

1. Write the content of the queue Q/ the set S/ keys  $d(v)$  after 5 iterations of the Dijkstra algorithm for the graph G below and source s (weights are on edges):

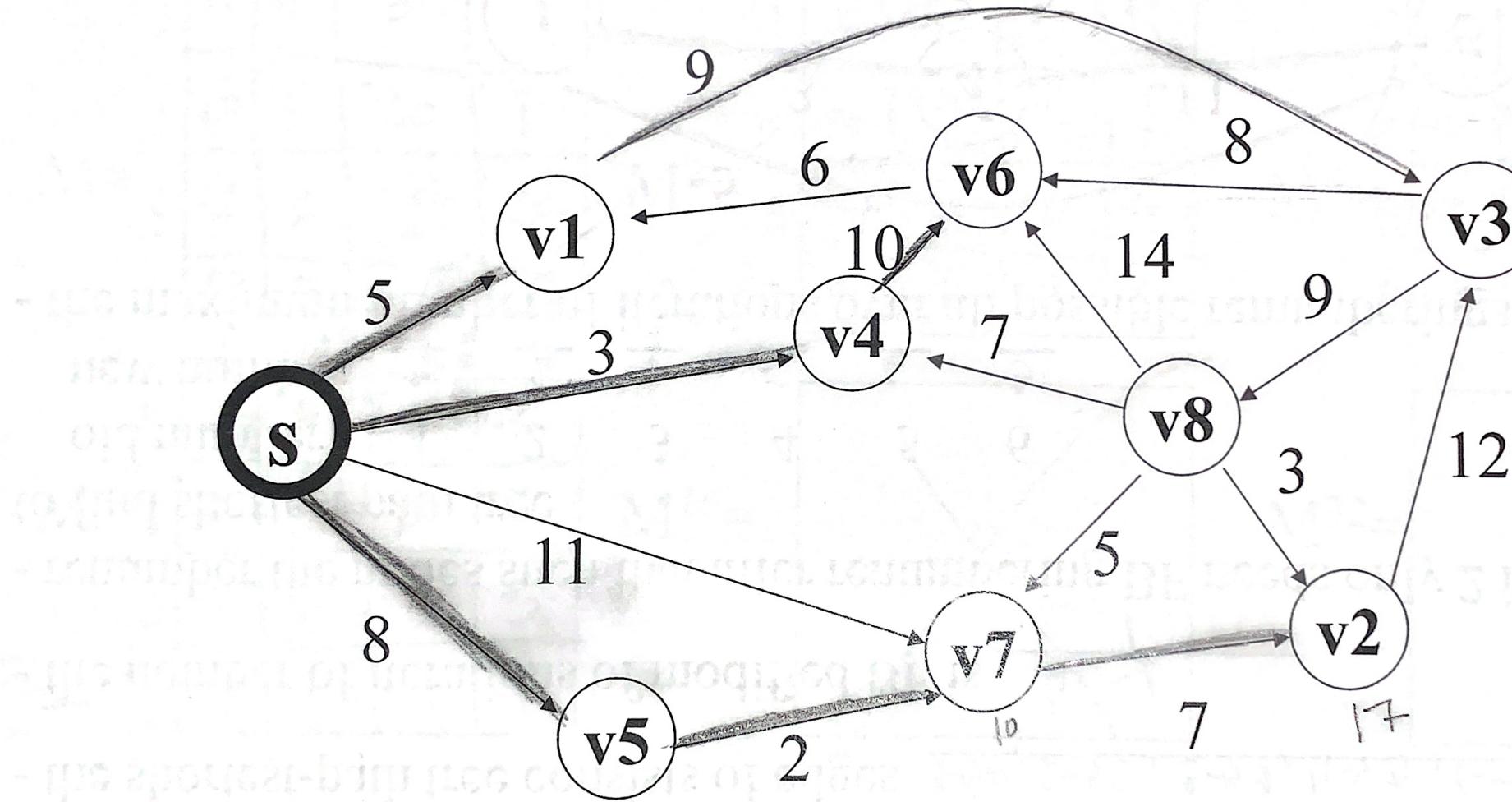
$$Q = \underline{V_2, V_3, V_6, V_8} \quad S = \underline{S, V_4, V_1, V_5, V_7}$$

20

$$d(s) = \underline{0} \quad d(v1) = \underline{5} \quad d(v2) = \underline{17} \quad d(v3) = \underline{14}$$

