1. Using Bellman-Ford, find the shortest path tree from the node 3

- the shortest-path tree consists of edges 1-5, 2-4, 2-6, 3-1, 5-2

- the number of iterations of max BF is 3+1 = 4

- renumber the nodes such that after renumbering BF needs only 2 iterations to find shortest path tree

3-1, 1-5, 5-2, 2-4, 26

old number	1	2	3	4	5	6
new number	3	4	1	5	3	6

- the maximum number of iterations over all possible renumberings is 5 ?

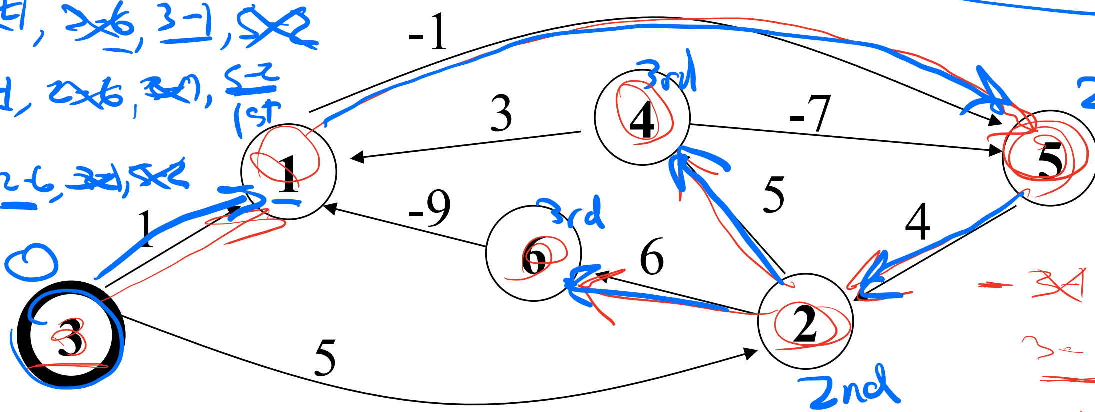
#nodes - 1

- ~~1-5~~, ~~2-4~~, ~~2-6~~, 3-1, ~~5-2~~

- 1-5, ~~2-4~~, ~~2-6~~, ~~3-1~~, 5-2

1st

- ~~1-5~~, 2-4, 2-6, ~~3-1~~, ~~5-2~~



1 2 3

- ~~3-4~~, 1-5, ~~5-2~~, 2-4, 2-6

3-1, ~~1-5~~, 5-2, ~~2-4~~, ~~2-6~~

4 5

3 → 1 → 5 → 2 → 4 → 6

1 2 3 4 5 6

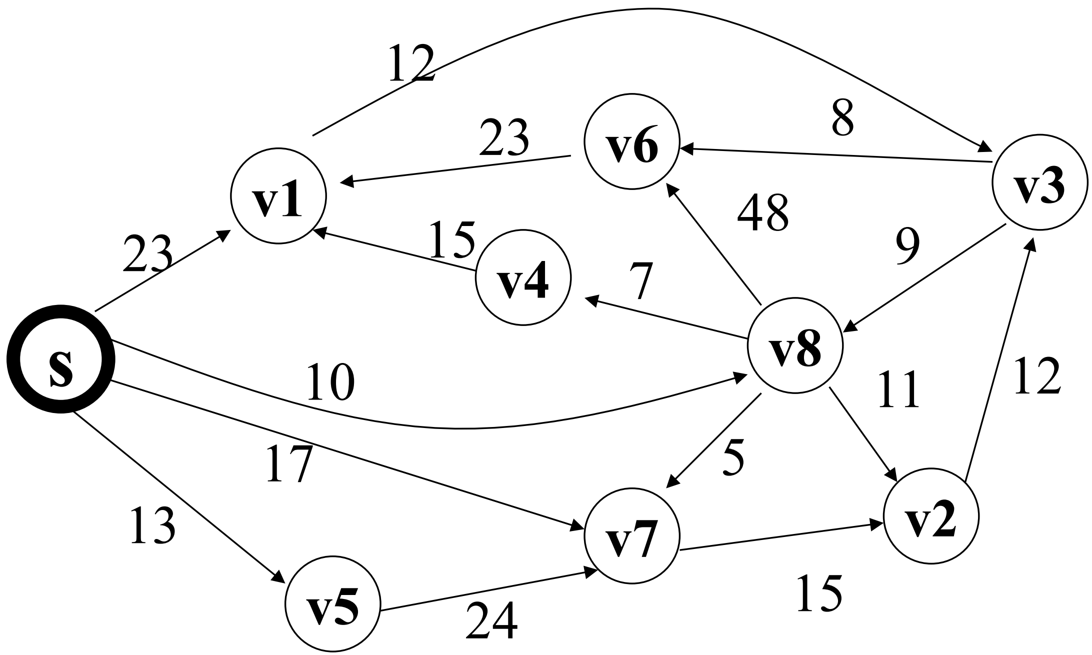
Quiz 3 CS4520 NAME:________________

7.Write the content of the queue Q/ the set S/ keys d(v) after 5 iterations of the Dijkstra algorithm for the graph G below and source s (weights are on edges):

