19기 정규세션

ToBig's 18기 강의자 이다인

Neural Network Basic

ontents

Unit 01 | Perceptron

Unit 02 | Backpropagation

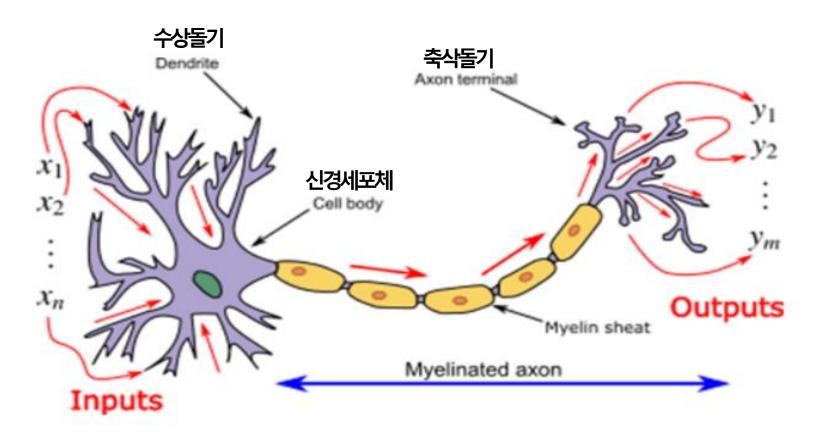
01 | Perceptron

인공신경망이란?

What is Artificial neural network(ANN)?

" 수학적 논리학이 아닌 <mark>인간의 두뇌를 모방</mark>하여 수많은 뉴런이 연결되어 있는 네트워크를 통해 문제를 해결하는 기계학습 모델 "

Neuron (신경세포)

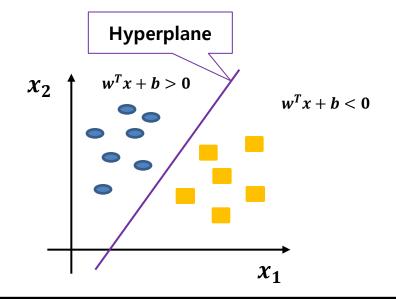


시냅스 (Synaptic terminals)



Perceptron

- Binary Classification (i.e., $y = \{-1, +1\}$)
- There exists a linear hyperplane that separates (i.e., data is linearly separable)
- 데이터가 linearly separable 하다는 것은 모든 데이터가 하나의 linear model을 통해서 완벽하게 분류될 수 있어야 한다는 것(error가 0이 되어야 함).



 $hypothesis class H = \{x: w^Tx + b = 0\}$

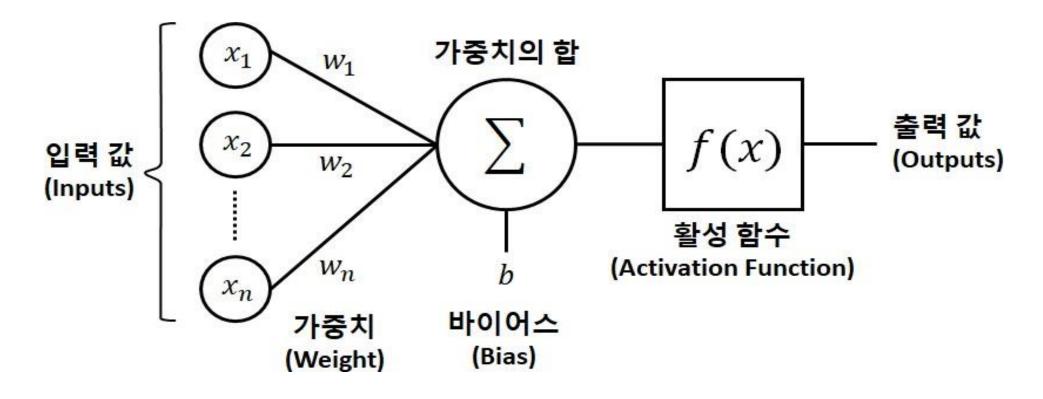
classifier $h(x) = sign(w^Tx + b)$

* x : 입력값

b: bias (편향)

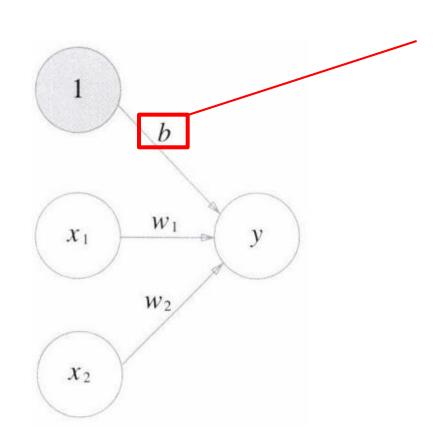
w : weight (가중치)

Perceptron



• 가중치(weight) : 각각의 입력에 대해 중요도를 부여 하는 수치

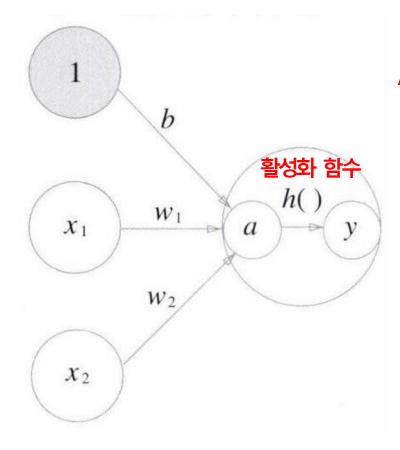
Perceptron



Bias(편향)

- 뉴런이 얼마나 쉽게 활성화 되느냐를 제어
- 노드의 민감도를 조정하거나 활성화를 조정하는 역할
- 가중치 만으로 세밀한 조정이 되지 않을 시 편향을 주어 조정이 가능하다.
- 일반적으로 입력값을 1로 고정하고 편향 b를 곱한 변수로 표현한다.

