GTM 73 Chapter1 Solutions

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1.0.1.

Solution. $\mathbb{N}^*$ and $\mathbb{N}$ □

1.0.2.

Solution. Nothing more than a easy verification. □

1.0.3.

Solution. No. A counterexample is

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \,\middle|\, a, b, c, d \in \mathbb{R} \right\}$$

with left identity

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and right inverse

$$\begin{pmatrix} a^{-1} & 0 \\ 0 & 0 \end{pmatrix}$$

However, it has other left identities

$$\begin{pmatrix} 1 & x \\ 0 & 0 \end{pmatrix}, \ \forall x \in \mathbb{R}$$

whence it is not a group. □