Q = ____________ S = ___________________

d(s) = ______ d(v1)=_____ d(v2)= _____ d(v3)= _____

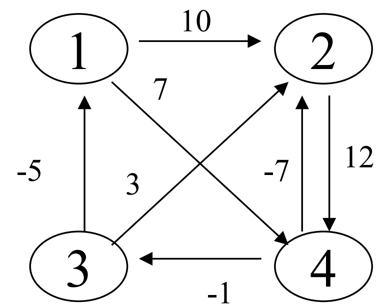
d(v4)= _____ d(v5)= _____ d(v6)= ____ d(v7)= ____ d(v8)= ____.



Quiz2 CS4520 NAME:____

3. Find all shortest path weights with the matrix multiplication method for the graph on the right side.

- give all matrices that are obtained on the way,
- are there any negative cycles in the graph?



$M=$

$M^2=$

$M^4=$

$M^8=$

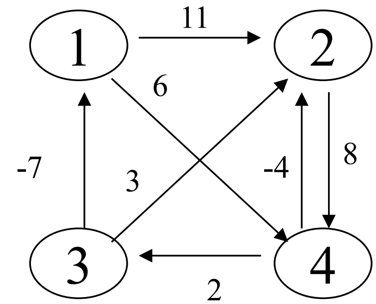
$M^{16}=$

$M^{32}=$

Quiz 3 CS4520 NAME:____

4. Find all shortest path weights with the Floyd-Warshall method for the graph on the right side.

- give all matrices that are obtained on the way



$D^0 =$

$D^1 =$

$D^2 =$

$D^3 =$

$D^4 =$

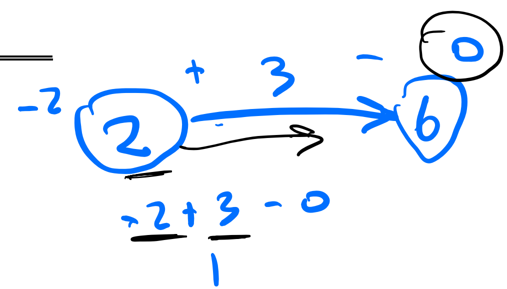
NAME:______________________________

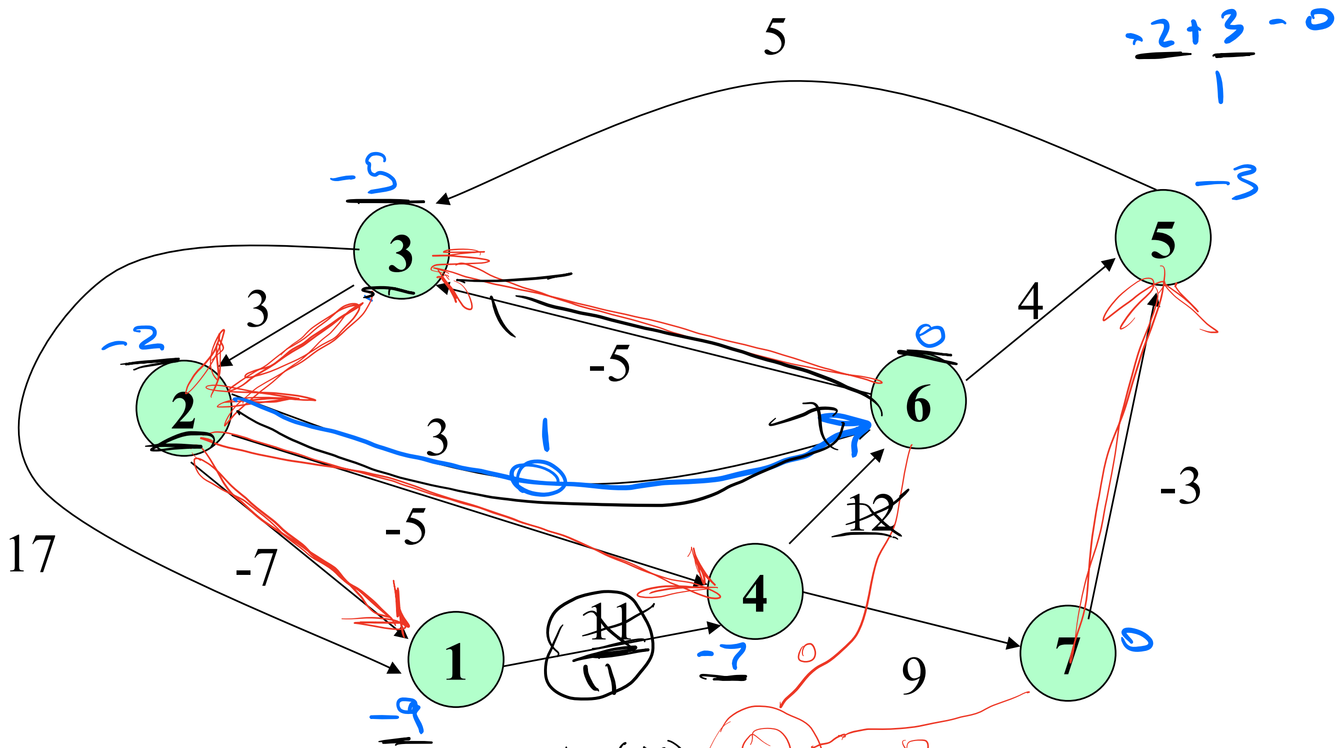
5. Johnson's algorithm is applied to the graph below. (20pts)