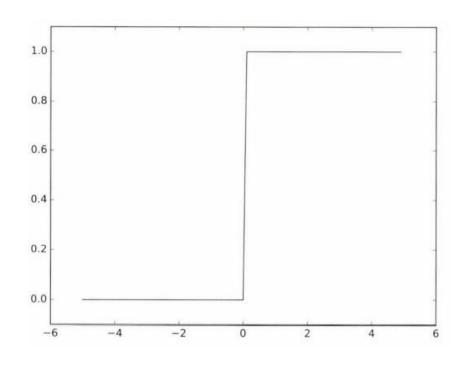
활성화 함수



Activation Function (활성화 함수)

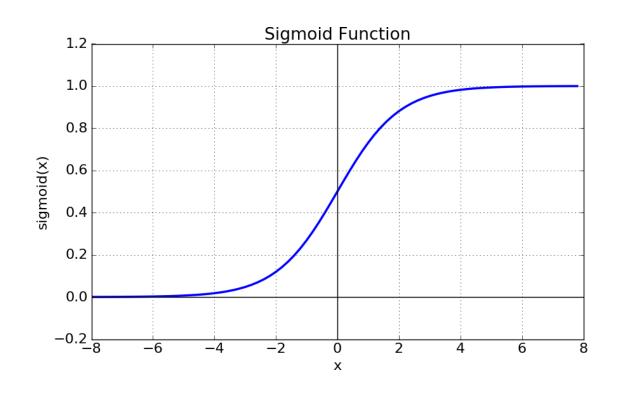
- 입력신호의 총합을 출력 신호로 변환하는 함수
- 활성화 함수로는 Step function, Relu,
 Sigmoid, tanh 등 여러함수존재

활성화 함수 – Step function



- 출력이 0 또는 1
- 활성화할지 말 지 여부만 반환

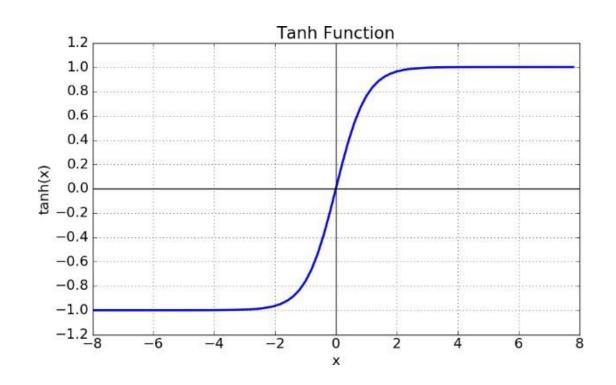
활성화 함수 – Sigmoid



$$h(x) = \frac{1}{1 + \exp(-x)}$$

- 0에서 1 사이의 값 출력
- 활성화 여부가 아닌, 활성화 정도를 반환
- 1에 가까울수록 많이 활성화됐다는 뜻

활성화 함수 - Tanh

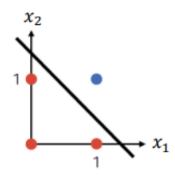


$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- -1에서 1 사이의 값 출력
- 시그모이드 함수보다 범위가 넓어 출력값의 변화폭이 더 크고 이로인해 기울기 소실 증상이 적음.

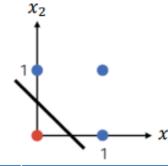
Perceptron 연산

AND



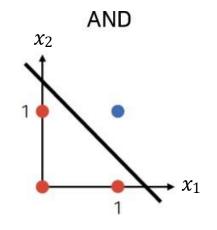
x_1	x_2	у
1	1	1
1	0	0
0	1	0
0	0	0

OR



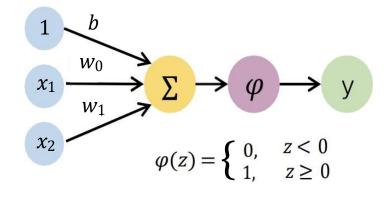
x_1	x_2	\mathcal{Y}
1	1	1
1	0	1
0	1	1
0	0	0

Perceptron 연산



$$w_0 = 1.0, w_1 = 1.0, b = -1.5$$

<i>x</i> ₁	<i>x</i> ₂	S	У
0	0	-1.5	0
0	1	-0.5	0
1	0	-0.5	0
1	1	0.5	1



$$\varphi(w_1x_1 + w_2x_2 + w_3) = y$$

①
$$(1.0 \times 0) + (1.0 \times 0) + (-1.5) = -1.5$$

 $\varphi((1.0 \times 0) + (1.0 \times 0) + (-1.5)) = 0$

②
$$(1.0\times0) + (1.0\times1) + (-1.5) = -0.5$$

 $\varphi((1.0\times0) + (1.0\times1) + (-1.5)) = 0$

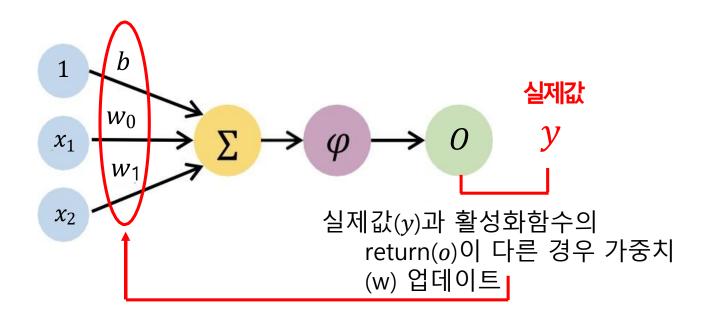
$$(1.0 \times 1) + (1.0 \times 0) + (-1.5) = -0.5$$

