Rotational motion

C 1. The moment of inertia of a body depends on

A. the angular velocity.
B. the angular acceleration.
C. the mass distribution.
D. the torque acting on the body.

D 2. To increase the moment of inertia of a body about an axis, you must

A. increase the angular acceleration.
B. increase the angular velocity.
C. decrease the angular velocity.
D. place part of the body farther from the axis.

B 3. A woman sits on a spinning stool with her arms folded. When she extends her arms, which of the following occurs?

A. She increases her moment of inertia, thus increasing her angular speed.
B. She increases her moment of inertia, thus decreasing her angular speed.
C. She decreases her moment of inertia, thus increasing her angular speed.
D. She decreases her moment of inertia, thus decreasing her angular speed.

D 4. Two points, A and B, are on a disk that rotates about an axis. Point A is three times as far from the axis as point B. If the speed of point B is v, then what is the speed of point A?

A. v B. 9v C. v/3 D. 3v

B 5. Rotational inertia is ___________.

A. equivalent to inertia.
B. the property of a rotating body that causes it to continue to turn until an outside torque changes its rotational motion.
C. when an object at rest begins to rotate when a linear force is applied to it.
D. the property of a moving body that causes it to continue it its linear path until an outside force changes its linear speed.

1. The radius of the circle traced out by the second hand on a clock is 6.00 cm. In a time *t*, the tip of the second hand moves through an arc length of 24.0 cm. Determine the value of *t* in seconds.

$r = 6.00\text{ cm}$

$s = 24.0\text{ cm}$

$\omega = \frac{2\pi\text{ rad}}{60\text{ s}} = 0.11\text{ rad/s}$

$\theta = \frac{s}{r} = \frac{24.0\text{ cm}}{6.00\text{ cm}} = 4.0\text{ rad}$

$\omega = \frac{\theta}{t}$

$t = \frac{\theta}{\omega} = \frac{4.00\text{ rad}}{0.11\text{ rad/s}} = \boxed{36.36\text{ s}}$

2. A rotating object starts from rest at t = 0.0 s and has a constant acceleration. At a time of t = 7.0 s the object has an angular velocity of 16 rad/s. What is the angular velocity at a time of t = 14 s?

$\omega_1 = 0$

$\omega_2 = 16\text{ rad/s}$

$t_1 = 0$

$t_2 = 7.0\text{ s}$

$t_3 = 14\text{ s}$

$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{16\frac{\text{rad}}{\text{s}} - 0}{7\text{ s}} = 2.3\text{ rad/s}$

$\omega_3 = \omega_1 + \alpha t_3 = 0 + 2.3\frac{\text{rad}}{\text{s}^2}(14\text{ s}) = \boxed{32.2\frac{\text{rad}}{\text{s}}}$

or

$\omega_3 = \omega_2 + \alpha t = 16\frac{\text{rad}}{\text{s}} + (2.3\frac{\text{rad}}{\text{s}})($ ... $= 32.1\frac{\text{ra}}{\text{s}}$

3. A gymnast is performing a floor routine. In a tumbling run she spins through the air, increasing her angular velocity from 3.00 rev/s to 5.00 rev/s while rotating through one-half of a revolution. How much time does this maneuver take?

$\omega_0 = 3.00\frac{\text{rev}}{\text{s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 18.8\frac{\text{rad}}{\text{s}}$

$\omega = 5.00\frac{\text{rev}}{\text{s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 31.4\text{ rad/s}$

$\theta = 0.500\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 3.14\text{ rad}$

$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{\left(31.4\frac{\text{rad}}{\text{s}}\right)^2 - \left(18.8\frac{\text{rad}}{\text{s}}\right)^2}{2(3.14\text{ rad})} = 101\frac{\text{rad}}{\text{s}^2}$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{31.4\frac{\text{rad}}{\text{s}} - 18.8\text{r}}{101\text{ rad/s}^2} = \boxed{0.13\text{ s}}$

4. The shaft of a pump starts from rest and has an angular acceleration of 3.00 rad/s² for 18.0 s.
   a. At the end of this interval, what is the shaft's angular speed?

$\omega_0 = 0$

$\alpha = 3.00 \frac{rad}{s^2}$

$t = 18.0 s$

$\omega = ?$

$\omega = \omega_0 + \alpha t$

$= 0 + \left(3.00 \frac{rad}{s^2}\right)(18.0 s)$

$= \boxed{54.0 \frac{rad}{s}}$

   b. At the end of this interval, what is the angle through which the shaft has turned?

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= \frac{1}{2}\left(3.00 \frac{rad}{s^2}\right)(18.0 s)^2 = \boxed{486 \text{ rad}}$

5. The wheels of a bicycle have an angular velocity of +20.0 rad/s. Then, the brakes are applied. In coming to rest, each wheel makes an angular displacement of +15.92 revolutions.
   a. How much time does it take for the bike to come to rest?

$\omega_0 = 20.0 \frac{rad}{s}$

$\omega = 0$

$\theta = 15.92 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 100 \text{ rad}$

$t = ?$

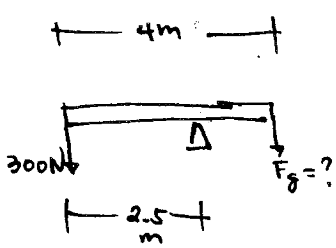
$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{0 - (20 \text{ rad/s})^2}{2(100 \text{ rad})} = -2.00$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 20.0 \text{ rad/s}}{-2.00 \text{ rad/s}^2} = \boxed{10.0 s}$

   b. What is the angular acceleration of each wheel?

$\alpha = -2.00 \frac{rad}{s^2}$

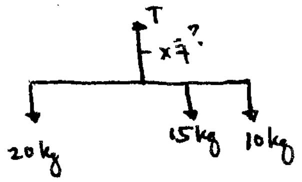
6. The fulcrum of a seesaw 4.0 m long is located 2.5 m from one end. A person weighing 300 N sits on the long end. What must the weight of the person sitting at the short end be to balance the seesaw? Ignore the weight of the seesaw.



$\Sigma\tau = 300N(2.5m) - F_g(1.5m) = 0$

$F_g = \frac{300N(2.5m)}{1.5m} = \boxed{500N}$

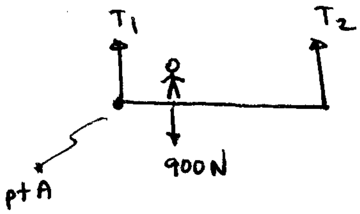
7. A 1.0-m-long horizontal, weightless rod hangs from the ceiling by a string tied to its middle. A 20-kg mass hangs from one end of the rod and a 10-kg mass hangs from the other end. Where should a third 15-kg mass hang relative to the center string to balance the rod?



$\Sigma\tau = (20\text{ kg})(9.8\text{ m/s}^2)(0.50\text{ m}) - 15\text{ kg}(9.8\frac{\text{m}}{\text{s}^2})(x) - 10\text{ kg}(9.8\frac{\text{m}}{\text{s}^2})(0.50$

$x = 0.33\text{ m}$ from the center on the 10 kg side

8. A 3.0-m-long weightless beam is supported at each end by cables. A painter weighing 900 N stands 1.0 m from the left cable. Calculate the tension in each cable.



around pt A:

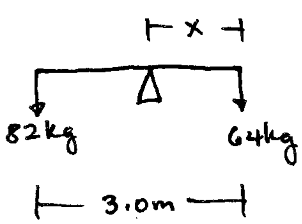
$\Sigma\tau = -900\text{ N}(1.0\text{ m}) + T_2(3.0\text{ m}) = 0$

$T_2 = \dfrac{900\text{ N}(1.0\text{ m})}{3.0\text{ m}} = 300\text{ N}$

$\Sigma F_y = T_1 + T_2 - 900\text{ N} = 0$

$T_1 = 900\text{ N} - T_2 = 900\text{ N} - 300\text{ N} = \boxed{600\text{ N}}$

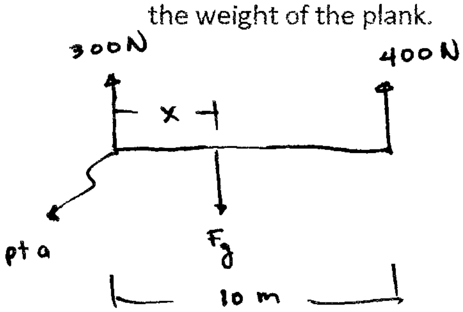
9. A 3.0-m-long weightless beam has an 82-kg mass resting on one end and a 64-kg mass on the other end. How far from the 64-kg mass should a fulcrum be located to balance the beam?



$\Sigma\tau = 82\text{ kg}(9.8\frac{\text{m}}{\text{s}^2})(3.0\text{ m} - x) - (64\text{ kg})(9.8\frac{\text{m}}{\text{s}^2})(x) = 0$

$x = \dfrac{82\text{ kg}(9.8\text{ m/s}^2)(3.0\text{ m})}{82\text{ kg}(9.8\text{ m/s}^2) + (64\text{ kg})(9.8\text{ m/s}^2)} = \boxed{1.7\text{ m}}$

10. A plank 10 m long is supported by cables from each end. The left cable has a tension of 300 N and the right cable, 400 N. Calculate the weight and location of a person sitting on the plank. Ignore the weight of the plank.



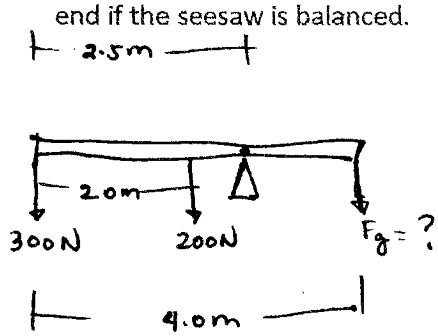
$\Sigma F_y = 300\text{ N} + 400\text{ N} - F_g = 0$

$\boxed{F_g = 700\text{ N}}$

$\Sigma\tau = 400\text{ N}(10\text{ m}) - F_g(x) = 0$

$x = \dfrac{400\text{ N}(10\text{ m})}{700\text{ N}} = \boxed{5.71\text{ m}}$

11. The fulcrum of a uniform seesaw that is 4.0 m long and weighs 200 N is located 2.5 m from one end. A person weighing 300 N sits on the long end. Determine the weight of a person at the other end if the seesaw is balanced.

