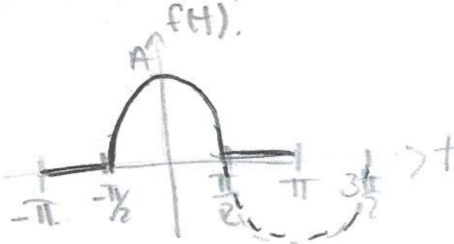


$$h(t) = \begin{cases} A\cos w_0 t & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & \text{en otro caso} \end{cases}$$

Ahora sabemos que el periodo $T_0 = 2\pi$ por lo que $W_0 = 1$ y $T_1 = \pi$ & $W_1 = 2$

Además la función $h(t)$ es par, por ello:

$b_n = 0$; entonces:

$$a_n = \frac{4}{T_0}\int_0^{\pi/2} f(t)\cos n w_0 t\, dt = \frac{4 \cdot A}{2\pi}\int_0^{\pi/2} \cos t \cdot \cos nt\, dt = \frac{2A}{\pi}\int_0^{\pi/2} \cos t \cdot \cos nt\, dt$$

$$\cos x * \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]; \ldots (1)$$

usando (1), tenemos

$$= \frac{2A}{\pi}\int_0^{\pi/2} \frac{1}{2}\left[\cos(t+nt) + \cos(t-nt)\right]dt = \frac{A}{\pi}\left[\int_0^{\pi/2} \cos(t(1+n))\,dt + \int_0^{\pi/2}\cos(t(1-n))\,dt\right]$$

$u = t(1+n)$

$du = (1+n)dt \Rightarrow dt = \frac{du}{(1+n)}$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \cos u \frac{du}{(1+n)} + \int_0^{\pi/2}\cos u \frac{du}{(1-n)}\right] = \frac{A}{\pi(1+n)}\left[\operatorname{sen}(u)\right]_0^{\pi/2(1+n)}$$

$$+ \frac{A}{\pi(1-n)}\left[\operatorname{sen}(u)\right]_0^{\pi/2(1-n)}$$

$$= \frac{A}{\pi(1+n)}\left[\operatorname{Sen}(t(1+n))\right]_0^{\pi/2} + \frac{A}{\pi(1-n)}\left[\operatorname{sen}(t(1-n))\right]_0^{\pi/2}$$

$$= \frac{A}{\pi(1+n)}\left[\operatorname{sen}\left(\frac{\pi}{2} + \frac{\pi n}{2}\right)\right] + \frac{A}{\pi(1-n)}\left[\operatorname{sen}\left(\frac{\pi}{2} - \frac{\pi n}{2}\right)\right]$$

Ahora: usando (2):

$$\operatorname{Sen}(x \pm y) = \operatorname{Sen} x * \cos y \pm \cos x * \operatorname{Sen} y \ldots (2)$$

$$= \frac{A}{\pi(1+n)}\left[\underbrace{\operatorname{sen}\frac{\pi}{2}}_{1}\cos\frac{\pi n}{2} + \cancel{\cos\frac{\pi}{2}}^{0} * \operatorname{sen}\frac{\pi n}{2}\right] + \frac{A}{\pi(1-n)}\left[\underbrace{\operatorname{sen}\frac{\pi}{2}}_{1}\cos\frac{\pi n}{2} - \cancel{\cos\frac{\pi}{2}}^{0} * \operatorname{sen}\frac{\pi n}{2}\right]$$

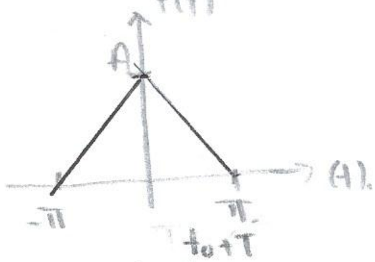
$$= \frac{A}{\pi(1+n)}\left[\cos\frac{\pi n}{2}\right] + \frac{A}{\pi(1-n)}\left[\cos\frac{\pi n}{2}\right] = \frac{A}{\pi}\cos\frac{\pi n}{2}\left(\frac{1}{1+n} + \frac{1}{1-n}\right)$$

$$= \frac{A}{\pi}\cos\frac{\pi n}{2}\left(\frac{2}{1-n^2}\right) = \frac{2A}{\pi(1-n^2)}\cos\frac{\pi n}{2} \quad \forall n \neq \pm 1.$$

$$a_1 = \lim_{n\to 1} \frac{\frac{d}{dn} 2A\cos\frac{\pi n}{2}}{\frac{d}{dn}(\pi - \pi n^2)} = \lim_{n\to 1} \frac{+\frac{2A\pi}{2}\operatorname{sen}\frac{n\pi}{2}}{-2\pi n} = \lim_{n\to 1}\frac{A\pi \operatorname{sen}\frac{\pi}{2}}{2\pi} = \frac{A}{2}$$

$$a_0 = \frac{2}{T_0}\int_0^{\pi/2} f(t)\,dt = \frac{2A}{2\pi}\int_0^{\pi/2}\cos t\,dt = \frac{A}{\pi}(\operatorname{Sen} t)\Big|_0^{\pi/2} = \frac{A}{\pi}\left[\operatorname{sen}\frac{\pi}{2} + 0\right] = \frac{A}{\pi}$$

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\cos t + \sum_{n=2}^{\infty}\left[\frac{A}{\pi(1-n^2)}\cos\frac{\pi n}{2} * \cos nt\right].$$



Se sabe que f(t) es Par, por ello $b_n = 0$

$$f(t) = \begin{cases} \frac{A}{\pi}(t+\pi) & -\pi < t \le 0. \quad T_1 = \pi \quad W_1 = \frac{2\pi}{T} = 2 \\ -\frac{A}{\pi}(t-\pi) & 0 < t \le \pi \quad T_2 = \pi \quad W_2 = 2 \\ T = 2\pi \quad W_0 = 1 \end{cases}$$

$$a_n = \frac{2}{T}\int f(t)\cos nW_0 t\,dt = \frac{A}{\pi^2}\int_{-\pi}^{0} t\cos nt\,dt + \frac{A}{\pi}\int_{-\pi}^{0}\cos nt\,dt - \frac{A}{\pi^2}\int_{0}^{\pi} t\cos nt\,dt$$

$$-\frac{A}{\pi}\int_{0}^{\pi}\cos nt\,dt.$$

$$= \frac{A}{\pi^2}\left[\frac{1}{n^2}\cos nt + \frac{t}{n}\operatorname{sen} nt\right]_{-\pi}^{0} + \frac{A}{\pi}\left[\cancel{\frac{1}{n}\operatorname{sen} nt}\right]_{-\pi}^{0} - \frac{A}{\pi^2}\left[\frac{1}{n^2}\cos nt + \frac{t}{n}\operatorname{sen} nt\right]_{0}^{\pi}$$

$$-\frac{A}{\pi}\left[\cancel{\frac{1}{n}\operatorname{sen} nt}\right]_{0}^{\pi}$$

$$= \frac{A}{\pi^2}\left[\frac{1}{n^2} - \frac{1}{n^2}\cos n\pi\right] - \frac{A}{\pi^2}\left[\frac{1}{n^2}\cos n\pi - \frac{1}{n^2}\right]$$