$$\varphi((1.0 \times 1) + (1.0 \times 0) + (-1.5)) = 0$$

(4)
$$(1.0 \times 1) + (1.0 \times 1) + (-1.5) = 0.5$$

 $\varphi((1.0 \times 1) + (1.0 \times 1) + (-1.5)) = 1$

Perceptron 학습



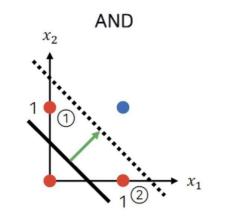
가중치 조정 식

$$w_i \leftarrow w_i + \eta(y - o)x_i$$

학습률(learning rate)

너무 작으면 학습 속도가 매우 느리고 너무 크면 가중치를 미세하게 조정하지 못하기 때문에 최적의 가중치를 찾기 어려움

Perceptron 학습



$$w_1 = 0.55, w_2 = 0.55, b = -0.65$$

x_1	<i>x</i> ₂	0	у
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

$$w_i \leftarrow w_i + \eta(y - o)x_i \qquad \eta = 0.05$$

$$b \leftarrow b + 0.05(0 - 1) \times 1 \qquad b \leftarrow -0.65 + 0.05(0 - 1) \times 1 = -0.7$$

$$w_1 \leftarrow w_1 + 0.05(0 - 1) \times 0 \qquad w_1 \leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55$$

$$w_2 \leftarrow w_2 + 0.05(0 - 1) \times 1 \qquad w_2 \leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5$$

①
$$w_1 \leftarrow w_1 + 0.05(0-1) \times 0$$
 $w_1 \leftarrow 0.55 + 0.05(0-1) \times 0 = 0.55$ $w_2 \leftarrow w_2 + 0.05(0-1) \times 1$ $w_2 \leftarrow 0.55 + 0.05(0-1) \times 1 = 0.5$

$$b \leftarrow b + 0.05(0 - 1) \times 1$$

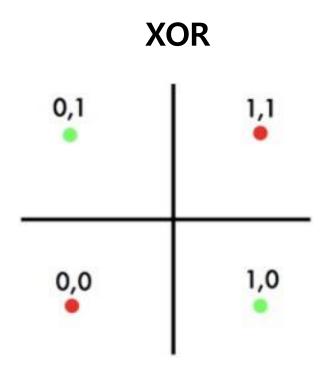
$$2 \quad w_1 \leftarrow w_1 + 0.05(0 - 1) \times 1$$

$$w_2 \leftarrow w_2 + 0.05(0 - 1) \times 0$$

$$b \leftarrow -0.65 + 0.05(0 - 1) \times 1 = -0.7$$

 $w_1 \leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5$
 $w_2 \leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55$

Perceptron의 한계



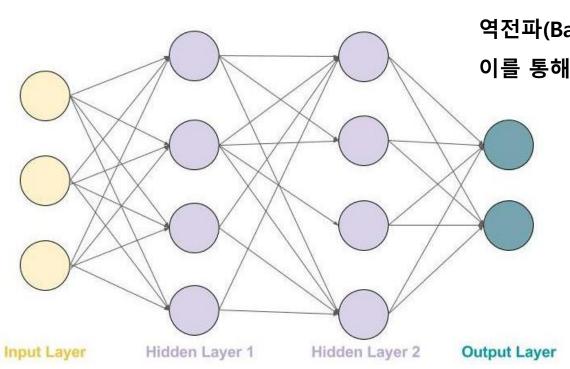
1969년 MIT AI랩 창립자 Minskey& Papert '현재의 퍼셉트론으로는 XOR 문제를 해결할 수 없다!'

Perceptron의 한계



02 | Backpropagation

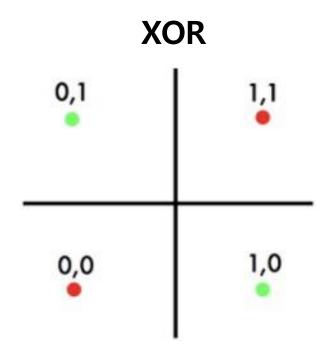
Multi Layer Perceptron

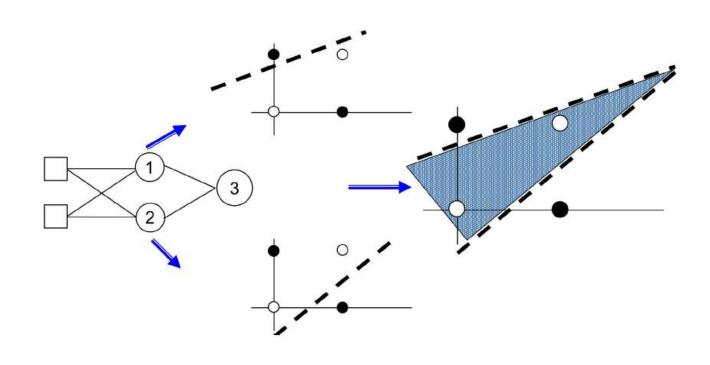


1986년 제프리 힌튼이 다층 퍼셉트론(Multi Layer Perceptron), 역전파(Back Propagation)을 실험적으로 증명. 이를 통해 오랜 AI의 겨울을 불러 온 XOR 문제 해결

