Homework 3

Three guys, John, Paul, and Peter are on death row in GA. Governor decided to pardon one of them randomly chosen. He told the name of the pardoned to the guard. The night before the execution, each prisoner is in their own cell, and none are supposed to know who will be pardoned. John asks the guard to give him the name of the other two who will be executed. Guard answers that Peter will be executed. John gets happier - before the guard gave him the name, his probability of surviving was 1/3, and now when he knows that either him or Paul will be pardoned, his probability of surviving is higher = by 1/2. Is it correct? Should John be happy?

(Give YOUR EXPLANATION for the probabilities for John to be pardoned)

The answer is that prisoner John did not gain any information about his own fate, since he already knew that the guard would give him the name of someone else. Prisoner John, prior to hearing from the guard, estimates his chances of being pardoned as 1/3, the same as both Paul and Peter. As the warden says Peter will be executed, it is either because Paul will be pardoned (1/3 chance), or John will be pardoned (1/3 chance) *and* the coin to decide whether to name Paul or Peter the guard flipped came up Peter (1/2 chance; for an overall 1/2 × 1/3 = 1/6 chance Peter was named because John will be pardoned). Hence, after hearing that Peter will be executed, the estimate of John's chance of being pardoned is half that of Paul. This means his chances of being pardoned, now knowing Peter is not, again are 1/3, but Paul has a 2/3 chance of being pardoned.

The explanation above may be summarized in the following table. As the guard is asked by John, he can only answer Peter or Paul to be executed (or "not pardoned").

|  |  |  |  |
| --- | --- | --- | --- |
| **Being pardoned** | **Guard: "not Peter"** | **Guard: "not Paul"** | **Sum** |
| John | 1/6 | 1/6 | 1/3 |
| Peter | 0 | 1/3 | 1/3 |
| Paul | 1/3 | 0 | 1/3 |

As the warden has answered that Peter will not be pardoned, the solution comes from the second column "not Peter". It appears that the odds for John vs. Paul to be pardoned are 1:2.

Call ***John***, ***Paul,*** and ***Peter*** the events that the corresponding prisoner will be pardoned, and ***b*** the event that the guard tells John that prisoner Peter is to be executed, then, using Bayes' theorem, the posterior probability of John being pardoned, is:

P(John|*b*) = P(*b*|John)P(John) / P(*b*|John)P(John) + P(*b*|Peter)P(Peter) + P(*b*|Paul)P(Paul)

= 1/2 × 1/3 / 1/2 × 1/3 + 0 × 1/3 + 1 × 1/3

= 1/3.

So, in the end John should not be getting happy and his probability of surviving was incorrect!