$$= \frac{A}{\pi^2 n^2} - \frac{A}{\pi^2 n^2}\cos n\pi - \frac{A}{\pi^2 n^2}\cos n\pi + \frac{A}{n^2\pi^2} = \frac{2A}{\pi^2 n^2}\left[1 - \cos n\pi\right]$$

$$a_1 = \lim_{n\to 0}\frac{\frac{d}{dn}(2A - 2A\cos n\pi)}{\frac{d\,\pi^2 n^2}{dn}} = \frac{2A\pi\operatorname{sen} n\pi}{2\pi^2 n} = \frac{n\operatorname{sen} n\pi}{\pi n} = 0$$

$$b_n = \frac{A}{\pi^2}\int_{-\pi}^{0} t\operatorname{sen} nt\,dt + \frac{A}{\pi}\int_{-\pi}^{0}\operatorname{sen} nt\,dt - \frac{A}{\pi^2}\int_{0}^{\pi} t\operatorname{sen} nt\,dt + \frac{A}{\pi}\int_{0}^{\pi}\operatorname{sen} nt\,dt.$$

$$= \frac{A}{\pi^2}\left[\frac{1}{n^2}\operatorname{sen} nt - \frac{t}{n}\cos nt\right]_{-\pi}^{0} - \frac{A}{\pi n}\left[\cos nt\right]_{-\pi}^{0}$$

$$-\frac{A}{\pi^2}\left[\frac{1}{n^2}\operatorname{sen} nt - \frac{t}{n}\cos nt\right]_{0}^{\pi} - \frac{A}{\pi n}\left[\cos nt\right]\Big|_{0}^{\pi}$$

$$= -\frac{A}{\pi^2}\left[\frac{\pi}{n}\cos(n\pi)\right] - \frac{A}{\pi n} + \frac{A}{\pi n}\cos n\pi + \frac{A}{\pi^2}\left[\frac{\pi}{n}\cos n\pi\right] - \frac{A}{\pi n}\cos n\pi$$

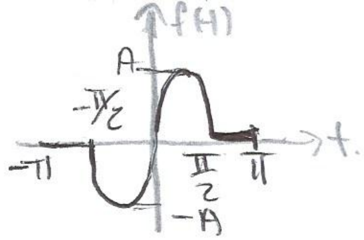
$$+\frac{A}{\pi n} = 0$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}\frac{A}{\pi}t\,dt + A\int_{-\pi}^{0}dt - \int_{0}^{\pi}\frac{A}{\pi}t\,dt + A\int_{0}^{\pi}dt\right] = \frac{A}{2\pi^2}\left[\frac{t^2}{2}\right]_{-\pi}^{0} + \frac{A}{2\pi}\left[t\right]_{-\pi}^{0} - \frac{A}{2\pi^2}\left[\frac{t^2}{2}\right]\Big|_{0}^{\pi}$$

$$+\frac{A}{2\pi}\left[t\right]_{0}^{\pi} = -\frac{A}{4} + \frac{A}{2} - \frac{A}{4} + \frac{A}{2} = \frac{A}{2}$$

Finalmente

$$f(t) = \frac{A}{2} + \sum_{n=0}^{\infty}\frac{2A}{\pi^2 n^2}\left[1 - \cos n\pi\right]\cos nt$$



Al ser impar $f(t)$ tenemos que $a_0 = a_n = 0$.

$$f(t) = \begin{cases} A\,\mathrm{Sen}\,2t & -\pi < t < \pi \quad T_1 = \pi \quad \omega_1 = 2 \\ 0 & \text{en otro caso} \quad T = 2\pi \quad \omega_0 = 1. \end{cases}$$

$$b_n = \frac{4}{T_0}\int_0^{T/2} f(t)\,\mathrm{Sen}\,n\omega_0 t\,dt = \frac{2A}{\pi}\int_0^{\pi/2} \mathrm{Sen}\,2t\,\mathrm{sen}\,nt\,dt.$$

$$= -\frac{A}{\pi}\int_0^{\pi/2} \cos\underbrace{(2t+nt)}_{(t(2+n))} - \cos\underbrace{(2t-nt)}_{(t(2-n))}\,dt.$$

$u = t(2+n)$
$du = (2+n)du \Rightarrow dt = \frac{du}{(2+n)}$

$u_1 = t(2-n)$
$du_1 = (2-n)dt \Rightarrow dt = \frac{du_1}{(2-n)}$

$$= -\frac{A}{\pi}\left[\int_0^{\pi/2} \cos u\,\frac{du}{(2+n)} - \int_0^{\pi/2} \cos u_1\,\frac{du_1}{(2-n)}\right] = -\frac{A}{\pi}\left\{\left[\frac{1}{(2+n)}\mathrm{Sen}\,u\right]_0^{\frac{\pi}{2}(2+n)} - \left[\frac{1}{(2-n)}\mathrm{Sen}\,u_1\right]_0^{\frac{\pi}{2}(2-n)}\right\}$$

$$= -\frac{A}{\pi}\left[\frac{1}{(2+n)}\mathrm{Sen}\underbrace{(t(2+n))}_{2t+tn}\Big|_0^{\pi/2} - \frac{1}{(2-n)}\mathrm{Sen}(t(2-n))\Big|_0^{\pi/2}\right] = -\frac{A}{\pi}\left[\frac{1}{(2+n)}\mathrm{Sen}\left(\pi + \frac{\pi n}{2}\right) - \frac{1}{(2-n)}\mathrm{Sen}\left(\pi - \frac{n\pi}{2}\right)\right]$$

$$= -\frac{A}{\pi}\left[\frac{1}{(2+n)}\left[\cancel{\mathrm{sen}\,\pi}^{\,0} * \cos\frac{\pi n}{2} + \cancel{\cos\pi}^{\,-1} * \mathrm{sen}\frac{\pi n}{2}\right] - \frac{1}{(2-n)}\left[\cancel{\mathrm{sen}\,\pi}^{\,0} * \cos\frac{\pi n}{2} - \cancel{\cos\pi}^{\,-1} * \mathrm{sen}\frac{n\pi}{2}\right]\right].$$

$$= -\frac{A}{\pi}\left[\frac{1}{(2+n)}\left[-\mathrm{sen}\frac{\pi n}{2}\right] - \frac{1}{(2-n)}\left[\mathrm{sen}\frac{n\pi}{2}\right]\right] = -\frac{A}{\pi}\left[-\frac{1}{(2+n)}\mathrm{sen}\frac{\pi n}{2} - \frac{1}{(2-n)}\mathrm{sen}\frac{\pi n}{2}\right]$$

$$= +\frac{A}{\pi}\mathrm{sen}\frac{\pi n}{2}\left[\frac{1}{(2+n)} + \frac{1}{(2-n)}\right] = \frac{A}{\pi}\mathrm{sen}\frac{\pi n}{2}\left[\frac{4}{4-n^2}\right] = \frac{4A}{\pi(4-n^2)}\mathrm{sen}\frac{\pi n}{2} \quad \forall n \neq \pm 2$$

