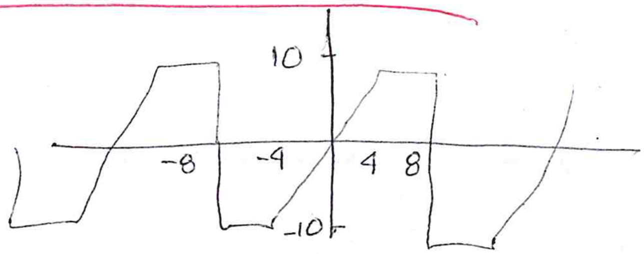
1 — Departamental
Teoría de Comunicaciones y Señales

# PROBLEMA 1




S.T.F

$T = 16 \therefore \omega_0 = \frac{2\pi}{16}$

$\omega_0 = \frac{\pi}{8}$

La función es impar $\therefore$

$a_0 = a_n = 0$

&

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\,\text{Sen}\, n\omega_0 t\,dt$$

$$b_n = \frac{4}{16}\int_0^{8} f(t)\,\text{Sen}\,\frac{n\pi}{8}t\,dt$$

$$b_n = \frac{1}{4}\int_0^{4}\frac{5}{2}t\cdot\text{Sen}\,\frac{n\pi}{8}t\,dt + \frac{1}{4}\int_4^{8}10\,\text{Sen}\,\frac{n\pi}{8}t\,dt$$

$$b_n = \frac{5}{8}\int_0^{4}t\cdot\text{Sen}\,\frac{n\pi}{8}t\,dt + \frac{5}{2}\int_4^{8}\text{Sen}\,\frac{n\pi}{8}t\,dt$$

$u = t \quad dv = \text{sen}\,\frac{n\pi}{8}t\,dt$

$du = dt \quad v = \frac{-8}{n\pi}\cos\frac{n\pi}{8}t$

$$b_n = \frac{5}{8}\left\{\frac{-8t}{n\pi}\cos\frac{n\pi}{8}t\Big|_0^4 + \frac{8}{n\pi}\int_0^4\cos\frac{n\pi}{8}t\,dt\right\}$$

$$-\left(\frac{5}{2}\right)\left(\frac{8}{n\pi}\right)\text{Cos}\,\frac{n\pi}{8}t\Big|_4^8$$

$$= \frac{5}{8}\left\{\frac{-32}{n\pi}\cos\frac{n\pi}{2} + \phi + \frac{64}{n^2\pi^2}\text{sen}\,\frac{n\pi}{8}t\Big|_0^4\right\}$$

$$-\frac{20}{n\pi}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$$

$$b_n = \frac{5}{8}\left\{\frac{-32}{n\pi}\cos\frac{n\pi}{2} + \frac{64}{n^2\pi^2}\text{sen}\,\frac{n\pi}{2}\right\}$$

$$-\frac{20}{n\pi}\cos n\pi + \frac{20}{n\pi}\cos\frac{n\pi}{2}$$

$$b_n = \cancel{-\frac{20}{n\pi}\cos\frac{n\pi}{2}} + \frac{40}{n^2\pi^2}\text{sen}\,\frac{n\pi}{2}$$

$$-\frac{20}{n\pi}\cos n\pi \cancel{+\frac{20}{n\pi}\cos\frac{n\pi}{2}}$$

$$b_n = \frac{40}{n^2\pi^2}\text{Sen}\,\frac{n\pi}{2} - \frac{20}{n\pi}\cos n\pi$$

$$b_n = \frac{20}{n\pi}\left[\frac{2}{n\pi}\text{sen}\,\frac{n\pi}{2} - (-1)^n\right]$$

Finalmente

$$f(t) = \sum_{n=1}^{\infty}\frac{20}{n\pi}\left[\frac{2}{n\pi}\text{sen}\,\frac{n\pi}{2} - (-1)^n\right]\cdot\text{Sen}\,\frac{n\pi}{8}$$

PROBLEMA 2. TIPO "B"

A partir de la Serie obtenida en el Problema anterior, deduzca la Serie exponencial de Fourier de f(t)

SOL: Si $C_n = \frac{1}{2}(a_n - i b_n)$

& $b_n = \frac{40}{n^2\pi^2}\,\text{Sen}\,\frac{n\pi}{2} - \frac{20}{n\pi}(-1)^n$

$\therefore$ $C_n = \frac{-j}{2}\left(\frac{40}{n^2\pi^2}\,\text{Sen}\,\frac{n\pi}{2} - \frac{20}{n\pi}(-1)^n\right)$

