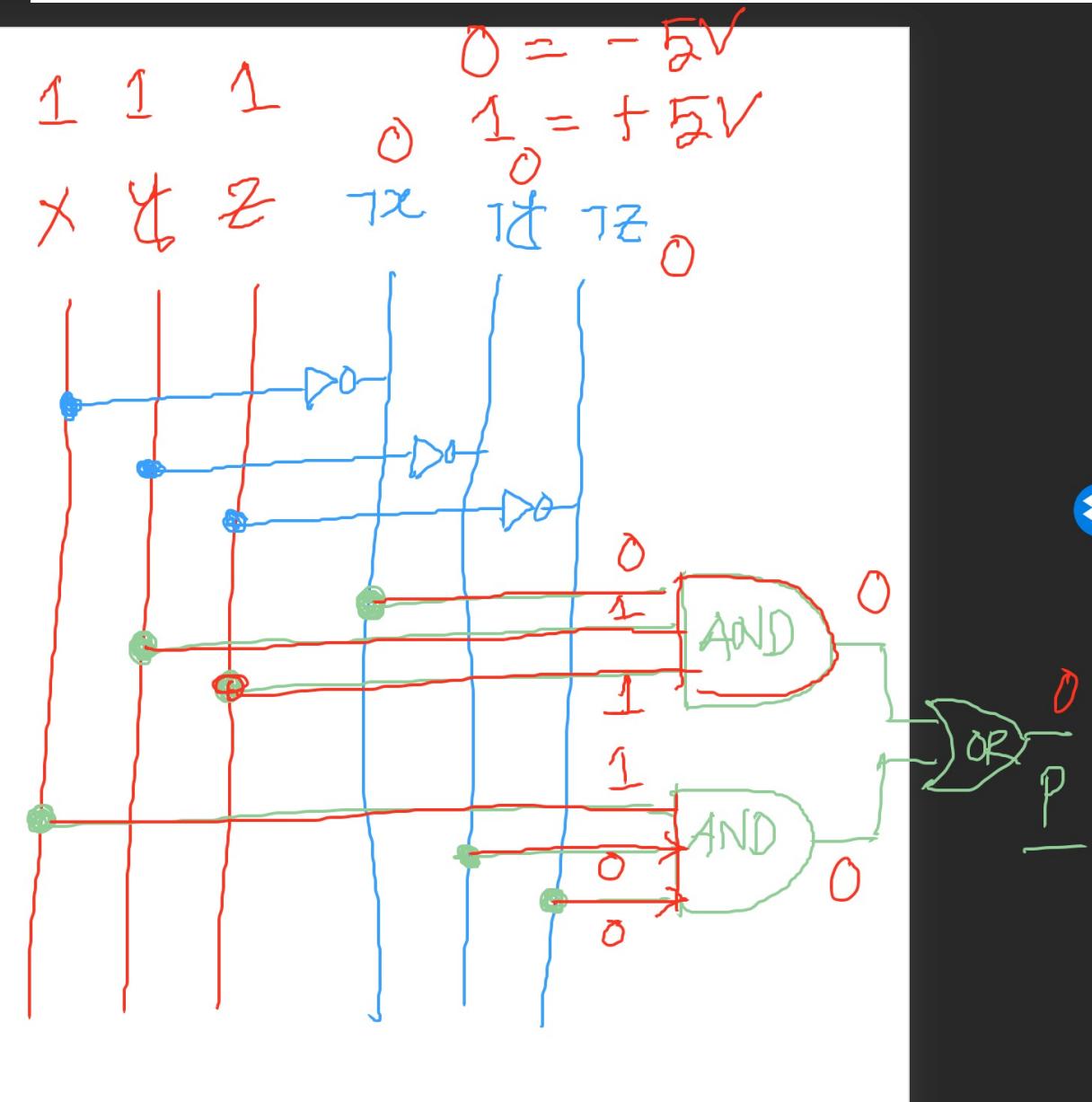
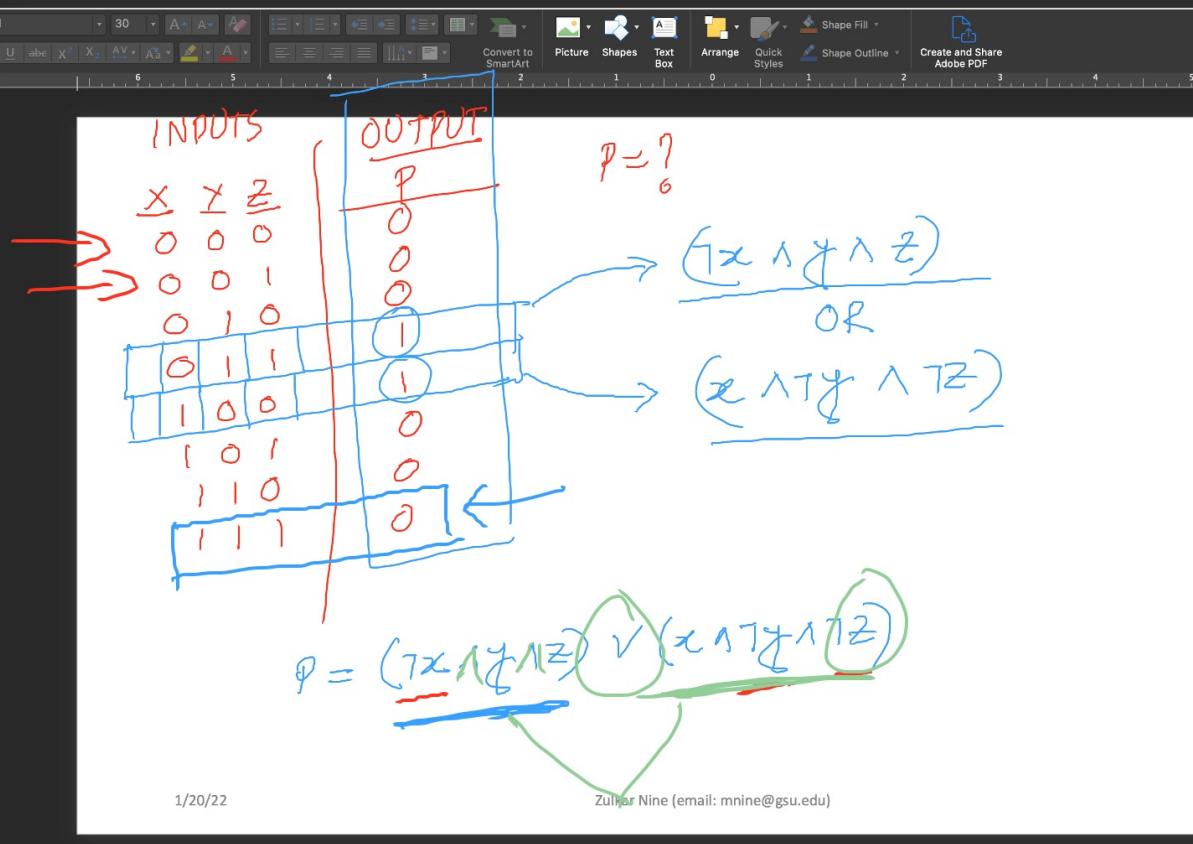
X	$\neg X$	Y	$\neg X \vee Y$
F	T	F	T
F	T	T	T
T	F	F	F
T	F	T	T

