Supplementary materials:

Spin echo dynamics under an applied drift field in graphene nanoribbon supperlattices

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To verify that the evolution operator (Eq. 8 in this paper) is exact, we provide an example associated with the Hamiltonian of spin-1/2 particle in an effective magnetic field having a known solution:[1]

$$H(t) = \Omega\cos(\omega t)\, s_x + \Omega \sin(\omega t)\, s_y + \Delta s_z. \tag{1}$$

The energy eigenvalues of (1) are $\varepsilon_\pm = \pm\hbar\delta/2$, where $\delta = \left[\Delta^2+\Omega^2\right]^{1/2}$. We construct a normalized orthogonal set of eigenspinors of Hamiltonian (1) as:

$$\chi_+(t) = \frac{1}{\sqrt{N}}\begin{pmatrix}\Omega \\ (\delta-\Delta)\exp\{i\omega t\}\end{pmatrix}, \tag{2}$$

$$\chi_-(t) = \frac{1}{\sqrt{N}}\begin{pmatrix}(\delta-\Delta)\exp\{-i\omega t\} \\ -\Omega\end{pmatrix}, \tag{3}$$

where

$$N^2 = \Omega^2 + (\delta-\Delta)^2. \tag{4}$$

Since the Hamiltonian (1) is time dependent, the general time dependent Schrödinger equations can be written as

$$\partial_t a(t) = -\frac{i}{2}\left(\Delta a + \Omega\exp\{-i\omega t\}\, b\right), \tag{5}$$

$$\partial_t b(t) = -\frac{i}{2}\left(\Omega\exp\{i\omega t\}\, a - \Delta b\right). \tag{6}$$

The exact solution of time dependent Schrödinger Eqs. (5) and (6) can be written as:[1]

$$a(t) = \frac{\Omega}{\sqrt{N}}\left\{\cos\left(\frac{\lambda' t}{2}\right) - \frac{i}{\lambda'}(\delta-\omega)\sin\left(\frac{\lambda' t}{2}\right)\right\}\exp\left(-\frac{i\omega t}{2}\right), \tag{7}$$

$$b(t) = \frac{\delta-\Delta}{\sqrt{N}}\left\{\cos\left(\frac{\lambda' t}{2}\right) - \frac{i}{\lambda'}(\delta+\omega)\sin\left(\frac{\lambda' t}{2}\right)\right\}\exp\left(\frac{i\omega t}{2}\right), \tag{8}$$

where

$$\lambda' = \sqrt{\Omega^2 + (\Delta-\omega)^2}. \tag{9}$$

Expressing (7) and (8) as a linear combination of $|\chi_+\rangle$ and $|\chi_-\rangle$, we have

$$\psi(t) = \left\{\cos\left(\frac{\lambda' t}{2}\right) - \frac{i}{\lambda'}\left(\delta - \frac{\omega\Delta}{\delta}\right)\sin\left(\frac{\lambda' t}{2}\right)\right\}\exp\left(-\frac{i\omega t}{2}\right)\chi_+(t) + i\frac{\omega\Omega}{\lambda'\delta}\sin\left(\frac{\lambda' t}{2}\right)\exp\left(\frac{i\omega t}{2}\right)\chi_-(t). \tag{10}$$

Clearly, one can immediately write the transition probability:

$$|\langle\chi_-(t)|\psi(t)\rangle|^2 = \left(\frac{\omega\Omega}{\lambda'\delta}\right)^2\sin^2\left(\frac{\lambda' t}{2}\right), \tag{11}$$

provided that

$$|\langle\chi_-(t)|\psi(t)\rangle|^2 + |\langle\chi_+(t)|\chi(t)\rangle|^2 = 1. \tag{12}$$

Next, we apply the Feynman disentangling operator scheme and find the exact evolution operator of the Hamiltonian (1) which initially takes the form of (Eq.8, see the paper) but with three different coupled Riccati equations (evolved during the disentangling operator scheme of the Hamiltonian (1)). These are:

$$\frac{d\alpha}{dt} = -\frac{i}{\hbar}\left\{\frac{\Omega}{2}\exp(-i\omega t) + \Delta\alpha - \frac{\Omega}{2}\exp(i\omega t)\,\alpha^2\right\}, \tag{13}$$

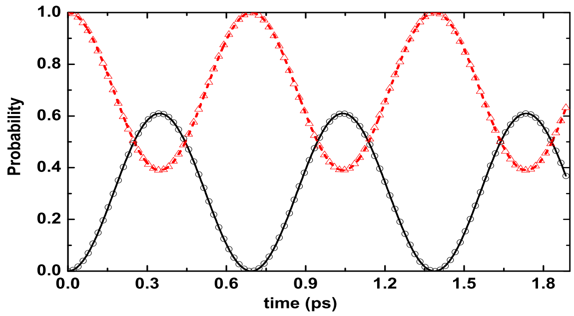

FIG. 1. (Color online) Transition probability vs time. Here we chose $\omega = 10$ THz and $\Omega = \Delta = 1$ THz. Transition probabilities obtained from Eqs. (11) and (12) (solid and dashed lines, respectively) are seen to be in excellent agreement to the ones obtained from the Feynman disentangling operator scheme. The transition probabilities are given by $|\langle \chi_+ (t) | U (t, 0) \chi (0) \rangle|^2$ (circles) and $|\langle \chi_- (t) | U (t, 0) \chi (0) \rangle|^2$ (triangles).

$$\frac{d\beta}{dt} = -\frac{i}{\hbar} \left\{ \Delta - \Omega \exp\left(i\omega t\right) \alpha \right\}, \quad (14)$$

$$\frac{d\gamma}{dt} = -\frac{i}{\hbar} \frac{\Omega}{2} \exp\left\{ \beta + i\omega t \right\}. \quad (15)$$

Usually, an exact solution of such Riccati differential equations does not exist. However, in this case, it is possible to find the exact solution of Eqs. (13), (14) and (15) as:

$$\alpha(t) = \frac{\exp\left(-i\lambda t\right) - \exp\left(-i\omega t\right)}{n_2 - n_1 \exp\left(-i\varpi t\right)}, \quad (16)$$

$$\beta(t) = \frac{\exp\left(-i\lambda t\right) \left(n_2 - n_1\right)^2}{\left\{ n_2 - n_1 \exp\left(-i\varpi t\right) \right\}^2}, \quad (17)$$

$$\gamma(t) = \frac{\exp\left(-i\varpi t\right) - 1}{n_2 - n_1 \exp\left(-i\varpi t\right)}, \quad (18)$$

where $\lambda = \varpi + \omega$, $\varpi = \Omega \left(n_2 - n_1\right)/2$,

$$n_1 = \frac{\Delta - \omega}{\Omega} + \frac{\left[\left(\Delta - \omega\right)^2 + \Omega^2 \right]^{1/2}}{\Omega}, \quad (19)$$

$$n_2 = \frac{\Delta - \omega}{\Omega} - \frac{\left[\left(\Delta - \omega\right)^2 + \Omega^2 \right]^{1/2}}{\Omega}. \quad (20)$$

In Fig. 1, the transition probability obtained from Eq. (10) (solid and dashed lines) is seen to be in excellent agreement to the one obtained from the disentangling scheme (circles and triangles). Thus, we have demonstrated that the evolution operator obtained from the disentangling operator scheme is exact. Finding an exact unitary operator is one of the requirements for quantum computing and is one of the motivations of the present work.

[1] D. J. Griffiths, *Introduction to Quantum Mechanics* (Pearson Education Inc., 1995).