①

$$H(t) = \Delta\sigma_z + \frac{\hbar}{\tau} x(t)\,\sigma_x \quad — ①$$

$$= \Delta\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{\hbar}{\tau} x(t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \cancel{\Delta} \begin{pmatrix} \Delta & \frac{\hbar}{\tau} x(t) \\ \frac{\hbar}{\tau} x(t) & -\Delta \end{pmatrix}$$

$$i\hbar \frac{\partial \chi}{\partial t} = \begin{pmatrix} \Delta & \frac{\hbar}{\tau} x(t) \\ \frac{\hbar}{\tau} x(t) & -\Delta \end{pmatrix} \chi$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix} = \begin{pmatrix} \Delta & \frac{\hbar}{\tau} x(t) \\ \frac{\hbar}{\tau} x(t) & -\Delta \end{pmatrix}\begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix}$$

$$\left.\begin{aligned} \dot{\chi}_+ &= \frac{1}{i\hbar}\left(\Delta \chi_+ + \frac{\hbar}{\tau} x(t)\,\chi_-\right) \\ \dot{\chi}_- &= \frac{1}{i\hbar}\left(\frac{\hbar}{\tau} x(t)\,\chi_+ - \Delta \chi_-\right) \end{aligned}\right] \text{Two coupled } \underline{\text{eqns.}}$$

$\underline{\text{Find}}$ $\chi(t) = \begin{pmatrix} \chi_+(t) \\ \chi_-(t) \end{pmatrix}$ ✓

~~we~~ At $t=0$, we write ① as

$$H(t=0) = \Delta\sigma_z + \frac{\hbar}{\tau} X(t=0)\,\sigma_x$$

$\underline{\text{symmetric}} \rightarrow$ $X(t=0) = \frac{\theta}{2} + \left(a - \frac{\theta}{2}\right) - a = x_0$

$\underline{\text{asymmetric}}$ $X(t=0) = \frac{\theta}{2} + \left(a - \frac{\theta}{2}\right) - a = x_0$

both are $\underline{\text{equal}}$

$$H(t=0) = \Delta\sigma_z + \frac{\hbar}{\tau} x_0 \sigma_x$$

Find eigenvalues

$$\begin{pmatrix} \Delta-\lambda & \frac{\hbar}{\tau}x_0 \\ \frac{\hbar}{\tau}x_0 & -\Delta-\lambda \end{pmatrix} = 0$$

$$\text{or, } (\Delta-\lambda)(\Delta+\lambda) - \left(\frac{\hbar}{\tau}\right)^2 x_0^2 = 0$$

$$\text{or, } \Delta^2 - \lambda^2 = \left(\frac{\hbar}{\tau}\right)^2 x_0^2$$

$$\lambda^2 = \Delta^2 - \left(\frac{\hbar}{\tau}\right)^2 x_0^2$$

$$\lambda = \pm\sqrt{\Delta^2 - \left(\frac{\hbar}{\tau}\right)^2 x_0^2} = \pm\delta$$

Construct normalized eigenspinors

$$(H_0 - \lambda_1 I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \Delta-\delta & \frac{\hbar}{\tau}x_0 \\ \frac{\hbar}{\tau}x_0 & -\Delta-\delta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$(\Delta-\delta)x + \frac{\hbar}{\tau}x_0 y = 0 \quad (1)$$

$$\left(\frac{\hbar}{\tau}x_0\right)x - (\Delta+\delta)y = 0 \quad (2)$$

From (1): $(\Delta-\delta)x + \frac{\hbar}{\tau}x_0 y = 0$

$y = $ [illegible] $(\Delta-\delta)\tau x$ [illegible] $\left(\frac{\delta-\Delta}{[illegible]}\right)\tau x$

(2)

Let $x = N_1$

$$\vec{x} = \begin{pmatrix} N_1 \\ \left(\frac{\delta - \Delta}{\hbar x_0}\right) \tau N_1 \end{pmatrix}$$

