

Durret 5th Chapter3 Solutions

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October 30, 2025

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3.1.1.

Solution. By the Taylor's expansion, the logritham of the sum is

$$\begin{aligned}\sum_{j=1}^n \ln(1 + c_{j,n}) &= \sum_{j=1}^n (c_{j,n} - \frac{1}{(1 + c_{j,n})^2} c_{j,n}^2) \\ &= \sum_{j=1}^n c_{j,n} - \sum_{j=1}^n \frac{1}{(1 + c_{j,n})^2} c_{j,n}^2\end{aligned}$$

where $\sum_{j=1}^n c_{j,n} \rightarrow \lambda$. And letting $c_n := \max_{1 \leq j \leq n} |c_{j,n}| < 1$, for large n such that $c_n < 1$

$$\left| \sum_{j=1}^n \frac{1}{(1 + c_{j,n})^2} c_{j,n}^2 \right| \leq \left| \frac{1}{(1 - c_n)^2} \right| |c_n| \sum_{j=1}^n |c_{j,n}| \rightarrow 0.$$

The desired result follows. □

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A Related Theorem Details

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