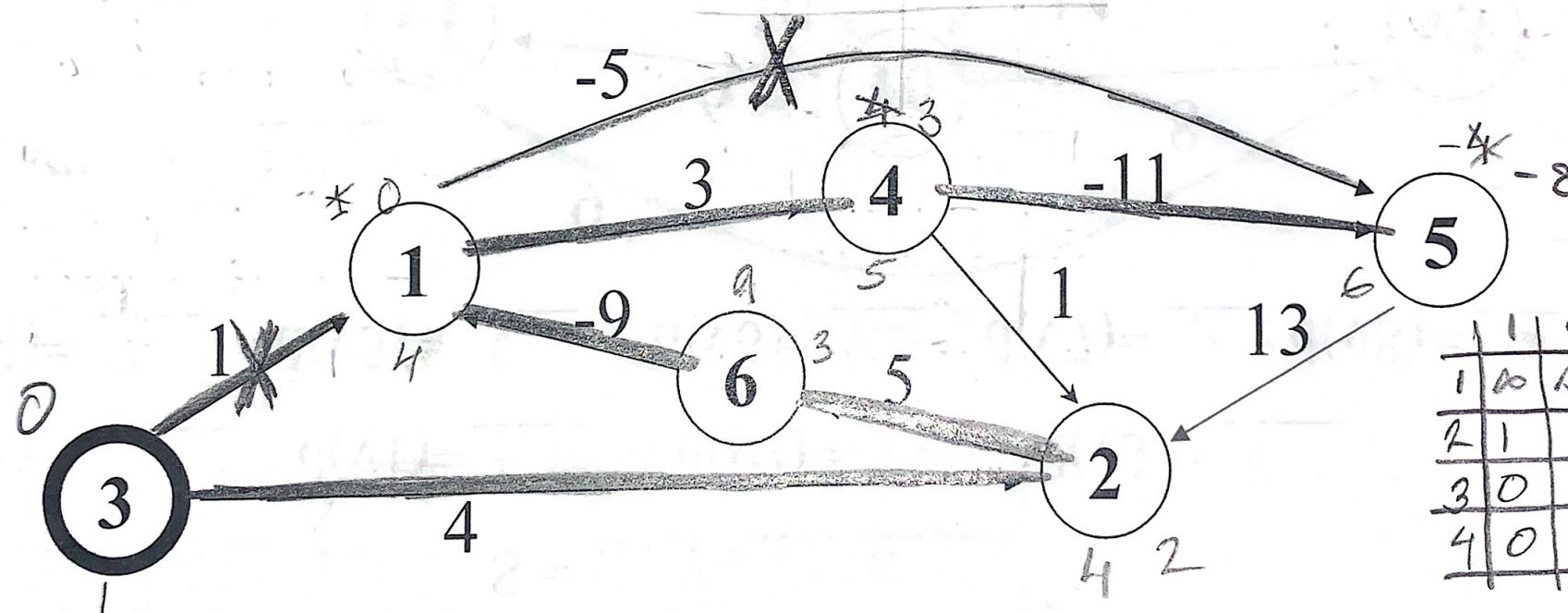
$$d(v4) = \underline{3} \quad d(v5) = \underline{8} \quad d(v6) = \underline{13} \quad d(v7) = \underline{10} \quad d(v8) = \underline{10}$$

86



2. Using Bellman-Ford, find the shortest path tree from the node 3

- the shortest-path tree consists of edges  $1 \rightarrow 4, 2 \rightarrow 6, 3 \rightarrow 2, 4 \rightarrow 5, 6 \rightarrow 1$  15
- the number of iterations of modified BF is 4
- renumber the nodes such that after renumbering BF needs only 2 iterations to find shortest path tree
- old number    1    2    3    4    5    6  
new number    4    1    2    5    6    3
- the maximum number of iterations over all possible renumberings is 5?



	1	2	3	4	5	6
1	0	∞	0	∞	∞	0
2	1	4	0	4	-5	9
3	0	4	0	4	-8	9
4	0	4	0	3	-8	9

4. Find all shortest path weights with the matrix multiplication method for the graph on the right side.

- give all matrices that are obtained on the way,
- are there any negative cycles in the graph? No

	1	2	3	4
1	0	4	3	7
2	0	0	0	1
3	-2	2	0	0
4	0	0	-1	0

$M =$

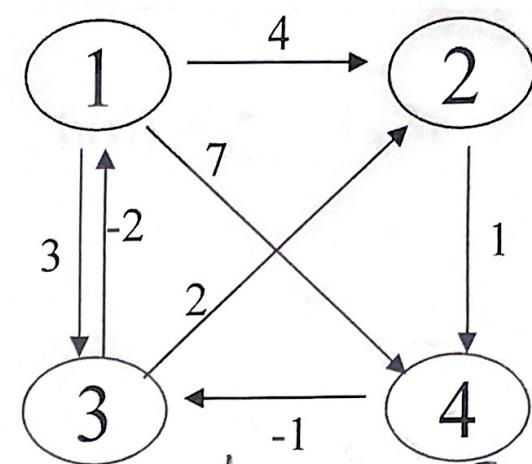
	1	2	3	4
1	0	4	3	5
2	0	0	0	1
3	-2	2	0	3
4	-3	1	-1	0