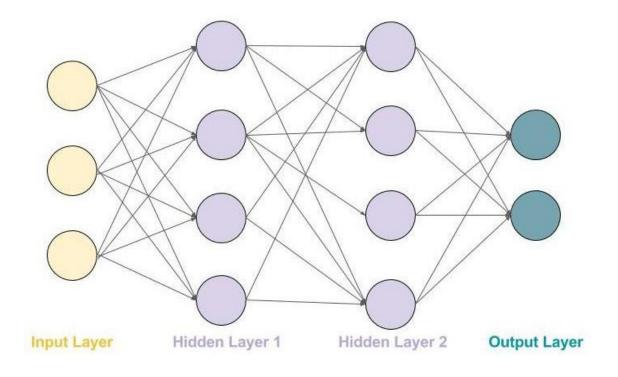
Multi Layer Perceptron

하나의 퍼셉트론으로는 해결하지 못했던 xor문제도 여러 퍼셉트론을 쌓는다면 해결 가능





Multi Layer Perceptron

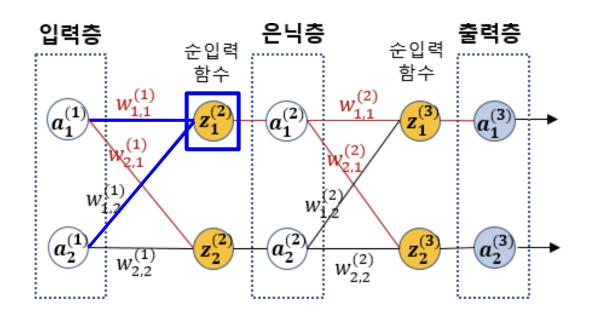


MLP를 학습시키는 방 법

: 오류 역전파

역전파(Backpropagation)

순전파(Feedforward) 알고리즘에서 발생한 오차를 줄이기 위해 새로운 가중치를 업데이트하고, 새로운 가중치로 다시 학습하는 과정



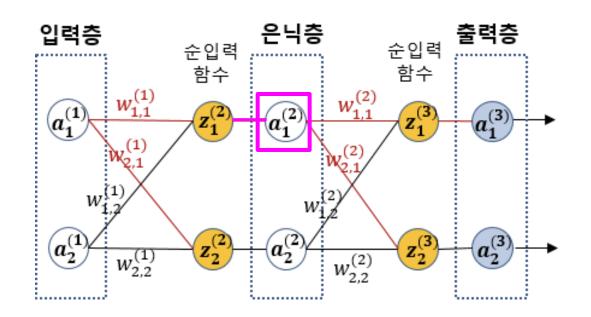
$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$



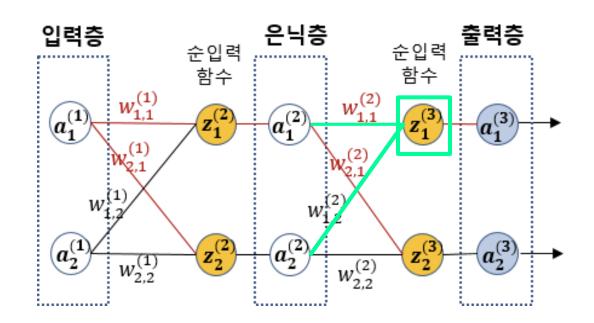
$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$



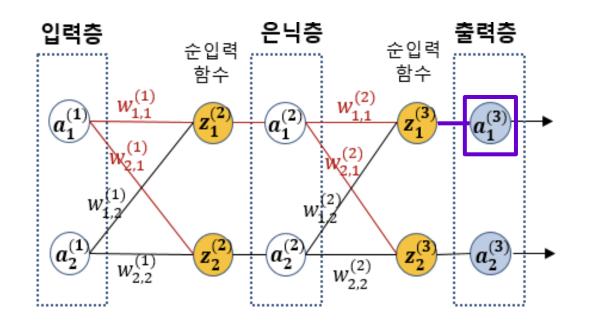
$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$



$$\phi(z) = \frac{1}{1 + e^{-z}}$$

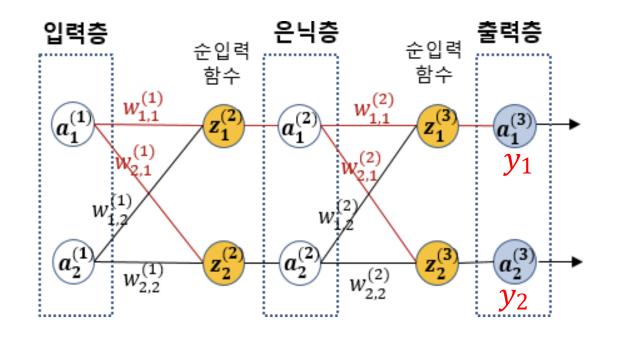
$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$

손실함수(Cost Function)

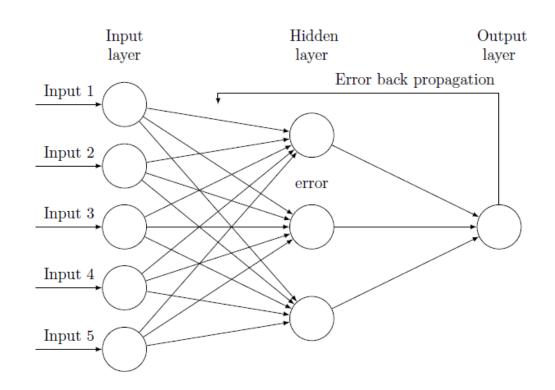


MSE =
$$\frac{1}{2N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$J_1 = \frac{1}{2}(a_1^{(3)} - y_1)^2$$

$$J_2 = \frac{1}{2} (a_2^{(3)} - y_2)^2$$

역전파(Backpropagation)



