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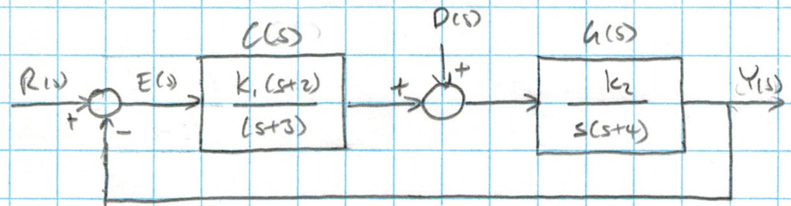
ECE 311 - ASSIGNMENT 4

1.



$e_D(\infty)$ FOR $D(s) = \frac{D_0}{s}$ $= -1.2e-5$

$e_R(\infty)$ FOR $R(s) = \frac{R_0}{s^2}$ $= 0.003$

$$E(s) = \frac{1}{1+CG} R(s) - \frac{G}{1+CG} D(s)$$

$$= \frac{1}{1+\frac{k_1k_2(s+2)}{s(s+3)(s+4)}} \cdot \frac{R_0}{s^2} - \frac{\frac{k_2}{s(s+4)}}{1+\frac{k_1k_2(s+2)}{s(s+3)(s+4)}} \frac{D_0}{s}$$

$$= \underbrace{\frac{R_0(s+3)(s+4)}{s^2(s+3)(s+4)+k_1k_2 s(s+2)}}_{E_R(s)} - \underbrace{\frac{D_0k_2(s+3)}{s^2(s+3)(s+4)+k_1k_2 s(s+2)}}_{E_D(s)}$$

BY SUPERPOSITION:

$$E_R(s) = \frac{R_0(s+3)(s+4)}{s^2(s+3)(s+4)+k_1k_2 s(s+2)}$$

$$e_R(\infty) = \lim_{s\to 0} s E_R(s) = \lim_{s\to 0} \frac{R_0(s+3)(s+4)}{s(s+3)(s+4)+k_1k_2(s+2)} = \frac{6R_0}{k_1k_2} = 0.003$$

$$E_D(s) = -\frac{D_0k_2(s+3)}{s^2(s+3)(s+4)+k_1k_2 s(s+2)}$$

$$e_D(\infty) = \lim_{s\to 0} s E_D(s) = \lim_{s\to 0} \frac{D_0k_2(s+3)}{s(s+3)(s+4)+k_1k_2(s+2)} = -\frac{3D_0k_2}{2k_1k_2} = -1.2e-5$$

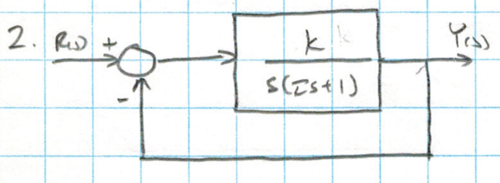
SINCE r IS UNIT RAMP AND d IS UNIT STEP $\rightarrow R_0 = 1,\ D_0 = 1$

$$\frac{3}{2k_1} = 1.2e-5 \qquad \frac{6}{k_1k_2} = 0.003$$

$$k_1 = 125000 \qquad k_2 = 0.016$$

$$\boxed{\begin{aligned} \therefore\ k_1 &= 125000 \\ k_2 &= 0.016 \end{aligned}}$$

2.



$r(t) = R_0 t \cdot \mathbb{1}(t)$

$R(s) = \frac{R_0}{s^2}$

$$G_{cl} = \frac{G}{1+G} = \frac{k}{\tau s^2 + s + k}$$

$$E(s) = R(s) - Y(s)$$

$$= R(s) - \frac{k}{\tau s^2 + s + k} R(s)$$

$$= \frac{s(\tau s+1)}{\tau s^2+s+k} \cdot \frac{R_0}{s^2}$$

$$= \frac{(\tau s+1) R_0}{s(\tau s^2+s+k)}$$

If $\tau, k > 0$, roots of $\tau s^2 + s + k$ in OLHP

FVT:

$$e(\infty) = \lim_{s \to 0} s E(s) = \lim_{s \to 0} \frac{(\tau s+1) R_0}{\tau s^2+s+k}$$