# Boolean Operations: Truth Tables

- Example:  $Z = ?$

X	Y	$\neg Y$	Z
F	F	T	F
F	T	F	F
T	F	T	T
T	T	F	F

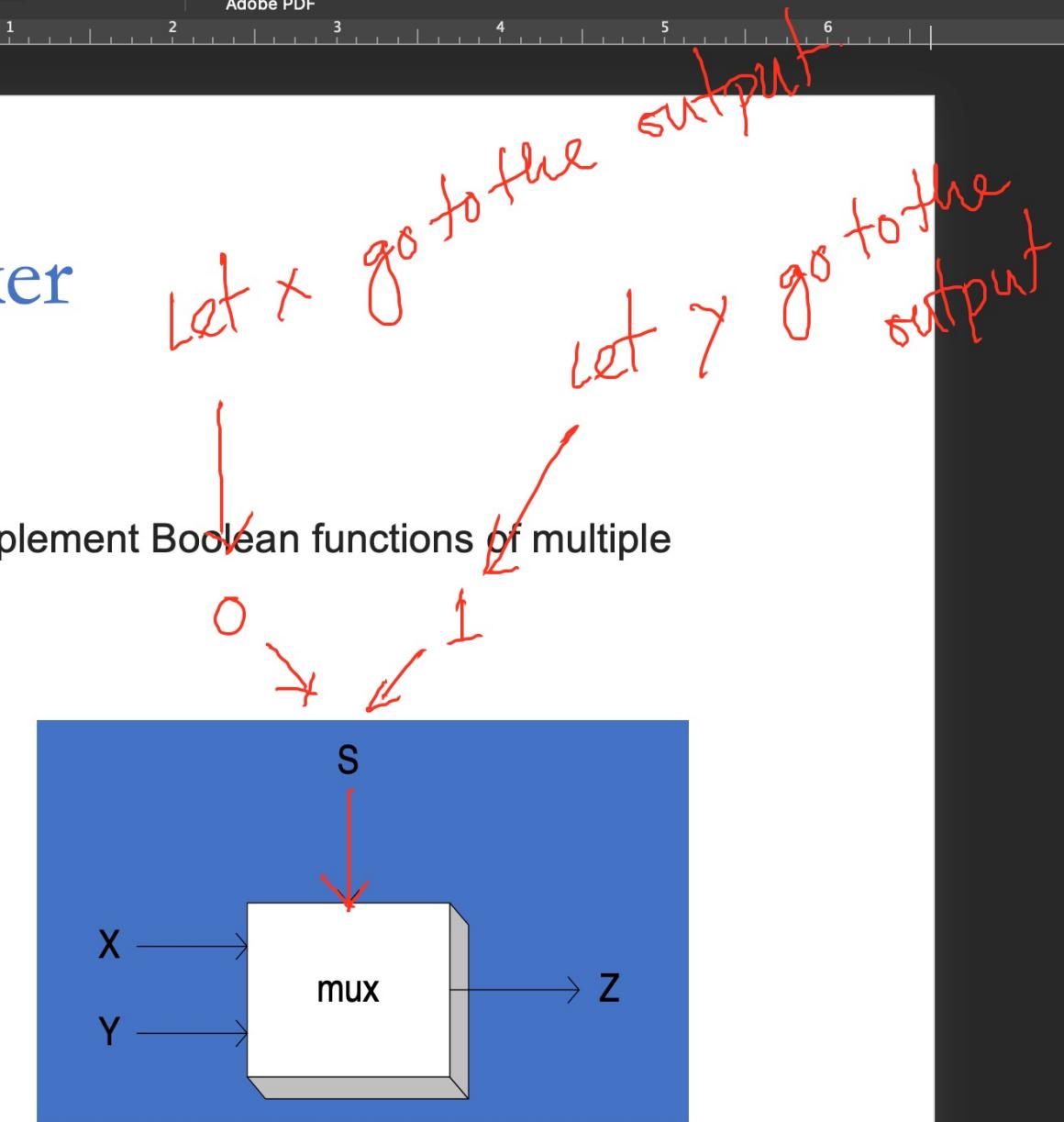




# Boolean Operations: Multiplexer

## Multiplexer:

- A **data selector**. Multiplexers can also be used to implement Boolean functions of multiple variables.
- Multiple-input, single-output
- A multiplexer (or **mux**) is a common digital circuit
- Two-input multiplexer

