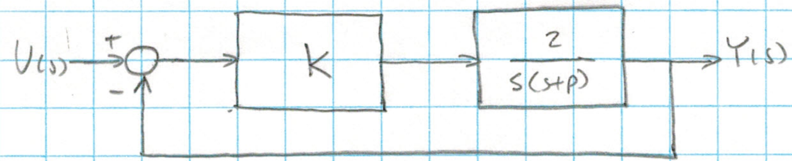
BINGRAN LI - 1004191910

ECE311 - ASSIGNMENT 3 - TIME RESPONSE

1. a)



$$G(s) = \frac{2k}{s(s+p)}\left(\frac{s(s+p)}{s(s+p)+2k}\right) = \frac{2k}{s^2+ps+2k} \rightarrow \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$T_s \leq 0.8 \rightarrow \frac{4}{\zeta\omega_n} \leq 0.8$

$p = \zeta\omega_n \geq \frac{4}{0.8}$

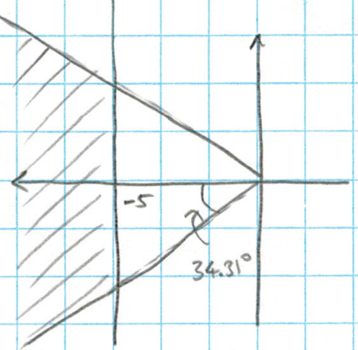
$p \geq 5$

$\%OS \leq 0.01$

$\zeta \geq \frac{-\ln(0.01)}{\sqrt{\pi^2+\ln^2(0.01)}}$

$\zeta \geq 0.826 = \cos\theta_d$

$\theta_d = 34.31°$



b) CHOOSE $p=5$ AND $\zeta = 0.9$

$p = \zeta\omega_n$
$5 = 0.9(\omega_n)$
$\omega_n = 5.55$

$2k = \omega_n^2$
$k = \frac{(5.55)^2}{2}$
$k = 15.43$

$\boxed{\therefore\ p = 5,\ k = 15.43}$

2. a) $T_r < 0.5$

$\omega_n > \frac{1.8}{0.5}$

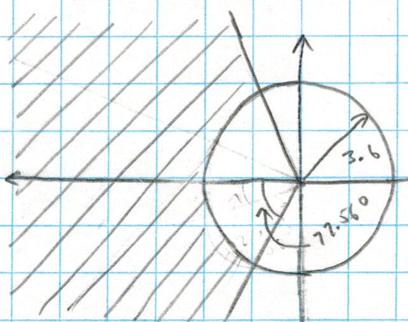
$\omega_n > 3.6$

$\%OS \leq 0.5$

$\zeta \geq \frac{-\ln(0.5)}{\sqrt{\pi^2+\ln^2(0.5)}}$

$\zeta \geq 0.215 = \cos\theta_d$

$\theta_d = 77.56°$



b) $$G(s) = \frac{k}{s^2+6s-7+k} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

BIBO STABLE: $\frac{-6 \pm \sqrt{36-4(6)(k-7)}}{2} < 0 \Rightarrow 36-4(6)(k-7) < 36 \quad k-7 > 0 \Rightarrow k > 7$

$2\zeta\omega_n = 6$
$\zeta\omega_n = 3$

CHOOSE $\omega_n = 4 > 3.6$
$\omega_n^2 = k-7$
$k = 21$

$\zeta\omega_n = 3$
$\zeta = 0.75 > 0.215$

$\boxed{\therefore\ k = 21 \text{ works}}$

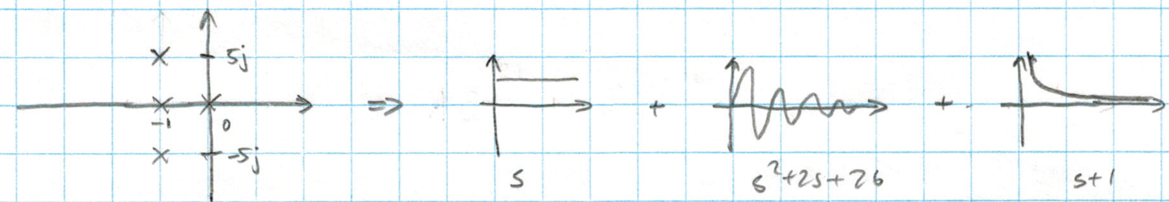
3. $G(s) = \dfrac{s+2}{(s+1)(s^2+2s+26)}$