MST

Give the **modified** weight of the edge (2,6) 1

The shortest path from 2 to 3 is 2-6, 6-3



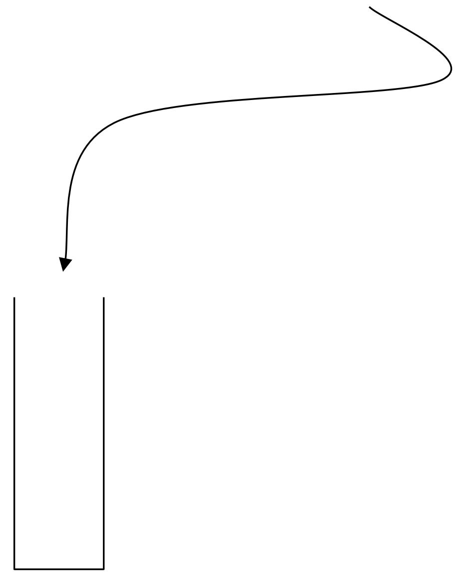


$-9+11-(-7)$

$2+7=11$

Quiz 3 CS4520 NAME:

1. Give the final content of the stack in Graham's algorithm for convex hull for these 3 points A, B and C (check the order!):
A=(190,291), B=(300,400) and C=(200,300)



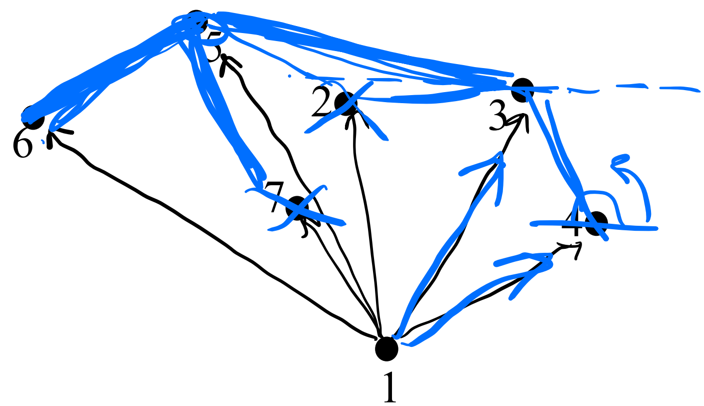
2. For Graham's scan finding convex hull of the point set given below:

- Give the sorted sequence of points for Graham scan 1, 4, 3, 2, 5, 7, 6

- Show the content of the stack after each change

							7		6	
				2		5	5	5	5	
			3	3	3	3	3	3	3	
		4	4	4	4	4	4	4	4	
empty	1	1	1	1	1	1	1	1	1	

- Give the convex hull of this point set 1, 4, 3, 5, 6



Quiz 4 CSC 4520 Sample

3. Below given a point set in the **rectilinear** metric (the height/width of any cell=1) where the closest pair of points should be found using divide and conquer. Show

- the first partition of the point set (draw a line)

