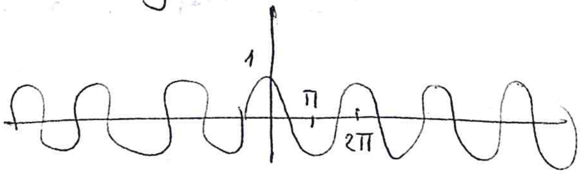
1er. Examen Departamental — TIPO "A"

PROBLEMA 1. Diga cuál es la serie trigonométrica de Fourier de la siguiente función f(t)

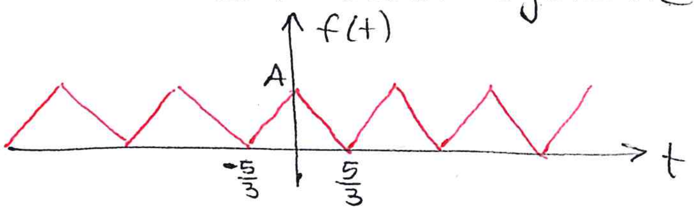


SOLUCION:

$f(t) = \cos t$, Así su S.T.F de la función cos t, es ella misma.

$$f(t) = \cos t$$

PROBLEMA 2. Encuentre la S.T.F de la función siguiente



SOLUCION:

$$f(t) = \begin{cases} \frac{3A}{5}\left(t+\frac{5}{3}\right) & -\frac{5}{3} < t \le 0 \\ -\frac{3A}{5}\left(t-\frac{5}{3}\right) & 0 < t \le \frac{5}{3} \\ f\left(t+\frac{10}{3}\right) & \text{otro caso} \end{cases} \qquad T = \frac{10}{3}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\omega_0 = 2\pi \cdot \frac{3}{10} = \frac{6\pi}{10}$$

$$\omega_0 = \frac{3\pi}{5}$$

f(t) es par

$\therefore \; b_n = 0$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos n\omega_0 t\, dt$$

$$a_n = \frac{4\cdot 3}{10}\int_0^{\frac{5}{3}} -\frac{3A}{5}\left(t-\frac{5}{3}\right)\cos \frac{3\pi}{5}nt\, dt$$

$$a_n = -\frac{18}{25}A\int_0^{\frac{5}{3}}\left(t-\frac{5}{3}\right)\cos\frac{3\pi}{5}nt\, dt$$

$u = t - \frac{5}{3}$ $\qquad dv = \cos\frac{3\pi}{5}nt\, dt$

$du = dt$ $\qquad v = \frac{5}{3\pi n}\operatorname{Sen}\frac{3\pi}{5}nt$

$$a_n = -\frac{18}{25}A\left\{\frac{5}{3\pi n}\left(t-\frac{5}{3}\right)\operatorname{Sen}\frac{3\pi}{5}nt\Big|_0^{\frac{5}{3}} - \frac{5}{3\pi n}\int_0^{\frac{5}{3}}\operatorname{sen}\frac{3\pi}{5}nt\, dt\right\}$$

$$a_n = -\frac{18}{25}A\left\{\frac{25}{9n\pi}\operatorname{Sen}\emptyset + \frac{25}{9\pi^2n^2}\cos\frac{3\pi}{5}n\right.$$

$$a_n = -\frac{18}{25}A\left\{\frac{25}{9\pi^2 n^2}\left[\underbrace{\cos n\pi}_{(-1)^n} - \underbrace{\cos\emptyset}_{1}\right]\right\}$$

$$a_n = \frac{-2A}{n^2\pi^2}\left[(-1)^n - 1\right]$$

Calculando a_0

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\, dt$$

$$a_0 = 2\cdot\frac{3}{10}\int_0^{\frac{10}{6}} f(t)\, dt$$

$$a_0 = \frac{3}{5}\int_0^{\frac{5}{3}}\left[-\frac{3A}{5}\left(t-\frac{5}{3}\right)\right]dt$$

$$a_0 = \frac{3}{5}\left\{-\frac{3A}{5}\right\}\left\{\frac{t^2}{2} - \frac{5}{3}t\right\}\Bigg|_0^{\frac{5}{3}}$$

$$a_0 = -\frac{9A}{25}\left\{\frac{1}{2}\left(\frac{25}{9}\right) - \frac{5}{3}\cdot\frac{5}{3} - \emptyset\right\}$$

$$a_0 = -\frac{9A}{25}\left\{\frac{25}{18} - \frac{25}{9}\right\} = -\frac{9A}{25}\left\{\frac{-25}{18}\right\}$$

$$a_0 = \frac{A}{2}$$

Finalmente:

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty}\frac{2A}{n^2\pi^2}\left(1-(-1)^n\right)\cos\frac{3\pi}{5}nt$$

TIPO "A"

PROBLEMA 3 A partir de la S.T.F obtenida en el problema 2, deduzca la Serie Exponencial de fourier

SOL:

Sabiendo que

$$C_n = \frac{1}{2}(a_n - j b_n)$$

& si

$$a_n = \frac{2A}{n^2\pi^2}\left[1-(-1)^n\right] \quad \forall n \neq 0$$

$$\& \quad a_0 = \frac{A}{2}$$

$\therefore$

$$C_n = \frac{1}{2}\left\{\frac{2A}{n^2\pi^2}\left(1-(-1)^n\right)\right\} \quad \forall n \neq 0$$

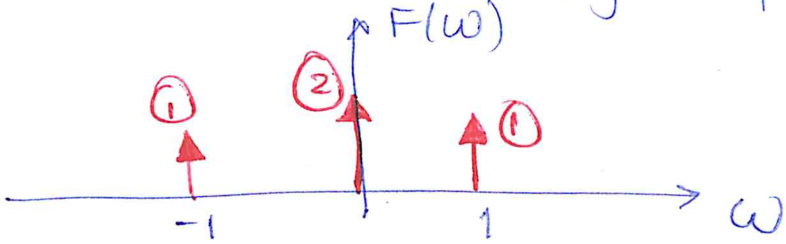
$$C_n = \frac{A}{n^2\pi^2}\left(1-(-1)^n\right) \quad \& \quad C_0 = \frac{A}{2}$$

Finalmente:

$$f(t) = \frac{A}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{A}{n^2\pi^2}\left[1-(-1)^n\right] e^{j\frac{3\pi}{5}nt}$$

PROBLEMA 4

Encuentre la transformada inversa de Fourier de la siguente función y grafiquela



SOLUCION:

ALTERNATIVA (A)

Por propiedades de la Transform.

$$F(\omega) = \delta(\omega+1) + 2\delta(\omega) + \delta(\omega-1)$$

Si $\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(-\omega)$

$\frac{1}{2\pi} \longleftrightarrow \delta(\omega)$

$\frac{1}{\pi} \longleftrightarrow 2\delta(\omega)$

$\frac{1}{2\pi} e^{jt} \longleftrightarrow \delta(\omega-1)$

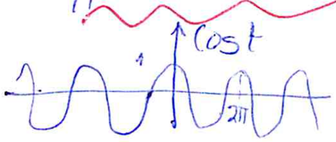
$\frac{1}{2\pi} e^{-jt} \longleftrightarrow \delta(\omega+1)$

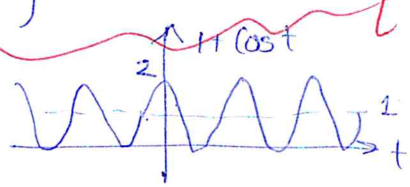
Finalmente, sumando las parejas:

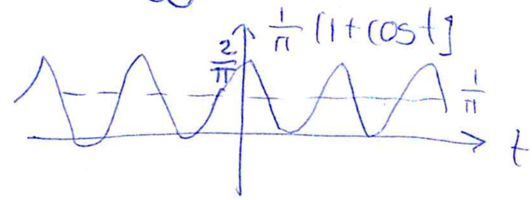
$$\frac{1}{\pi} + \frac{1}{2\pi} e^{jt} + \frac{1}{2\pi} e^{-jt} \longleftrightarrow F(\omega)$$

$$\frac{1}{\pi} + \frac{1}{\pi}\cos t \longleftrightarrow F(\omega)$$

$$\frac{1}{\pi}(1 + \cos t) \longleftrightarrow F(\omega)$$







ALTERNATIVA (B)