$u(t) = 1(t) \qquad U(s) = \frac{1}{s}$

$Y(s) = \dfrac{s+2}{s(s+1)(s^2+2s+26)} = \dfrac{s+2}{s(s+1)(s+1\pm 5j)}$

ZERO AT $p=-2$

POLES AT $p=0, -1, -1\pm 5j$



$s^2+2\zeta\omega_n+\omega_n^2$

$\omega_n = \sqrt{26} = 5.1$

$\zeta = 0.2$

FOR DECAYING SINUSOID

$T_r \simeq \dfrac{1.8}{\omega_n} = 0.35s$

$T_s \simeq \dfrac{4}{\zeta\omega_n} = 2s$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.63s$

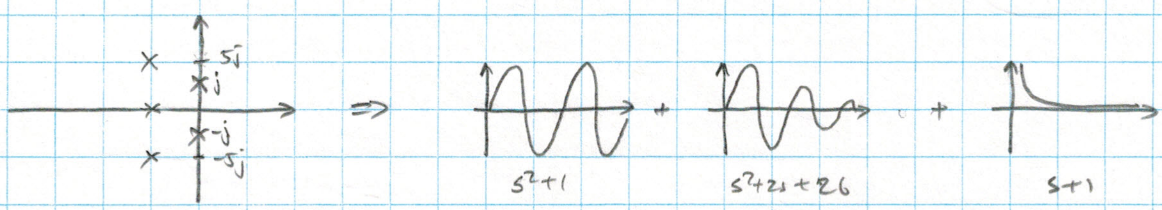
$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 52.66\%$

$u(t) = \sin(t) \qquad U(s) = \dfrac{1}{s^2+1}$

$Y(s) = \dfrac{s+2}{(s^2+1)(s+1)(s^2+2s+26)} = \dfrac{s+2}{(s+1)(s\pm j)(s^2+1\pm 5j)}$

ZERO AT $p=-2$

POLES AT $p=-1, \pm j, -1\pm 5j$



FOR DECAYING SINUSOID

$T_r \simeq 0.35s$

$T_s \simeq 2s$

$T_p = 0.63s$

$\%OS = 52.66\%$

4. $G_1(s) = \dfrac{1}{(s+2)^2+1}$ $\quad u(t) = \mathbb{1}(t)$ $\quad U(s) = 1/s$ $\quad Y_1(s) = \dfrac{1}{s\left((s+2)^2+1\right)}$

$(s+\sigma)^2 + \omega_d^2$

POLES:

$p = 0, -2 \pm j$

$P_{1,2} = -2 \pm j$

$|P_{1,2}| = \sqrt{2^2+1^2} = \sqrt{5}$

$T_r = \dfrac{1.8}{|P_{1,2}|} = 0.8\,s$ $\qquad \%OS = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$T_s = \dfrac{4}{\sigma} = 2\,s$ $\qquad \%OS = 0.19\,\%$

$G_2(s) = \dfrac{20}{(s+20)\left((s+2)^2+1\right)}$ $\quad u(t) = \mathbb{1}(t)$ $\quad U(s) = 1/s$ $\quad Y_2(s) = \dfrac{20}{s(s+20)\left((s+2)^2+1\right)}$

$y_1(t) = \frac{1}{5} - \frac{1}{5}e^{-2t}(\cos t + 2\sin(t))$ $\qquad y_2(t) = \frac{1}{5} - \frac{64}{325}e^{-2t}\left(\cos t + \frac{37}{16}\sin t\right) - \frac{1}{325}e^{-20t}$

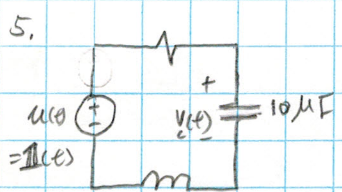
$y_1 - y_2 = -0.0031\,e^{-2t}\cos t + 0.0554\,e^{-2t}\sin t + 0.0031\,e^{-20t}$

$|y_1 - y_2| = |-0.0031|\,|e^{-2t}|\,|\cos t| + |0.0554|\,|e^{-2t}|\,|\sin t| + |0.0031|\,|e^{-20t}|$

$= 0.0031 + 0.0554 + 0.0031 = 0.0616$

$\boxed{\therefore \text{ UPPER BOUND FOR } |y_2 - y_2| \text{ IS } 0.0616}$

5.