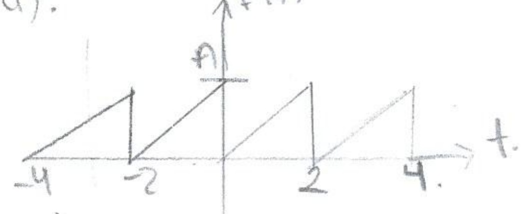
$$b_1 = \lim_{n\to 2}\frac{\frac{d}{dn}\left(4A\,\mathrm{sen}\frac{\pi n}{2}\right)}{\frac{d}{dn}(4\pi - \pi n^2)} = \lim_{n\to 2}\frac{2A\pi\cos\frac{\pi n}{2}}{-2\pi n} = \lim_{n\to 2}\frac{A\cos\pi^{\,-1}}{-2} = \frac{A}{2}.$$

$$f(t) = \sum_{\substack{n=0\\ n\neq 2}}^{\infty}\left[\frac{4A}{\pi(4-n^2)}\mathrm{sen}\frac{\pi n}{2}\right]\mathrm{Sen}\,nt + \frac{A}{2}\mathrm{Sen}\,2t$$

$$= \frac{A}{2}\mathrm{Sen}\,2t + \sum_{\substack{n=0\\ n\neq 2}}^{\infty}\left[\frac{4A}{\pi(4-\pi^2)}\mathrm{sen}\frac{\pi n}{2}\right]\mathrm{Sen}\,nt$$

Problema 2

a).



donde:

$$f(t)=\begin{cases}\frac{A}{2}t \;;\; 0<t\leq 2.\\ 0 \text{ en otro caso.}\end{cases} \qquad T=2 \qquad w_0=\pi$$

Entonces

$$a_n=\frac{2}{2}\int_0^2\frac{A}{2}t\cos n\pi t\,dt=\frac{A}{2}\int_0^2 t\cos n\pi t\,dt=\frac{A}{2}\left[\frac{\cos n\pi t}{n^2\pi^2}+\frac{t\,\text{sen}\,n\pi t}{n\pi}\right]_0^2$$

$$=\frac{A}{2}\left[\frac{\overset{\nearrow 1}{\cancel{\cos 2n\pi}}}{n^2\pi^2}+\frac{2\,\overset{\nearrow 0}{\cancel{\text{sen}\,2n\pi}}}{n\pi}-\left(\frac{1}{n^2\pi^2}\right)\right]=0.$$

$$b_n=\frac{2}{2}\int_0^2\frac{A}{2}t\,\text{sen}\,n\pi t\,dt=\frac{A}{2}\int_0^2 t\,\text{sen}\,n\pi t\,dt=\frac{A}{2}\left[\frac{\text{sen}\,n\pi t}{n^2\pi^2}-\frac{t\cos n\pi t}{n\pi}\right]_0^2$$

$$=\frac{A}{\cancel{2}}\left[\frac{\overset{\nearrow 0}{\cancel{\text{sen}\,2n\pi}}}{n^2\pi^2}-\frac{\cancel{2}\overset{\nearrow 1}{\cos 2n\pi}}{n\pi}\right]=-\frac{A}{n\pi}$$

$$a_0=\frac{1}{2}\int_0^2\frac{A}{2}t\,dt=\frac{A}{4}\int_0^2 t\,dt=\frac{A}{4}\left[\frac{t^2}{2}\right]_0^2=\frac{A}{4}\left[\frac{4}{2}\right]=\frac{A}{2}.$$

Entonces

$$f(t)=\frac{A}{2}-\frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\text{sen}\,n\pi t)$$

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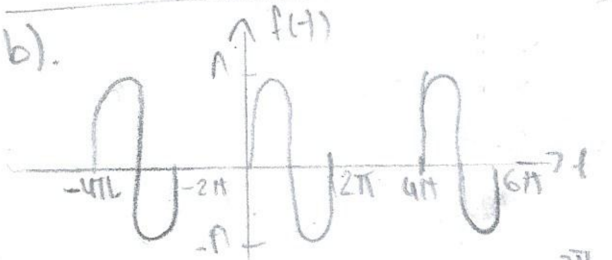
b).



$$f(t)=\begin{cases}A\,\text{sen}\,w_1 t \Rightarrow A\,\text{sen}\,t. \quad 0<t\leq 2\pi.\\ T_1=2\pi \quad w_1=1\\ 0 \text{ en otro caso.}\end{cases}$$

$$T=4\pi. \quad W_0=\frac{1}{2}$$

$$a_n=\frac{1}{2\pi}\int_{t_0}^{t_0+T_1}f(t)\cos\frac{n}{2}t\,dt=\frac{A}{2\pi}\int_0^{2\pi}\text{sen}\,t\cos\frac{n}{2}t\,dt=\frac{A}{4\pi}\left[\int_0^{2\pi}\text{sen}(t(1+\tfrac{n}{2}))+\text{sen}(t(1-\tfrac{n}{2}))\,dt\right]$$

$$u=t(1+\tfrac{n}{2}) \qquad\qquad u_1=t(1-\tfrac{n}{2}).$$

$$du=(1+\tfrac{n}{2})dt\Rightarrow dt=\frac{du}{(1+\frac{n}{2})} \qquad du_1=(1-\tfrac{n}{2})dt\Rightarrow dt=\frac{du_1}{(1-\frac{n}{2})}.$$

$$=\frac{A}{4\pi}\left\{\frac{1}{(1+\frac{n}{2})}(-\cos u)\Big|_0^{2\pi(1+\frac{n}{2})}-\frac{1}{(1-\frac{n}{2})}\cos u_1\Big|_0^{2\pi(1-\frac{n}{2})}\right\}.$$

$$=\frac{A}{4\pi}\left[-\frac{1}{(1+\frac{n}{2})}\cos(t+\tfrac{tn}{2})\Big|_0^{2\pi}-\frac{1}{(1-\frac{n}{2})}\cos(t-\tfrac{tn}{2})\Big|_0^{2\pi}\right].$$

$$=\frac{A}{4\pi}\left\{-\frac{1}{(1+\frac{n}{2})}\left[\cos(2\pi+\pi n)-1\right]-\frac{1}{(1-\frac{n}{2})}\left[\cos(2\pi-\pi n)-1\right]\right\}.$$

$$=\frac{A}{4\pi}\left\{-\frac{1}{(1+\frac{n}{2})}\left[\overset{\nearrow 1}{\cancel{\cos 2\pi}}\ast\cos\pi n-1\right]-\frac{1}{(1-\frac{n}{2})}\left[\cos(\pi n)-1\right]\right.$$

$$=\frac{A}{4\pi}\left[-\frac{\cos n\pi}{(1+\frac{n}{2})}+\frac{1}{(1+\frac{n}{2})}-\frac{\cos\pi n}{(1-\frac{n}{2})}+\frac{1}{(1-\frac{n}{2})}\right]$$

$$= -\frac{A}{4\pi}\left[\cos n\pi\left(\frac{1}{(1+\frac{n}{2})}+\frac{1}{(1-\frac{n}{2})}\right)+\frac{2}{1-\frac{n^2}{4}}\right] = -\frac{A}{4\pi}\left[\cos n\pi\left(\frac{2}{1-\frac{n^2}{4}}\right)+\frac{2}{1-\frac{n^2}{4}}\right]$$