Normalization condition

$$\sqrt{N_1^2 + \left(\frac{\delta - \Delta}{\hbar x_0}\right)^2 \tau^2 N_1^2} = 1$$

$$N_1^2 \left[1 + \left(\frac{\delta - \Delta}{\hbar x_0}\right)^2 \tau^2 \right] = 1$$

$$N_1^2 = \frac{(\hbar x_0)^2}{(\hbar x_0)^2 + (\delta - \Delta)^2 \tau^2}$$

$$N_1 = \frac{\hbar x_0}{\sqrt{(\hbar x_0)^2 + (\delta - \Delta)^2 \tau^2}}$$

$$\vec{x}_1 = \frac{1}{\sqrt{(\hbar x_0)^2 + (\delta - \Delta)^2 \tau^2}} \begin{pmatrix} \hbar x_0 \\ \frac{\delta - \Delta}{\cancel{\hbar x_0}} \tau \cancel{\hbar x_0} \end{pmatrix}$$

$$= \frac{1}{\sqrt{(\hbar x_0)^2 + (\delta - \Delta)^2 \tau^2}} \begin{pmatrix} \hbar x_0 \\ (\delta - \Delta) \tau \end{pmatrix}$$

(29)

Let $\lambda = -\delta$

$$\begin{pmatrix} \Delta+\delta & \frac{\hbar}{\tau}x_0 \\ \frac{\hbar}{\tau}x_0 & -\Delta+\delta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$(\Delta+\delta)x + \frac{\hbar}{\tau}x_0 y = 0 \quad -① $$

$$\frac{\hbar}{\tau}x_0 x - (\Delta-\delta)y = 0 \quad -② $$

From ①

$$x = -\frac{\hbar x_0}{\tau(\Delta+\delta)}y$$

Let $y = -N_1$

$$\therefore X_2 = \begin{pmatrix} N_1 \frac{\hbar x_0}{\tau(\Delta+\delta)} \\ -N_1 \end{pmatrix}$$

Normalization

$$\sqrt{N_1^2\left(\frac{\hbar x_0}{\tau}\right)^2\frac{1}{(\Delta+\delta)^2} + N_1^2} = 1$$

$$N_1^2 = \frac{1}{\frac{(\hbar x_0)^2}{\tau^2}\frac{1}{(\Delta+\delta)^2}+1} = \frac{(\Delta+\delta)^2\tau^2}{(\hbar x_0)^2+\tau^2(\Delta+\delta)^2}$$

$$N_1 = \frac{(\Delta+\delta)\tau}{\sqrt{(\hbar x_0)^2+\tau^2(\Delta+\delta)^2}}$$

From ②

$$x = \frac{\Delta-\delta}{\hbar x_0}\tau y$$

Let $y = -N_1$

$$X_2 = \begin{pmatrix} -\frac{\Delta-\delta}{\hbar x_0}\tau N_1 \\ -N_1 \end{pmatrix}$$

Norm $\sqrt{\left(\frac{\Delta-\delta}{\hbar x_0}\right)^2\tau^2 N_1^2 + N_1^2} = 1$

$$N_1^2 = \frac{1}{\left(\frac{\Delta-\delta}{\hbar x_0}\right)^2\tau^2+1}$$

$$N_1^2 = \frac{(\hbar x_0)^2}{(\hbar x_0)^2+(\Delta-\delta)^2 \ldots}$$

③

$$x_2 = \frac{\hbar x_0}{\sqrt{(\hbar x_0)^2 + (\Delta - \delta)^2 \tau^2}} \begin{pmatrix} \frac{\delta - \Delta}{\hbar x_0} \tau \\ -1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{(\hbar x_0)^2 + (\Delta - \delta)^2 \tau^2}} \begin{pmatrix} (\delta - \Delta) \tau \\ -\hbar x_0 \end{pmatrix}$$

Two orthogonal spinors

$$\chi_+(0) = \frac{1}{\sqrt{(\hbar x_0)^2 + (\delta - \Delta)^2 \tau^2}} \begin{pmatrix} \hbar x_0 \\ (\delta - \Delta) \tau \end{pmatrix}$$

$$\chi_-(0) = \frac{1}{\sqrt{(\hbar x_0)^2 + (\delta - \Delta)^2 \tau^2}} \begin{pmatrix} (\delta - \Delta) \tau \\ -\hbar x_0 \end{pmatrix}$$

prove that $\langle \chi_+ | \chi_- \rangle = 0$ ✓

$\langle \chi_+ | \chi_+ \rangle = 1$

$\langle \chi_- | \chi_- \rangle = 1$