$V_c(s) = \dfrac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

$\%OS = 0.15$ $\qquad T_s = 0.002$ $\qquad 2\zeta\omega_n = \dfrac{R}{L} = 4000$

$\zeta = \dfrac{-\ln(0.15)}{\sqrt{\pi^2 + \ln^2(0.15)}}$ $\qquad \dfrac{4}{\zeta\omega_n} = 0.002$ $\qquad \dfrac{1}{LC} = \omega_n^2 = (3869)^2$

$\zeta = 0.5169$ $\qquad \zeta\omega_n = 2000$

$\omega_n = 3869$ $\qquad L = \dfrac{1}{\omega_n^2 C} = \dfrac{1}{(3869)^2(1e\text{-}5)} = 6.7\,mH$

$R = 4000(6.7e\text{-}3) = 26.8\,\Omega$

$$\boxed{\begin{aligned} \therefore\ R &= 26.8\,\Omega \\ L &= 6.7\,mH \end{aligned}}$$

1. $$G'(s) = \frac{G(s)}{1+G(s)} = \frac{ks(s+2)}{(s^2-4s+8)(s+3)+ks(s+2)} = \frac{ks(s+2)}{s^3+(k-1)s^2+(2k-4)s+24}$$

ROUTH TABLE

s^3	1	$2k-4$
s^2	$k-1$	24
s^1	$\frac{2k^2-6k-20}{k-1}$	0
s^0	24	0

$k-1>0$

$k>1$

$$\frac{2k^2-6k-20}{k-1} > 0$$

$k^2-3k-10>0$

$(k-5)(k+2)>0$

$k>5$ or $k<-2$

$\boxed{\therefore\ k>5}$

2. $$G'(s) = \frac{G(s)}{1+G(s)} = \frac{k}{s(s+1)(s+2)(s+5)+k} = \frac{k}{s^4+8s^3+17s^2+10s+k}$$

ROUTH TABLE

s^4	1	17	k
s^3	8	10	0
s^2	$126/8$	$-k$	0
s^1	$10-\frac{32}{63}k$	0	0
s^0	k	0	0

$k>0$

$10-\frac{32}{63}k>0$

$k < 315/16 = 19.69$

$\boxed{\therefore\ 0<k<19.69}$

3. $\theta'' = -\frac{mgl}{I}\sin\theta + \frac{1}{I}u \qquad x = \begin{bmatrix}\theta \\ \theta'\end{bmatrix} = \begin{bmatrix}x_1 \\ x_2\end{bmatrix} \qquad y = \theta = x_1$

$x_1' = x_2$

$x_2' = -\frac{mgl}{I}\sin x_1 + \frac{1}{I}u$

LINEARIZE AT $\left(\begin{bmatrix}0\\0\end{bmatrix}, 0\right)$:

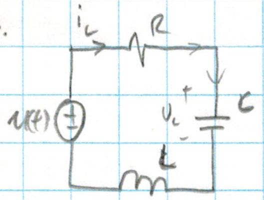
$$\begin{cases} x' = \begin{bmatrix} x_2 \\ -\frac{mgl}{I}\sin x_1 + \frac{1}{I}u \end{bmatrix} \\ y = x_1 \end{cases} \qquad \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -\frac{mgl}{I}\cos x_1 & 0 \end{bmatrix} \qquad \frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ \frac{1}{I} \end{bmatrix} \qquad \frac{\partial h}{\partial x} = \begin{bmatrix}1 & 0\end{bmatrix} \quad \frac{\partial h}{\partial u} = 0$$

$$G(s) = C\cdot(sI-A)^{-1}B = \begin{bmatrix}1 & 0\end{bmatrix}\cdot\begin{bmatrix} s & 1 \\ -\frac{mgl}{I} & s\end{bmatrix}^{-1}\cdot\begin{bmatrix}0 \\ \frac{1}{I}\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\end{bmatrix}\cdot\frac{1}{s^2+\frac{mgl}{I}}\cdot\begin{bmatrix} s & -1 \\ \frac{mgl}{I} & s\end{bmatrix}\cdot\begin{bmatrix}0 \\ \frac{1}{I}\end{bmatrix} = \frac{-\frac{1}{I}}{s^2+\frac{mgl}{I}}$$

$\boxed{\therefore \text{ UNSTABLE, ZEROS LIE ON IM. AXIS}}$

4.



$$i_L R + V_c + L\frac{di_L}{dt} = u \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} V_c \\ i_L \end{bmatrix}$$

$$i_L = C\frac{dV_c}{dt} \qquad y = V_c$$

$$x_1' = \frac{1}{C} x_2$$

$$x_2' = -\frac{1}{L} x_1 - \frac{R}{L} x_2 + \frac{1}{L} u$$

$$\begin{cases} x' = \begin{bmatrix} 0 & 1/L \\ -1/L & -R/L \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{cases}$$

$$G(s) = \frac{1/LC}{s^2 + R/L\, s + 1/LC}$$

POLES AT $p = \dfrac{-R/L \pm \sqrt{(R/L)^2 - 4(1)(1/LC)}}{2} \leftarrow \sqrt{(R/L)^2 - 4/LC} < R/L$ GIVEN $R, L, C > 0$

THUS $p < 0$, AND SYSTEM IS ASYMTOTICALLY STABLE