$$= -\frac{A}{4\pi}\cdot\frac{2}{1-\frac{n^2}{4}}\left[\cos n\pi+1\right] = -\frac{A}{2\pi(1-\frac{n^2}{4})}\left[\cos n\pi+1\right] \quad \forall n \neq \pm 2.$$

$$a_1 = \lim_{n\to 2} -\frac{A[\cos n\pi+1]}{\frac{d}{dn}\left(2\pi-\frac{\pi n^2}{2}\right)} = \lim_{n\to 2} -\frac{A[\cos n\pi+1]}{-\pi n} = \frac{A\,[\overset{1}{\cos 2\pi}+1]}{2\pi} = \frac{2A}{2\pi} = \frac{A}{\pi}$$

$$b_n = \frac{A}{2\pi}\int_0^{2\pi} \operatorname{sen} t \operatorname{sen}\frac{n}{2}t\,dt = -\frac{A}{4\pi}\left[\int_0^{2\pi}\left[\cos\left(t+\frac{tn}{2}\right)-\cos\left(t-\frac{tn}{2}\right)\right]dt\right]$$

$u = t(1+\frac{n}{2})$ $\qquad\qquad$ $u_1 = t(1-\frac{n}{2})$

$du = (1+\frac{n}{2})dt \Rightarrow dt = \frac{du}{(1+\frac{n}{2})}$ $\qquad$ $du_1 = (1-\frac{n}{2})dt \Rightarrow dt = \frac{du_1}{(1-\frac{n}{2})}$.

$$= -\frac{A}{4\pi}\left[\frac{1}{(1+\frac{n}{2})}\operatorname{sen} u\Big|_0^{2\pi(1+\frac{n}{2})} - \frac{1}{(1-\frac{n}{2})}\operatorname{sen} u_1\Big|_0^{2\pi(1-\frac{n}{2})}\right].$$

$$= -\frac{A}{4\pi}\left[\frac{1}{(1+\frac{n}{2})}\operatorname{sen}\left(t+\frac{tn}{2}\right)\Big|_0^{2\pi} - \frac{1}{(1-\frac{n}{2})}\operatorname{sen}\left(t-\frac{tn}{2}\right)\Big|_0^{2\pi}\right].$$

$$= -\frac{A}{4\pi}\left[\frac{1}{(1+\frac{n}{2})}\left(\operatorname{sen}(2\pi+\pi n)\right) - \frac{1}{(1-\frac{n}{2})}\left(\operatorname{sen}(2\pi-\pi n)\right)\right].$$

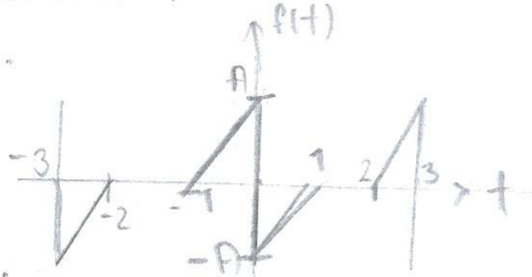
$$= -\frac{A}{4\pi}\left[\frac{1}{(1+\frac{n}{2})}\left[\cancel{\operatorname{sen}2\pi * \cos\pi n + \cos 2\pi * \operatorname{sen}\pi n}^{\,0}\right] - \frac{1}{(1-\frac{n}{2})}\left[0-0\right]\right] = 0$$

$$a_0 = \frac{A}{4\pi}\int_0^{2\pi}\operatorname{sen} t\,dt = \frac{A}{4\pi}\left[-\frac{\cos nt}{n}\right]_0^{2\pi} = -\frac{A}{4\pi n}\left[\overset{1}{\cos 2\pi n}-1\right] = 0.$$

Finalmente.

$$f(t) = \frac{A}{\pi}\cos t - \sum_{\substack{n=1\\ n\neq 2}}^{\infty}\frac{A}{2\pi(1-\frac{n^2}{4})}\left[\cos n\pi+1\right] * \cos\frac{nt}{2}$$

c).



$$f(t)\begin{cases} A(t+1) & -1<t\leq 0. \\ A(t-1), & 0<t\leq 1 \quad T_1=1 \quad w_1=2\pi. \\ 0 \text{ en otro caso.} \end{cases}$$

$$T_0=3 \qquad w_0=\frac{2}{3}\pi.$$

Sabemos que $f(t)$ es impar; por lo que $a_0=a_n=0$.

$$b_n=\frac{4}{T_0}\int_0^{T_1/2} f(t)\,\text{Sen}\,n w_0 t\,dt=\frac{4A}{3}\int_0^{1/2}(t-1)\,\text{Sen}\,\frac{2\pi n}{3}t\,dt=\frac{4A}{3}\int_0^{1/2}\left(t\,\text{sen}\,\frac{2\pi n}{3}t-\text{Sen}\,\frac{2\pi n}{3}t\right)dt.$$

$$=\frac{4A}{3}\left[\frac{\text{sen}\,\frac{2\pi n}{3}t}{\left(\frac{4\pi^2n^2}{9}\right)}-\frac{t\cos\frac{2\pi n}{3}t}{\left(\frac{2\pi n}{3}\right)}+\frac{\cos\frac{2\pi n}{3}t}{\left(\frac{2\pi n}{3}\right)}\right]_0^{1/2}$$

$$=\frac{4A}{3}\left[\frac{\text{sen}\,\frac{\pi n}{3}}{\frac{4\pi^2n^2}{9}}-\frac{\frac{1}{2}\cos\frac{\pi n}{3}}{\left(\frac{2\pi n}{3}\right)}+\frac{\cos\left(\frac{\pi n}{3}\right)}{\left(\frac{2\pi n}{3}\right)}-\frac{1}{\left(\frac{2\pi n}{3}\right)}\right]$$

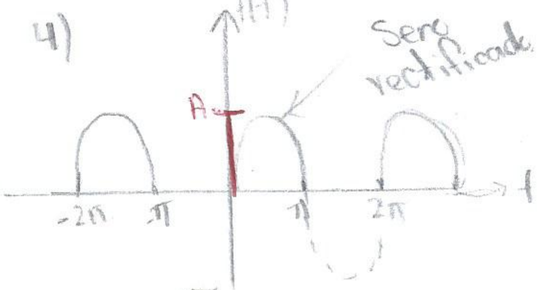
$$=\frac{4A}{3}\left[\frac{9\,\text{Sen}\,\frac{\pi n}{3}}{4\pi^2n^2}-\frac{3\cos\frac{\pi n}{3}}{4\pi n}+\frac{3\cos\frac{\pi n}{3}}{2\pi n}-\frac{3}{2\pi n}.\right]$$

$$=\frac{3A\,\text{Sen}\,\frac{\pi n}{3}}{\pi^2n^2}-\frac{A\cos\frac{\pi n}{3}}{\pi n}+\frac{2A\cos\frac{\pi n}{3}}{\pi n}-\frac{2A}{\pi n}=\frac{3A\,\text{sen}\,\frac{\pi n}{3}}{\pi^2n^2}+\frac{A\cos\frac{\pi n}{3}}{\pi n}-\frac{2A}{\pi n}.$$

