

Durrett 5th Chapter3 Solutions

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1 The De Moivre-Laplace Theorem

3.1.1. Generalize the proof of Lemma 3.1.1 to conclude that if $\max_{1 \leq j \leq n} |c_{j,n}| \rightarrow 0$, $\sum_{j=1}^n c_{j,n} \rightarrow \lambda$ and $\sup_n \sum_{j=1}^n |c_{j,n}| < \infty$ then $\prod_{j=1}^n (1 + c_{j,n}) \rightarrow e^\lambda$.

Solution. By the Taylor's expansion, the logarithm of the sum is

$$\begin{aligned} \sum_{j=1}^n \ln(1 + c_{j,n}) &= \sum_{j=1}^n \left(c_{j,n} - \frac{1}{(1 + c_{j,n})^2} c_{j,n}^2 \right) \\ &= \sum_{j=1}^n c_{j,n} - \sum_{j=1}^n \frac{1}{(1 + c_{j,n})^2} c_{j,n}^2 \end{aligned}$$

where $\sum_{j=1}^n c_{j,n} \rightarrow \lambda$. And letting $c_n := \max_{1 \leq j \leq n} |c_{j,n}| < 1$, for large n such that $c_n < 1$

$$\left| \sum_{j=1}^n \frac{1}{(1 + c_{j,n})^2} c_{j,n}^2 \right| \leq \left| \frac{1}{(1 - c_n)^2} \right| |c_n| \sum_{j=1}^n |c_{j,n}| \rightarrow 0$$

The desired result follows. □

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A Related Theorem Details

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