Experiment 2 documentation

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1 Introduction

In this experiment, we try to decompose the change in sea surface height into three major components: change in ocean floor, change in ocean mass and ocean steric change. Each of these changes represents an unobserved latent process. Then we link these process to data sets that they contribute to. For sea surface heigh, we have altimetry data and ocean mass from GRACE. GIA is constrained by a physical model solution as prior mean. There is no data or prior linked to the steric change but it is constrained by the change balance. Below shows an illustration of the framework.

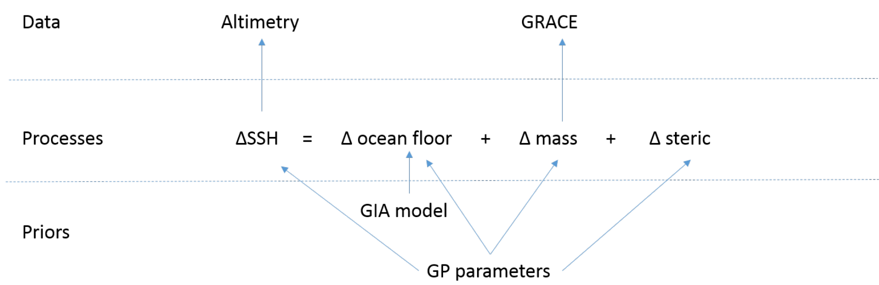

Figure 1:

We assume all processes are converted to the same unit mm/year of water equivalence; hence the data also need to be processed into the same format. For the moment, we also assume the change is time invariant and it is a trend over the period 2005 - 2015 and therefore the data are also processed into trend estiamtes with corresponding standard errors of the estimates as the measuremnt errors. Another issue is that data and processes are not in the same spatial unit and we need to link them to have the same unit.

In the following, we first define the Bayesian hierarchical model for this experiment formally. Then we demonstrate the estimation procedure step by step.

2 The Bayesian hierarchical model

Now we formally write down the BHM and the related assumptions. Denote the latent processes we want to solve by the following X_{ssh} for change in sea surface hight, X_{GIA} for GIA, X_{mass} for hight change due to mass change in the ocean and X_{steric} for the hight change due to steric change. Then for the data, denote the altimetry trend by Y_{alt} with measurement error ε_{alt} and the GRACE data by Y_{GRACE} with measurement errorε_{GRACE}. Denote the GIA foward model solution by m_{GIA}.

2.1 Data

2.1.1 Link altimetry to SSH

The altimetry is provided as gridded data at 1 degree resolution. We assume these altimetry values are the average of the grid and every point within the grid has the same value and therefore use a point to point map for altimetry and SSH

$$Y_{alt} = \mathcal{A}_{ssh} X_{ssh} + \varepsilon_{alt}$$

where A_{ssh} is an linear operator that maps the SSH value to an altimery grid where it falls in. ### 2.1.2 Link GRACE to mass change The GRACE data are provided as agrregation over large polygon grids, hence we assume a map that integrates X_{mass} values over the same polygon of the GRACE to produce the data. For the GRACE data $Y_{GRACE}(i)$ at a given polygon Δ_i, we have

$$Y_{GRACE}(i) = \left(\int_{\Delta_i} X_{mass}(s)\,\mathrm{d}s\right) + \varepsilon_{GRACE}(i)$$

A more general and covenient representation is to write the integration as a linear operation. Denote by $\mathcal{A}_i$ the linear operator on Δ_i, Then the above equation can be written as

$$Y_{GRACE}(i) = \mathcal{A}_{mass,i} X_{mass} + \varepsilon_{GRACE}(i)$$

And for all GRACE data in a vector form, we have

$$Y_{GRACE} = \mathcal{A}_{mass} X_{mass} + \varepsilon_{GRACE}$$

In practice, processes are approximated by GMRF on a given triangulation, the linear operator A_i sum up the average value of the triangle vertices times its area. Some processing functions need to written to find the centroid of the tiangle, the area of the triangle when it is completely inside the GRACE polygon; otherwise take the intersection.

2.2 Process

For all these process we assume they are stationary Gaussian processes with Mat'{e}rn covariance functions over all the oceans. And we use the following notations for modelling

$X_{ssh} \sim \mathcal{GP}(\mu_{ssh}, K(\theta_{ssh})) X_{GIA} \sim \mathcal{GP}(\mu_{GIA}, K(\theta_{GIA})) X_{mass} \sim \mathcal{GP}(\mu_{mass}, K(\theta_{mass})) X_{steric} \sim \mathcal{GP}(\mu_{steric}, K(\theta_{steric}))$

where the μs are the means of the processes and the θs are the hyper-parameters in the covariance functions.

2.3 Priors

2.3.1 measurement errors

The measrument errors are all assumed to be indepedent Gaussian noises. The variance of the Gaussian distributions are given by the standard error of the trend estimates.

2.3.2 Means

For SSH, mass and steric we all assume the means are zero: SSh and mass mean can be updated from the data, steric is regarded as residuals. The GIA signals are usually well represented in the GIA forward models although with modelling errors; therefore we set $\mu_{GIA} = m_{GIA}$.

2.3.3 Hyper-parameters in the covariance

The parameters in the covariance function defines the spatial charactersitic of the processes. One way to specify these parameters is to set them as fixed values learned from existing studies. Another way is to set vague priors on the values of these parameters. In practice, we combine both to make the computation feasible. In general, we set the priors all to be log normal distribution with mean equal to some sensible values and variance large enough.

2.3.3.1 SSH

Need to discuss the charateristics of this process and where to learn the hyper-parameters.

2.3.3.2 GIA

The hyper parameters represent the properties of the discrepancy process $\tilde{X}_{GIA} = X_{GIA} - m_{GIA}$. The variation should be small scaled and has short spatial correlation.

2.3.3.3 mass

Also need to decide where to learn.

2.3.3.4 steric

Same as above and it is constrained by the balance equation.

2.4 Final BHM all together

Write all the assuptions and modelling togther, we have the BHM as

$Y_{alt}|X_{ssh} \sim \mathcal{N}(A_{ssh}X_{ssh}, V_A)Y_{GRACE}|X_{mass} \sim \mathcal{N}(\mathcal{A}_{mass}X_{mass}, V_G)X_{ssh} \sim \mathcal{GP}(0, K(\theta_{ssh}))X_{mass} \sim \mathcal{GP}(0, K(\theta_{mass}))X_{steric}$

where V_A and V_G are the variance matrix for the Altimetry and GRACE measurement errors. Note here we have two data and one physical constraints to solve this system, hence we need at least one set of strong prior for the process hyper-parameters so that the model is identifiable.

3 Implementation

3.1 Data processing

3.2 Prior setting

3.3 Mesh

3.4 Bayesian inference

3.5 Plot results