Entonces:

$$f(t)=\sum_{n=1}^{\infty}\left[\frac{3A\,\text{sen}\,\frac{\pi n}{3}}{\pi^2n^2}+\frac{A\cos\frac{\pi n}{3}}{\pi n}-\frac{2A}{\pi n}\right]\text{Sen}\,\frac{2\pi}{3}t.$$

4)



$$f(t)\begin{cases} A\,\text{sen}\,w_1 t \Rightarrow A\,\text{sen}\,t & 0\le t\le \pi \\ T_1 = 2\pi \quad w_1 = 1 & \\ 0 \text{ en otro caso.} & \end{cases}$$

$$T = 2\pi \quad W_0 = 1$$

$$a_n = \frac{A}{\pi}\int_0^{\pi} \text{sen}\,t\cos nt\,dt = \frac{A}{2\pi}\left[\int_0^{\pi} \text{sen}(t+nt) + \text{sen}(t-nt)\,dt\right]$$

$$u=(1+n)t \quad du=(1+n)dt \quad dt=\frac{du}{(1+n)} \qquad u_1=(1-n)t \quad du_1=(1-n)dt \quad dt=\frac{du_1}{(1-n)}.$$

$$= \frac{A}{2\pi}\left[-\frac{1}{(1+n)}\cos u\Big|_0^{\pi(1+n)} - \frac{1}{(1-n)}\cos u\Big|_0^{\pi(1-n)}\right] = \frac{A}{2\pi}\left\{-\frac{1}{(1+n)}\left[\cos(\pi+\pi n)-1\right] - \frac{1}{(1-n)}\left[\cos(\pi-\pi n)-1\right]\right\}$$

$$= \frac{A}{2\pi}\left\{-\frac{1}{(1+n)}\left[\overbrace{\cancel{\cos\pi}}^{-1} * \cos\pi n - 1\right] - \frac{1}{(1-n)}\left[\overbrace{\cancel{\cos\pi}}^{-1} * \cos\pi n - 1\right]\right\}$$

$$= \frac{A}{2\pi}\left[\frac{1}{(1+n)}\cos\pi n + \frac{1}{(1+n)} + \frac{1}{(1-n)}\cos\pi n + \frac{1}{(1-n)}\right] = \frac{A}{2\pi}\left[\cos\pi n\left[\frac{1}{(1+n)}+\frac{1}{(1-n)}\right] + \frac{1}{(1+n)} + \frac{1}{(1-n)}\right]$$

$$= \frac{A}{\pi(1-n^2)}\left[\cos\pi n + 1\right]$$

$$a_1 = \lim_{n\to 1}\frac{\frac{d}{dn}A\cos\pi n + A}{\frac{d}{dn}\pi + \pi n^2} = \lim_{n\to 1}\frac{-\pi A\,\text{sen}\,\pi n}{2\pi n} = 0$$

$$b_n = \frac{A}{\pi}\int_0^{\pi}\text{sen}\,t\;\text{sen}\,nt\,dt = -\frac{A}{2\pi}\left[\int_0^{\pi}\cos(t+nt) - \cos(t-nt)\right]$$

$$u=(1+n)t \quad du=(1+n)dt \quad dt=\frac{du}{(1+n)} \qquad u_1=(1-n)t \quad du_1=(1-n)dt \quad dt=\frac{du_1}{(1-n)}$$

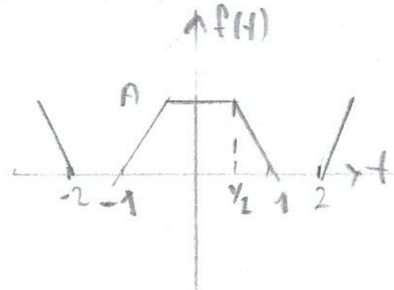
$$= -\frac{A}{2\pi}\left[\frac{1}{(1+n)}\text{sen}\,u\Big|_0^{\pi(1+n)} - \frac{1}{(1-n)}\text{sen}\,u\Big|_0^{\pi(1-n)}\right] = -\frac{A}{2\pi}\left\{\frac{1}{(1+n)}\left[\text{sen}(\pi+\pi n)\right] - \frac{1}{(1-n)}\left[\text{sen}(\pi-\pi n)\right]\right\}$$

$$= -\frac{A}{2\pi}\left\{\frac{1}{(1+n)}\left[\underbrace{\text{sen}\,\pi\cos\pi n + \cos\pi\,\text{sen}\,\pi n}_{\to 0}\right] - \frac{1}{(1-n)}\left[\underbrace{\text{sen}\,\pi\cos\pi n - \cos\pi\,\text{sen}\,\pi n}_{\to 0}\right]\right\} = 0$$

$$a_0 = \frac{A}{2\pi}\int_0^{\pi}\text{sen}\,t\,dt = -\frac{A}{2\pi}\left[\cos t\right]_0^{\pi} = -\frac{A}{2\pi}\left[\overbrace{\cancel{\cos\pi}}^{-1} - 1\right] = \frac{A}{\pi}$$

$$f(t) = \frac{A}{\pi} + \sum_{n=1}^{\infty}\frac{A}{\pi(1-n^2)}\left[\cos\pi n + 1\right] * \cos nt.$$

e)



$$f(t)=\begin{cases} A & -\tfrac{1}{2}<t\le \tfrac{1}{2} \\ 2A(t+1) & -1<t\le -\tfrac{1}{2} \\ -2A(t-1) & \tfrac{1}{2}<t\le 1 \\ 0 & \text{en otro caso} \end{cases}$$

$T=3 \quad W_0=\frac{2}{3}\pi$

Además sabemos que $f(t)$ es par; por ello $b_n=0$

$$a_n=\frac{4}{T}\int_0^{T/2} f(t)\cos n W_0 t\,dt=\frac{4}{3}\int_0^{3/2} f(t)\cos n\frac{2\pi n}{3}t\,dt=\frac{4}{3}\left[A\int_0^{1/2}\cos n\frac{2\pi n}{3}t\,dt-2A\int_{1/2}^{1}(t-1)\cos\frac{2\pi n}{3}t\,dt\right]$$

$$=\frac{4A}{3}\left[\frac{3}{2\pi n}\operatorname{sen}\frac{2\pi n t}{3}\right]_0^{1/2}-\frac{8A}{3}\left[\int_{1/2}^{1}t\cos\frac{2\pi n t}{3}\,dt-\int_{1/2}^{1}\cos\frac{2\pi n t}{3}\,dt\right]$$

$$=\frac{2A}{\pi n}\left[\operatorname{sen}\frac{\pi n}{3}\right]-\frac{8A}{3}\left[\frac{\cos\frac{2\pi n}{3}t}{\frac{4\pi^2 n^2}{9}}+\frac{t\operatorname{sen}\frac{2\pi n}{3}t}{\frac{2\pi n}{3}}-\frac{3}{2\pi n}\operatorname{sen}\frac{2\pi n t}{3}\right]_{1/2}^{1}$$

