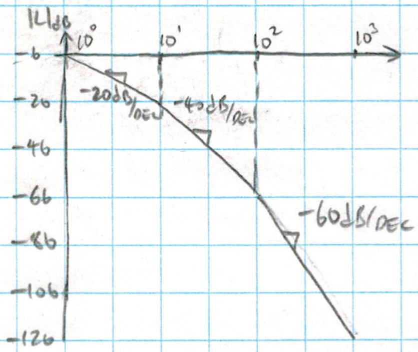
BINGRAN LI - 1004191910

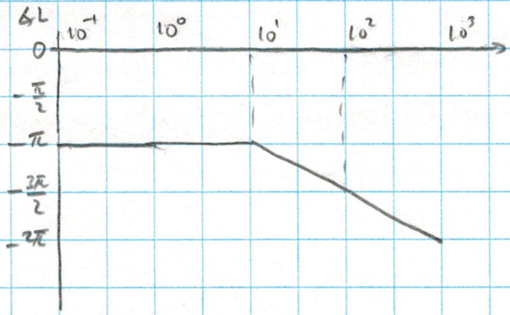
ECE311 - ASSIGNMENT 6

1. $$L = \frac{500}{s(s+10)(s+100)} = \frac{1}{2s\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$$

$$|L|_{dB} = -20\log 2 - 20\log|s| - 20\log\left|1+\frac{s}{10}\right| - 20\log\left|1+\frac{s}{100}\right|$$

$$\angle L = -\angle(2) - \angle|s| - \angle\left|1+\frac{s}{10}\right| - \angle\left|1+\frac{s}{100}\right|$$

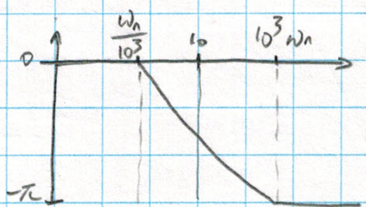




2. $$L = \frac{100(s+1)}{s(s^2+2s+100)} = \frac{(s+1)}{s\left(\left(\frac{s}{10}\right)^2 + \frac{2}{100}s + 1\right)} \leftarrow \omega_n = 10 \quad \zeta = 0.1$$

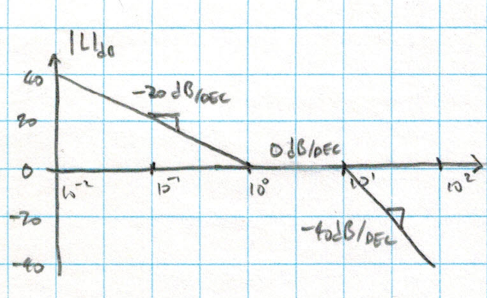
$$|L|_{dB} = -\log|s| + \log|s+1| - \log\left|\left(\frac{s}{10}\right)^2 + \frac{2}{100}s + 1\right| \leftarrow$$ TREAT LIKE DOUBLE POLE AT $s = 10$

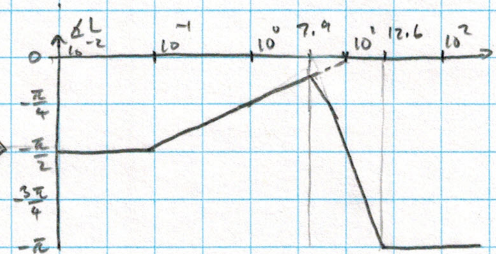
$$\angle L = -\angle(s) + \angle(s+1) - \angle\left(\left(\frac{s}{10}\right)^2 + \frac{2}{100}s + 1\right)$$