2-3

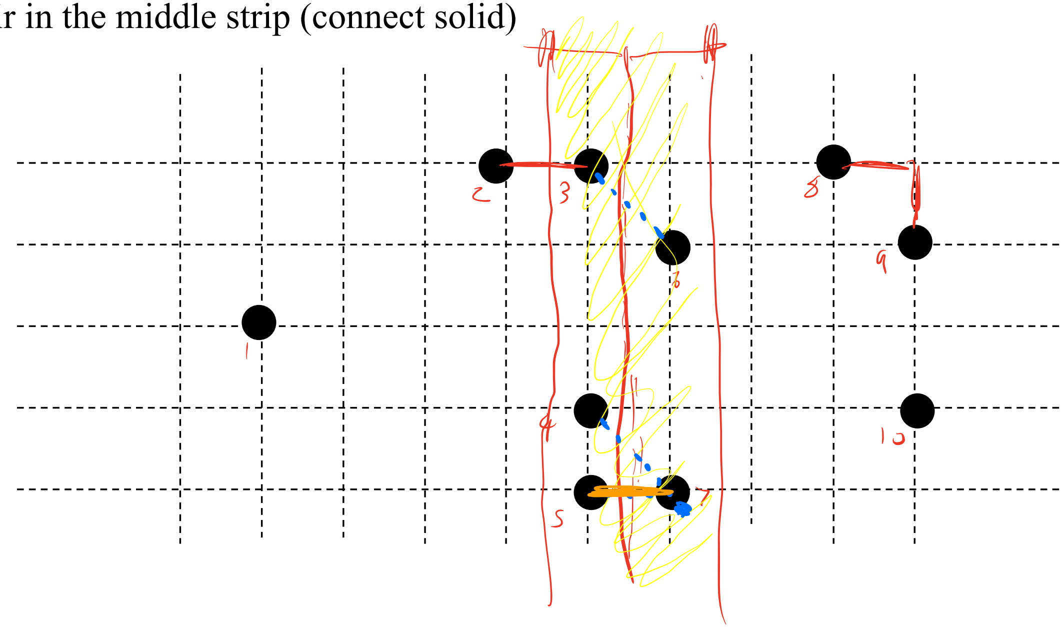
- the closest pair in the left part (connect solid), δ_{left} = 1,

and the right part (connect solid), δ_{right} = 2

8-9 Smaller of two is Delta

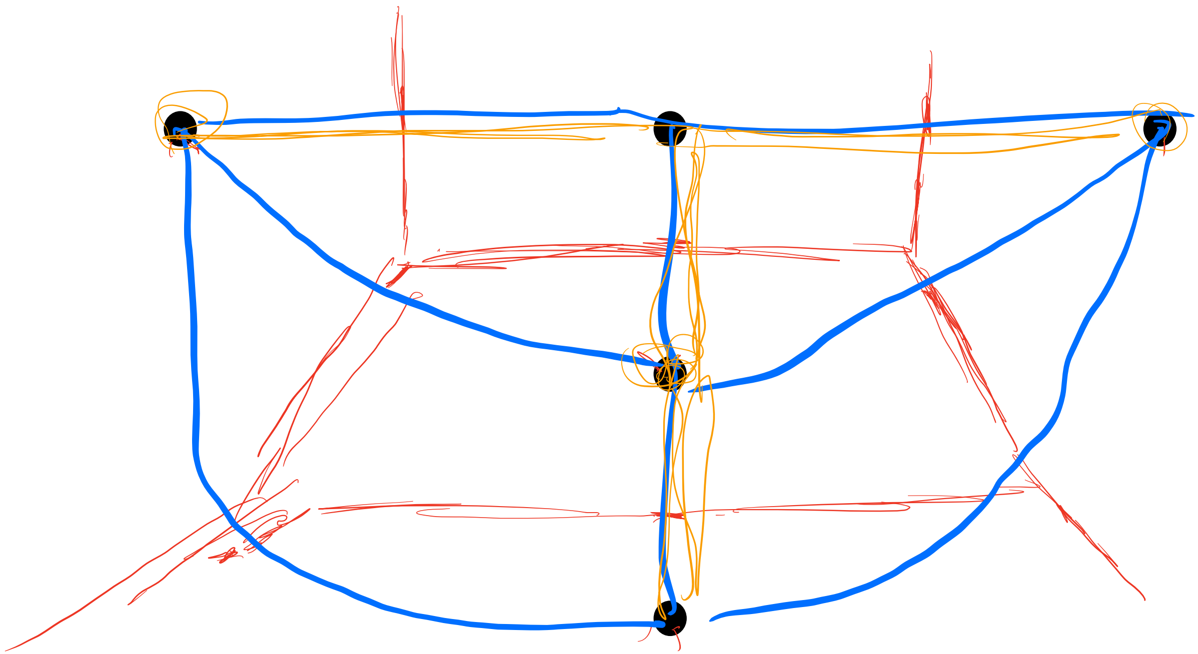
- the middle strip (shade) -go out delta from first partition
- pairs in the middle strip for which distances should be computed (connect dashed)
- closest pair in the middle strip (connect solid)