- Input과 output 값을 알고 있는 상태에서 신경망을 학습시키는 방법
- 출력부터 반대 방향으로 순차적으로 편미분을 수 행해 가면서 weight와 bias 값을 갱신시킴

$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

가중치 업데이트 식

편미분

다변수함수의 특정 변수를 제외한 나머지 변수를 상수로 생각하여 미분

$$z = f(x, y) = x^2 + xy + y^2$$

$$\frac{\partial z}{\partial x} = 2x + y, \qquad \frac{\partial z}{\partial y} = 2y + x$$

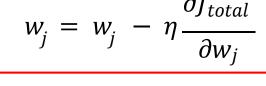
$$\Delta f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (2x + y, 2y + x)$$

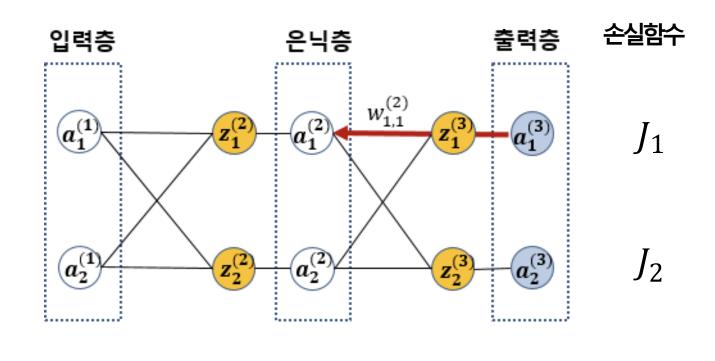
Chain Rule

연쇄 법칙, 합성 함수를 미분할 때의 계산 공식

$$f(g(x))' = f'(g(x))g'(x)$$
 $y = f(u), u = g(x)$ 일 때, $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x}$ 성립

역전파(Backpropagation)

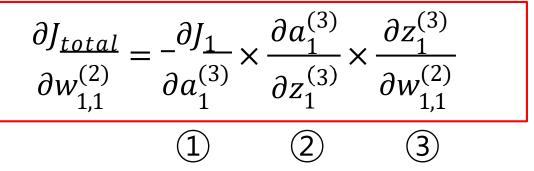


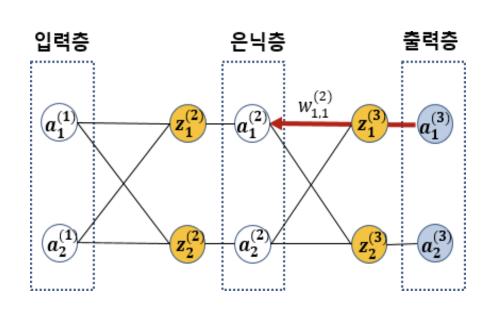


$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}}$$

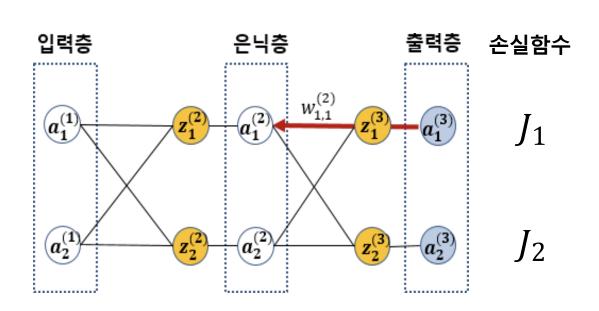
역전파의 출발노드인 $a_1^{(3)}$ 의 I_{otal} 은 I_1

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}$$





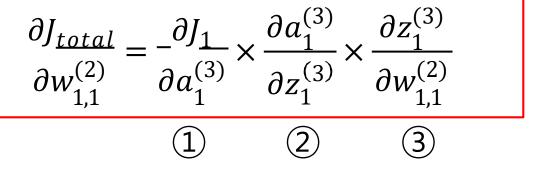
참고:
$$J_1 = \frac{1}{2} \left(a_1^{(3)} - y_1 \right)^2$$

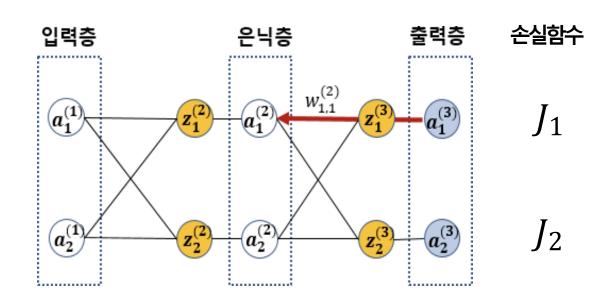


$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{-\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}$$

$$\boxed{1} \qquad \boxed{2} \qquad \boxed{3}$$

참고:
$$a_1^{(3)} = \phi(z_1^{(3)})$$
 $\sigma'(x) = \frac{\partial}{\partial x} \frac{1}{1+e^{-x}}$ $= \frac{e^{-x}}{(1+e^{-x})^2}$ $= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$ $= \sigma(x)(1-\sigma(x))$