Por definición, usando la Integral

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int^{(-1)^+}_{(-1)^-} \delta(\omega+1) e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{0^-}^{0^+} 2\delta(\omega) \underbrace{d\omega}_{e^{j\omega t}} + \frac{1}{2\pi}\int_{1^-}^{1^+} \delta(\omega-1) e^{j\omega t} d$$

$$= \frac{1}{2\pi} e^{-jt} + \frac{2}{2\pi} e^{0} + \frac{1}{2\pi} e^{jt}$$

$$= \frac{2}{2\pi} + \frac{1}{2\pi}(e^{jt} + e^{-jt})$$

$$= \frac{2}{2\pi} + \frac{1}{\pi}\cos t$$

$$= \frac{1}{\pi} + \frac{1}{\pi}\cos t$$

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{\pi}(1 + \cos t)$$

PROBLEMA 5

Usando las Propiedades de la transformada de Fourier, complete la pareja de transformadas siguientes:

$$-6\delta[5t-10]\cdot \cos 15t + \frac{1}{3-jt}\cdot t^2 + e^{j4t}(t-1) \longleftrightarrow ?$$

SOLUCION

① $-6\delta(5t-10)\cdot \cos 15t \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$\delta(t-10) \longleftrightarrow e^{-j10\omega}$

$\delta(5t-10) \longleftrightarrow \frac{1}{|5|} e^{-j10\frac{\omega}{5}}$

$\delta(5t-10) \longleftrightarrow \frac{1}{10}\left[e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$

$-6\delta(5t-10) \longleftrightarrow \frac{-3}{5}\left[e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$

② $\frac{1}{3-jt}\cdot t^2 \longleftrightarrow ?$

Si $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

$\frac{1}{a+jt} \longleftrightarrow 2\pi e^{-a(-\omega)} u(-\omega)$

$\frac{1}{3+jt} \longleftrightarrow 2\pi e^{3\omega} u(-\omega)$

$\frac{1}{3-jt} \longleftrightarrow 2\pi e^{-3\omega} u(\omega)$

$\frac{(-jt)^2}{3-jt} \longleftrightarrow 2\pi \frac{d^2}{d^2\omega}\left[e^{-3\omega} u(\omega)\right]$

$\frac{t^2}{3-jt} \longleftrightarrow -2\pi \frac{d^2}{d^2\omega}\left[e^{-3\omega} u(\omega)\right]$

③ $e^{j4t}(t-1) \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(-\omega)$

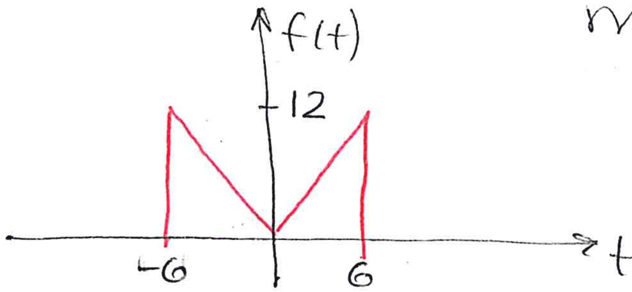
$-jt \longleftrightarrow 2\pi \frac{d}{d\omega}\delta(\omega)$

$t \longleftrightarrow 2\pi j \frac{d}{d\omega}\delta(\omega)$

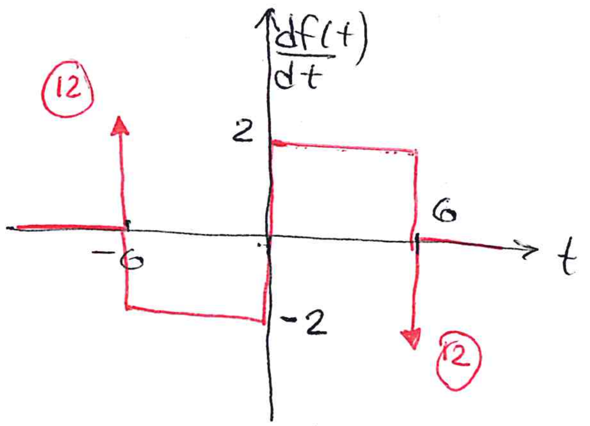
$t-1 \longleftrightarrow 2\pi j\cdot e^{-j\omega}\cdot \frac{d}{d\omega}\delta(\omega)$