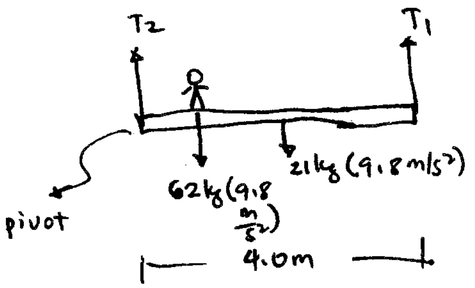


at the fulcrum:

$\Sigma\tau = 300\text{ N}(2.5\text{ m}) + (200\text{ N})(0.5\text{ m}) - F_g(1.5\text{ m}) = 0$

$F_g = \dfrac{300\text{ N}(2.5\text{ m}) + (200\text{ N})(0.5\text{ m})}{1.5\text{ m}} = \boxed{570\text{ N}}$

12. A 4.0-m-long uniform board of mass 21 kg is supported at each end by cables. A 62-kg painter stands 1.0 m from the left cable. Calculate the tension in each cable.

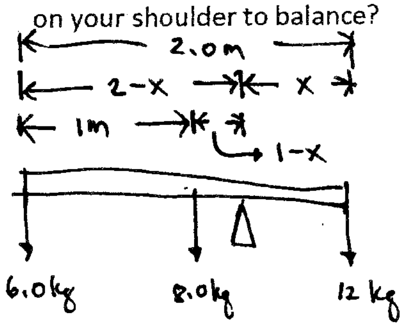


$$\Sigma \tau = -(62\,kg)(9.8\,\frac{m}{s^2})(1.0\,m) - (21\,kg)(9.8\,m/s^2)(2.0\,m) + T_1(4.0\,m) = 0$$

$$\boxed{T_1 = 250\,N}$$

$$\Sigma F_y = T_2 + T_1 - (62\,kg)(9.8\,m/s^2) - (21\,kg)(9.8\,m/s^2) = 0$$

13. A 2.0-m-long uniform beam of mass 8.0 kg supports a 12.0-kg bag of vegetables at one end and a 6.0-kg bag of fruit at the other end. At what distance from the vegetables should the beam rest on your shoulder to balance?



$$\Sigma \tau = (6.0\,kg)(9.8\,\frac{m}{s^2})(2-x) + (8.0\,kg)(9.8\,\frac{m}{s^2})(1-x) - 12\,kg(9.8\,\frac{m}{s^2})(x) = 0$$

$$\boxed{x = 0.77\,m}$$

Practice: Rotational motion

1. A rotating bicycle wheel has an angular speed of 3.00 rad/s at some instant of time. It is then given an angular acceleration of 1.50 rad/s^2. A chalk line drawn on the wheel is horizontal at t = 0. What angle does this line make with its original direction at t = 2.00 s?

$\omega_0 = 3.00 \frac{rad}{s}$

$\alpha = 1.50 \frac{rad}{s^2}$

$t = 2.00 s$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 3.00 \frac{rad}{s}(2.00 s) + \frac{1}{2}\left(1.50 \frac{rad}{s^2}\right)(2.00 s)^2$

$= \boxed{9.0 \text{ rad}}$

2. A record player is switched from 33 rpm to 45 rpm. The turntable changes speed in 2.0 s. Calculate its average angular acceleration.

$\omega_0 = 33 \frac{rev}{min}\left(\frac{2\pi rad}{1 rev}\right)\left(\frac{1 min}{60 s}\right) = 3.46 \frac{rad}{s}$

$\omega = 45 \frac{rev}{min}\left(\frac{2\pi rad}{1 rev}\right)\left(\frac{1 min}{60 s}\right) = 4.71 \frac{rad}{s}$

$t = 2.0 s$

$\alpha = ?$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{4.71 \frac{rad}{s} - 3.46 r}{2.0 s}$

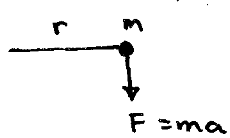
$= 0.63 \frac{rad}{s^2}$

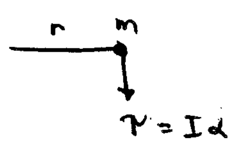
3. A 2.0-kg block attached to the end of a 0.80-m-long stick of negligible mass can rotate in a circular path on a horizontal frictionless surface. The mass starts at rest and is pushed by a 4.0-N force in a direction tangent to the circle. Calculate
   a. the angular acceleration of the block, and

$m = 2.0 kg$

$\omega_0 = 0$

$\alpha = ?$





$\tau = Fr$

$= mar$

$\tau = I\alpha$; $a = r\alpha$

$= I\left(\frac{a}{r}\right)$

$m\alpha r = I\frac{\alpha}{r}$

$I = mr^2$

$\alpha = \frac{\tau}{I} = \frac{4.0 N(\cancel{0.80 m})}{2.0 kg(0.80 m)^{\cancel{2}}} = 2.5 \frac{N}{kg}$

$= \boxed{2.5 \text{ r}}$

   b. the time needed for it to acquire a speed of 3.0 m/s.

$\omega_0 = 0$

recall: $v = r\omega$

$\omega = \frac{v}{r} = \frac{3.0 m/s}{0.80 m} = 3.75 \frac{rad}{s}$

$\alpha = \frac{\omega - \omega_0}{t}$

$t = \frac{\omega - \omega_0}{\alpha} = \frac{3.75 \frac{rad}{s} - 0}{2.5 rad/s^2}$

$= \boxed{1.5 S}$

4. A Ferris wheel with a moment of inertia of 2.0 x 10^5 kg·m^2 is to accelerate from rest to an angular velocity of 0.20 rad/s in 20 s. What is the minimum torque that its motor must provide to cause this acceleration?

$I = 2.0 \times 10^5 kg \cdot m^2$

$\tau = ?$

$\omega_0 = 0$

$\omega = 0.20 \frac{rad}{s}$

$t = 20 s$

$\tau = I\alpha$

$\alpha = \frac{\omega - \omega_0}{t}$

$= \frac{0.20 \frac{rad}{s} - 0}{20 s} = 0.01 \frac{rad}{s^2}$

$\tau = (2.0 \times 10^5 kg \cdot m^2)\left(0.01 \frac{rad}{s^2}\right)$

$= \boxed{2000 N \cdot m}$

5. Determine the average torque needed to accelerate the turbine of a jet engine from rest to an angular velocity of 160 rad/s in 25 s. The turbine's rotating part have a 32-kg·m^2 moment of inertia.

$\omega_0 = 0$

$\omega = 160 \frac{rad}{s}$

$t = 25 s$

$I = 32 kg \cdot m^2$

$\tau = I\alpha$

$\alpha = \frac{\omega - \omega_0}{t} = \frac{160 \frac{rad}{s} - 0}{25 s}$

$= 6.4 \frac{rad}{s^2}$

$\tau = 32 kg \cdot m^2 \left(6.4 \frac{rad}{s^2}\right)$

$= \boxed{204.8 N \cdot m}$

6. A merry-go-round, considered as a uniform disc of radius 5.0 m and mass 25000 kg, must have a new motor that will accelerate it form rest to 1.5 rad/s in 8.0 s. Calculate the torque that the motor must provide to the merry-go-round.

$r = 5.0m$
$m = 25000 kg$
$\omega_0 = 0$
$\omega = 1.5 rad/s$
$t = 8.0s$

$\tau = ?$
$\tau = I\alpha$
$\alpha = \frac{\omega - \omega_0}{t} = \frac{1.5\frac{rad}{s} - 0}{8.0s} = 0.19 rad/s^2$

$I = \frac{1}{2}MR^2 = \frac{1}{2}(25000kg)(5.0m)^2 = 312500 kg$

$\therefore \tau = (312500 kg \cdot m^2)(0.19 \frac{rad}{s^2})$
$= \boxed{59375 N \cdot m}$

7. A ballerina spins with an initial angular velocity of 1.5 rev/s when her arms and a leg are extended. As she draws her arms and leg in toward her body, her moment of inertia becomes 1.0 kg·m^2 and her angular velocity is 4.0 rev/s. Calculate her initial moment of inertia.

$\omega_0 = 1.5 \frac{rev}{s}\left(\frac{2\pi rad}{1 rev}\right) = 9.42 rad/s$
$\omega = 4.0 \frac{rev}{s}\left(\frac{2\pi rad}{1 rev}\right) = 25.13 \frac{rad}{s}$
$I = 1.0 kg \cdot m^2$
$I_0 = ?$

$I_0\omega_0 = I\omega$
$I_0 = \frac{I\omega}{\omega_0} = \frac{(1.0 kg \cdot m^2)(25.13 \frac{rad}{s})}{9.42 rad/s}$
$= \boxed{2.67 kg \cdot m^2}$

8. A 0.20-kg block moves at the end of a 0.50-m string in a circular path on a frictionless table. The block's initial angular velocity is 2.0 rad/s. As the block rotates, the string wraps around the stick at the axis of rotation. Calculate the final angular velocity and tangential speed of the block when the string is 0.20 m from the axis.

$\omega_0 = 2.0 rad/s$
$m = 0.20kg$
$r_0 = 0.50m$
$r = 0.20m$
$\omega_0 = ?$

$I_0 = mr_0^2$
$= 0.20kg(0.50m)^2$
$= 0.05 kg \cdot m^2$
$I = mr^2$
$= 0.20kg(0.20m)^2$
$= 0.008 kg \cdot m^2$

$I_0\omega_0 = I\omega$
$\omega = \frac{I_0\omega_0}{I}$
$= \frac{0.05 kg \cdot m^2 (2.0 rad/s)}{0.008 kg \cdot m^2}$
$= \boxed{12.5 \frac{rad}{s}}$

$V = r\omega$
$=(0.20m)$
$(12.5 r$
$= \boxed{2.5 \frac{m}{s}}$

9. A turntable whose moment of inertia is 1.0 x 10^{-3} kg·m^2 rotates on a frictionless air cushion with an angular velocity of 2.0 rev/s. A 1.0-g beetle falls to the center of the turntable and then walks 0.15 m to it edge. Calculate the angular velocity of the turntable with the beetle on the edge.