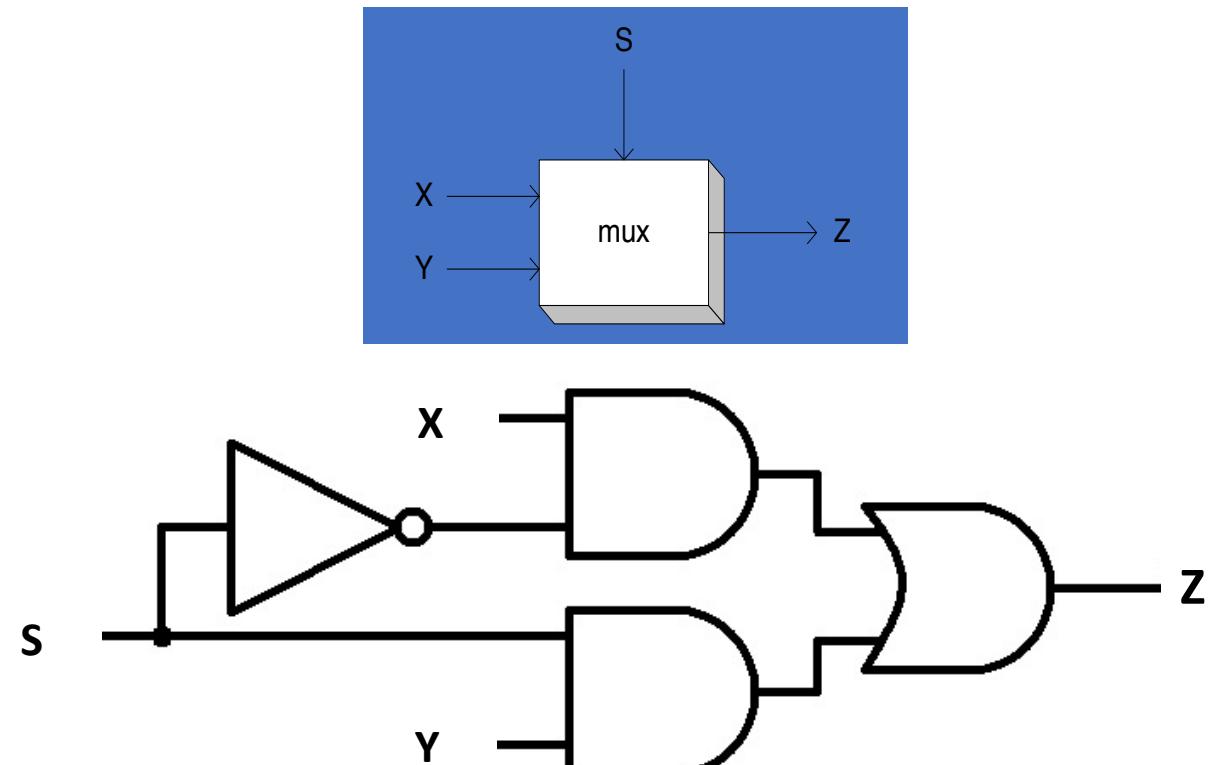


Two-input multiplexer



# Boolean Operations: Truth Tables (**Multiplexer**)

S	X	Y	Z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

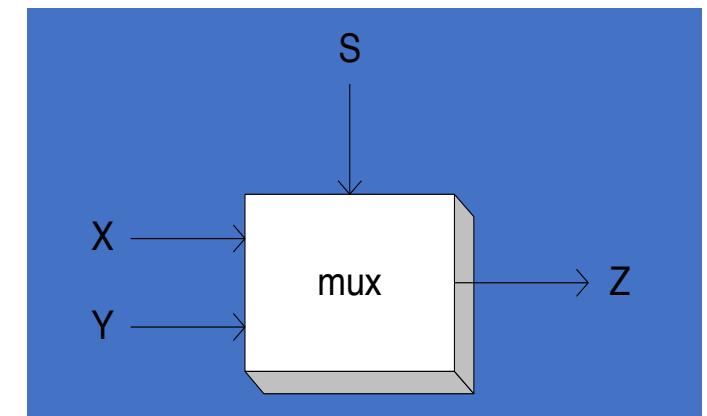


$$Z = (X \text{ and } \bar{S}) \text{ OR } (Y \text{ and } S)$$

# Boolean Operations: Truth Tables (**Multiplexers**)

- Example:  $(Y \wedge S) \vee (X \wedge \neg S)$

X	Y	S	$Y \wedge S$	$\neg S$	$X \wedge \neg S$	$(Y \wedge S) \vee (X \wedge \neg S)$
F	F	F	F	T	F	F
F	T	F	F	T	F	F