$$= \frac{R_0}{k}$$

$\boxed{\therefore \text{ Steady state tracking error is } \frac{R_0}{k}}$

3, a) $$Y(s) = \frac{k(s+a)(s+b)}{s^2(s^2+ks+kb) + k(s+a)(s+b)} R(s)$$ $a>0,\ b>0,\ k>0$

$$= \frac{k(s+a)(s+b)}{s^4 + ks^3 + kbs^2 + ks^2 + k(a+b)s + ab} R(s)$$

$$Y(s) = \frac{k(s+a)(s+b)}{s^4 + ks^3 + k(1+b)s^2 + k(a+b)s + ab} R(s)$$ is BIBO stable

Routh Table

s^4	1	$k(1+b)$	ab
s^3	k	$k(a+b)$	0
s^2	$k(1+b)-(a+b)$	ab	0
s^1	$k(a+b) - \frac{kab}{k(1+b)-(a+b)}$	0	0
s^0	ab	0	0

$k > 0$

$k(1+b) - (a+b) > 0$

$\boxed{k > \frac{a+b}{1+b}}$

$k(a+b) - \frac{kab}{k(1+b)-(a+b)} > 0$

$k^2(a+b)(1+b) - k(a+b)^2 > kab$

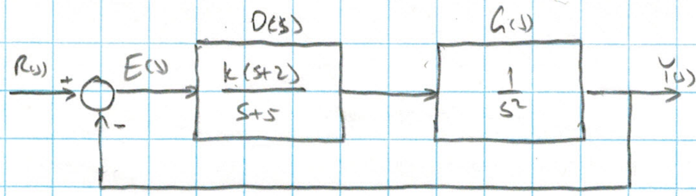
$\boxed{k > \frac{a^2+3ab+b^2}{a^2+ab+b-b^2}}$

$ab > 0$

b) $R(s) = \frac{1}{s}$

$$y_{ss} = \lim_{s\to 0} s\, Y(s) \cdot \frac{1}{s} = \lim_{s \to 0} Y(s) = \frac{abk}{ab} = k$$

$\boxed{\therefore y_{ss} = k}$

4. 

$k > 0$

$r(t) = \frac{1}{2}t^2 \cdot \mathbb{1}(t)$

$R(s) = \frac{1}{s^3}$

$e(\infty) = 0.01$

$$Y(s) = \frac{DG}{1+DG} \cdot R(s)$$

$$= \frac{k(s+2)}{s^2(s+5)} \cdot \frac{s^2(s+5)}{s^2(s+5)+k(s+2)} R(s)$$

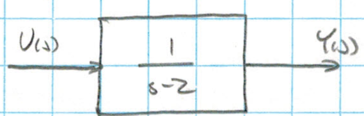
$$= \frac{k(s+2)}{s^2(s+5)+k(s+2)} R(s)$$

$$E(s) = R(s) - Y(s)$$

$$= \frac{s^2(s+5)}{s^2(s+5)+k(s+2)} \cdot \frac{1}{s^3}$$

$$e(\infty) = \lim_{s\to 0} s E(s) = \lim_{s \to 0} \frac{(s+5)}{s^2(s+5)+k(s+2)} = \frac{5}{2k} = 0.01, \quad k = 250$$

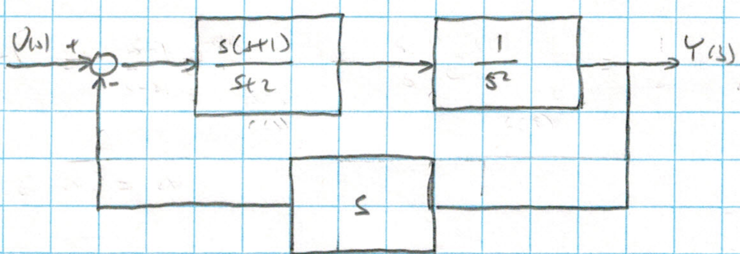
$$\boxed{\therefore\ k = 250}$$

5. a) 

$U(s) = \frac{1}{s}$ $\quad Y(s) = \frac{1}{s-2} U(s)$

Not BIBO stable, $e(\infty) = \infty$

$$\boxed{\therefore\ e(\infty) = \infty}$$

b) 