$I_0 = 1.0 \times 10^{-3} kg \cdot m^2$
$\omega_0 = 2.0 \frac{rev}{s}\left(\frac{2\pi rad}{1 rev}\right) = 12.57 \frac{rad}{s}$
$m = 1.0 \times 10^{-3} kg$
$r_0 = 0$
$r = 0.15m$
$\omega = ?$

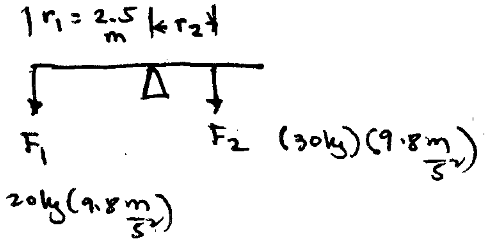
$I_0\omega_0 = I\omega$
$I = I_T + I_b = 1.0 \times 10^{-3} kg \cdot m^2 + mr^2$
$= 1.0 \times 10^{-3} kg \cdot m^2 + (1.0 \times 10^{-3} kg)(0.15m)^2$
$= 1.02 \times 10^{-3} kg \cdot m^2$
$\omega = \frac{I_0\omega_0}{I} = \frac{1.0 \times 10^{-3} kg \cdot m^2 (12.57 rad/s)}{1.02 \times 10^{-3} kg \cdot m^2}$
$= \boxed{12.32 \; rad}$

10. A force of 20 N is applied perpendicular to the end of a bar of length 0.50 m. Calculate the torque produced by the force.

$r = 0.50m$
$F = 20N$
$\theta = 90°$

$\tau = rF\sin\theta$
$= (0.50m)(20N)\sin 90$
$= \boxed{10 N \cdot m}$

11. A child of mass 20 kg is located 2.5 m from the fulcrum or pivot point of a seesaw. Where must a child of mass 30 kg sit on the seesaw in order to provide balance?



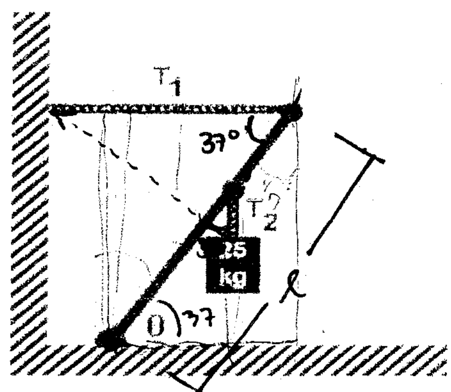
$\Sigma\tau = 0 = \tau_1 - \tau_2 = 0$

$F_1 r_1 - F_2 r_2 = 0$

$r_2 = \frac{F_1 r_1}{F_2} = \frac{20kg(9.8 m/s^2)(2.5m)}{30kg(9.8 m/s^2)} = \boxed{1.67 m}$

12. A 25.0-kg box is suspended 2/3 of the way up a uniform 100-N beam. If θ = 37.0°, what is the value of T_1?

$\tau = r_\perp F$



$\Sigma\tau = \tau_1 - \tau_2 - \tau_3 = 0$

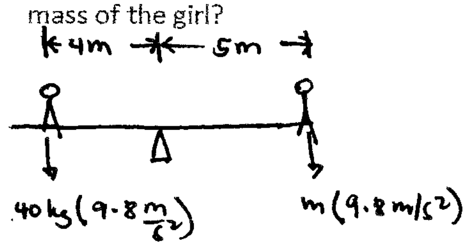
$T_1 \ell \sin 37 - T_2(\frac{2}{3}\ell)\sin 53 - T_3(\frac{1}{2}\ell)\sin 53$

$T_1 = \frac{\frac{2}{3}T_2 \cancel{\ell}\sin 53 + \frac{1}{2}T_3 \cancel{\ell}\sin 53}{\cancel{\ell}\sin 37}$

$= \frac{\frac{2}{3}(25kg)(9.8\frac{m}{s^2})\sin 53 + \frac{1}{2}(100N)\sin 53}{\sin 37}$

$= \boxed{283.10 N}$

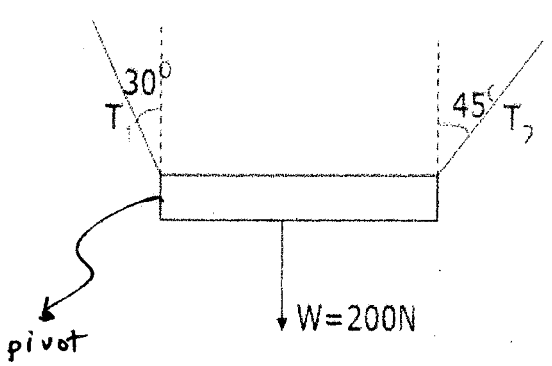
13. A see-saw is supported at the center. A 40 kg boy sits at the left, 4.0 m away from the center, and a girl sits 5.0 m at the right, 5.0 m away from the center. If the see-saw is balanced, what is the mass of the girl?



$\Sigma\tau = 40kg(\cancel{9.8\frac{m}{s^2}})(4.0m) - m(\cancel{9.8\frac{m}{s^2}})(5.0m) = 0$

$m = \frac{40kg(4.0m)}{5.0m} = \boxed{32 kg}$

14. A 200-N sign which is 3.50 meters long is held up by two ropes, one at a 30.0° angle from the vertical, and the other at 45.0°. What is the tension in each rope?



about the pivot:

$\Sigma\tau = T_2(3.50m)\sin 45 - 200N(1.75N) = 0$

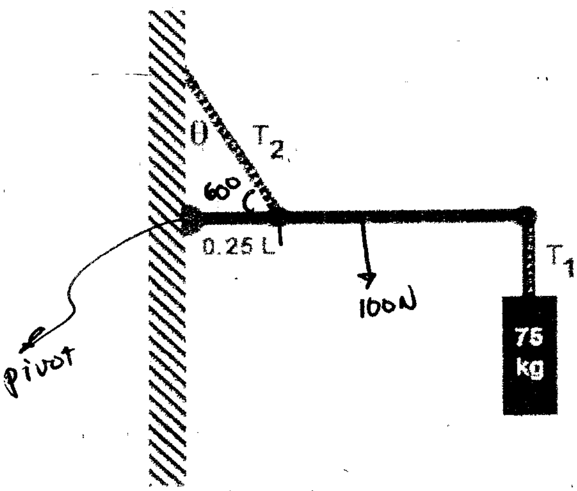
$T_2 = \frac{200N(1.75N)}{3.50m \sin 45} = \boxed{141 N}$

$\Sigma F_y = T_1\cos 30 + T_2\cos 45 - W = 0$

$T_1 = \frac{W - T_2\cos 45}{\cos 30} = \frac{200N - (141N)\cos 45}{\cos 30}$

$= \boxed{116 N}$

15. A 75.0-kg block is suspended from the end of a uniform 100-N beam. If θ = 30.0°, what is the value of T_2?

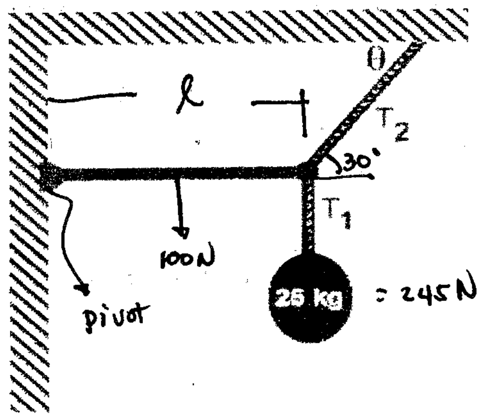


$$\Sigma\tau = T_2(\sin 60)(0.25\cancel{L}) - 100N(0.5\cancel{L}) - 75\,kg\left(9.8\frac{m}{s^2}\right)(\cancel{L}) = 0$$

$$T_2 = \frac{50.0N + 735N}{0.25\sin 60}$$

$$= \boxed{3.63\times10^3 N}$$

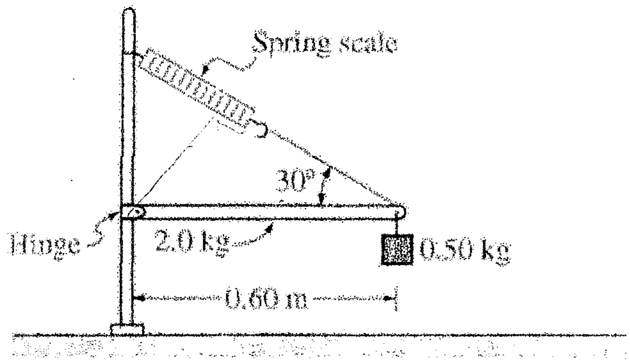
16. A 25.0-kg bag is suspended from the end of a uniform 100-N beam. If θ = 30.0°, what is the value of T_2?



$$\Sigma\tau = T_2\sin 30(\cancel{\ell}) - 100N\left(\frac{\ell}{2}\right) - 245N(\ell) =$$

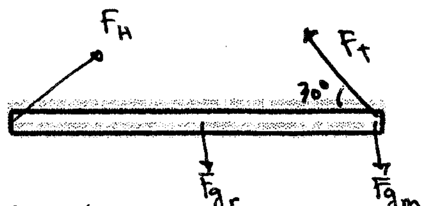
$$\boxed{T_2 = 590N}$$

Torque and Statics



2008M2. The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

a. On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each *force vector originating at its point of application.*



b. Calculate the reading on the spring scale.