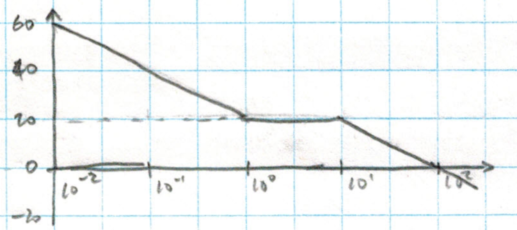
$$\frac{\omega_n}{10^\zeta} = 7.9$$

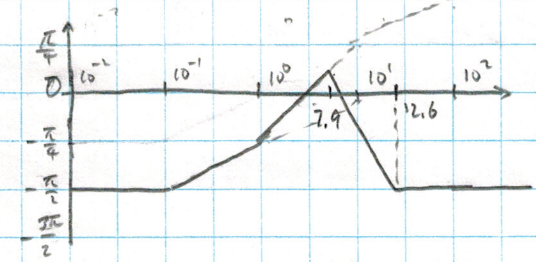
$$10^\zeta\omega_n = 12.6$$



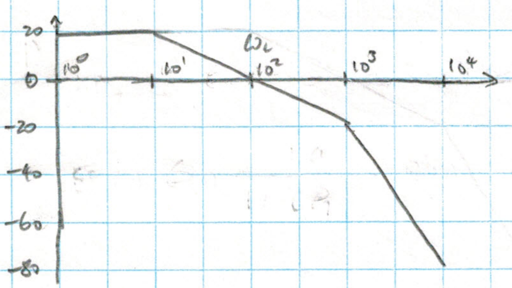


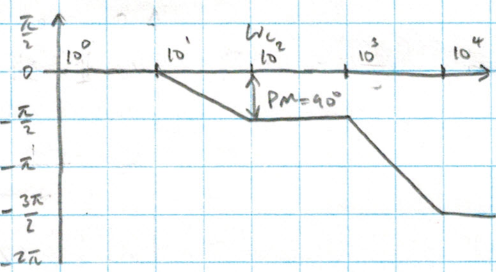
3. $L = \dfrac{100(s+1)(s+10)}{s(s^2+2s+100)} = \dfrac{10(s+1)(1+\frac{s}{10})}{s\left((\frac{s}{10})^2+\frac{2}{100}s+1\right)}$ $\leftarrow \omega_n = 10, \quad \zeta = 0.1$





4. $G(s) = \dfrac{10^8}{(s+10)(s+10^3)^2} = \dfrac{10}{(1+\frac{s}{10})(1+\frac{s}{1000})^2}$





$\omega_c = 100$

$C(s) = \dfrac{T_1 s + 1}{T_1 s}$ $\qquad \dfrac{1}{T_1} = 0.1\,\omega_c, \quad T_1 \geq 0.1$

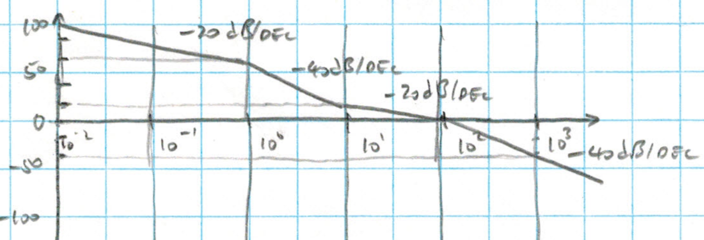
$C(s) = \dfrac{0.1s+1}{0.1s}$

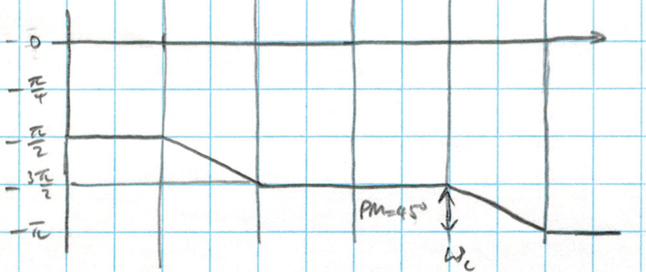
5. $PM = \tan^{-1}\left(\frac{2\zeta}{\sqrt{2\zeta + \sqrt{1+4\zeta^4}}}\right)$ $\quad \omega_{BW} = \omega_n \sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$ $\quad , \omega_c \approx 0.5\,\omega_{BW}$

i) $G(s) = \dfrac{s+10}{s(s+1)(s+100)}$, $\%OS = 1.44\%$, $T_s = 0.06\,s$

$\omega_n = 83$
$\zeta = 0.803$ $\rightarrow$ $PM = 70°$
$\omega_{BW} = 72$ rad/s , $\omega_c = 36$ rad/s

i) $r(t) = \mathbb{1}(t)$: IMP => $e(\infty) = 0$ AS LONG AS CLS IS BIBO
$r(t) = t\,\mathbb{1}(t)$: FUT => $C(\infty) = K = 10^4$ " "