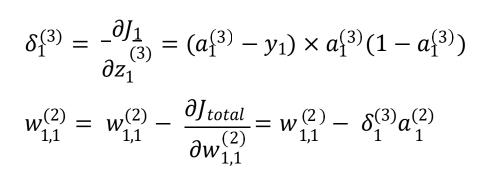


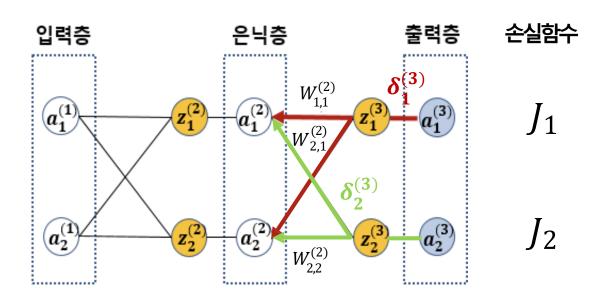
참고:
$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \begin{bmatrix} -\partial J_{1} \\ \partial a_{1}^{(3)} \end{bmatrix} \times \underbrace{\frac{\partial a_{1}^{(3)}}{\partial z_{1}^{(3)}}} \times \underbrace{\frac{\partial z_{1}^{(3)}}{\partial w_{1,1}^{(2)}}} = \underbrace{\begin{bmatrix} (a_{1}^{(3)} - y_{1}) \\ (a_{1}^{(3)} - y_{1}) \end{bmatrix}} \times \underbrace{\begin{bmatrix} a_{1}^{(3)} (1 - a_{1}^{(3)}) \\ a_{1}^{(2)} \end{bmatrix}} \times \underbrace{\begin{bmatrix} a_{1}^{(3)} - y_{1} \\ a_{1}^{(3)} \end{bmatrix}} \times \underbrace{\begin{bmatrix} a_{1}^{(3)} - y_{1} \\ a_{1}^$$

$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}$$

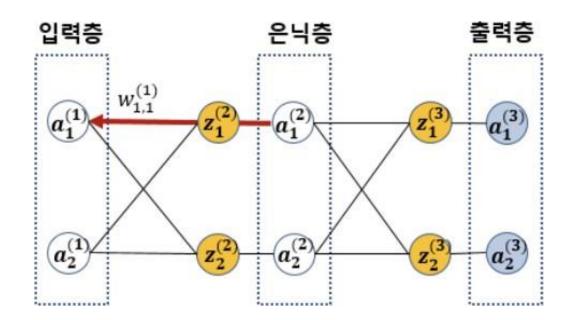
역전파(Backpropagation)





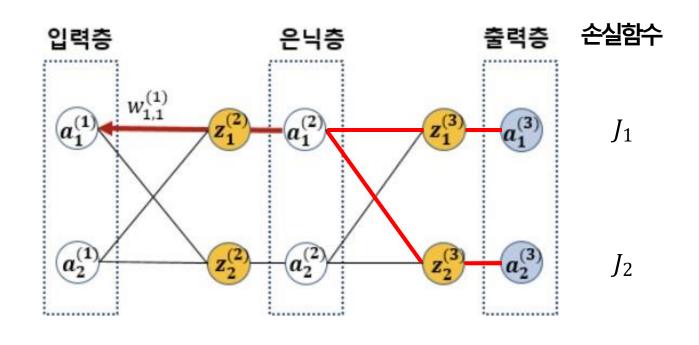
같은 방식으로

$$\begin{split} \delta_2^{(3)} &= \frac{\partial J_2}{\partial z_2^{(3)}} = \left(a_2^{(3)} - y_2 \right) \times a_2^{(3)} \left(1 - a_2^{(3)} \right) \\ w_{2,1}^{(2)} &= w_{2,1}^{(2)} - \delta_2^{(3)} a_1^{(2)} \\ w_{2,2}^{(2)} &= w_{2,2}^{(2)} - \delta_2^{(3)} a_2^{(2)} \end{split}$$



$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$



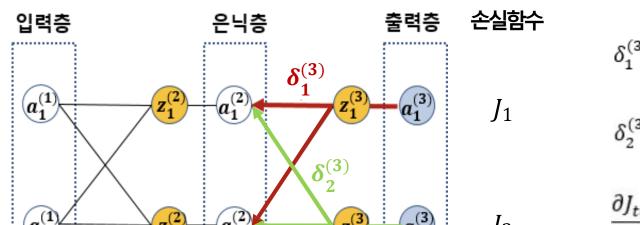
$$w_j = w_j - \eta \frac{\partial J_{tota}}{\partial w_j}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$a_{1}^{(2)} \stackrel{\mathbf{O}}{=} | J_{total} \stackrel{\mathbf{C}}{=} | J_{1} |_{2}$$

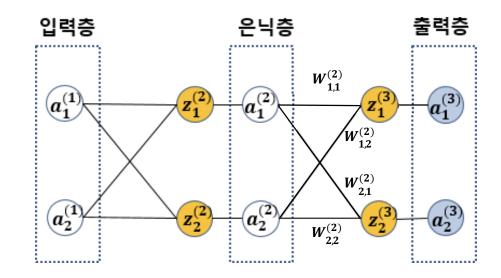
$$+J$$

$$\frac{\partial J_{total}}{\partial a_{1}^{(2)}} = \frac{\partial J_{1}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial a_{1}^{(2)}} = \frac{\partial J_{1}}{\partial z_{1}^{(3)}} \times \frac{\partial z_{1}^{(3)}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial z_{2}^{(3)}} \times \frac{\partial z_{2}^{(3)}}{\partial a_{1}^{(2)}}$$