Hinge as pivot:

$F_T(0.6)\sin 30 = 0.50(9.8)(0.60) + 2(9.8)0.3$

$F_T = \boxed{29.4\ N}$

The rotational inertia of a rod about its ~~center~~ end is ~~1/12~~ 1/3 ML^2, where M is the mass of the rod and L is its length.

c. Calculate the rotational inertia of the rod-block system about the hinge.

$I_T = I_r + I_m = \frac{1}{3}ML^2 + ML^2 = \frac{1}{3}(2.0\,kg)(0.60\,m)^2 + 0.50\,kg(0.60\,m)^2$

$= \boxed{0.42\ kg\cdot m^2}$

d. If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

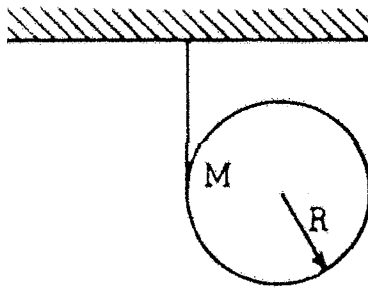
$\Sigma\tau = I\alpha$

$2(9.8)(0.3) + (0.5)(0.6)(9.8) = I\alpha$

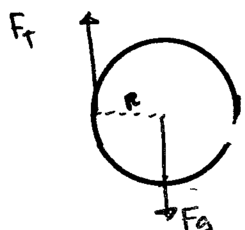
$\alpha = \frac{2(9.8)(0.3) + (0.5)(0.6)(9.8)}{0.42}$

$= \boxed{21\ \frac{rad}{s^2}}$

1976M2.



A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is 1/2 MR^2.



a. On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.

b. In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.

$\Sigma\tau = I\alpha$

$F_T(\cancel{R}) = \frac{1}{2}MR^{\cancel{2}}\alpha$

$F_T = \frac{1}{2}M\cancel{R}\left(\frac{a}{\cancel{R}}\right) = F_T = \frac{1}{2}Ma$

$\Sigma F = ma$

$Mg - F_T = Ma$

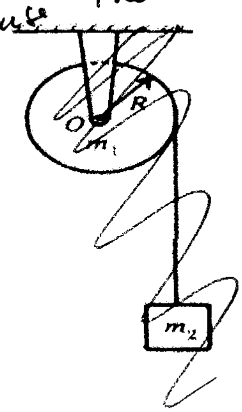
$\cancel{M}g - \frac{1}{2}\cancel{M}a = \cancel{M}a$

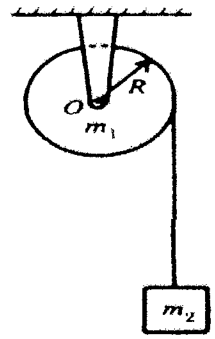
$\frac{3}{2}a = g$

$\boxed{a = \frac{2}{3}g}$

c. While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

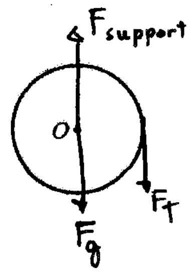
Straight down because their are no horizontal forces.

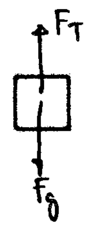




1983M2. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis *through O. The moment of inertia of the cylinder about the axis is $I = 1/2\ m_1R^2$. A block of mass m, suspended by a* cord wrapped around the cylinder as shown above, is released at time t = 0.

a. On the diagram below draw and identify all of the forces acting on the cylinder and on the block.





b. In terms of m_1, m_2, R, and g, determine each of the following.

i. The acceleration of the block

$\Sigma F = ma$

~~Block~~ Cylinder

$\Sigma \tau = I\alpha$

$F_T R = \frac{1}{2} m_1 R^2 \alpha$

$F_T \cancel{R} = \frac{1}{2} m_1 R^{\cancel{2}} \left(\frac{a}{\cancel{R}}\right) \rightarrow F_T = \frac{1}{2} m_1 a$

Block: $F_g - F_T = m_2 a$

$m_2 g - F_T = m_2 a$.

$m_2 g - \frac{1}{2} m_1 a = m_2 a$

$m_2 g = m_2 a + \frac{1}{2} m_1 a$

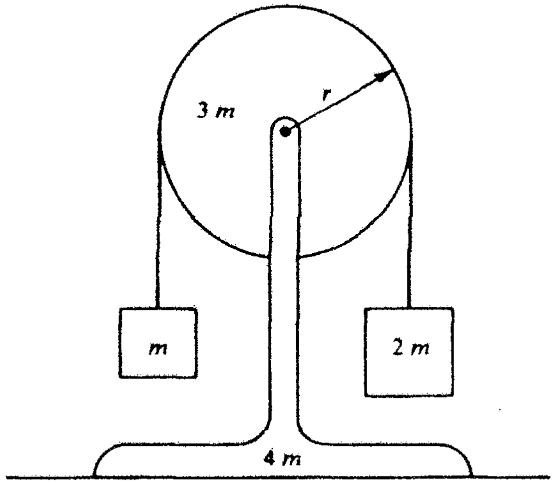
$$\boxed{a = \frac{m_2 g}{m_2 + \frac{1}{2} m_1} \quad \text{or} \quad \frac{2 m_2 g}{2 m_2 + m_1}}$$

ii. The tension in the cord

$$F_T = \frac{1}{2} m_1 \left[\frac{2 m_2 g}{2 m_2 + m_1}\right]$$

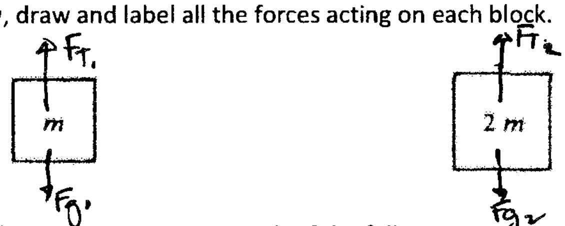
$$= \boxed{\frac{m_1 m_2 g}{2 m_2 + m_1}}$$

1985M3.



1985M3. A pulley of mass 3m and radius r is mounted on frictionless bearings and supported by a stand of mass 4m at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$. Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass 2m on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

a. On the diagrams below, draw and label all the forces acting on each block.



b. Use the symbols identified in part a. to write each of the following.

i. The equations of translational motion (Newton's second law) for each of the two blocks

$\Sigma F = ma$

$F_{T_1} - F_{g_1} = ma$ $\qquad$ $F_{g_2} - F_{T_2} = ma$

$\boxed{F_{T_1} - mg = ma}$ $\qquad$ $\boxed{2mg - F_{T_2} = 2ma}$

ii. The analogous equation for the rotational motion of the pulley

$\Sigma\tau = I\alpha$; $\boxed{(F_{T_2} - F_{T_1})r = I\alpha}$

c. Solve the equations in part b. for the acceleration of the two blocks.

$\alpha = \frac{a}{r}$

$\cancel{2mg - F_{T_2} = 2(F_{T_1} - mg)}$

$\cancel{= 2F_{T_1} - 2mg}$

$\cancel{4mg - F_{T_2} = 2F_{T_1}}$

$F_{T_1} - mg = ma$

$+\ 2mg - F_{T_2} = 2ma$

$F_{T_1} - F_{T_2} + mg = 3ma$

$F_{T_1} - F_{T_2} = 3ma - mg$

$F_{T_2} - F_{T_1} = mg - 3ma$

Subst. to torque eq

$(F_{T_2} - F_{T_1})r = I\alpha$

$(mg - 3ma)r = 1.5mr^2\left(\frac{a}{r}\right)$

$g - 3a = 1.5a$

$\boxed{a = \frac{g}{4.5}}$

d. Determine the tension in the segment of the cord attached to the block of mass m.

$F_{T_1} - mg = m\left(\frac{g}{4.5}\right)$

$F_{T_1} = \frac{mg}{4.5} + mg = \boxed{1.22\ mg \text{ or } \frac{2mg}{9} + mg = \frac{11}{9}mg}$

e. Determine the normal force exerted on the apparatus by the table while the blocks are in motion.

$F_N = 7mg + F_{T_1} + F_{T_2}$

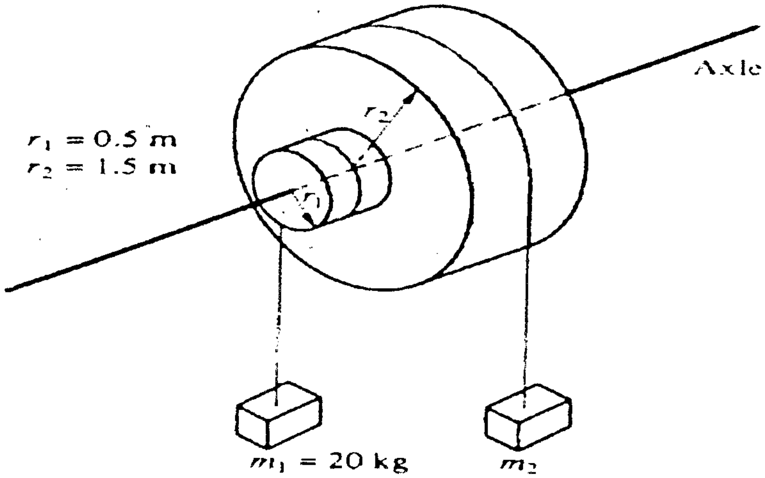
$\cancel{= 7mg +}$

$F_{T_2} = 2mg - 2ma$

$= 2mg - 2mq = 2mg - \frac{4mg}{9} = \frac{18mg}{9} - \frac{4mg}{9} = \frac{14mg}{9}$

$F_N = 7mg + \frac{11}{9}mg + \frac{14mg}{9} = \boxed{\frac{88mg}{9}}$

$a = r\alpha$ -



1991M2. Two masses, m_1 and m_2 are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45$ kg•m^2
Also $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

a. Determine m_2 such that the system will remain in equilibrium.

$\Sigma\tau = 0 \quad \tau_1 = \tau_2$

$m_1 g r_1 = m_2 g r_2$

$m_2 = \frac{m_1 r_1}{r_2} = \frac{20(0.5m)}{1.5m} = \boxed{6.67 \text{ kg}}$

The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.

