Ahlfors Chapter1 Solutions

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1

Although in the first several section, the book has not given the representation of trigonometric. While it seems trivial and non-benefit to avoid it, so it will be used.

1.1.1.

Solution.

$$(1+2i)^3 = (1 + 3\times 2i - 12 - 8i) = -11 - 2i$$

$$\frac{5}{-3+4i} = \frac{5(-3-4i)}{9+16} = \frac{-3-4i}{5}$$

$$\left(\frac{2+i}{3-2i}\right)^2 = \left(\frac{6+3i+4i-2}{13}\right)^2 = \frac{-33+56i}{169}$$

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}}\left((\cos\frac{\pi}{4} + \sin\frac{\pi}{4}i)^n + (\cos\frac{\pi}{4} + \sin -\frac{\pi}{4}i)^n\right) = 2^{1+\frac{n}{2}}\cos\frac{n\pi}{4}$$

□

2

1.2.1.

Solution.

$$\sqrt{i} = \sqrt{\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}} = \cos\left(k\pi + \frac{\pi}{4}\right) + i\sin\left(k\pi + \frac{\pi}{4}\right) = \pm(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i)$$

$$\sqrt{-i} = i\sqrt{i} = \pm(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i)$$

$$\sqrt{1+i} = \sqrt{2}\sqrt{\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}} = \sqrt{2}\left(\cos\left(k\pi + \frac{\pi}{8}\right) + i\sin\left(k\pi + \frac{\pi}{8}\right)\right) = \pm\sqrt{2}\left(\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)\right)$$

$$\sqrt{\frac{1-i\sqrt{3}}{2}} = \sqrt{\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)} = \pm(\frac{\sqrt{3}}{2} - \frac{i}{2})$$

□