T	F	F	F	T	T	T
T	T	F	F	T	T	T
F	F	T	F	F	F	F
F	T	T	T	F	F	T
T	F	T	F	F	F	F
T	T	T	T	F	F	T



Two-input multiplexer

# Boolean Operations: Truth Tables (**Multiplexer**)

- For combinational logic, if/else is implemented as a 2:1 multiplexer.

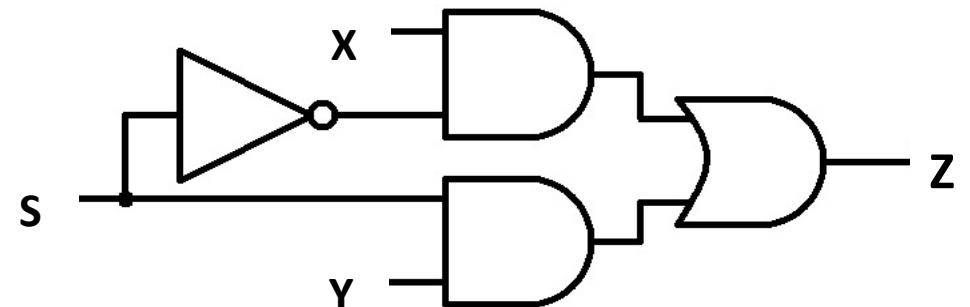
- If (S)

```
Z= X;  
else  
Z= Y;
```

- In Boolean algebra, this would be:

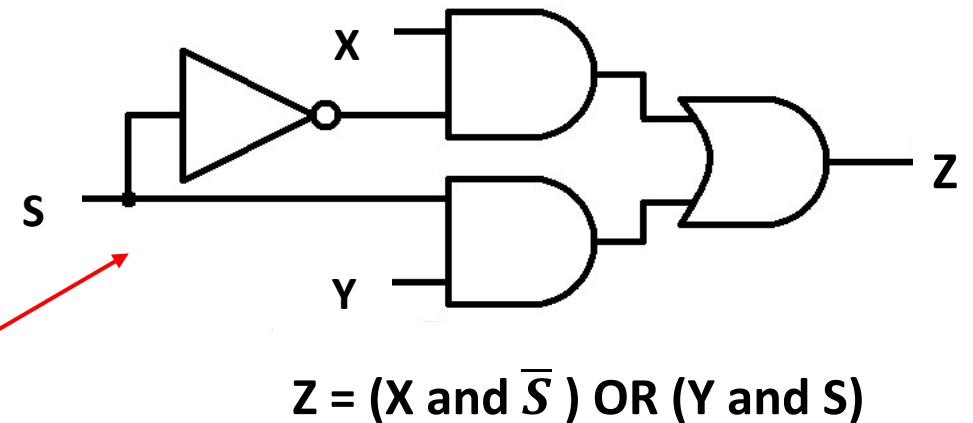
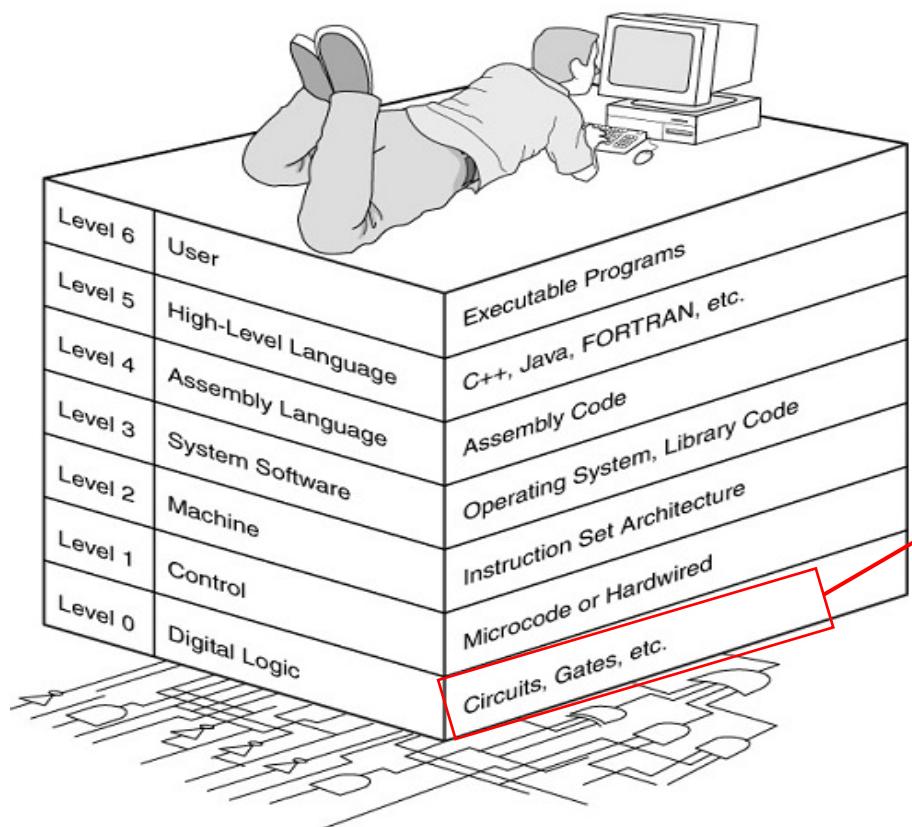
$$Z = (X \text{ and } \bar{S}) \text{ OR } (Y \text{ and } S) \text{ where:}$$