$\cancel{\Sigma\tau = I\alpha}$

$\cancel{m_1 g r_1}$

$\cancel{\alpha = \frac{m_1 g r_1}{I} = \frac{20(9.8)(0.5)}{45 \text{ kg m}^2} =}$

$\Sigma\tau = I\alpha$

$F_T r_1 = 45(\alpha)$

$\Sigma F_T = F_g - F_T = ma$

$F_g - \frac{45\alpha}{r} = m r \alpha$

$\alpha = \frac{F_g}{\frac{45}{r_1} + m r_1}$

$= \frac{20(9.8)}{\frac{45}{0.5} + (20)(0.5)}$

$= \boxed{1.96 \frac{rad}{s^2}}$

c. Determine the tension in the cable supporting m_1.

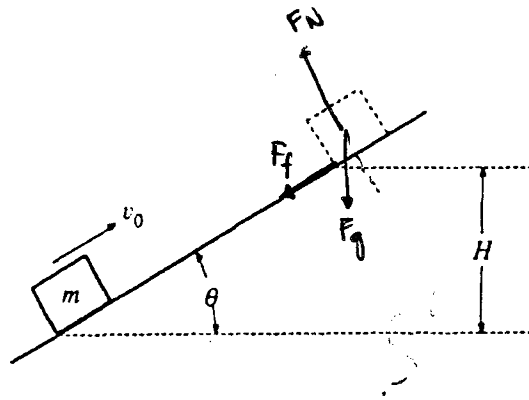
$F_T = \frac{45(1.96 \text{ rad/s}^2)}{0.5} = \boxed{176.4 \text{ N}}$

d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.

$V_0 = 0$
$y = 1.0m$
$v = ?$

$a = r\alpha$
$= 0.5(1.96 \frac{rad}{s^2})$
$= 0.98 \frac{m}{s^2}$

$v^2 = \cancel{v_0^2} + 2ay$
$v = \sqrt{2ay}$
$= \sqrt{2(0.98)(1.0m)}$
$= \boxed{1.4 \text{ m/s}}$



1990M2. A block of mass m slides up the incline shown above with an initial speed v_0 in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.

$$K_0 + \cancel{U_0}^{0} = \cancel{K}^{0} + U$$

$$\tfrac{1}{2}\cancel{m}v_0^2 = \cancel{m}gH$$

$$\boxed{H = \frac{v_0^2}{2g}}$$

b. If the incline is rough with coefficient of sliding friction μ, determine the maximum height to which the block will rise in terms of H and the given quantities.

$$K_0 + \cancel{U_0}^{0} - W_{F_f} = \cancel{K}^{0} + U$$

$$\tfrac{1}{2}mv_0^2 + \mu F_N d = mgh$$

$$F_N = F_g\cos\theta = mg\cos\theta$$

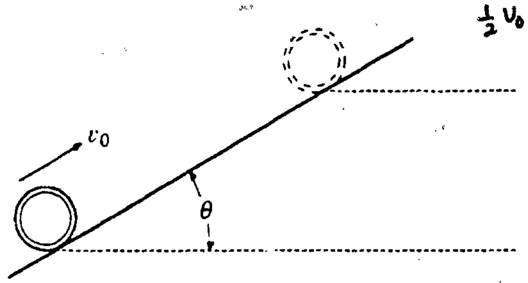
$$d = \frac{h}{\sin\theta}$$

$$\tfrac{1}{2}\cancel{m}v_0^2 - \mu\cancel{m}g\cos\theta\left(\frac{h}{\sin\theta}\right) = \cancel{m}gh$$

$$\tfrac{1}{2}v_0^2 - \mu g h\cot\theta = gh$$

$$\tfrac{1}{2}v_0^2 = gh + \mu gh\cot\theta$$

$$h = \frac{v_0^2}{2g(1+\mu\cot\theta)} = \frac{H}{1+\mu\cot\theta}$$



A thin hoop of mass m and radius R moves up the incline shown above with an initial speed v_0 in the position shown.

c. If the incline is rough and the hoop rolls up the incline without slipping, determine the maximum height to which the hoop will rise in terms of H and the given quantities. $I = mR^2$ $\omega = \frac{v}{R}$

$$\cancel{U_0}^{0} + K_{0_{trans}} + K_{0_{rot}} = U + \cancel{K_{trans}}^{0} + \cancel{K_{rot}}^{0}$$

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega^2 = mgh$$

$$\tfrac{1}{2}\cancel{m}v_0^2 + \tfrac{1}{2}\cancel{m}\cancel{R^2}\left(\frac{v_0}{\cancel{R}}\right)^2 = \cancel{m}gh$$

$$h = \frac{v_0^2}{g} = \boxed{2H}$$

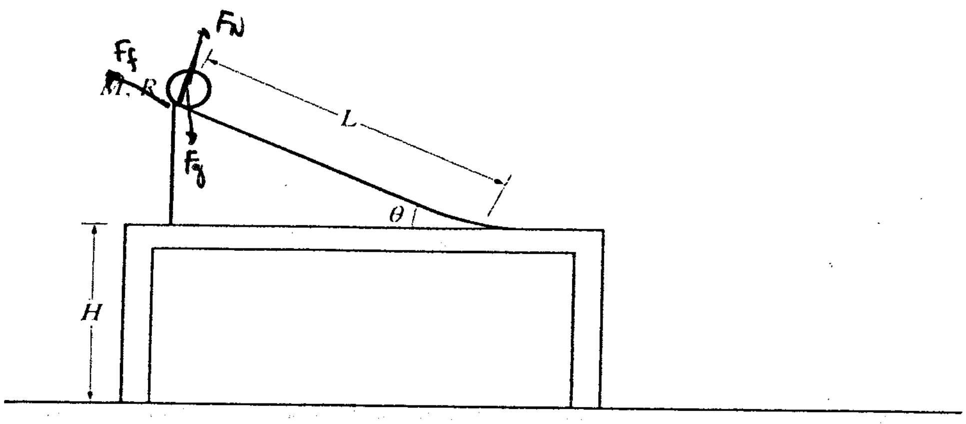
d. If the incline is frictionless, determine the maximum height to which the hoop will rise in terms of H and the given quantities.

$$K_{rot} = \text{constant}$$

$$K_{0_{trans}} = U$$

$$\tfrac{1}{2}\cancel{m}v_0^2 = \cancel{m}gh$$

$$h = \frac{v_0^2}{2g} = \boxed{H}$$



2006M3. A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to the horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

$a = r\alpha$

$\alpha = \frac{a}{r}$

$v = r\omega$

$\omega = \frac{v}{r}$

a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.

$\Sigma F = F_g \sin\theta - F_f = ma$

$\Sigma\tau = F_f R = I\alpha = I\frac{a}{R}$

$F_f \cancel{R} = MR^{\cancel{2}}\frac{a}{\cancel{R}}$

$F_f = Ma$

$F_g \sin\theta - Ma = Ma$

$F_g \sin\theta = 2Ma$

$a = \frac{F_g \sin\theta}{2M} = \frac{\cancel{M}g\sin\theta}{2\cancel{M}} = \boxed{\frac{g\sin\theta}{2}}$

b. Derive the expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

$U_0 + \cancel{K_{t_0}}^{0} + \cancel{K_{r_0}}^{0} = \cancel{U}^{0} + K_t + K_r$

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$\cancel{M}gL\sin\theta = \frac{1}{2}\cancel{M}\cancel{R^2}\left(\frac{v}{\cancel{R}}\right)^2 + \frac{1}{2}\cancel{M}v^2$

$gL\sin\theta = \frac{1}{2}v^2 + \frac{1}{2}v^2 = v^2$

$\boxed{v = \sqrt{gL\sin\theta}}$

c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.

$x = vt$

$v_{0y} = 0$

$-H = \cancel{v_{0y}t}^{0} + \frac{1}{2}a_y t^2$

$= \frac{1}{2}(-g)t^2$

$t = \sqrt{\frac{2H}{g}}$

$x = \sqrt{gL\sin\theta} \cdot \sqrt{\frac{2H}{g}}$

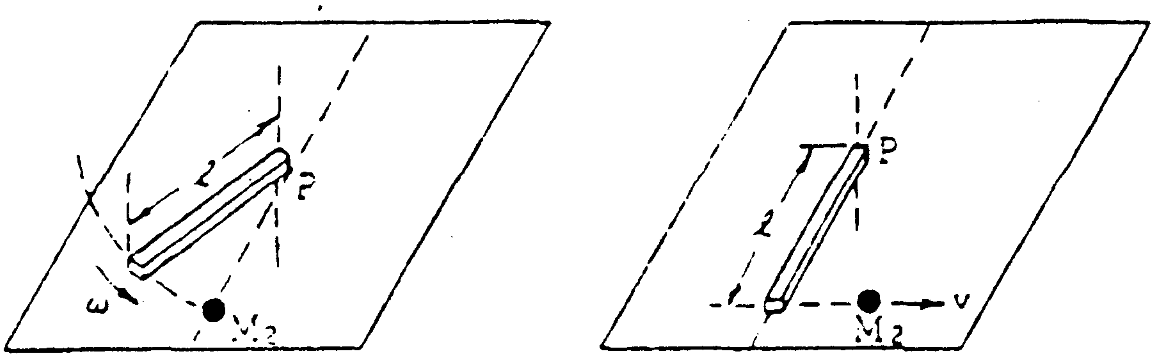
$= \sqrt{\cancel{g}L\sin\theta \cdot \frac{2H}{\cancel{g}}} = \sqrt{2HL\sin\theta}$

d. Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part c. for the hoop?

____less than ____the same as __✓__greater than

Briefly justify your response.

For solid disk, $I = \frac{1}{2}MR^2$ so K_{rot} at the bottom would be less than with that of the hoop. This makes K_t at the bottom greater which results to greater v at the bottom. Since $x = vt$, the x will be greater.