least vert



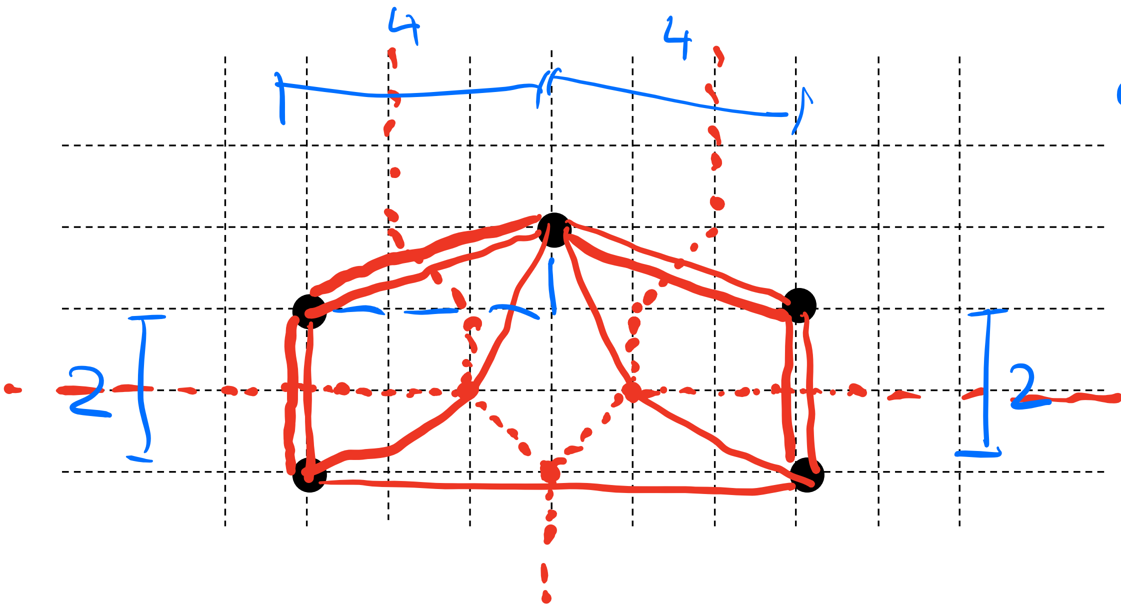
4. Below given a point set in the **Euclidean** metric. Draw

- Voronoi regions (dashed edges) – barriers between points
- Voronoi graph / Delanau triangulation (solid edges) – connections with one barrier
- minimum spanning tree (double edges) – closest path



5. Below given a point set in the **rectilinear** metric (the height/width of any cell=1) . Draw

- Voronoi regions(dashed lines) - barriers
- Voronoi graph / Delanau triangulation (solid edges) - on barrier connection
- minimum spanning tree (double edges) - shortest path
- the length of the minimum spanning tree is 12



4+4+2+2=12

Quiz 4 CSC 4520 Sample

6. Prove, that the k-tree problem (finding minimum weight tree subgraph with k vertices) is in class NP

a) Optimization formulation

b) Decision formulation

c) Polynomial-size certificate

d) Polynomial time verification algorithm

7. For the 3-CNF

$$f = (x' + y + z) \& (x + y' + z') \& (x + y + z') \& (x' + y' + z) \& (x' + y + z') \& (x' + y' + z')$$

