Assignment 3: Dependencies, Decompositions, Normal forms

1. **a) Step 1 - Split the RHSs to get our initial set of FDs, S1:**
   M -> I
   M -> J
   M -> L
   J -> L
   J -> I
   JN -> K
   JN -> M
   M -> J
   KLN -> M
   K -> I
   K -> J
   K -> L
   IJ -> K

   Step 2 - For each FD, try to reduce the LHS:

   No singleton LHS will yield to anything, so we can ignore those one.

   JN -> K, JN -> M:
   J+ = IJKL, N+ = N
   Therefore, can't reduce anything

   KLN -> M:
   K+ = IJKL, L+ = L, N+ = N
   Therefore, we can eliminate L resulting in KN -> M

   IJ -> K:
   I+ = I, J+ = IJKL
   Therefore, we can eliminate I resulting in J -> K

   Our new set of FDs, let's call it S2:
   (a) M -> I
   (b) M -> J
   (c) M -> L
   (d) J -> L
   (e) J -> I
   (f) JN -> K
   (g) JN -> M
   (h) KN -> M
   (i) K -> I
   (j) K -> J

(k) K -> L
(l) J -> K

Step 3 - Eliminate Redundancies:

(a) $M^{+}_{S2\text{-}(a)}$ = IJKL. (Remove)
(b) $M^{+}_{S2\text{-}(b)}$ = ML. (Need)
(c) $M^{+}_{S2\text{-}(c)}$ = IJKLM. (Remove)
(d) $J^{+}_{S2\text{-}(d)}$ = IJKL. (Remove)
(e) $J^{+}_{S2\text{-}(e)}$ = IJKL. (Remove)
(f) $JN^{+}_{S2\text{-}(f)}$ = IJKLNM. (Remove)
(g) $JN^{+}_{S2\text{-}(g)}$ = IJKLNM. (Remove)
(h) $KN^{+}_{S2\text{-}(h)}$ = IJKLN. (Need)
(i) $K^{+}_{S2\text{-}(i)}$ = JKL. (Need)
(j) $K^{+}_{S2\text{-}(j)}$ = IKL. (Need)
(k) $K^{+}_{S2\text{-}(k)}$ = JKI. (Need)
(l) $J^{+}_{S2\text{-}(l)}$ = J. (Need)

Therefore, the minimal basis:

M -> J, KN -> M, K -> L, K -> J, K -> I, J -> K

b) All Keys for R

- The only letter that only appears on LHS is N which means it must be in every key
- OP doesn't appear in any of the FDs, therefore it must appear in every key
- IJKLM appear on both the RHS and LHS and as a result we would need to check every letter individually

Step 1: Check Singletons:

- $NOPI^{+}$ = NOPI (Not a Key)
- $NOPJ^{+}$ = IJKLMNOP (Key)
- $NOPK^{+}$ = IJKLMNOP (Key)
- $NOPL^{+}$ = NOPL (Not a Key)
- $NOPM^{+}$ = IJKLMNOP (Key)

Step 2: Append Either J, K, or M to get a key involving I or L:

- $NOPLM^{+}$ = IJKLMNOP
- $NOPLK^{+}$ = IJKLMNOP
- $NOPLJ^{+}$ = IJKLMNOP
- $NOPIM^{+}$ = IJKLMNOP
- $NOPIK^{+}$ = IJKLMNOP
- $NOPIJ^{+}$ = IJKLMNOP

Therefore, all the possible keys are NOPLM, NOPLK, NOPLJ, NOPIM, NOPIK, NOPIJ, NOPM, NOPK, NOPJ.

c) Step 1: use Minimal Basis obtained in a) and merge RHSs where necessary:

M -> J, KN -> M, K -> IJL, J -> K

Step 2: Group up into a set of relations:

R1(MJ), R2(KNM), R3(KIJL), R4(JK)

Step 3: Remove R4 since JK appears in R3 as well and add attributes OP as its own Relation:

Therefore, the final decomposition is:

R1(MJ), R2(KNM), R3(KIJL), R5(OP)

d) Step 1: Project each FD onto each relation to find if a relation violates BCNF:

J -> K projects onto relation R3, J^+ = KIJL.

This means that J is not a super key.

Therefore, redundancy is allowed in this schema

2. **a)** J -> FGI, F -> D, DEI -> F

b) Step 1 - Decompose DEI -> F:
DEI^+ = DEFI

S1(DEFI) and S2(CDGHJ)

Step 2 - Project FDs onto S1:

D^+ = D
E^+ = E
F^+ = FD

F -> D which violates BCNF

Step 3 - Decompose S1 Further:

S3(FD) and S4(EFI)

Step 4 - Project FDs onto S3:

D^+ = D
F^+ = FD

F -> D, this means that F is a super key of S3.

Therefore, the relation satisfies BCNF

Step 4 - Project FDs onto S4:

$E^+ = E$
$F^+ = FD$
$I^+ = I$
$EF^+ = EFD$
$EI^+ = EI$
$FI^+ = FDI$

Therefore, this relation satisfies BCNF

Step 5 - Project FDs onto S2:

$C^+ = CDEFGHIJ$
$D^+ = D$
$G^+ = G$
$H^+ = H$
$J^+ = JFGID$

C -> EH is a super key.
J -> FGI violates BCNF.

Step 6 - Decompose S2 Further:

S5(GJ) and S6(CDHJ)

Step 7 - Project FDs onto S5:

$G^+ = G$
$J^+ = JFGI$

J -> FGI is a super key.

Therefore, this relation satisfies BCNF.

Step 8 - Project FDs onto S6:

$C^+ = CDEFGHIJ$
$D^+ = D$
$G^+ = G$

$H^+ = H$
$J^+ = JFGI$
$DJ^+ = DDFGIJ$
$DH^+ = DH$
$HJ^+ = JFGIDH$

Therefore, this relation satisfies BCNF.

Step 9 - Solution:

The following relations all satisfy BCNF:

S3(FD): F -> D

S4(EFI): No FDs

S5(GJ): J -> FGI

S6(CDHJ): No FDs