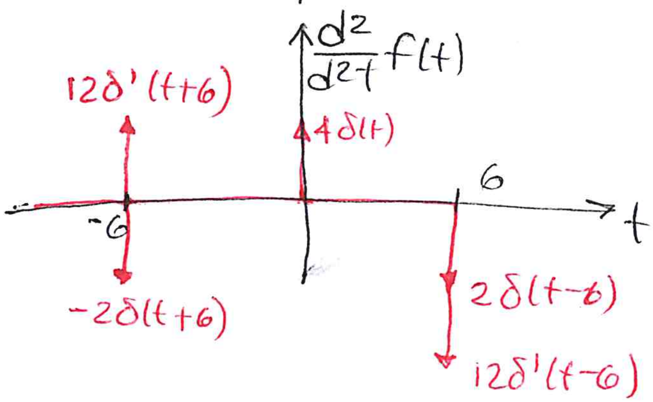
$e^{j4t}(t-1) \longleftrightarrow 2\pi j\, e^{-j(\omega-4)} \frac{d}{d\omega}\delta(\omega-4)$

PROBLEMA 6. Usando la Propiedad de Diferen. en tiempo, encuentre la transformada de f(t)



SOLUCION:





$$\frac{d^2}{d^2t} f(t) = 12\delta'(t+6) - 2\delta(t+6) + 4\delta(t) - 2\delta(t-6) - 12\delta'(t-6)$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t} f(t)\right\} = 12\,\mathcal{F}\{\delta'(t+6)\} - 2\mathcal{F}\{\delta(t+6)\} + 4\mathcal{F}\{\delta(t)\} - 2\mathcal{F}\{\delta(t-6)\} - 12\mathcal{F}\{\delta'(t-6)\}$$

Si $\delta(t) \longleftrightarrow 1$

$\delta(t-6) \longleftrightarrow e^{-i6\omega}$

$\delta'(t-6) \longleftrightarrow j\omega e^{-i6\omega}$

$$\mathcal{F}\left\{\frac{d^2}{d^2t} f(t)\right\} = 12 \cdot j\omega e^{i6\omega} - 2e^{i6\omega} + 4 - 2e^{-i6\omega} - 12 \cdot j\omega e^{-i6\omega}$$

simbólicamente:

$$\frac{d^2}{d^2t} f(t) \longleftrightarrow 12j\omega e^{j6\omega} - 2e^{i6\omega} + 4 - 2e^{-i6\omega} - 12j\omega e^{-i6\omega}$$

$$\frac{d^2}{d^2t} f(t) \longleftrightarrow 12i\omega(e^{i6\omega} - e^{-i6\omega}) - 2(e^{i6\omega} + e^{-i6\omega}) + 4$$

$$\frac{d^2}{d^2t} f(t) \longleftrightarrow -24\omega \operatorname{Sen} 6\omega - 4\cos 6\omega + 4$$

$$\frac{d^2}{d^2t} f(t) \longleftrightarrow 4(1-\cos 6\omega) - 24\omega \operatorname{Sen} 6\omega$$

De la Prop. de diferenc. en t.

Si $f(t) \longleftrightarrow F(\omega)$

$$\frac{d^2}{d^2t} f(t) \longleftrightarrow (j\omega)^2 F(\omega)$$

$$\therefore (j\omega)^2 F(\omega) = 4(1-\cos 6\omega) - 24\omega \operatorname{Sen} 6\omega$$

$$F(\omega) = \frac{-4}{\omega^2}(1-\cos(6\omega)) + \frac{24}{\omega}\operatorname{Sen} 6\omega$$

$$F(\omega) = 144 \operatorname{Sen} 6\omega - 72 Sa^2 3\omega$$