← USE LEAD CONTROLLER TO ADD PHASE AT $\omega_c^{DESIRED}$ AND ADD $\phi_{MAX} = 70 - 45° = 35°$

$G_1(s) = \dfrac{T_1 s + 1}{\alpha_1 T_1 s + 1}$, $\alpha_1 = \dfrac{1-\sin\phi_{MAX}}{1+\sin\phi_{MAX}} = 0.4$

$36 = \dfrac{T_1}{T_1\sqrt{0.4}}$, $T_1 = 0.04$

$C_1(s) = \dfrac{0.04s+1}{0.016s+1}$

$10\log\left(\frac{1}{\alpha_1}\right) = 3.98$, $20\log\left(\frac{1}{\alpha_1}\right) = 7.96$ ← USE LAG COMPENSATION TO DECREASE GAIN BY 12dB AT HIGH FREQUENCIES.

$C_2(s) = \dfrac{T_2 s + 1}{\alpha T_2 s + 1}$, $\alpha_2 > 1$

$-20\log(\alpha_2) = -12\,dB$, $\alpha_2 = 4$

$\dfrac{1}{T_2} = 0.1\,\omega_c$, $T_2 = 0.28$

$C_2(s) = \dfrac{0.28s+1}{1.12s+1}$

$C(s) = K\,C_1(s)\,C_2(s)$

6. a) $\omega_c \simeq 30$ rad/s $\rightarrow$ PM $\simeq 80^\circ$

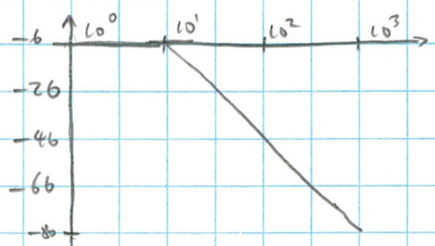
b) $C(s) = k\left(\frac{T_1 s+1}{T_1 s}\right)$

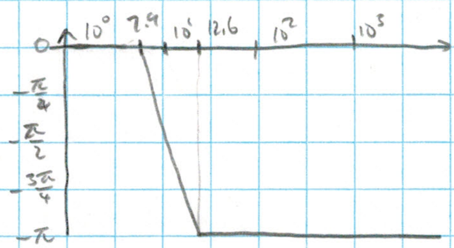
Pick $k=1 \Rightarrow C(s) = \frac{T_1 s+1}{T_1 s}$

$\frac{1}{T_1} = 0.1\omega_c \rightarrow T_1 = \frac{1}{3}$ rad/s

$C(s) = 3\,\frac{\frac{1}{3}s+1}{s}$

7. $G(s) = \frac{50}{s^2+2s+100} = \frac{1}{2\left(\left(\frac{s}{10}\right)^2+\frac{2}{100}s+1\right)}$





$\phi_{MAX} = 30.3^\circ$

$C_1(s) = k\frac{T_1 s+1}{\alpha_1 T_1 s+1}$ $\quad \frac{1}{T_1} = 0.1\omega_c \quad T_1 = 1$, $\alpha = \frac{1}{100}$

$C_1(s) = \frac{s+1}{\frac{1}{100}s+1}$

$C_2(s) = \alpha_2\frac{T_2 s+1}{\alpha_2 T_2 s+1}$ $\quad \alpha_2 = 20$, $\frac{1}{T_2} = 0.1\omega_c = 4.73$ rad/s

$C_2(s) = 20\,\frac{0.473s+1}{20(0.473)s+1} = 20\,\frac{0.473s+1}{9.46s+1}$

$C(s) = C_1(s)\,C_2(s) = \left(\frac{s+1}{\frac{1}{100}s+1}\right)\left(20\,\frac{0.473s+1}{9.46s+1}\right)$