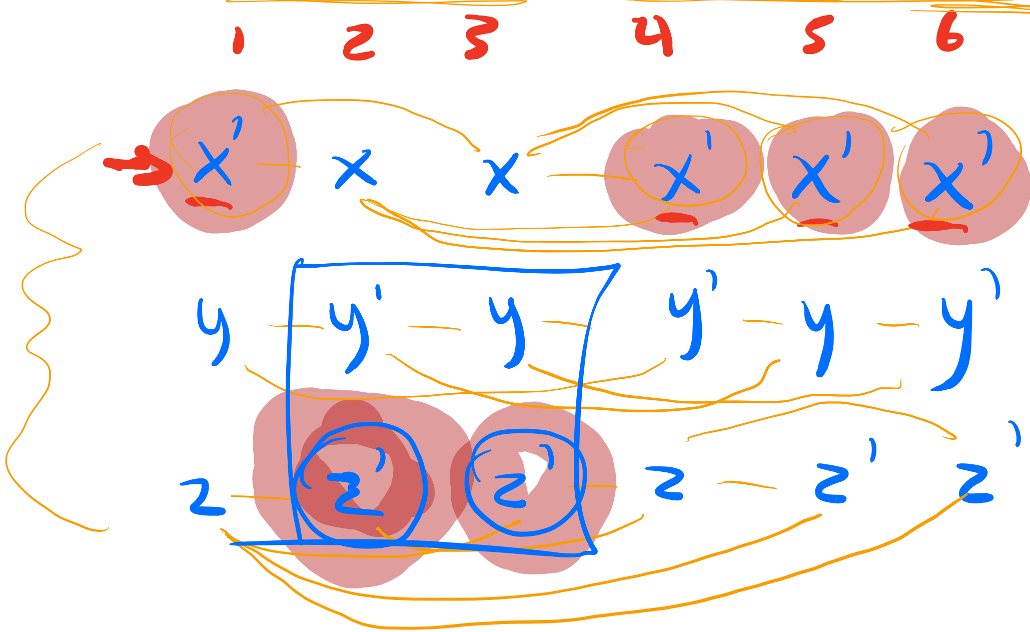
x = 0
z = 0
y = 0

- give 0-1 assignment to variables such that f=1 $x' = 1, z' = 1$

1. - give 0-1 assignment to variables such that f=0 $x = y = z' = 0$

x = 0
y = 0
z = 1

-Draw the corresponding graph and mark the maximum independent set



Prim → not prim

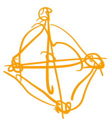
8. In the following graph find

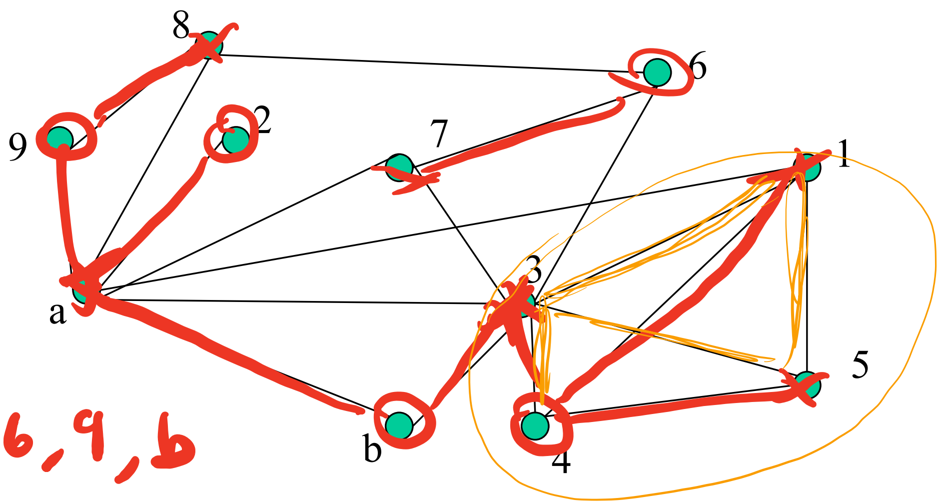
- Maximum Independent Set 2, 4, 6, 9, b <coms