$U(s) = \frac{1}{s}$

$Y(s) = \frac{G_1}{1+G_1G_2}$

$G_1 = \frac{s+1}{s(s+2)}$

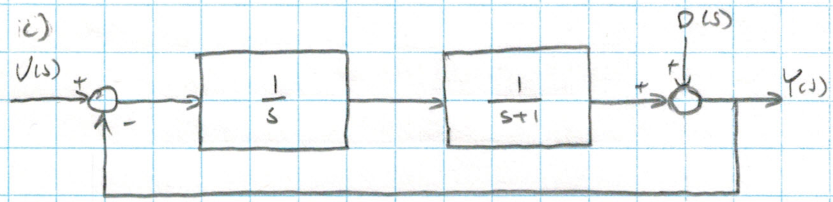
$G_2 = s$

$$E(s) = U(s) - Y(s) = 1 - \frac{s+1}{s(s+2)} \cdot \frac{(s+2)+(s+1)}{s+1} = \frac{s^2+2s-2s-3}{s(s+2)} = \frac{s^2-3}{s(s+2)}$$

$$e(\infty) = \lim_{s\to 0} s E(s) = \lim_{s \to 0} \frac{s^2-3}{s+2} = -\frac{3}{2}$$

$$\boxed{\therefore\ e(\infty) = -3/2}$$

ii)



$V(s) = \frac{1}{s}$ $\qquad D(s) = \frac{1}{s}$

$C(s) = \frac{1}{s(s+1)}$ $\qquad G(s) = 1$

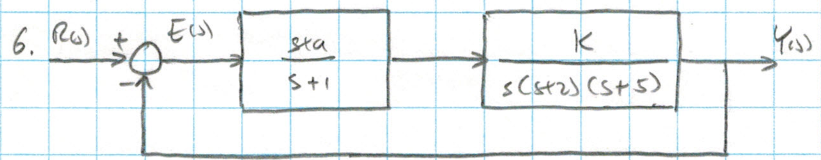
$$E(s) = \frac{1}{1+CG} R(s) - \frac{G}{1+CG} D(s)$$

$$= \frac{1}{1+C} R(s) - \frac{1}{1+C} D(s)$$

$$= \frac{1}{1+C}\left(\frac{1}{s} - \frac{1}{s}\right) = 0 \qquad e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} 0 = 0$$

$$\boxed{\therefore\ e(\infty) = 0}$$

6.



i) $$G_{CL} = \frac{CG}{1+CG} = \frac{k(s+a)}{s(s+1)(s+2)(s+5) + k(s+a)} = \frac{k(s+a)}{s^4 + 8s^3 + 17s^2 + (10+k)s + ka}$$

ROUTH TABLE

s^4	1	17	ka
s^3	8	10+k	0
s^2	$\frac{126-k}{8}$	ka	0
s^1	$\frac{(k+10)(126-k) - 64ka}{126-k}$	0	
s^0	ka		

$126 - k > 0$

$$\boxed{k < 126}$$

$$\boxed{ka > 0}$$

$$\boxed{(k+10)(126-k) - 64ka > 0}$$

ii) $e(\infty) \le 0.25 R_0$ $\quad r(t) = R_0 t \cdot \mathbb{1}(t)$ $\quad R(s) = \frac{R_0}{s^2}$

$$E(s) = R(s) - \frac{k(s+a)}{s(s+1)(s+2)(s+5) + k(s+a)} R(s)$$

$$= \frac{s(s+1)(s+2)(s+5)}{s^4 + 8s^3 + 17s^2 + (10+k)s + ka} \cdot \frac{R_0}{s^2}$$

$$e(\infty) = \lim_{s\to 0} s E(s) = \lim_{s \to 0} \frac{(s+1)(s+2)(s+5) R_0}{s^4 + 8s^3 + 17s^2 + (10+k)s + ka} = \frac{10 R_0}{ka}$$

$$\frac{10 R_0}{ka} < 0.25 R_0$$

$$ka \ge 40$$

$$\boxed{\therefore ka \ge 40}$$

iii) $a = 2$

$ka \ge 40$

$k \ge 20$

$-k^2 - 12k + 1260 > 0$

$-42 < k < 30$

$k > 0$

$$\boxed{\therefore 20 \le k < 30}$$