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ECE311 - Assignment 2

1. $y''' + 3y'' - 2y' + y = u'' - 3u' + 2u$, assuming all ICs are 0

$$s^3Y + 3s^2Y - 2sY + Y = s^2U - 3sU + 2U$$

$$\frac{Y}{U} = \frac{s^2-3s+2}{s^3+3s^2-2s+1} = G(s)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -3 & 1 \end{bmatrix}, \quad D = 0$$

$\therefore$ TF: $\quad G(s) = \dfrac{s^2-3s+2}{s^3+3s^2-2s+1}$

State-space:

$$\begin{cases} x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 2 & -3 & 1 \end{bmatrix} x \end{cases}$$

2. $$\begin{cases} x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{cases}$$

$$G = C(sI-A)^{-1}B$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 3 & s+2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \frac{1}{s^3+2s^2+3s+1} \begin{bmatrix} s^2+2s+3 & s+2 & 1 \\ -1 & s^2+2s & s \\ -s & -3s-1 & s^2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^3+2s^2+3s+1} \begin{bmatrix} s^2+2s+3 & s+2 & s \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{s^2+3s+6}{s^3+2s^2+3s+1}$$

$$\therefore G(s) = \frac{s^2+3s+6}{s^3+2s^2+3s+1}$$

3. Part 1: $u = \sin t \cdot \mathbb{1}(t)$
$y = 0.5\,(e^{-t} + \sin t - \cos t)\cdot \mathbb{1}(t)$

$$U(s) = \frac{1}{s^2+1}$$

$$Y(s) = \frac{1}{2}\left(\frac{1}{s+1} + \frac{1}{s^2+1} - \frac{s}{s^2+1}\right)$$

$$= \frac{s^2+1 + s+1 - s(s+1)}{2(s+1)(s^2+1)} = \frac{2}{2(s+1)(s^2+1)}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s^2+1)}\left(\frac{s^2+1}{1}\right)$$

$$\therefore G(s) = \frac{1}{s+1}$$

Part 2: $\begin{cases} x_1' = x_2 + u \\ x_2' = u \\ y = x_1 + x_2 + u \end{cases} \Rightarrow \begin{cases} x' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + u \end{cases}$

$$G(s) = C(sI - A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \frac{1}{s^2}\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1$$

$$= \frac{1}{s^2}\begin{bmatrix} s & 1+s \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 1$$

$$\therefore G(s) = \frac{2s+1}{s^2} + 1$$

4. $\dfrac{X(s)}{F(s)} = \dfrac{s+2}{s^3+8s^2+9s+15}$, ASSUMING IC's ARE 0

$(s^3+8s^2+9s+15)X = (s+2)F$

$\therefore\ x''' + 8x'' + 9x' + 15x = f' + 2f$ IS THE DIFF. EQN

5. $\begin{cases} x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{cases}$

$G(s) = C(sI-A)^{-1}B$

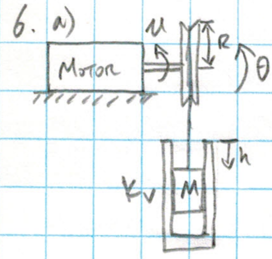
$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3 & 2 & s+5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

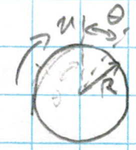
$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \dfrac{1}{s^3+5s^2+2s+4} \begin{bmatrix} s^2+5s+2 & s+5 & 1 \\ -8 & s^2+5s & s \\ -3s & -2s-3 & s^2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

$= \dfrac{1}{s^3+5s^2+2s+3} \begin{bmatrix} s^2+5s+2 & s+5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

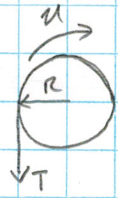
$\therefore\ G(s) = \dfrac{10}{s^3+5s^2+2s+3}$

6. a)





FBD:



$h = -R\theta$ ①

$Mh'' = Mg - k_v h' - T$

$T = Mg - k_v h' - Mh''$ ②

$I\theta'' = u - TR$ ③

Sub ② into ③: $I\theta'' = u - R(Mg - k_v h' - Mh'')$ ④

Sub ① into ④: $I\theta'' = u - RMg + R^2 k_v \theta' + R^2 M \theta''$

$\therefore (I - R^2 M)\theta'' - R^2 k_v \theta' = u - RMg$

b) $\tilde{u} = u - RMg$ $\quad \theta = -\frac{1}{R}h$, IC's are 0

$\left(RM - \frac{I}{R}\right)h'' + Rk_v h' = \tilde{u}$

Let $\mathcal{L}\{\tilde{u}\} = U(s)$
$\mathcal{L}\{h\} = H(s)$

$\left(RM - \frac{I}{R}\right)s^2 H + Rk_v s H = U$

$$\therefore \frac{H(s)}{U(s)} = \frac{1}{\left(RM - \frac{I}{R}\right)s^2 + Rk_v s}$$

c) $(I - R^2 M)\theta'' - R^2 k_v \theta' = \tilde{u}$

Let $\mathcal{L}\{\tilde{u}\} = U(s)$
$\mathcal{L}\{\theta'\} = \Theta(s)$

$(I - R^2 M)s\Theta - R^2 k_v \Theta = U$

$$\therefore \frac{\Theta(s)}{U(s)} = \frac{1}{(I - R^2 M)s - R^2 k_v}$$