Before Collision After Collision

1978M2. A system consists of a mass M_2 and a uniform rod of mass M_1 and length l. The rod is initially rotating with an angular speed ω on a horizontal frictionless table about a vertical axis fixed at one end through point P. The moment of inertia of the rod about P is $Ml^2/3$. The rod strikes the stationary mass M_2. As a result of this collision, the rod is stopped and the mass M_2 moves away with speed v.

a. Using angular momentum conservation determine the speed v in terms of M_1, M_2, l, and ω.

$L = rmv$

$$I_o \omega_o = mvr$$
$$\frac{Ml^2}{3}\omega = M_2 v l$$
$$\boxed{v = \frac{Ml\omega}{3M_2}}$$

b. Determine the linear momentum of this system just before the collision in terms of M_1, l, and ω.

p = momentum of center of mass of rod

$= M_1 v_{cm} = \boxed{\frac{M_1 l \omega}{2}}$

$v = r\omega$
$= \frac{l}{2}\omega$

c. Determine the linear momentum of this system just after the collision in terms of M_1, l, and ω.

$$p = M_2 v = \cancel{M_2}\left(\frac{M_1 l \omega}{3\cancel{M_2}}\right) = \boxed{\frac{M_1 l \omega}{3}}$$

d. What is responsible for the change in the linear momentum of this system during the collision?

The force on the pivot point. Linear momentum is NOT conserved.

e. Why is the angular momentum of this system about point P conserved during the collision?

The net external force acts ~~at~~ point P, which means that net torque at P is zero. Since net torque is zero, the angular momentum is conserved.

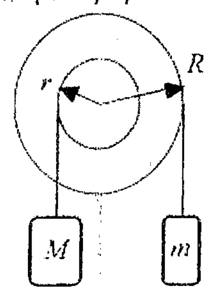
Name: ____ Class Period: __________

Classwork: Rotational motion

November 19, 2018

Multiple Choice. On the space, write the letter of the correct answer.

__D__ 1. The pulley system consists of two solid disks of different radii fastened together coaxially, with two ddifferent masses connected to the pulleys as shown. Under what condition will this pulley system be in static equilibrium?



A. m = M B. rm = RM C. $r^2m = R_2M$ D. rM = Rm

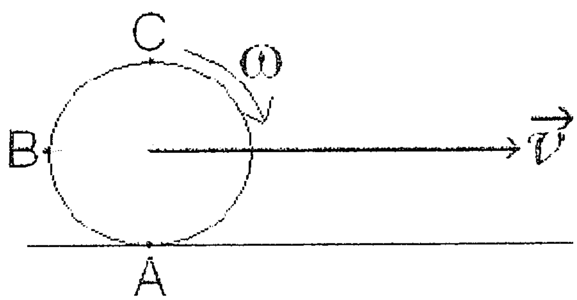
$$\Sigma\tau = rMg - Rmg = 0$$
$$rM\cancel{g} = Rm\cancel{g}$$
$$rM = Rm$$

__D__ 2. Whenever a constant non-zero net torque acts on a rigid object, it produces a

A. constant angular velocity. C. change in angular acceleration.

B. constant angular momentum. D. change in angular velocity.

For the next three questions refer to the following situation: A rigid wheel of radius R rolls without slipping on a horizontal road. The linear velocity of the center of the wheel with respect to the road is v and the angular speed is ω.



__B__ 3. The direction of the instantaneous velocity of point B with respect to the road is roughly:

A. ↘ B. ↗ C. ↓ D. ↑

__B__ 4. Which one of the following statements is true about the speed of the points at the rim of the wheelwith respect to the road?

A. It is larger for points at the bottom of the wheel than for points at the top of it.

B. It is smaller for points at the bottom of the wheel than for points at the top of it.

C. It is the same for all points, with v = ω R.

D. It is the same for all points, with v = 2 ωR.

__D__ 5. Rank the speeds of points A, B, C at the rim of the wheel with respect to the road, largest first.

A. $v_A = v_B = v_C$ B. $v_A > v_B > v_C$ C. $v_C > v_A > v_B$ D. $v_C > v_B > v_A$

Solve the following problems. Show your work to earn the credit. Each problem is worth 5 points. 1 point – correctly label ALL given values, each value MUST have the correct unit, 2 points – write the correct equation and plug-in the correct values, 2 points – correct numerical answer, with correct sign, unit or direction.

1. Romeo and Juliet are playing at a children's playground. It has a merry-go-round. The merry-go-round is a large, flat, uniform circular disc that can spin around its center. You can assume it spins without friction. Its radius is 4.25 m and its mass is 300 kg. Use I = ½ mr^2.
   a. Calculate the merry-go-round's moment of inertia.

$$I = \frac{1}{2} m r^2$$
$$= \frac{1}{2}(300 kg)(4.25 m)^2$$
$$= \boxed{2709.38 \ kg \cdot m^2}$$

   b. If Romeo pushes the merry-go-round with a force that produces a torque of 850 N·m, calculate the merry-go-round's angular acceleration.

$\Sigma\tau = 850 \ N \cdot m$

$\Sigma\tau = I\alpha$

$$\alpha = \frac{\Sigma\tau}{I} = \frac{850 \ N\cdot m}{2709.38 \ kg \cdot m^2} = \boxed{0.31 \ rad/s^2}$$

   c. What is the magnitude of the angular velocity of the merry-go-round after 15 s? The merry-go-round was initially at rest.

$\omega_0 = 0$

$t = 15 s$

$\omega = ?$

$$\alpha = \frac{\omega - \omega_0}{t}$$
$$\omega = \alpha t + \omega_0 = 0.31 \frac{rad}{s^2}(15 s) + 0 = \boxed{4.65 \frac{rad}{s}}$$

2. The diameter of a tire is 64.8 cm. A tack is embedded in the tread of the right rear tire. What is the magnitude of the tack's angular velocity vector if the vehicle is traveling at 10.0 km/h?

$r = 0.324 \ m$

$v = 10.0 \frac{km}{h} = 2.78 \ m/s$

$$V = r\omega$$
$$\omega = \frac{v}{r} = \frac{2.78 \ m/s}{0.324 \ m} = 8.58 \frac{rad}{s}$$

3. A propeller (arm length 1.2 m) starts from rest and begins to rotate counterclockwise with a constant angular acceleration of size 2.7 rad/s^2.
   a. How long does it take for the propeller's angular speed to reach 5.7 rad/s?

$\omega_0 = 0$

$\alpha = 2.7 \ rad/s^2$

$\omega = 5.7 \ rad/s$

$t = ?$

$$\alpha = \frac{\omega - \omega_0}{t}$$
$$t = \frac{\omega - \omega_0}{\alpha} = \frac{5.7 \frac{rad}{s} - 0}{2.7 \ rad/s^2} = \boxed{2.11 s}$$

b. How many revolutions does it take for the propeller's angular speed to reach 5.7 rad/s?

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \frac{1}{2}\left(2.7\ \frac{rad}{s^2}\right)(2.11\ s)^2 = 6.01\ rad = \boxed{0.96\ rev}$$

c. What is the linear speed of the tip of the propeller at 5.7 rad/s?

$$v = r\omega$$

$$= (1.2\ m)\left(5.7\ \frac{rad}{s}\right)$$

$$= \boxed{6.84\ m/s}$$

d. What is the linear acceleration of the tip of the propeller at this point?

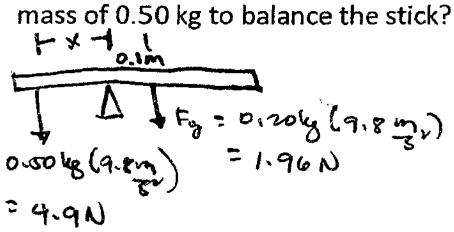
$$a = \frac{\Delta v}{t} = \frac{\Delta(r\omega)}{t} = \frac{r\Delta\omega}{t}$$

$$a = \frac{1.2m\left(5.7\ \frac{rad}{s} - 0\right)}{2.11s} = \boxed{3.24\ m/s^2}$$

or

$$a = r\alpha = 1.2m\left(2.7\ \frac{rad}{s^2}\right) = 3.24\ m/s^2$$

4. A uniform meterstick of mass 0.20 kg is pivoted at the 40 cm mark. Where should one hang a mass of 0.50 kg to balance the stick?



$$\Sigma\tau = 4.9N(x) - 1.96N(0.10m) = 0$$

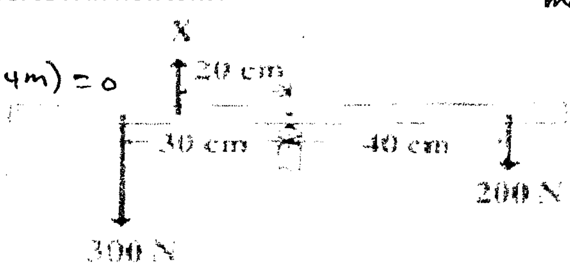
$$x = \frac{1.96N(0.10m)}{4.9N} = 0.04m$$

the mass should hang at 0.04m to the left of fulcrum, at 36 cm mark.

