Assignment 1
Due April 18, 2020 at 11.59 PM via ilearn Assignment-1 Submission folder

IMAGES. All images may be found in the TEST IMAGES folder on iLearn or in the folder uploaded specific to this assignment.

Independent Reading. Please read through these papers. They are related to topics covered in class; they will provide more depth in some of the topics.
1. https://www.dropbox.com/s/qeb26hj5tkttafg/Additional%20Reading%20-%20Assignment%201.pdf?dl=0
2. https://web.eecs.umich.edu/~ahowens/eecs504/w20/readings/simoncelli05a-preprint.pdf

Problem 1. [2 pts]

(a) On the `house.tif` image, compute the 2D Discrete Fourier transform and display.

(b) Apply Gaussian smoothing to the image, sample the smoothed image, and try to reconstruct the original image (of the same size) from the samples. Repeat for different choices of the Gaussian filter width and number of samples. What do you observe in terms of the relationship between the reconstruction error and the number of samples?

Problem 2. [2 pts] Plot the 4×4 DFT and DCT basis images. Compute the DCT and DFT transformed image of `gonazalezwoods725.png` and display. Which transform will work better for image compression? Explain briefly.

Problem 3. [3 pts] Prove the 2D Fourier Transform properties related to Linearity, Convolution and Energy Conservation in the image below. Please read independent reading materials (1) mentioned above for help.

TABLE 2.4 Properties and Examples of Fourier Transform of Two-Dimensional Sequences

Property	Sequence	Transform
	$x(m, n), y(m, n), h(m, n), \cdots$	$X(\omega_1, \omega_2), Y(\omega_1, \omega_2), H(\omega_1, \omega_2), \cdots$
Linearity	$a_1 x_1(m, n) + a_2 x_2(m, n)$	$a_1 X_1(\omega_1, \omega_2) + a_2 X_2(\omega_1, \omega_2)$
Conjugation	$x^*(m, n)$	$X^*(-\omega_1, -\omega_2)$
Separability	$x_1(m) x_2(n)$	$X_1(\omega_1) X_2(\omega_2)$
Shifting	$x(m \pm m_0, n \pm n_0)$	$\exp[\pm j(m_0 \omega_1 + n_0 \omega_2)] X(\omega_1, \omega_2)$
Modulation	$\exp[\pm j(\omega_{01} m + \omega_{02} n)] x(m, n)$	$X(\omega_1 \mp \omega_{01}, \omega_2 \mp \omega_{02})$
Convolution	$y(m, n) = h(m, n) \circledast x(m, n)$	$Y(\omega_1, \omega_2) = H(\omega_1, \omega_2) X(\omega_1, \omega_2)$
Multiplication	$h(m, n) x(m, n)$	$\left(\frac{1}{4\pi^2}\right) H(\omega_1, \omega_2) \circledast X(\omega_1, \omega_2)$
Spatial correlation	$c(m, n) = h(m, n) \bigstar x(m, n)$	$C(\omega_1, \omega_2) = H(-\omega_1, -\omega_2) X(\omega_1, \omega_2)$
Inner product	$I = \sum\sum_{m, n = -\infty}^{\infty} x(m, n) y^*(m, n)$	$I = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(\omega_1, \omega_2) Y^*(\omega_1, \omega_2) \, d\omega_1 \, d\omega_2$
Energy conservation	$\mathscr{E} = \sum\sum_{m, n = -\infty}^{\infty} \lvert x(m, n) \rvert^2$	$\mathscr{E} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \lvert X(\omega_1, \omega_2) \rvert^2 \, d\omega_1 \, d\omega_2$
	$\sum\sum_{m, n = -\infty}^{\infty} \exp[j(m \omega_{01} + n \omega_{02})]$	$4\pi^2 \delta(\omega_1 - \omega_{01}, \omega_2 - \omega_{02})$
	$\delta(m, n)$	$\frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \exp[-j(\omega_1 m + \omega_2 n)] \, d\omega_1 \, d\omega_2$

Figure 1: Problem 3. 2D Fourier Transform Properties

Problem 4. [3 pts] `jump_noisy.png` is a noisy version of the image `jump.png`.

(a) Using an image pyramid with Gaussian filtering, create a multi-resolution decomposition of the noisy image (at least two levels).

(b) De-noise the image by reconstructing from the multi-resolution decomposition by ignoring some of the high-frequency components. Also explain how you reduced the impact of the high frequency components. Reconstructed image should be similar to Figure 2.

Figure 2: Problem 4 (b). Reconstructed image should be similar to the image above.

Submission Protocol. You should submit codes as well as explanations to each problem (if required). You should add comments to your codes to make them reader friendly. All coding related problems should be in separate scripts named after the problem number. For e.g. Problem 1 has two parts (a) and (b). The codes corresponding to them should be in Problem1a.x and Problem1b.x (x = m if you are using MATLAB or x = py if you are using python.). If you may require to call functions for a problem, you may do so, but include them in your submission. Keep all the images necessary to run a code in the same folder as the code, while you are submitting. You MUST also include a report written electronically (using the likes of LaTeXor MS Word). It should contain explanations, images, etc (as required). The total length of the report cannot be more than 5 pages with 11 point font and single spacing.

Each student must do the assignment independently, although you may discuss prior to that. While discussion is allowed, we will be particularly careful about any plagiarism, whether from each other or from other sources.