$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$\begin{split} \delta_{1}^{(3)} &= \frac{\partial J_{1}}{\partial z_{1}^{(3)}} = \left(a_{1}^{(3)} - y_{1}\right) \times a_{1}^{(3)} \left(1 - a_{1}^{(3)}\right) \\ \delta_{2}^{(3)} &= \frac{\partial J_{2}}{\partial z_{2}^{(3)}} = \left(a_{2}^{(3)} - y_{2}\right) \times a_{2}^{(3)} \left(1 - a_{2}^{(3)}\right) \\ \frac{\partial J_{total}}{\partial a_{1}^{(2)}} &= \frac{\partial J_{1}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial a_{1}^{(2)}} = \boxed{\frac{\partial J_{1}}{\partial z_{1}^{(3)}}} \times \frac{\partial z_{1}^{(3)}}{\partial a_{1}^{(2)}} + \boxed{\frac{\partial J_{2}}{\partial z_{2}^{(3)}}} \times \frac{\partial z_{2}^{(3)}}{\partial a_{1}^{(2)}} \\ &= \boxed{\delta_{1}^{(3)} w_{1,1}^{(2)} + \delta_{2}^{(3)} w_{2,1}^{(2)}} \end{split}$$

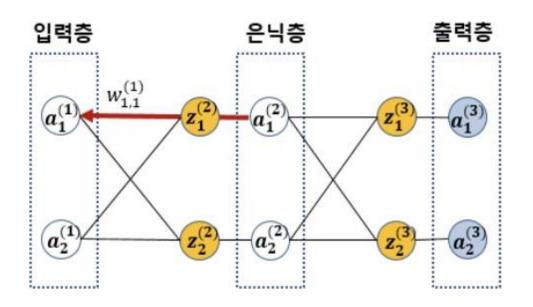


$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$z_2^{(3)} = w_{2,1}^{(2)} a_1^{(2)} + w_{2,2}^{(2)} a_2^{(2)}$$

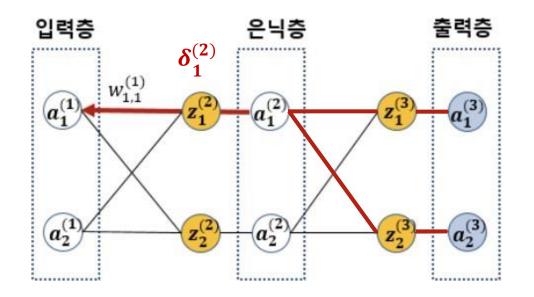
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$\begin{split} \frac{\partial J_{total}}{\partial a_{1}^{(2)}} &= \frac{\partial J_{1}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial a_{1}^{(2)}} = \frac{\partial J_{1}}{\partial z_{1}^{(3)}} \times \underbrace{\frac{\partial z_{1}^{(3)}}{\partial a_{1}^{(2)}}} + \frac{\partial J_{2}}{\partial z_{2}^{(3)}} \times \underbrace{\frac{\partial z_{2}^{(3)}}{\partial a_{1}^{(2)}}} \\ &= \delta_{1}^{(3)} w_{1,1}^{(2)} + \delta_{2}^{(3)} w_{2,1}^{(2)} \end{split}$$



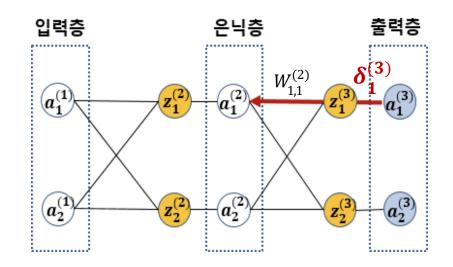
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \left(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)}\right) \times \underline{a_1^{(2)} \left(1 - a_1^{(2)}\right)} \times \underline{a_1^{(1)}}$$

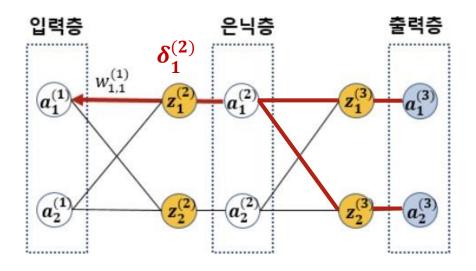


$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\left(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)}\right) \times a_1^{(2)} \left(1 - a_1^{(2)}\right) \times a_1^{(1)}}{\delta_1^{(2)} = \left(\delta_1^{(3)} w_{1,1}^{(2)} - \delta_2^{(3)} w_{2,1}^{(2)}\right) \times a_1^{(2)} \left(1 - a_1^{(2)}\right)}$$

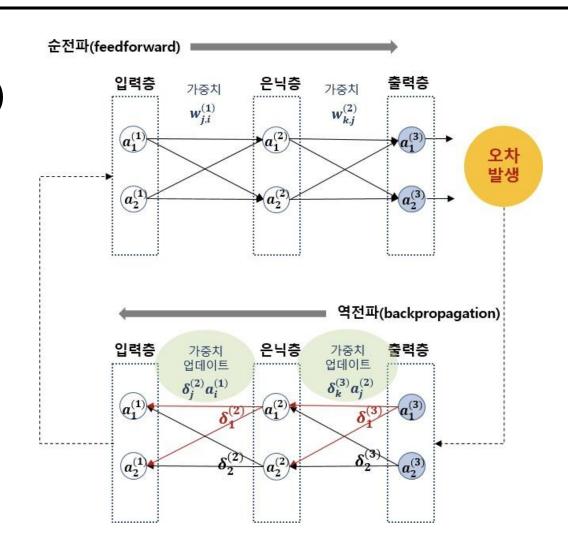
$$w_{1,1}^{(1)} = w_{1,1}^{(1)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = w_{1,1}^{(1)} - \delta_1^{(2)} a_1^{(1)}$$