- **S** is the input fed by the **if** condition,
- **X** is the input fed by the **then** subexpression,
- **Y** is the input fed by the **else** subexpression, and
- **Z** is the output of **the expression**.

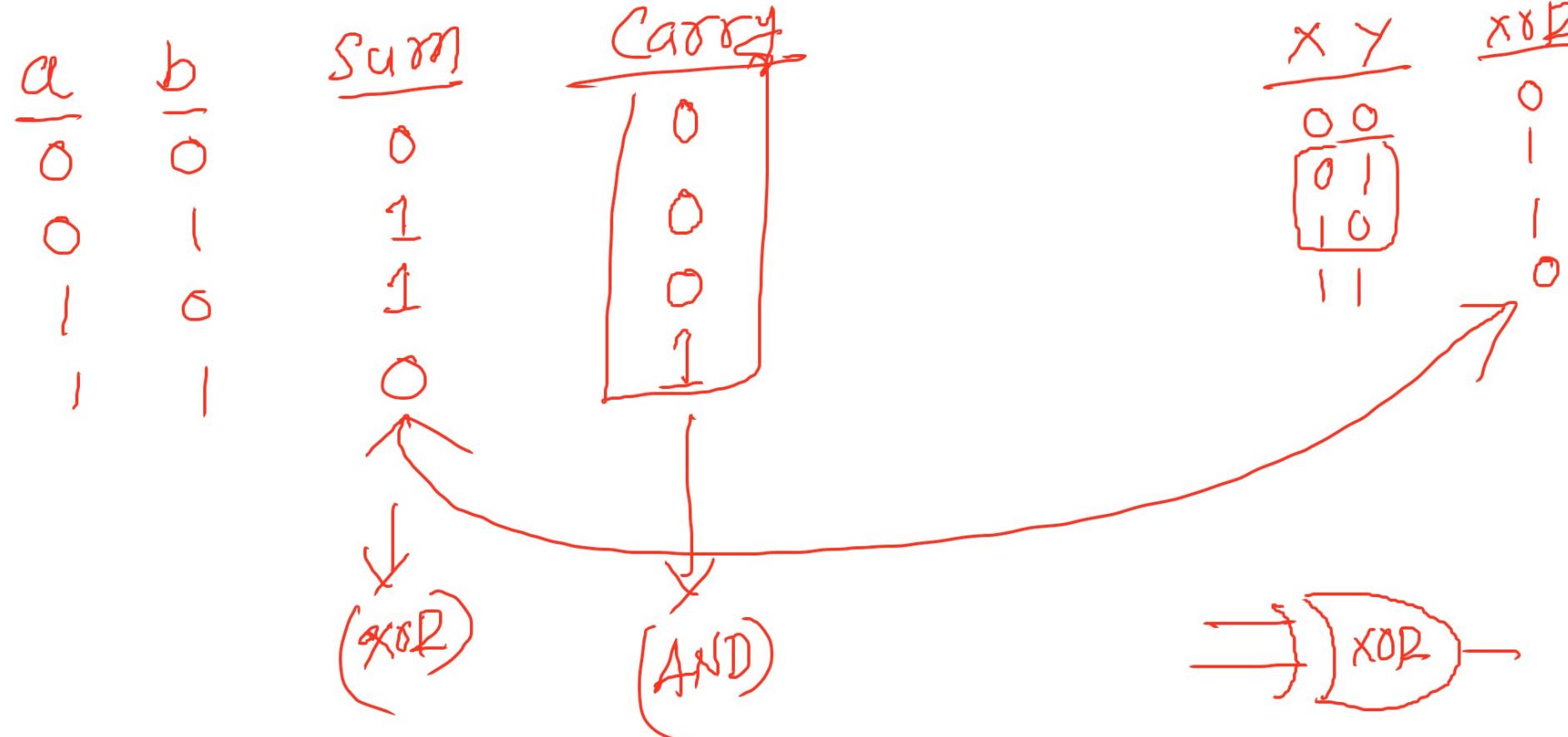


$$Z = (X \text{ and } \bar{S}) \text{ OR } (Y \text{ and } S)$$

# Boolean Operations: Truth Tables (**Multiplexer**)



# Boolean Operations: Half and Full Adders

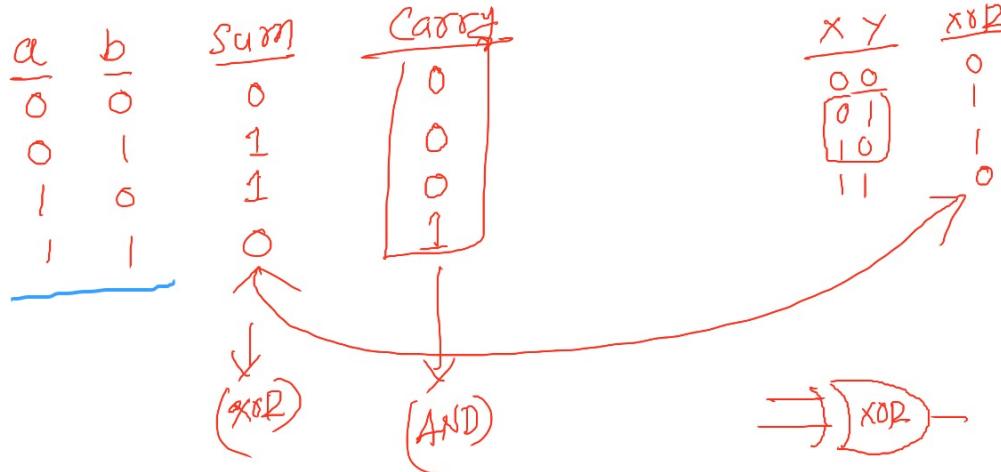