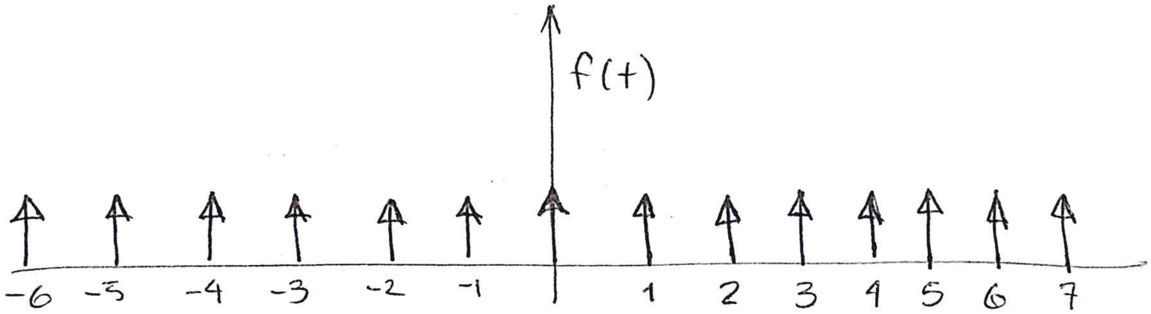
$C_n = -j\left(\frac{20}{n^2\pi^2}\,\text{sen}\,\frac{n\pi}{2} - \frac{10}{n\pi}(-1)^n\right)$

$C_n = j\,\frac{10}{n\pi}\left[(-1)^n - \frac{2}{n\pi}\,\text{Sen}\,\frac{n\pi}{2}\right]$

Finalmente:

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{j10}{n\pi}\left[(-1)^n - \frac{2}{n\pi}\,\text{Sen}\,\frac{n\pi}{2}\right]\cdot e^{\frac{jn\pi}{8}t}$$

**PROBLEMA 3.** Encuentre la transformada de Fourier de $f(t)$ y grafique su espectro




SOLUCION:

Como $f(t)$ es una función periódica, su transformada es de la forma:

$$\mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \,\delta(\omega - n\omega_0)$$

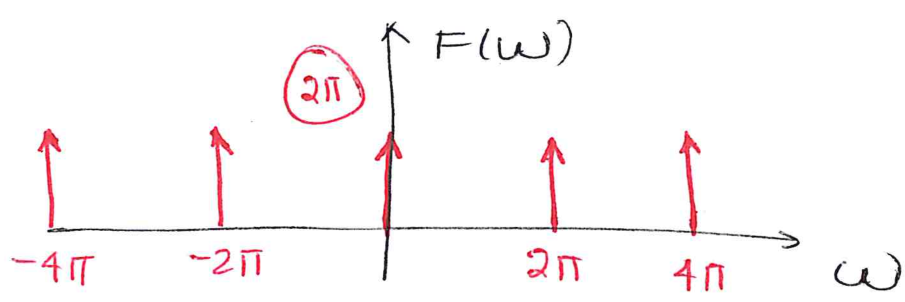
Donde $\omega_0 = \frac{2\pi}{T} \therefore \omega_0 = \frac{2\pi}{1} = 2\pi$ & $C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t}\, dt$

$$C_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(t) e^{-jn2\pi t}\, dt$$

$$C_n = \int_{0^-}^{0^+} \delta(t) e^{-j2\pi n t}\, dt \quad \therefore \quad C_n = e^0 = 1$$

Finalmente

$$F(\omega) = \mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$



# PROBLEMA 4

Usando las Propiedades de la transformada de Fourier, complete la pareja de transformadas siguientes:

$$-6\delta[5t-10]\cdot \cos 15t + \frac{1}{3-jt}\cdot t^2 + e^{j4t}(t-1) \longleftrightarrow ?$$

SOLUCION

① $-6\delta(5t-10)\cdot \cos 15t \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$\delta(t-10) \longleftrightarrow e^{-j10\omega}$

$\delta(5t-10) \longleftrightarrow \frac{1}{|5|} e^{-j10\frac{\omega}{5}}$

$\delta(5t-10) \longleftrightarrow \frac{1}{10}\left[e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$

$-6\delta(5t-10) \longleftrightarrow \frac{-3}{5}\left[e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$

② $\frac{1}{3-jt}\cdot t^2 \longleftrightarrow ?$

Si $e^{-at}u(t) \longleftrightarrow \frac{1}{a+i\omega}$

$\frac{1}{a+jt} \longleftrightarrow 2\pi e^{-a(-\omega)} u(-\omega)$

$\frac{1}{3+jt} \longleftrightarrow 2\pi e^{3\omega} u(-\omega)$

$\frac{1}{3-jt} \longleftrightarrow 2\pi e^{-3\omega} u(\omega)$

$\frac{(-jt)^2}{3-jt} \longleftrightarrow 2\pi \frac{d^2}{d^2\omega}\left[e^{-3\omega} u(\omega)\right]$

$\frac{t^2}{3-jt} \longleftrightarrow -2\pi \frac{d^2}{d^2\omega}\left[e^{-3\omega} u(\omega)\right]$

③ $e^{j4t}(t-1) \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(-\omega)$

$-jt \longleftrightarrow 2\pi \frac{d}{d\omega}\delta(\omega)$

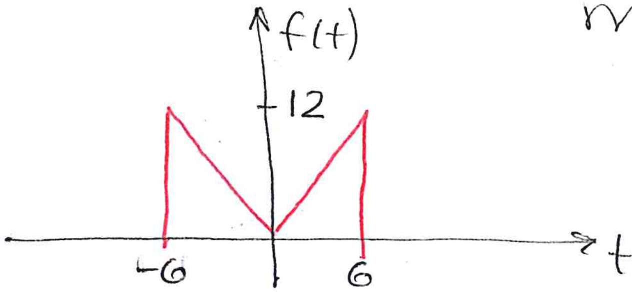
$t \longleftrightarrow 2\pi j \frac{d}{d\omega}\delta(\omega)$