$$=\frac{2A}{\pi n}\operatorname{sen}\frac{\pi n}{3}-\frac{6A}{\pi^2 n^2}\left[\cos\frac{2\pi n}{3}-\cos\frac{\pi n}{3}\right]-\frac{4A}{\pi n}\left[\operatorname{sen}\frac{2\pi n}{3}-\frac{1}{2}\operatorname{sen}\frac{\pi n}{3}\right]$$

$$+\frac{4A}{\pi n}\left[\operatorname{sen}\frac{2\pi n}{3}-\operatorname{sen}\frac{\pi n}{3}\right]$$

$$=\frac{2A}{\pi n}\operatorname{sen}\frac{\pi n}{3}-\frac{6A}{\pi^2 n^2}\left[\cos\frac{2\pi n}{3}-\cos\frac{\pi n}{3}\right]-\frac{4A}{\pi n}\operatorname{sen}\frac{2\pi n}{3}+\frac{2A}{\pi n}\operatorname{sen}\frac{\pi n}{3}$$

$$+\frac{4A}{\pi n}\operatorname{sen}\frac{2\pi n}{3}-\frac{4A}{\pi n}\operatorname{sen}\frac{\pi n}{3}$$

$$=-\frac{6A}{\pi^2 n^2}\left[\cos\frac{2\pi n}{3}-\cos\frac{\pi n}{3}\right]$$

$$a_0=\frac{2}{T}\int_0^{T/2} f(t)\,dt=\frac{4}{3}\left[\int_0^{1/2}A\,dt-2A\int_{1/2}^{1}(t-1)\,dt\right]=\frac{4A}{3}\left[t\right]_0^{1/2}-\frac{8A}{3}\left[\int_{1/2}^{1}t\,dt-\int_{1/2}^{1}dt\right]$$

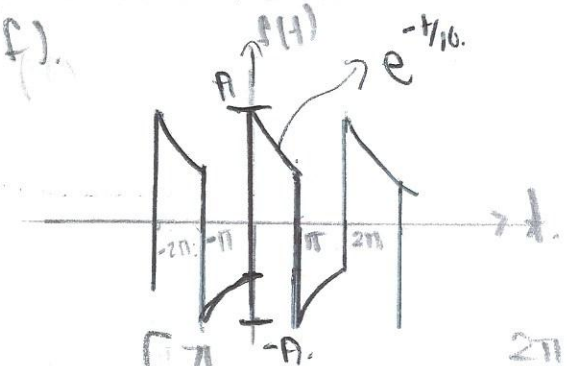
$$=\frac{4A}{3}\left[\frac{1}{2}\right]-\frac{8A}{3}\left[\frac{1}{2}t^2\right]_{1/2}^{1}+\frac{8A}{3}\left[t\right]\Big|_{1/2}^{1}=\frac{2A}{3}-\frac{4A}{3}\left[1-\frac{1}{4}\right]+\frac{8A}{3}\left[1-\frac{1}{2}\right]$$

$$=\frac{2A}{3}-\frac{4A}{3}\left[\frac{3}{4}\right]+\frac{8A}{3}\left[\frac{1}{2}\right]=\frac{2A}{3}-A+\frac{4A}{3}=2A-A=\underline{A}$$

Finalmente

$$f(t)=A-\frac{6A}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\frac{2\pi n}{3}-\cos\frac{\pi n}{3}\right]\cos\frac{2n\pi}{3}t$$

f).



$T = 2\pi \quad w_0 = 1$

$$f(t) = \begin{cases} Ae^{-t/10} & 0 < t \leq \pi \\ -Ae^{-t/10} & \pi < t \leq 2\pi \end{cases}$$

$$a_n = \frac{1}{\pi}\left[A\int_0^{\pi} e^{-t/10}\cos nt\,dt - A\int_{\pi}^{2\pi} e^{-t/10}\cos nt\,dt\right] =$$

$$= \frac{A}{\pi}\left[\frac{e^{-t/10}}{\frac{1}{100}+n^2}\left(-\frac{1}{10}\cos(nt) + n\,\text{sen}(nt)\right)\right]\Bigg|_0^{\pi} - \frac{A}{\pi}\left[\frac{e^{-t/10}}{\frac{1}{100}+n^2}\left(-\frac{1}{10}\cos nt + n\,\text{sen}\,nt\right)\right]_{\pi}^{2\pi}$$

$$= \frac{A}{\pi}\left[\frac{e^{-\frac{\pi}{10}}}{\frac{1}{100}+n^2}\left(-\frac{1}{10}\cos n\pi\right) - \frac{1}{\frac{1}{100}+n^2}\left(-\frac{1}{10}\right)\right] - \frac{A}{\pi}\left[\frac{e^{-\frac{\pi}{5}}}{\frac{1}{100}+n^2}\left(-\frac{1}{10}\right) - \frac{e^{\frac{\pi}{10}}}{\frac{1}{100}+n^2}\left(-\frac{1}{10}\cos n\pi\right)\right]$$

$$= \frac{A}{\pi}\left[\frac{-10e^{-\frac{\pi}{10}}\cos n\pi}{1+100n^2} + \frac{1+100n^2}{1000}\right] - \frac{A}{\pi}\left[\frac{10e^{-\frac{\pi}{5}}}{1+100n^2} + \frac{10e^{\frac{\pi}{10}}\cos n\pi}{1+100n^2}\right]$$

$$= -\frac{10A}{\pi(1+100n^2)}e^{-\frac{\pi}{10}}\cos n\pi + \frac{A}{1000\pi}(1+100n^2) - \frac{10A}{\pi(1+100n^2)}e^{-\frac{\pi}{5}} - \frac{10A}{\pi(1+100n^2)}\left(e^{\frac{\pi}{10}}\cos n\pi\right)$$

Problema 3:

a): $f(t)=\{A\ ;\ -\frac{1}{2}<t\leq\frac{1}{2}$ $T=\frac{5}{2}$ $w_0=\frac{4\pi}{5}$

$$C_n=\frac{2A}{5}\int_{-1/2}^{1/2}e^{-\frac{4\pi nit}{5}}dt=\frac{2A}{5}\left[-\frac{5}{4\pi ni}e^{-\frac{4\pi nit}{5}}\right]_{-1/2}^{1/2}=-\frac{A}{2\pi ni}e^{-\frac{4\pi nit}{5}}\Big|_{-1/2}^{1/2}$$

$$=-\frac{A}{2\pi ni}\left[e^{-\frac{2\pi ni}{5}}-e^{\frac{2\pi ni}{5}}\right]=\frac{A}{\pi n}\left[\frac{e^{\frac{2\pi ni}{5}}-e^{-\frac{2\pi ni}{5}}}{2i}\right]=\frac{A}{\pi n}Sen\left(\frac{2\pi n}{5}\right)$$

$$C_0=\lim_{n\to 0}\frac{\frac{d}{dn}A\,Sen\frac{2\pi n}{5}}{\frac{d}{dn}\pi n}=\lim_{n\to 0}\frac{\frac{2A\pi}{5}Cos\frac{2\pi n}{5}}{\pi}=\frac{\frac{2A\pi}{5}}{\pi}=\frac{2A}{5}$$

$$f(t)=\frac{2A}{5}+\sum_{\substack{-\infty\\ n\neq 0}}^{\infty}\frac{A}{\pi n}Sen\frac{2\pi n}{5}e^{in\frac{4\pi}{5}t}$$