<u>X</u>	<u>Y</u>	<u>XOR</u>
0	0	0
0	1	1
1	0	1
1	1	0

AND

<u>X</u>	<u>Y</u>	<u>AND</u>
0	0	0
0	1	0
1	0	0
1	1	1

## Boolean Operations: Half and Full Adders

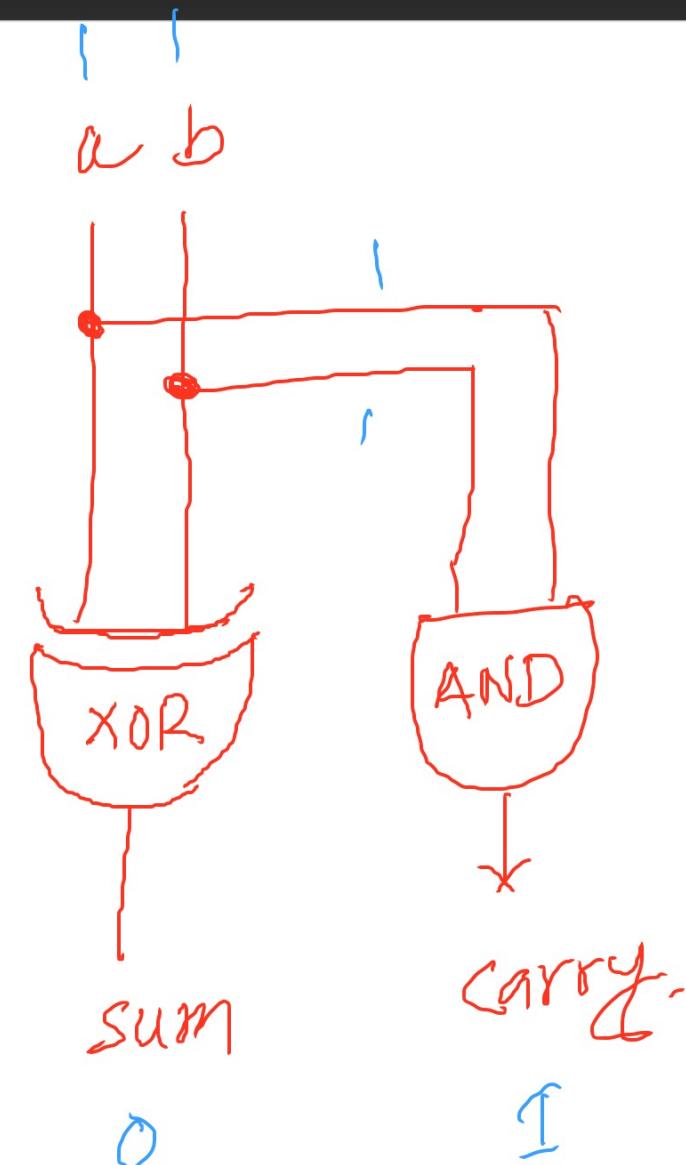


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<u>AND</u>
0 0
0 1
1 0
1 1



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# Boolean Operations: Half and Full Adders