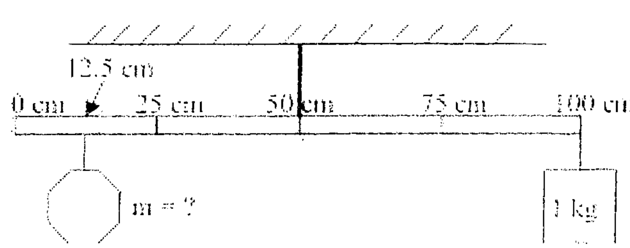
5. A uniform meterstick is balanced at its midpoint with several forces applied as shown below. If the stick is in equilibrium, what is the magnitude of the force X in newtons?

$$\Sigma\tau = 300N(0.3m) - x(0.20m) - (200N)(0.4m) = 0$$

$$\boxed{X = 50\ N}$$



6. What is the mass of the rock shown in the figure?

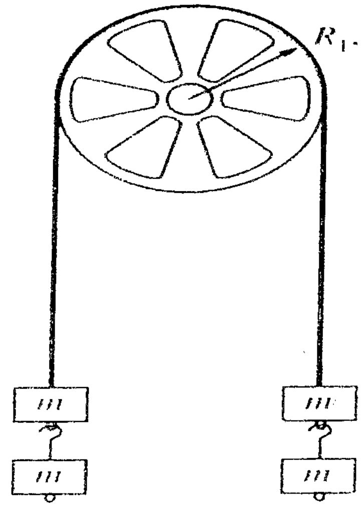


$$\Sigma\tau = m\left(9.8\ \frac{m}{s^2}\right)(0.375m) - 1kg\left(9.8\ \frac{m}{s^2}\right)(0.5m) = 0$$

$$m = \frac{1kg\left(9.8\ \frac{m}{s^2}\right)(0.5m)}{(9.8\ m/s^2)(0.375m)}$$

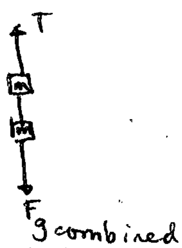
$$= \boxed{1.33\ kg}$$

7.



2000M3. A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley. Determine the answers to parts a and b in terms of m, R_1, I_1 and fundamental constants.

a. Determine the tension in the cord.

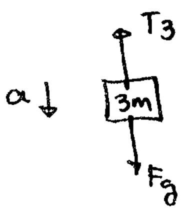


$$T - F_{g\,\text{combined}} = 0$$

$$T = F_{g\,\text{combined}} = \boxed{2mg}$$

b. One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration g/3. Determine the following:

i. The tension T_3 in the section of cord supporting the three blocks on the left.



$$\Sigma F_y = F_g - T_3 = ma$$

$$T_3 = F_g - ma$$

$$= 3mg - \cancel{3}m\left(\frac{g}{\cancel{3}}\right)$$

$$= \boxed{2mg}$$

ii. The tension T_1 in the section of cord supporting the single block on the right.

$$T_1 - F_g = ma$$

$$T_1 = ma + F_g$$

$$= m\left(\frac{g}{3}\right) + mg = \boxed{\frac{4}{3}mg}$$

iii. The rotational inertia of the pulley.

$$T_3R_1 - T_1R_1 = I\alpha$$

$$\alpha = \frac{a}{r} = \frac{g}{3r}$$

$$I = \frac{T_3R_1 - T_1R_1}{\alpha}$$

$$= \frac{2mgR_1 - \frac{4}{3}mgR_1}{\frac{g}{3R}}$$

$$\frac{\frac{6}{3}m\cancel{g}R_1 - \frac{4}{3}m\cancel{g}R_1}{\frac{\cancel{g}}{3R_1}}$$

$$= 6mR_1^2 - 4mR_1^2$$

$$= \boxed{2mR_1^2}$$

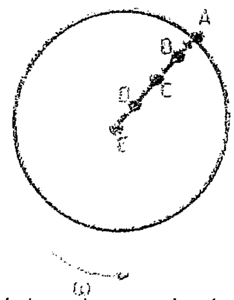
Name: ______________________________ Class Period: __________

Classwork: Rotational motion

November 20, 2018

On the space, write the letter that corresponds to the correct answer.

For Questions 1-3, refer to the following diagram:



__A__ 1. A solid disk with a radius R rotates at a constant rate ω. Which of the following correctly describes the angular displacements of the points?

A. A = B = C = D

B. A > B > C > D

C. A < B < C < D

D. Cannot be determined. Not enough information is known.

__A__ 2. A solid disk with a radius R rotates at a constant rate ω. Which of the following correctly describes their angular velocity of the points?

A. A = B = C = D

B. A > B > C > D

C. A < B < C < D

D. Cannot be determined. Not enough information is known.

__B__ 3. A solid disk with a radius R rotates at a constant rate ω. Which of the following correctly describes the tangential speed of the points?

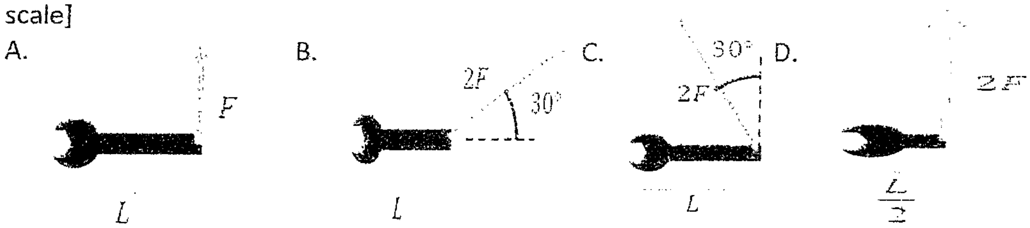
A. A = B = C = D

B. A > B > C > D

C. A < B < C < D

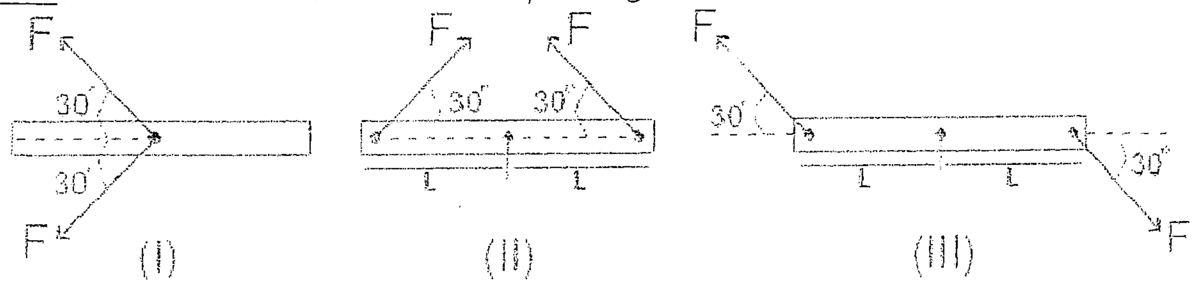
D. Cannot be determined. Not enough information is known.

__C__ 4. A series of wrenches of different lengths is used on a hexagonal bolt, as shown below. Which combination of wrench length and force applies the greatest torque to the bolt? [Diagram not drawn on scale]



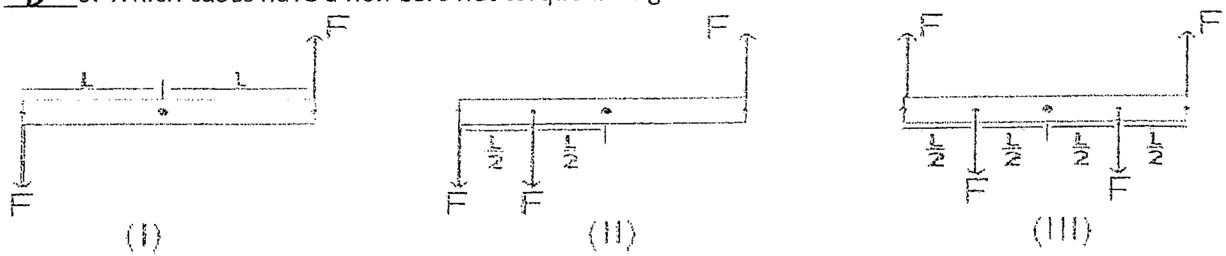
The figures in the next four questions show three cases in which a rigid rod of length 2L is acted upon by some forces. All forces labeled F have the same magnitude.

C 5. Which cases have a non-zero net torque acting on the rod about its center?



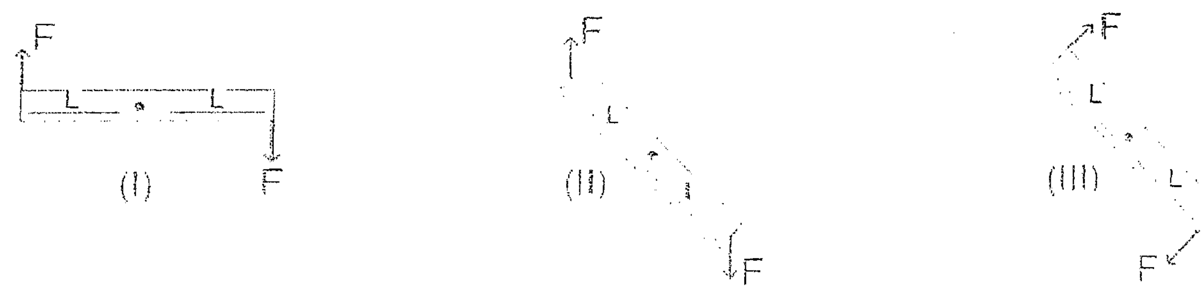
A. (I) only.   B. (II) only.   C. (III) only.   D. (I) and (II) only.

D 6. Which cases have a non-zero net torque acting on the rod about its center?



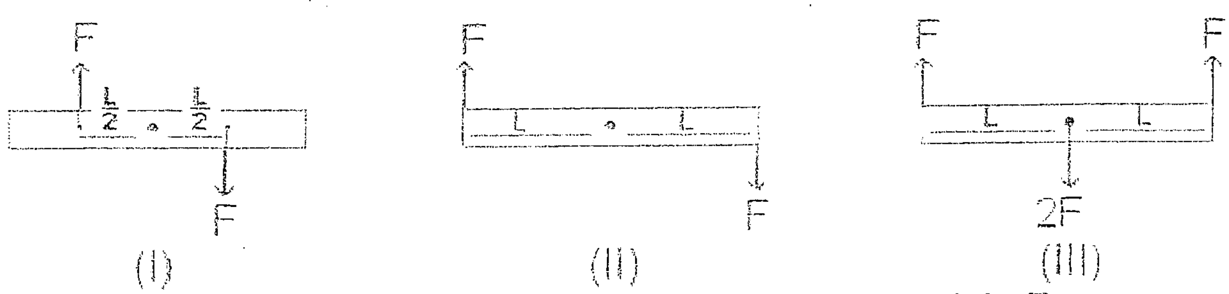
A. (I) only.   B. (II) only.   C. (III) only.   D. (I) and (II) only.