$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}$$

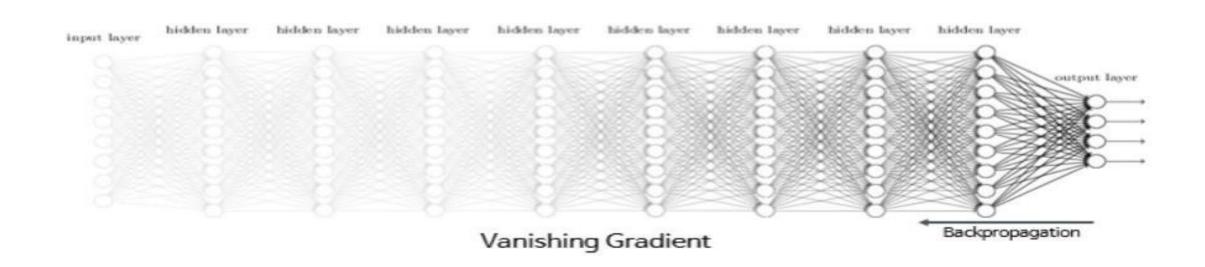


$$w_{1,1}^{(1)} = w_{1,1}^{(1)} - \delta_1^{(2)} a_1^{(1)}$$



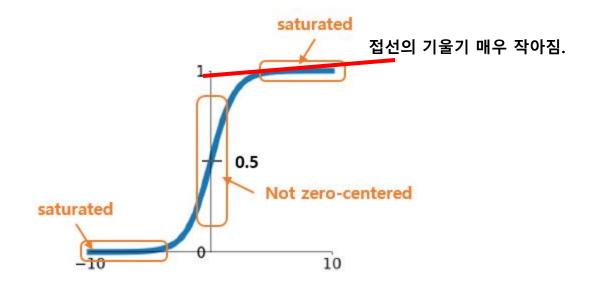
Vanishing Gradient Problem

깊이가 깊은 심층신경망에서는 역전파 알고리즘이 입력층으로 전달됨에 따라 그래디언트가 점점 작아져 결국 가중치 매개변수가 업데이트 되지 않는 경우가 발생



Vanishing Gradient Problem - sigmoid

Sigmoid



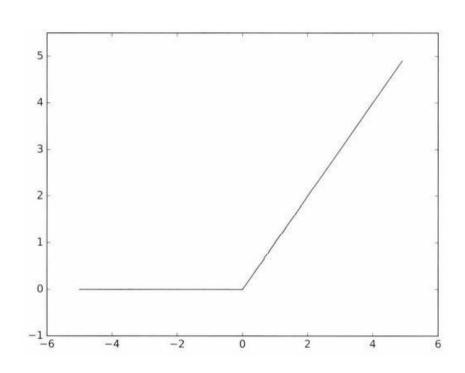
- 기울기가 작아지는 좌우 부분은 미분하면 0이 됨
- 역전파를 이용하여 편미분할 때 $\frac{dj_{total}}{dw}$ 가 0이되어 가중치 업데이트가 없어지는 현상이 saturated 현상

Vanishing Gradient Problem



크고 복잡한 데이터를 다루기 위해서는 히든 레이어를 여러 개 연결해야 하는데, MLP와 역전파 방법으로는 한계 => 2차 인공지능의 겨울

활성화함수 - ReLU 함수



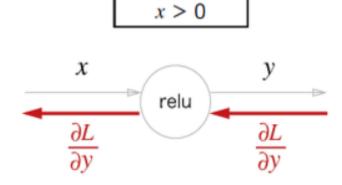
$$h(x) = \begin{cases} x & (x > 0) \\ 0 & (x \le 0) \end{cases}$$

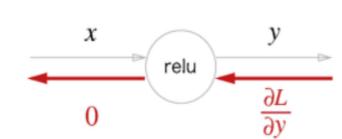
- Vanishing gradient problem 해결!
- 입력이 0 넘으면 값이 클수록 많이 활성화
- 입력이 0보다 작으면 무조건 비활성화
- 다른 활성화 함수보다 계산 복잡도가 낮아서 학습 속도가 빠름
- 비선형함수

활성화함수 - ReLU 함수

$$y = \left\{egin{array}{ll} x & (x>0) \ 0 & (x\leq 0) \end{array}
ight.$$

$$\frac{\partial y}{\partial x} = \begin{cases} 1 & (x > 0) \\ 0 & (x \le 0) \end{cases}$$





 $x \leq 0$

Unitㅣ과제

"week3_NeuralNetworkBasic_assignment.pdf" 파일의 문제들을 상세한 풀이과정과 함께 풀어주세요.

"week3_NeuralNetworkBasic_실습.ipynb" 노트북 파일에서 코드 실습을 진행해 주세요.

Reference

참고자료

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- <u>신경망의 기본 구조 (velog.io)</u>
- <u>텐서플로우 딥러닝 강의 12-2 ReLU 활성함수 YouTube</u>

- <u>딥러닝 Neural Network AND 함수, XOR 문제 해결 방법</u> (tistory.com)
- <u>퍼셉트론(Perceptron) (tistory.com)</u>
- <u>실체가 손에 잡히는 딥러닝(3) "이것만은 꼭 알아두자!</u> <u>딥러닝의 꽃 - 가중치, 편향, 활성화 함수, 역전파" | Popit</u>
- [기계학습] Neural Networks 1 (velog.io)
- 활성화 함수 종료 : [ML] <u>활성화 함수(Activation Function)</u> 종류 정리 (tistory.com)
- Vanishing Gradient Problem(기울기 소멸 문제) 창의 컴퓨팅(Creative Computing)

Q&A

들어주셔서 감사합니다.