$M^2 =$

	1	2	3	4
1	0	4	3	5
2	-2	0	0	1
3	-2	2	0	3
4	-3	1	-1	0

$M^8 =$

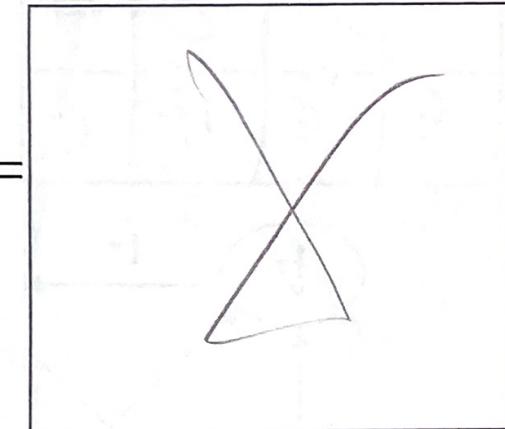

NAME: \_\_\_\_\_



20

	1	2	3	4
1	0	4	3	5
2	-2	0	0	1
3	-2	2	0	3
4	-3	1	-1	0

$M^4 =$



$M^{32} =$

5. Find all shortest path weights with the Floyd-Warshall method for the graph on the right side.

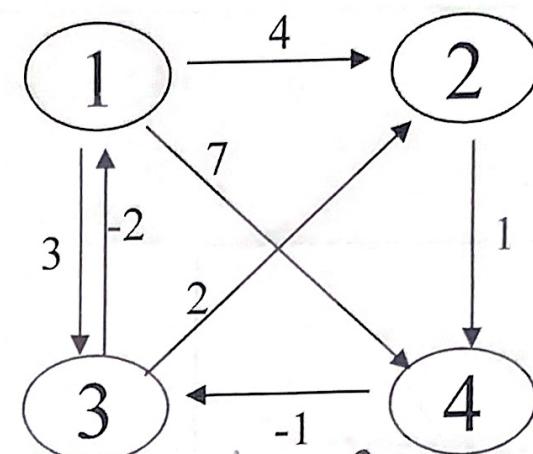
- give all matrices that are obtained on the way

	1	2	3	4
1	0	4	3	7
2	$\infty$	0	4	1
3	-2	2	0	$\infty$
4	$\infty$	$\infty$	-1	0

	1	2	3	4
1	0	4	3	5
2	$\infty$	0	$\infty$	1
3	-2	2	0	3
4	-3	1	-1	0

	1	2	3	4
1	0	4	3	7
2	$\infty$	0	$\infty$	1
3	-2	2	0	5
4	$\infty$	$\infty$	-1	0

	1	2	3	4
1	0	4	3	5
2	-2	0	0	1
3	-2	2	0	3
4	-3	3	-1	0



	1	2	3	4
1	0	4	3	5
2	$\infty$	0	$\infty$	1
3	-2	2	0	3
4	$\infty$	$\infty$	-1	0

$$C^0 = \begin{array}{|c|c|c|c|} \hline & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 3 & 7 \\ \hline 2 & \infty & 0 & 4 & 1 \\ \hline 3 & -2 & 2 & 0 & \infty \\ \hline 4 & \infty & \infty & -1 & 0 \\ \hline \end{array}$$

$$C^1 =$$

	1	2	3	4
1	0	4	3	5
2	$\infty$	0	$\infty$	1
3	-2	2	0	3
4	-3	1	-1	0

$$C^4 =$$

6. Johnson's algorithm is applied to the graph below.

(a) Give  $h(1) = \underline{0}$ ,  $h(2) = \underline{-2}$ ,  $h(3) = \underline{-9}$ ,  $h(4) = \underline{-1}$ ,  $h(5) = \underline{-9}$ ,  $h(6) = \underline{0}$ ,  $h(7) = \underline{-3}$

Give the modified weight of

(b) the shortest 2-to-3-path  $\underline{1 \rightarrow 2 \rightarrow 1 \rightarrow 5 \rightarrow 4 \rightarrow 3}$   
 $8 + -9 + 8 + -8 = -1$

(c) the edge  $(3,2)$   $\underline{-3}$

