88/100

1\. Give asymptotic upper and lower bounds for T(n)

.25

6

(1) $T(n) = 26T(n/3) + n^3$

$a = 26$, $b = 3$, $f(n) = n^3$

$n^{\log_3 26} = n^x$ $4 > x > 3$

case 1: $f(n) = O(n^{\log_3 26 - \epsilon}) \rightarrow T(n) = \Theta(n^{\log_3 26})$ ✗

(2). $T(n) = 125T(n/25) + n^{1.51}$

$a = 125$, $b = 25$, $f(n) = n^{1.51}$

$n^{\log_{25} 125} = n^{1/2 \log_5 5^3} = n^{\log_5 5^{3/2}} = n^{3/2} < n^{1.51}$

$3 \times 3 = 9 \times 3 = 27$

case 3: $f(n) = \Omega(n^{\log_{25} 125 + \epsilon}) \rightarrow T(n) = n^{1.51}$ ✓

$27 \times 3 = 81$

(3). $T(n) = 81T(n/3) + n^3$

$a = 81$, $b = 3$, $f(n) = n^3$

$n^{\log_3 81} = n^{\log_3 3^4} = n^4$

case 1: $f(n) = O(n^{\log_3 81 - \epsilon}) \rightarrow \underline{T(n) = n^{\log_3 81} = n^4}$ ✗

(4). $T(n) = T(n\text{-}9) + n^3/5$

$= T(n-18) + (n-9)^{3/5} + n^{3/5}$

$= T(9) + \underbrace{9^{3/5} + 18^{3/5} \ldots n^{3/5}}$ $= n^{8/5}$ case 3: $T(n) = n^{8/5}$ ✗

2. For Quicksort (from slides = left side for the pivot) for the sequence
28, 15, 41, 71, 45, 25, 37

the last swap is 71⟷45 ✓, the first swap is 28⟷15 ✓

12 the number of swaps is 7 ✓, the number of comparisons is 21 ✗

28 15 41 71 45 25 37

25 15 | 41 71 45 28 37

15 | 25 | 37 28 | 45 71 41

15 | 25 | 28 | 37 | 41 | 71 45

15 | 25 | 28 | 37 | 41 | 45 | 71

#C	#S
7	1
2+5	3
5	2
2	1
21	7

3. For Insertion sort for the sequence

28, 15, 41, 71, 45, 25, 37

the last swap is 41 ⟷ 37 ✓, the first swap is 28 ⟷ 15 ✗

9

the number of swaps is 4 ✗, the number of comparisons is 14 ✗

+3

	#c	#s
28 15, 41 71 45 25 37	1	1
15 28 41, 71 45 25 37	1	0
15 28 41 71, 45 25 37	1	0
15 28 41 ^ 71 45, 25 37	2	1
15 28 ^ 41 45 71 25, 37	5	1
15 25 28 ^ 41 45 71 37,	4	1
15 25 28 37 41 45 71	14	4

4. Given a set of 12 balls among which there are exactly 3 radioactive. During each test you put any number of balls in a box, push a button and a bulb will be on if there is at least one radioactive ball in the box and will be off otherwise. Using lower bound method tell what is the minimum number of tests is necessary (you cannot do better) to find all 3 radioactive balls.

20

# of answers = $\binom{12}{3}$

# of outcomes = 2

$\log_2 \binom{12}{3}$

EC. Show how to find at least 1 radioactive ball out of 3 radioactive balls from total of 10 balls in 3 tests

15

Test 1: choose 5 out of 10 balls in the box. If the light turns on, choose this group for test 2. If it doesn't choose the other.

Test 2: choose 2 out of 5.