- Minimum Vertex Cover 1, 3, 5, 7, 8, a

- Maximum Clique 1, 3, 4, 5

1, 3, 4, 5





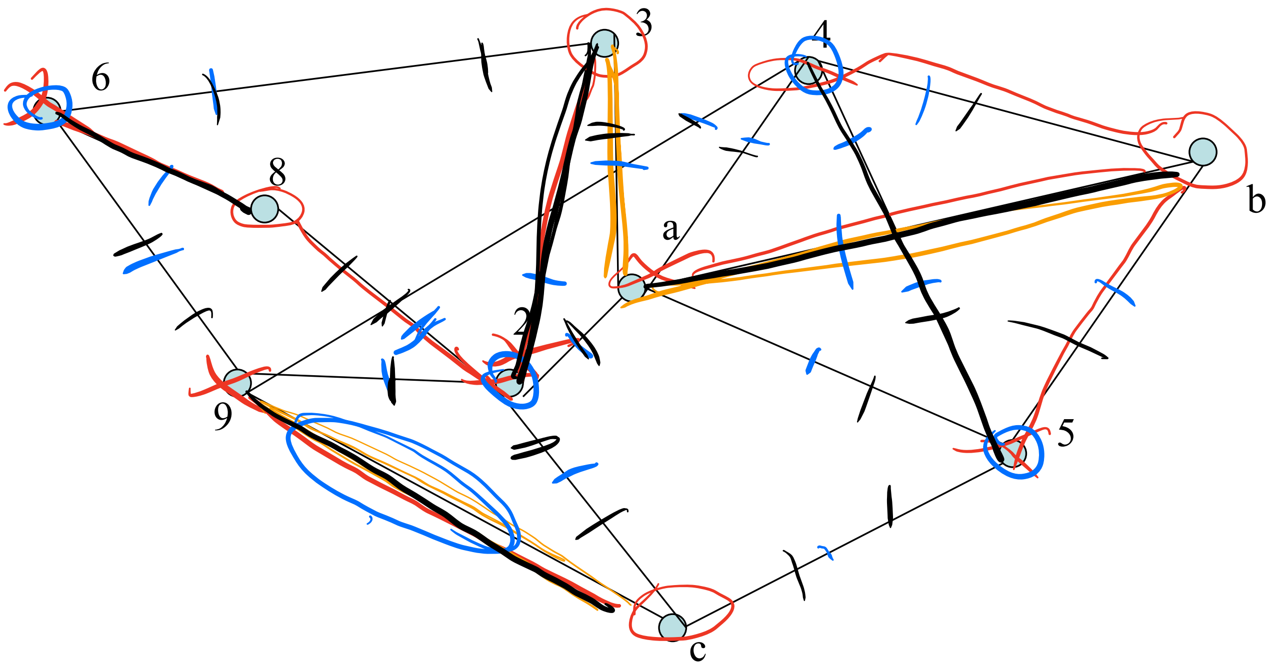
2, 4, 6, 9, b

1. For minimum vertex cover problem in the following graph give

2. greedy solution = nodes 2, 4, 5, 6, a, 9

3. 2-VC solution = nodes 2 - 3, 4 - 5, 6 - 8, 9 - c, a - b

1. Optimal solution = nodes 2, 3, 4, 5, 6, 9, a



Quiz 5 CSc 4520 NAME____________________________

2. For the following graph, find

1. Optimal TSP tour 1, 3, 7, 2, 9, 8, 6, a, 4, 5, 1 length = 48

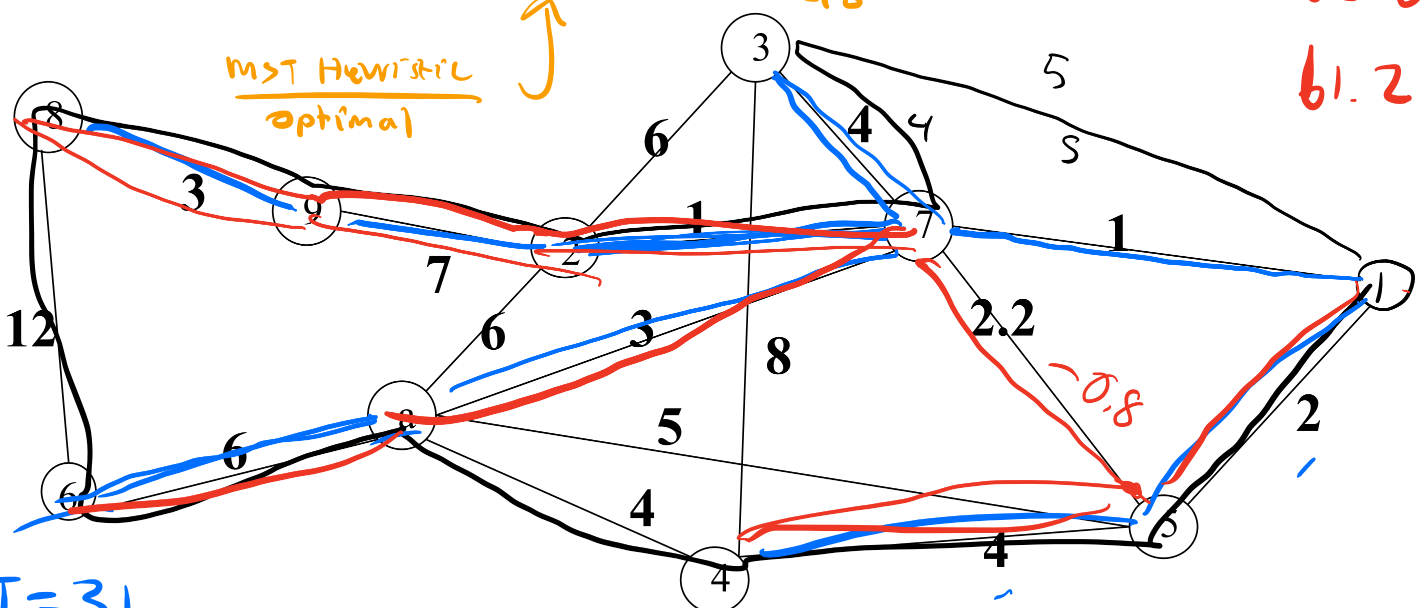
2. Double MST tour 1, 5, 4, 5, 1, 7, 2, 9, 8, 9, 2, 7, 3, 7, a, 6, a, 7, 1 length = 62