$t-1 \longleftrightarrow 2\pi j \cdot e^{-j\omega}\cdot \frac{d}{d\omega}\delta(\omega)$

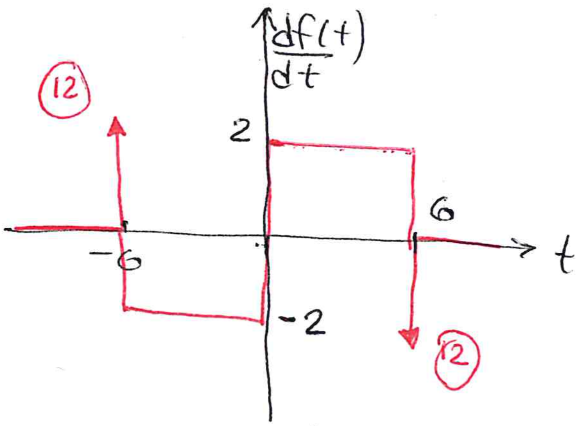
$e^{j4t}(t-1) \longleftrightarrow 2\pi i\, e^{-i(\omega-4)} \frac{d}{d\omega}\delta(\omega-4)$

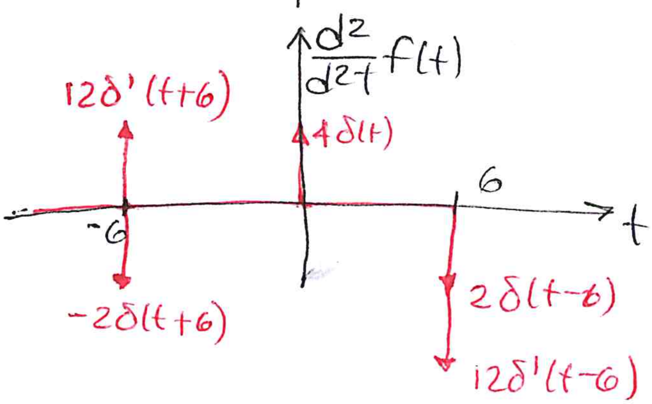
## PROBLEMA 5

Usando la Propiedad de Diferen. en tiempo, encuentre la transformada de f(t)




SOLUCION:




$$\frac{d^2}{d^2t}f(t) = 12\delta'(t+6) - 2\delta(t+6) + 4\delta(t) - 2\delta(t-6) - 12\delta'(t-6)$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}f(t)\right\} = 12\mathcal{F}\left\{\delta'(t+6)\right\} - 2\mathcal{F}\left\{\delta(t+6)\right\} + 4\mathcal{F}\left\{\delta(t)\right\} - 2\mathcal{F}\left\{\delta(t-6)\right\} - 12\mathcal{F}\left\{\delta'(t-6)\right\}$$

Si $\delta(t) \longleftrightarrow 1$

$\delta(t-6) \longleftrightarrow e^{-j6\omega}$

$\delta'(t-6) \longleftrightarrow j\omega e^{-j6\omega}$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}f(t)\right\} = 12\cdot j\omega e^{j6\omega} - 2e^{j6\omega} + 4 - 2e^{-j6\omega} - 12\cdot j\omega e^{-j6\omega}$$

simbólicamente:

$$\frac{d^2}{d^2t}f(t) \longleftrightarrow 12j\omega e^{j6\omega} - 2e^{j6\omega} + 4 - 2e^{-j6\omega} - 12j\omega e^{-j6\omega}$$

$$\frac{d^2}{d^2t}f(t) \longleftrightarrow 12j\omega\left(e^{j6\omega} - e^{-j6\omega}\right) - 2\left(e^{j6\omega} + e^{-j6\omega}\right) + 4$$

$$\frac{d^2}{d^2t}f(t) \longleftrightarrow -24\omega\,\text{Sen}\,6\omega - 4\cos 6\omega + 4$$

$$\frac{d^2}{d^2t}f(t) \longleftrightarrow 4(1-\cos 6\omega) - 24\omega\,\text{Sen}\,6\omega$$

De la Prop. de diferenc. en t.

Si $f(t) \longleftrightarrow F(\omega)$

$$\frac{d^2}{d^2t}f(t) \longleftrightarrow (j\omega)^2 F(\omega)$$

$$\therefore (j\omega)^2 F(\omega) = 4(1-\cos 6\omega) - 24\omega\,\text{Sen}\,6\omega$$

$$F(\omega) = \frac{-4}{\omega^2}(1-\cos 6\omega) + \frac{24}{\omega}\text{Sen}\,6\omega$$

$$F(\omega) = 144\,\text{Sen}\,6\omega - 72\,Sa^2\,3\omega$$