- **Half Adder:**

- The generation of the **sum** and **carry** bits may be performed by very simple electronic circuits.
- The generation of the sum and carry requires an **and** gate and an **xor** gate.
- These may be obtained together in a **circuit** called a **half adder**

a	b	Sum
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive or

a	b	Carry
0	0	0
0	1	0
1	0	0
1	1	1

and

a	b	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

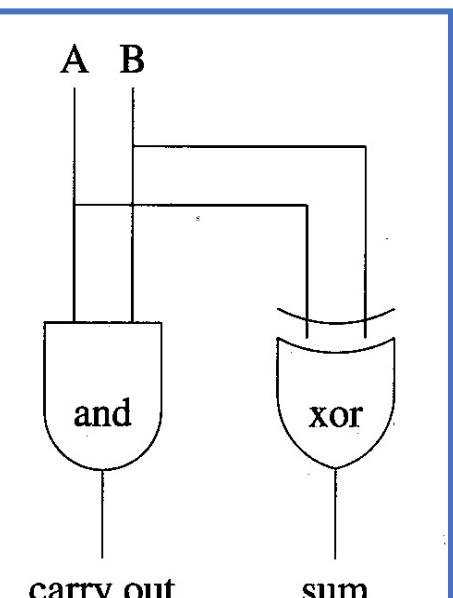


Figure 4.1: Half Adder

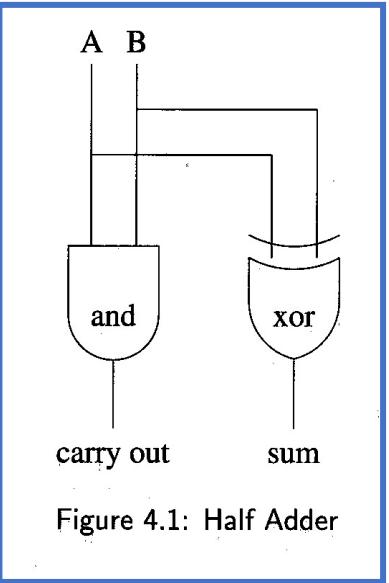


Figure 4.1: Half Adder

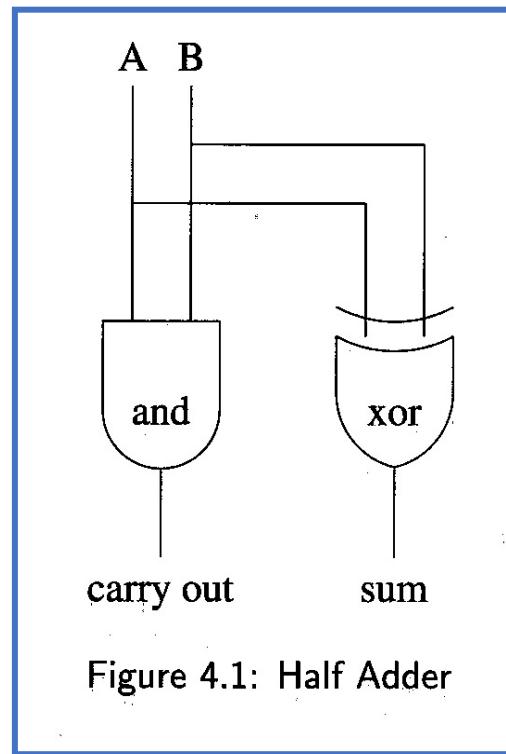


Figure 4.1: Half Adder

# Boolean Operations: Half and Full Adders

- **Full Adder:**

- Two half adders may then be combined together with an or gate to add two inputs, A and B, with a carry in, to produce a sum and a carry out.
- Such a circuit is called a full adder