Test 3: choose 1 out of 2. If it lights up, then thats radioactive; otherwise it's the other one

5. How many comparisons is necessary for the selection algorithm (from slides = pivot is the leftmost) to find median out of the sequence

0

28, 15, 41, 71, 45, 25, 37

25 15 41 | 71 45 28 37

71 45 28 37

37 28 | 45 71

37 28

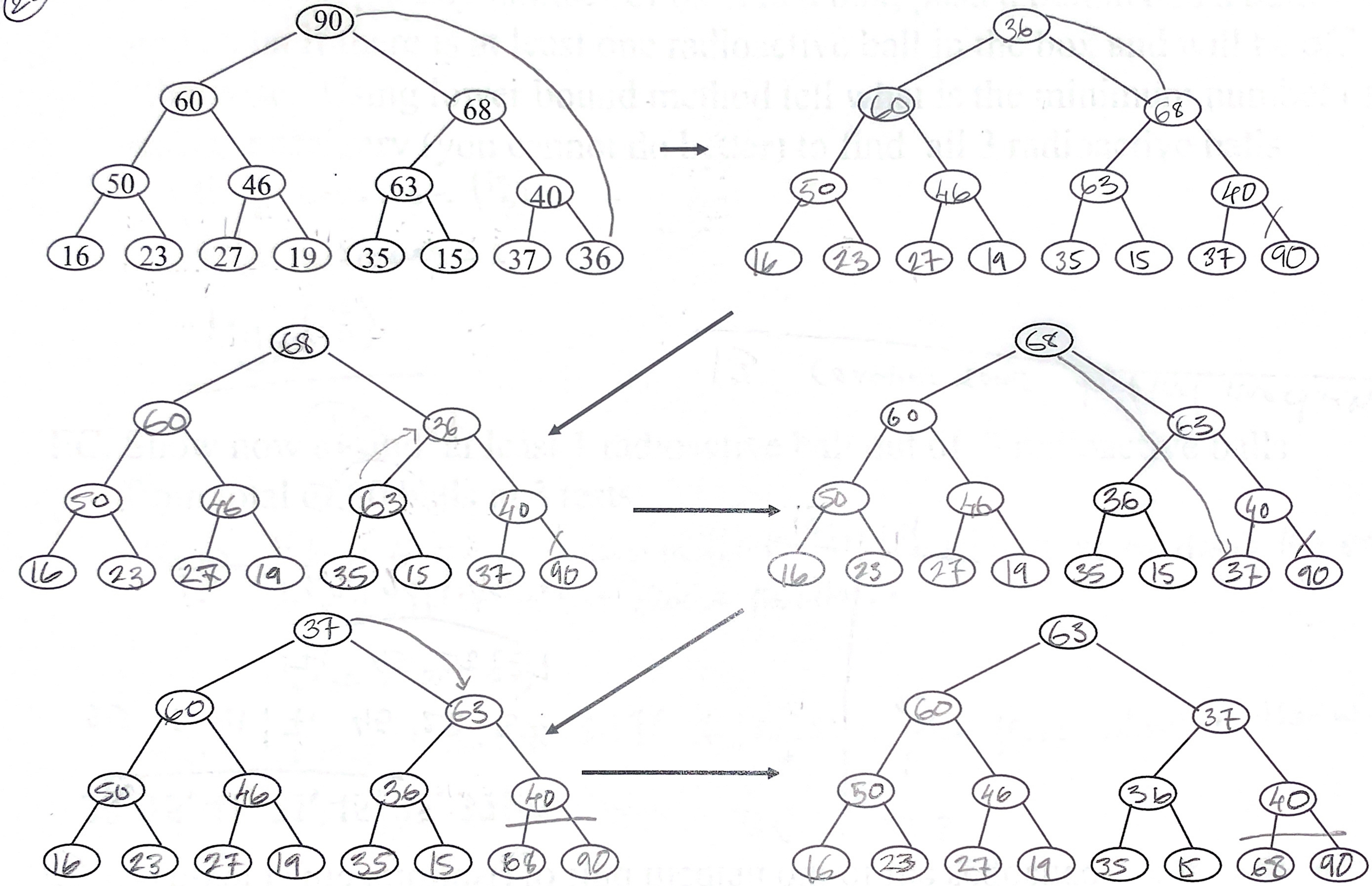
28 37

#C	#S
7	1
4	2
2	1
13	4

13 comparisons to find median

6. Show first 5 swaps of heapsort (deletions of max) with the input heap below

20



Quiz 2 CS4520 NAME:________________________________