Partiendo del valor de $C_n=\frac{A}{\pi n}Sen\left(\frac{2\pi n}{5}\right)$; multiplicamos tanto en numerador y denominador por $\frac{2}{5}$; entonces:

$$\frac{\frac{2A}{5}Sen\frac{2\pi n}{5}}{\frac{2\pi n}{5}}=\frac{2A}{5}Sa.\frac{2\pi n}{5}\ \ldots (2);$$

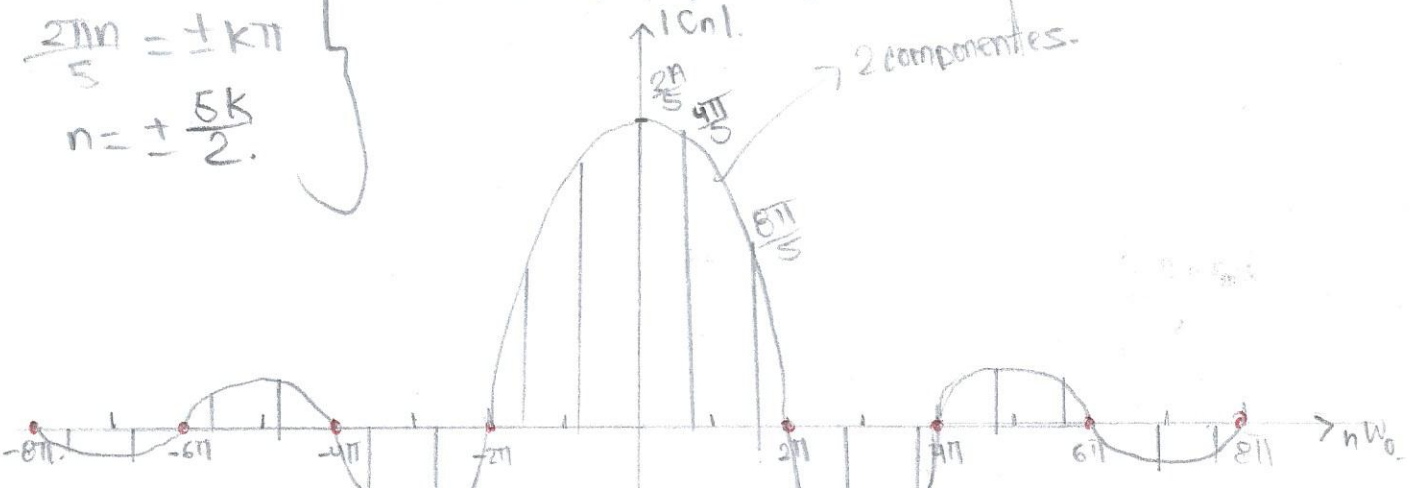
Nota; hay que notar que C_n, es puramente real, por ello tenemos solo espectro de magnitud.

Sacando los pts. de cruce, basandonos en (2).

$$\frac{2A}{5}Sa\frac{2\pi n}{5}=0.$$
$$Sen\frac{2\pi n}{5}=0$$
$$\frac{2\pi n}{5}=\pm k\pi$$
$$n=\pm\frac{5k}{2}.$$

$n=\pm\frac{5}{2},\pm 5;\pm\frac{15}{2};\pm 10;\pm\ldots$

$nW_0=\pm 2\pi;\pm 4\pi;\pm 6\pi;\pm 8\pi$



6.

b). $f(t)\begin{cases} A(t+1) & -1<t\leq 0 \\ -A(t-1) & 0<t\leq 1 \\ 0 \text{ en otro caso.} \end{cases}$ $\quad T=4 \quad \omega_0=\frac{\pi}{2}$

$$C_n=\frac{A}{4}\int_{-1}^{0}(t+1)e^{-in\frac{\pi}{2}t}dt-\frac{A}{4}\int_{0}^{1}(t-1)e^{-in\frac{\pi}{2}t}dt=\frac{A}{4}\left[\int_{-1}^{0}te^{-in\frac{\pi}{2}t}dt+\int_{-1}^{0}e^{-in\frac{\pi}{2}t}dt\right]$$

$$-\frac{A}{4}\left[\int_{0}^{1}te^{-in\frac{\pi}{2}t}dt-\int_{0}^{1}e^{-in\frac{\pi}{2}t}dt\right].$$

$$=\frac{A}{4}\left[\frac{te^{\frac{-in\pi t}{2}}}{\left(\frac{-in\pi}{2}\right)}-\frac{e^{\frac{-in\pi t}{2}}}{\left(-\frac{in\pi}{2}\right)^2}+\frac{2}{in\pi}e^{\frac{-in\pi t}{2}}\right]_{-1}^{0}$$

$$-\frac{A}{4}\left[\frac{te^{\frac{-in\pi t}{2}}}{\left(-\frac{in\pi}{2}\right)}-\frac{e^{\frac{-in\pi t}{2}}}{\left(-\frac{in\pi}{2}\right)^2}+\frac{2}{in\pi}e^{\frac{-in\pi t}{2}}\right]_{0}^{1}$$

$$=-\frac{A}{2in\pi}\left[te^{\frac{in\pi}{2}}\right]+\frac{A}{n^2\pi^2}\left[1-e^{\frac{in\pi}{2}}\right]-\frac{A}{2in\pi}\left[1-e^{\frac{in\pi}{2}}\right]$$

$$+\frac{A}{2in\pi}\left[e^{\frac{-in\pi}{2}}\right]-\frac{A}{n^2\pi^2}\left[e^{\frac{-in\pi}{2}}-1\right]-\frac{A}{2in\pi}\left[e^{\frac{-in\pi}{2}}-1\right].$$

$$=-\frac{A}{2in\pi}e^{\frac{in\pi}{2}}+\frac{A}{n^2\pi^2}-\frac{A}{n^2\pi^2}e^{\frac{in\pi}{2}}-\frac{A}{2in\pi}+\frac{A}{2in\pi}e^{\frac{in\pi}{2}}$$

$$+\frac{A}{2in\pi}e^{\frac{-in\pi}{2}}-\frac{A}{n^2\pi^2}e^{\frac{-in\pi}{2}}+\frac{A}{n^2\pi^2}-\frac{A}{2in\pi}e^{\frac{-in\pi}{2}}+\frac{A}{2in\pi}$$

$$=\frac{2A}{n^2\pi^2}-\frac{A}{n^2\pi^2}\left[e^{\frac{in\pi}{2}}+e^{\frac{-in\pi}{2}}\right]\cdot\frac{2}{2}=\frac{2A}{n^2\pi^2}-\frac{2A}{n^2\pi^2}\left[\cos\frac{n\pi}{2}\right].$$