3. MST-heuristic tour (with shortcuts) 1, 5, 4, 7, 2, 9, 8, 3, a, 6, 1 length = 61.2

4. The error of the MST-heuristic is 27.5% $\left(\frac{61.2}{48} - 1\right) \times 100$

62−0.8

61.2



MST=31

31x2=62

Quiz 5 CSc 4520 NAME________________________________

3. For the following graph, find

1. Christofides heuristic matching (3,4)(6,8) length = 20

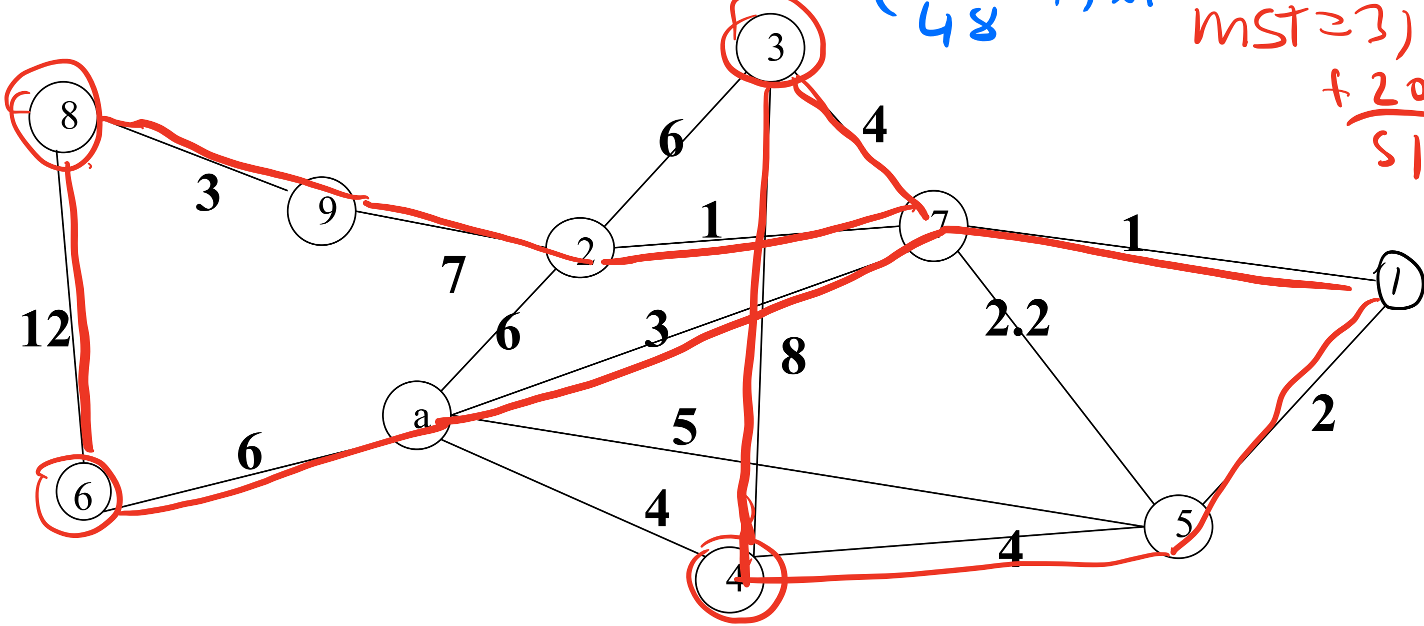
MST+matching tour 1,5,4,3,7,2,9,8,6,a,7,1 length = 51

Christofides heuristic tour (w/shorts) 1,5,4,3,7,2,9,8,6,a,1 length = 51

The error of the Christofides heuristic is 6.25 %

$\left(\frac{51}{48} - 1\right) \times 100$

$$\begin{aligned} MST &= 31 \\ +\ &20 \\ \hline &51 \end{aligned}$$

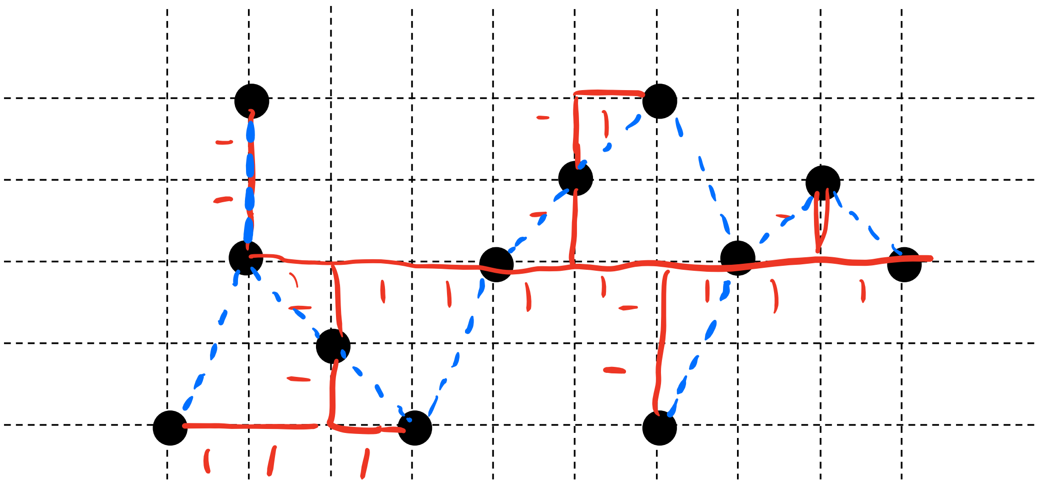


Quiz 5 CSc 4520 NAME______________________________

4 . In the rectilinear metric for points given below find

- The minimum Steiner tree (bold), its length is 9+12 = 21
- The minimum spanning tree (dashed), its length is 14+12 = 26
- the approximation ratio of the MST heuristic in this case is $\frac{26}{21}$
- Approximation error in % 24%

$\frac{\text{span}}{\text{stein}}$



Quiz 5 CSc 4520 NAME

8. Given the following stable marriage instance:

B1's preferences: 1st choice=G1, 2nd=G3, next – G4, last – G2

B2: G2,G3,G4,G1

B3: G4,G2,G1,G3

B4: G4,G1,G3,G2

G1: B2,B1,B3,B4

G2: B4,B3,B1,B2

G3: B3,B2,B1,B4

G4: B1,B2,B3,B4

Boys' best stable marriage is ____________________

Girls' best stable marriage is ____________________