D 7. Rank order the three cases according to the angular acceleration of the rod about an axis passing through its center and perpendicular to the paper.



A. (III) > (II) > (I).   B. (III) = (II) > (I).   C. (II) > (III) > (I).   D. (I) = (III) > (II).

C 8. Rank order the three cases according to the angular acceleration of the rod about an axis passing through its center and perpendicular to the paper, largest first.



A. (I) = (II) > (III).   B. (I) > (II) > (III).   C. (II) > (I) > (III).   D. (II) > (III) > (I).

Solve the following problems. Show your work to earn the credit. Each problem is worth 5 points. 1 point – correctly label ALL given values, each value MUST have the correct unit, 2 points – write the correct equation and plug-in the correct values, 2 points – correct numerical answer, with correct sign, unit or direction.

Where the moment of inertia needs to be calculated, use the following equations:

$I_{sphere} = \frac{2}{5}mr^2$ $\quad I_{solid\ disk\ or\ cylinder} = \frac{1}{2}mr^2$ $\quad I_{wheel\ or\ hoop} = mr^2$

Moment of inertia by a point mass located r distance from the axis of rotation: $I = mr^2$

1. An airplane propeller is rotating at 1900 rev/min.
   a. Compute the propeller's angular velocity in rad/s.

$$\omega = 1900 \frac{rev}{min} \left(\frac{2\pi\, rad}{1\, rev}\right)\left(\frac{1\, min}{60\, s}\right) = 198.97 \frac{rad}{s}$$

   b. How long in seconds does it take for the propeller to turn through 30.0 degrees?

$$\Delta\theta = 30^\circ \left(\frac{2\pi\, rad}{360^\circ}\right) = 0.52\ rad$$

$$\omega = \frac{\Delta\theta}{t}$$

$$t = \frac{\Delta\theta}{\omega} = \frac{0.52\ rad}{198.97\ rad/s} = \boxed{2.61 \times 10^{-3} s}$$

2. A disk with a 1.0-m radius reaches a maximum angular speed of 18 rad/s before it stops 35 revolutions (220 rad) after attaining the maximum speed. How long did it take the disk to stop?

$r = 1.0 m$

$\omega_0 = 18\ rad/s$

$\omega = 0$

$\theta - \theta_0 = 220 rad$

$t = ?$

$$\theta - \theta_0 = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$t = \frac{2(\theta - \theta_0)}{\omega + \omega_0} = \frac{2\,(220\ rad)}{18 \frac{rad}{s} + 0} = \boxed{24.44 s}$$

3. A wagon with wheels having diameter d = 123.0 cm accelerates from rest to 14.5 km/h in 13.3 s. What is the angular acceleration of the wheels?

$r = 0.615 m$

$V_0 = 0$

$V = 14.5 \frac{km}{h} \doteq 4.03 m/s$

$t = 13.3 s$

$\omega_0 = 0$

$$\omega = \frac{V}{r} = \frac{4.03 m/s}{0.615 m} = 6.55 \frac{rad}{s}$$

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.55 \frac{rad}{s} - 0}{13.3 s} = \boxed{0.49\ rad/s^2}$$

4. How many revolutions does a disc make in in 5.44 seconds accelerating from 0 to 7.598 thousand rpm?

$t = 5.44 s$

$\omega_0 = 0$

$\omega = 7.598 \times 10^3 \frac{rev}{min}$

$= 795.66 \frac{rad}{s}$

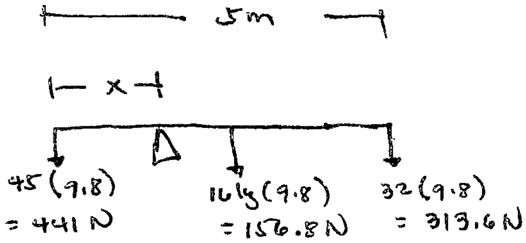
$$\theta - \theta_0 = \left(\frac{\omega + \omega_0}{2}\right)t$$

$$= \left(\frac{795.66 \frac{rad}{s} + 0}{2}\right)(5.44 s)$$

$$= 2164.20\ rad$$

$$= \boxed{344.44\ rev}$$

5. A boy and a girl have masses of 45 kg and 32 kg respectively. Both are balanced on opposite ends of a 5.0 m long wooden plank with a mass of 16 kg. At what point along the plank does the pivot point have to be?



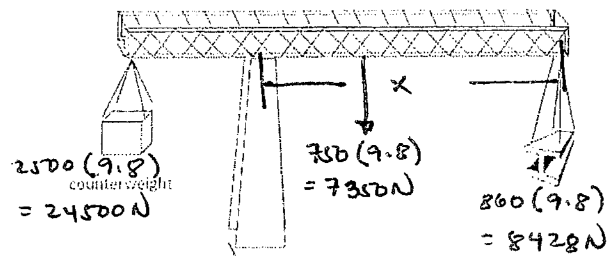
$\Sigma\tau = -313.6\,N(5-x) - 156.8\,N(2.5-x) + 441\,N(x) = 0$

$x = 2.15$ m from the end where the 45 kg mass is hanging.

6. Construction cranes are able to balance heavy loads by using a counterweight. The crane operator can slide the entire torque arm horizontally in order to balance the load. If the torque arm has a total length of 56m and a mass of 750 kg, and the counterweight has a mass of 2500 kg, where should the operator position the pivot point to balance an 860 kg object?

$\Sigma\tau = 24500(56-x) - 7350(x-28) - 8428(x) = 0$

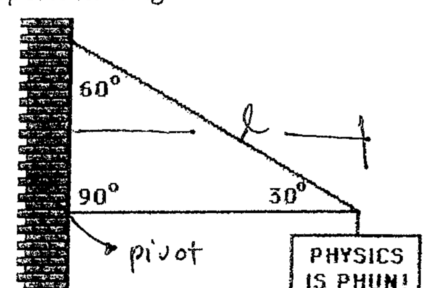
$\boxed{x = 39.17\,m}$



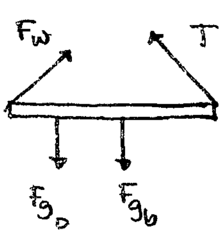
7. The sign below hangs outside the physics classroom, advertising the most important truth to be found inside. The sign is supported by a diagonal cable and a rigid horizontal bar. If the sign has a mass of 50 kg, then determine the tension in the diagonal cable that supports its weight.

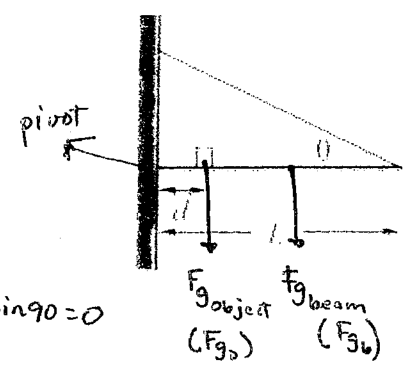
$\Sigma\tau = -50(9.8)\,\ell \sin 90 + T\,\ell \sin 30 = 0$

$\boxed{T = 980\,N}$



8. A uniform steel beam of mass m_1= 200 kg is held up by a steel cable that is connected to the beam a distance L = 5.0 m from the wall, at an angle θ = 30° as shown in the diagram. The beam is bolted to the wall with an unknown force F exerted by the wall on the beam. An object of mass m_2 = 60 kg, resting on top of the beam, is placed a distance d = 1.0 m from the wall.
Calculate the tension in the cable.

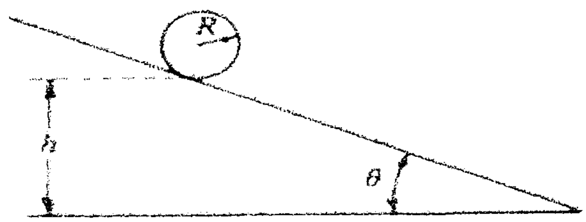




$\Sigma\tau = T(5.0m)\sin 30 - 1960N(2.5m)\sin 90 - 588N(1.0m)\sin 90 = 0$

$\boxed{T = 2195.2\,N}$

9.



1986M2. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height h above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express your answers in terms of M, R, h, g and θ.

a. Determine the following for the sphere when it is at the bottom od the plane:

i. Its translational kinetic energy

$U = K_t + K_r$

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$\cancel{M}gh = \frac{1}{2}\cancel{M}v^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5}\cancel{M}\cancel{R^2}\right)\left(\frac{v^2}{\cancel{R^2}}\right)$

$gh = \frac{5}{10}v^2 + \frac{2}{10}v^2 = \frac{7}{10}v^2$

$v^2 = \frac{10}{7}gh$

$K_t = \frac{1}{2}Mv^2$

$= \frac{1}{2}M\left(\frac{10gh}{7}\right)$

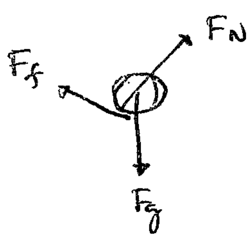
$\boxed{K_t = \frac{5}{7}Mgh}$

ii. Its rotational kinetic energy

$K_r = \frac{1}{2}I\omega^2$

$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5}MR^2\right)\frac{v^2}{R^2} = \frac{1}{5}M\left(\frac{10gh}{7}\right) = \boxed{\frac{2}{7}Mgh}$

b. Determine the sphere's linear acceleration when it is on the plane.



$\Sigma\tau = I\alpha = F_f R$

$F_f \cancel{R} = \left(\frac{2}{5}M\cancel{R^2}\right)\left(\frac{a}{\cancel{R}}\right)$; $\alpha = \frac{a}{r}$

$F_f = \frac{2}{5}Ma$

$\Sigma F = F_g \sin\theta - F_f = Ma$

$Mg\sin\theta - \frac{2}{5}Ma = Ma$

$\cancel{M}g\sin\theta = \cancel{M}a + \frac{2}{5}\cancel{M}a$

$g\sin\theta = \frac{7}{5}a$

$\boxed{a = \frac{5}{7}g\sin\theta}$