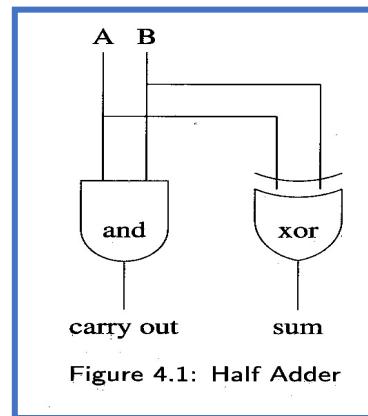


Figure 4.1: Half Adder

**$1 + 1 + 1 = 1$  and carry 1**  
Half adder cannot  
handle this addition?

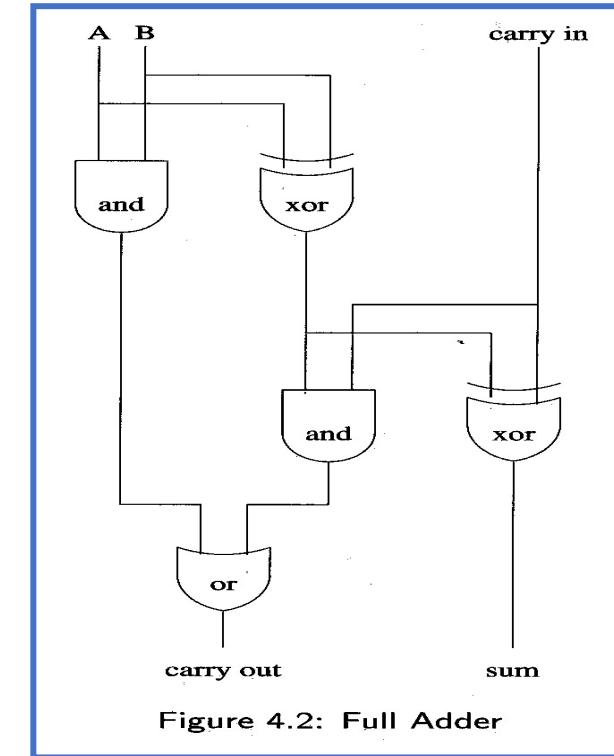
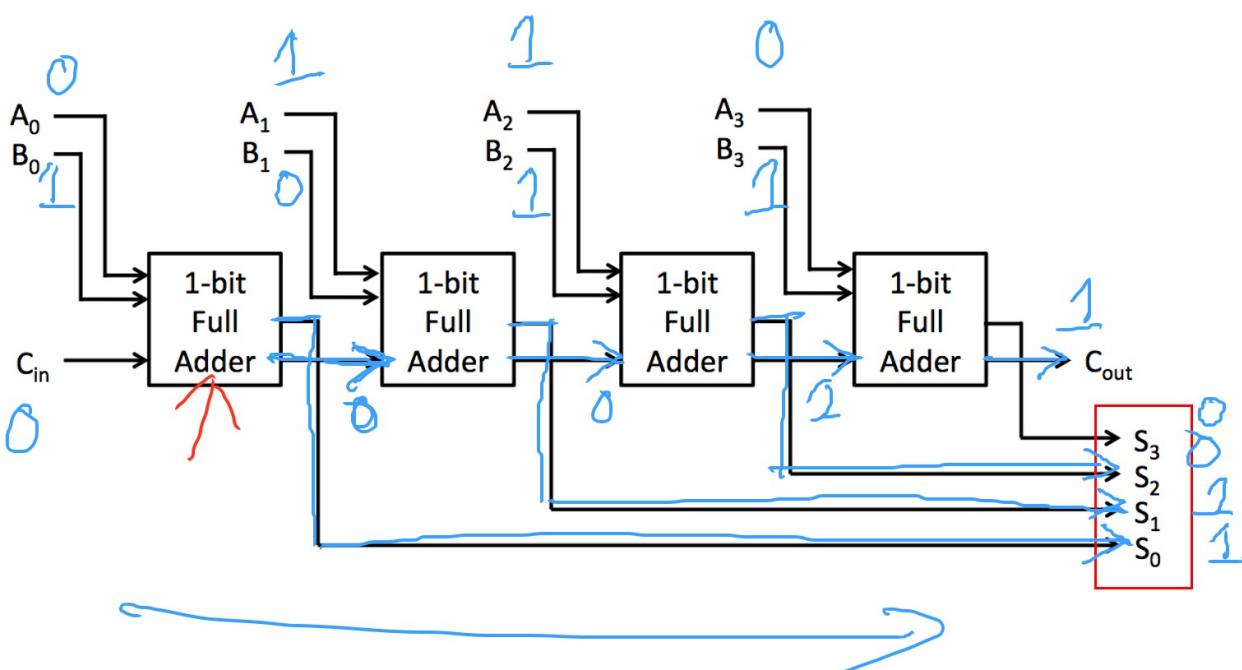
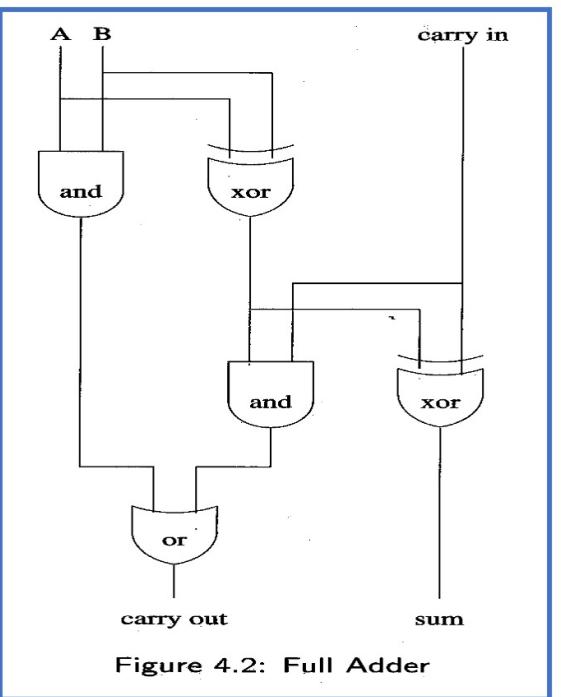


Figure 4.2: Full Adder

$A_2 \ A_2 \ A_1 \ A_0$   
 $A =$   
 $B =$   
 $B_3 \ B_2 \ B_1 \ B_0$

## Boolean Operations: Half and Full Adders

- Full Adder:



# Summary

- Assembly language helps you learn **how software is constructed at the lowest levels**
- Assembly language has **one-to-one relationship with machine language**
- Each **layer** in a computer's architecture is an abstraction of a machine
  - layers can be hardware or software
- Boolean expressions are **essential to the design of computer hardware and software**