Homework 1

Given five rocks of different weights and lever scales, in 7 tests, sort the rocks by weight. Show that you will find the correct order for all outcomes of comparisons by the 7th comparison.

Explanation 1:

#1: Group two rocks together. See which is heavier. Comparisons: 1.

#2: Group another two rocks together. See which is heavier. Comparisons: 2.

#3: See whether the heaviest rock from the first set is heavier than the heaviest rock from the second set. Comparisons: 3.

So now we have a partial ordering: A > B > C, where A is the heaviest rock of the four, B is the other rock from #3, and C is the rock B was initially paired with. D is the other one from the four rocks we've used so far, and E's the other one.

#4: Let's take E and see where it goes. Compare it with B first. Then we'll have one more comparison to find out where it goes. Comparisons: 5.

We now have an ordering that includes E, for example, A > B > E > C.

#5: Let's find out where D goes. We know it's not heavier than A, so we need to know how it compares with B, C, and E. If E is heavier than A, we need to check B and C; otherwise, check whichever's in the middle of B, C, and E first. That will take two more comparisons (or maybe one if E's heavier than A and we're lucky). Comparisons: 7.

Explanation 2:

Test 1: Compare two rocks, see which is heavier given a lever scale.

Test 2: Compare two more rocks to see which is heavier, not repeating a rock from Test 1.

Test 3: After noting the heavier of the two pairs, determine which of two is heavier, this helps us to determine a base to begin from.

Test 4: From this point, we know A (the heaviest from Test 3) is greater than B (the rock compared with A in step 3) which is greater than C (hypothetically could represent both rocks that were lighter in the first two tests) So now, we bring the 5th rock, E, to compare with rock B (the middle between A and C). This gives us a placement for E, let’s say A>B>E>C

Test 5: Now for placing D, essentially splitting C into the two rocks from test 1 and 2. We are certain it is not heavier than A, so depending on how it compares to B E and C will determine the order. Depending on where we begin, if E is greater than A, then it goes at the beginning of the order and then we would only need to compare B and C, whereas is A is greater than E, E will need to be compared with the weight between B C and E.

Test 6: From that point, we only need to make one more comparison if E is greater than A, then we will need the 7th test if A is greater.

Test 7: The final test between the two rocks left.