Assignment 7

Due June 9. No late submissions allowed.

Please answer all questions. Students are expected to solve the problems on their own. No discussions allowed. The majority of the grading will be based on whether you are approaching the problem in the right way, even if you are not able to completely answer the question.

1. (2 pts.) Given training data $\{x_i\}_{i=1}^{L}$, we want to learn the parameters $\theta = \{\lambda_k, \mu_k, \Sigma_k\}_{k=1}^{K}$ of a Mixture of Gaussians model using the EM algorithm. Derive the update rules for θ.

2. (2 pts.) Consider the multiclass logistic regressions problem in Sec. 4.3.4 of Pattern Recognition and Machine Learning by Bishop. It provides an outline of an iterative algorithm for the multi-class classification problem using the softmax function. Complete the derivation of (4.110) by filling in the details in the outline.

3. (2 pts.) Let $(\tilde{x}, \tilde{y})$ represent the image plane coordinates, whose corresponding representation in homogeneous coordinates is (x, y, z). Consider a camera model where $\tilde{x} = x$ and $\tilde{y} = \frac{y}{z}$. Prove that the projection of a 3D line on the image plane of this camera is a hyperbola.

4. (2 pts.) Consider the stereo system as defined in class, but with the world origin at the center of projection of the left camera and all distances represented with respect to this coordinate system. This implies $\mathbf{M}_l = \mathbf{M}_{l,int}\,[I_{3\times3}|\mathbf{0}]$ and $\mathbf{M}_r = \mathbf{M}_{r,int}\,[\mathbf{R}|\mathbf{t}]$. Prove that the fundamental matrix $\mathbf{F} = \hat{\mathbf{e}}_r \mathbf{M}_r \mathbf{M}_l^{+}$. What is the center of projection of the right camera in this coordinate system?

5. (2 pts.) Consider a pair of corresponding points, represented as $\mathbf{x}_i$ and $\mathbf{x}_i^{'}$, in homogeneous coordinates. Let the transformation between them be represented by a 3×3 matrix H, i.e. $\mathbf{x}_i^{'} = H\mathbf{x}_i$. If $\mathbf{h}$ is a nine-dimensional vector of the components of the matrix H, prove that $\mathbf{h}$ can be obtained as the solution of a linear system of equations of the form $A_i\mathbf{h} = 0$. Derive the explicit forms of A_i. (Hint: $\mathbf{x}_i^{'}$ and $H\mathbf{x}_i$ are collinear, hence their cross-product is zero.)
   a) What is the rank of A_i?
   b) What is the minimum number of corresponding points required to solve for $\mathbf{h}$?
   c) Describe, very briefly, how you would solve for $\mathbf{h}$.