



**INSTITUTO POLITECNICO NACIONAL
ESCUELA SUPERIOR DE CÓMPUTO
METODOS CUANTITATIVOS PARA LA TOMA DE
DECISIONES**



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Grupo: 3CM7

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Multiplicadores LaGrange

2.-

$$f(x, y, z) = x^2 + y^2 + z^2 \rightarrow \text{Maximizar}$$

s.a

$$x + y + z = 1 \quad (1)$$

$$x + 2y + 3z = 6 \quad (2)$$

$$L(x, y, z, \lambda_1, \lambda_2) = x^2 + y^2 + z^2 - \lambda_1(x + y + z - 1) - \lambda_2(x + 2y + 3z - 6)$$

$$= x^2 + y^2 + z^2 - \lambda_1 x - \lambda_1 y - \lambda_1 z + \lambda_1 - \lambda_2 x - 2\lambda_2 y - 3\lambda_2 z + 6\lambda_2$$

$$\frac{\partial L}{\partial x} = 2x - \lambda_1 - \lambda_2 = 0$$

$$\frac{\partial L}{\partial y} = 2y - \lambda_1 - 2\lambda_2 = 0$$

$$\frac{\partial L}{\partial z} = 2z - \lambda_1 - 3\lambda_2 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -x - y - z + 1 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = -x - 2y - 3z + 6 = 0$$

$$x = \frac{\lambda_1 + \lambda_2}{2}$$

$$y = \frac{\lambda_1 + 2\lambda_2}{2}$$

$$z = \frac{\lambda_1 + 3\lambda_2}{2}$$

$$x + y + z = 1$$

$$x + 2y + 3z = 6$$

$$\frac{\lambda_1 + \lambda_2}{2} + \frac{\lambda_1 + 2\lambda_2}{2} + \frac{\lambda_1 + 3\lambda_2}{2} = 1$$

$$\frac{\lambda_1 + \lambda_2}{2} + 2\left(\frac{\lambda_1 + 2\lambda_2}{2}\right) + 3\left(\frac{\lambda_1 + 3\lambda_2}{2}\right) = 6$$

$$\frac{3\lambda_1 + 6\lambda_2}{2} = 1$$

$$\frac{6\lambda_1 + 14\lambda_2}{2} = 6$$

$$3\lambda_1 + 6\lambda_2 = 2$$

$$6\lambda_1 + 14\lambda_2 = 12$$

$$3\lambda_1 + 6\lambda_2 = 2$$

$$-3\lambda_1 - 7\lambda_2 = -6$$

$$-\lambda_2 = -4$$

$$\lambda_2 = 4$$

$$3\lambda_1 + 6(4) = 2$$

$$3\lambda_1 = 2 - 24$$

$$\lambda_1 = -\frac{22}{3}$$

$$x = -\frac{5}{3} \quad y = \frac{1}{3} \quad z = \frac{7}{3}$$

$$f(x, y, z) = \left(-\frac{5}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{7}{3}\right)^2 = \frac{25}{3}$$

| x | y | z | f(x, y, z) |
|----------------|---------------|---------------|----------------|
| $-\frac{5}{3}$ | 0 | 0 | $\frac{25}{9}$ |
| 0 | $\frac{1}{3}$ | 0 | $\frac{1}{9}$ |
| 0 | 0 | $\frac{7}{3}$ | $\frac{49}{9}$ |
| $-\frac{5}{3}$ | $\frac{1}{3}$ | $\frac{7}{3}$ | 6 |
| $-\frac{5}{3}$ | 0 | $\frac{7}{3}$ | 10 |
| $-\frac{5}{3}$ | $\frac{1}{3}$ | 0 | 25 |
| $-\frac{5}{3}$ | 0 | 0 | 25 |
| $-\frac{5}{3}$ | $\frac{1}{3}$ | 0 | 19 |
| $-\frac{5}{3}$ | 0 | 0 | 3 |

Max
f(x, y, z) = 19

N.S.

N.S.

N.S.

N.S.

N.S.

N.S.

N.S.

N.S.

N.S.

N.S.

3.

$$f(x, y) = -2x^2 - y^2 + xy + 8x + 3y \quad \text{Max}$$

s.d

$$3,000x + 1,000y = 100,000 \rightarrow 1,000(3x + y) = (100)1,000 \rightarrow 3x + y = 100$$

$$\begin{aligned} L(x, y, \lambda_1) &= -2x^2 - y^2 + xy + 8x + 3y - \lambda_1(3x + y - 100) \\ &= -2x^2 - y^2 + xy + 8x + 3y - 3\lambda_1 x - \lambda_1 y + \lambda_1 100 \end{aligned}$$

$$\frac{\partial L}{\partial x} = -4x + y + 8 - 3\lambda_1 = 0$$

$$f(x, y) = -2\left(\frac{699}{28}\right)^2 - \left(\frac{703}{28}\right)^2 + \left(\frac{699}{28}\right)\left(\frac{703}{28}\right) + 8\left(\frac{699}{28}\right) + 3\left(\frac{703}{28}\right)$$

$$= -974.982 \rightarrow -974.982$$

$$\frac{\partial L}{\partial y} = -2y + x + 3 - \lambda_1 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -3x - y + 100 = 0$$

$$+3\lambda_1 = -4x + y + 8$$

$$\lambda_1 = \frac{-4x + y + 8}{3}$$

$$\lambda_1 = x - 2y + 3$$

$$x - 2y + 3 = \frac{-4x + y + 8}{3}$$

$$3x - 6y + 9 = -4x + y + 8$$

$$7x - 7y = -1$$

$$3x + y = 100$$

$$y = 100 - 3x$$

$$7x - 7(100 - 3x) = -1$$

$$7x - 700 + 21x = -1$$

$$-28x = -1 + 700$$

$$x = \frac{699}{28} \quad y = \frac{703}{28}$$

| x | y | f(x, y) |
|----|-----|----------|
| 0 | 100 | -9700000 |
| 25 | 25 | -975000 |
| 33 | 1 | -1879000 |
| 1 | 97 | -9015000 |
| 15 | 56 | -2545000 |
| 20 | 40 | -1320000 |
| 5 | 85 | -6555000 |
| 10 | 70 | -4110000 |
| 30 | 10 | -1330000 |