$$=\frac{2A}{n^2\pi^2}\left[1-\cos\frac{n\pi}{2}\right].$$

$$C_0=\lim_{n\to 0}\frac{\frac{d}{dn}\left[2A-2A\cos\frac{n\pi}{2}\right]}{\frac{d}{dn}n^2\pi^2}=\lim_{n\to 0}\frac{A\pi\operatorname{sen}\frac{n\pi}{2}}{2n\pi^2}=0$$

$$f(t)=\sum_{-\infty}^{\infty}\frac{2A}{n^2\pi^2}\left[1-\cos\frac{n\pi}{2}\right]e^{in\frac{\pi}{2}t}$$

c) $f(t) = \begin{cases} -A(t-1) & 0 \le t \le 1 \\ 0 & \text{en otro caso.} \end{cases}$ $T = 1$ $W_0 = 2\pi.$

$$C_n = -A\int_0^1 (t-1) e^{-in2\pi t} dt = -A\int_0^1 t e^{-in2\pi t} dt + A\int_0^1 e^{-in2\pi n t} dt$$

$$= \frac{A}{in2\pi}\left[t e^{-in2\pi t}\right]_0^1 - \frac{A}{4n^2\pi^2}\left[e^{-in2\pi t}\right]\Big|_0^1 - \frac{A}{in2\pi n}\left[e^{-in2\pi n t}\right]_0^1$$

$$= \frac{A}{in2\pi}\left[e^{-in2\pi}\right] - \frac{A}{4n^2\pi^2}\left[e^{-in2\pi} - 1\right] - \frac{A}{in2\pi}\left[e^{-in2\pi} - 1\right]$$

$$= \cancel{\frac{A}{in2\pi} e^{-in2\pi}} - \frac{A}{4n^2\pi^2} e^{-in2\pi} + \frac{A}{4n^2\pi^2} - \cancel{\frac{A}{in2\pi} e^{-in2\pi}} + \frac{A}{in2\pi}$$

$$= \frac{A}{in2\pi} + \frac{A}{4\pi^2 n^2}\left[1 - e^{-in2\pi}\right]$$

$$C_0 = \lim_{n\to 0} \frac{A}{\frac{d}{dn} in2\pi} + \frac{\frac{d}{dn} A - A\cos 2n\pi}{\frac{d}{dn} 4\pi^2 n^2} = \lim_{n\to 0} \frac{A 2\pi \operatorname{Sen} 2n\pi}{8n\pi^2} = \lim_{n\to 0} \frac{A \operatorname{Sen} 2\pi n}{4n\pi} = 0.$$

$$f(t) = \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \left\{\frac{A}{in2\pi} + \frac{A}{4\pi^2 n^2}\left[1 - e^{-in2\pi}\right]\right\} e^{in2\pi t}.$$

$f(t)$

d). $f(t) = \begin{cases} A\cos w_1 t & 0 < t < \pi/2 \\ T_1 = 2\pi \quad W_1 = 1 \\ A\cos t & 0 < t \le \pi/2 \end{cases}$ $T = \pi$ $W_0 = 2$

$$C_n = \frac{A}{\pi}\int_0^{\pi/2} \cos t\, e^{-in2t} dt = \frac{A}{\pi}\left[-\frac{e^{-in2t}}{4n^2+1}\left(-2in\cos t + \operatorname{sen} t\right)\right]_0^{\pi/2}$$

$$= \frac{A}{\pi}\left[\frac{2in}{4n^2+1} e^{-in2t}\cos t - \frac{1}{4n^2+1}\operatorname{sen} t\, e^{-in2t}\right]_0^{\pi/2}$$

$$= \frac{2Ain}{\pi(4n^2+1)}(-1) - \frac{A}{\pi(4n^2+1)}\left[e^{-in\pi}\right]$$

$$= -\frac{A}{\pi(4n^2+1)}\left[2in + e^{-in\pi}\right] = -\frac{A}{\pi(4n^2+1)}\left[2in + \cos n\pi\right]$$

$$f(t) = \sum_{-\infty}^{\infty} -\frac{A}{\pi(4n^2+1)}\left[2in + \cos n\pi\right] e^{in2t}$$

e).

$$f(t)=\begin{cases} A\cos w_1 t & \Rightarrow A\cos\frac{\pi}{2}t \quad -1\le t\le 1 \\ T_1=4 \quad w_1=\frac{\pi}{2} \end{cases}$$

$T=2 \quad w_0=\pi$.

$$C_n=\frac{A}{2}\int_{-1}^{1}\cos\frac{\pi}{2}t\, e^{-in\frac{\pi}{2}t}\,dt = \frac{A}{2}\left[\frac{e^{-\frac{in\pi}{2}t}}{-\frac{n^2\pi^2}{4}+\frac{\pi^2}{4}}\left(-\frac{in\pi}{2}\cos\frac{\pi}{2}t+\frac{\pi}{2}\operatorname{sen}\frac{\pi}{2}t\right)\right]_{-1}^{1}$$

$$=\frac{A}{2}\left[\frac{4e^{-\frac{in\pi}{2}t}}{\pi^2(1-n^2)}\left(-\frac{in\pi}{2}\cos\frac{\pi}{2}t+\frac{\pi}{2}\operatorname{sen}\frac{\pi t}{2}\right)\right]_{-1}^{1}=\frac{A}{2}\left[-\frac{2in}{\pi(1-n^2)}e^{-\frac{in\pi t}{2}}\cos\frac{\pi t}{2}\right.$$

$$\left.+\frac{2}{\pi(1-n^2)}e^{-\frac{in\pi t}{2}}\operatorname{sen}\frac{\pi t}{2}\right]_{-1}^{1}=\frac{A}{\pi(1-n^2)}\left[e^{-\frac{in\pi}{2}}+e^{\frac{in\pi}{2}}\right]=\frac{2A}{\pi(1-n^2)}\cos\frac{n\pi}{2}.$$

$$C_1=\lim_{n\to 1}\frac{\frac{d}{dn}2A\cos\frac{n\pi}{2}}{\frac{d}{dn}\pi-\pi n^2}=\lim_{n\to 1}\frac{A\pi\operatorname{sen}\frac{n\pi}{2}}{2n\pi}=\frac{A\pi}{2\pi}=\frac{A}{2}$$

$$C_{-1}=\lim_{n\to -1}\frac{\frac{d}{dn}2A\cos\frac{n\pi}{2}}{\frac{d}{dn}\pi-\pi n^2}=\lim_{n\to -1}\frac{A\pi\operatorname{sen}\frac{n\pi}{2}}{2n\pi}=\frac{A}{2}$$

$$f(t)=\frac{A}{2}e^{i\pi t}+\frac{A}{2}e^{-i\pi t}+\sum_{\substack{-\infty\\ n\ne 1\\ n\ne -1}}^{\infty}\frac{2A}{\pi(1-n^2)}\cos\frac{n\pi}{2}.e^{in\pi t}.$$

$$f(t)=2A\cos\pi t+\sum_{\substack{n=-\infty\\ n\ne 1;-1}}^{\infty}\left[\frac{2A}{\pi(1-n^2)}\cos\frac{n\pi}{2}\right]e^{in\pi t}$$

f) $f(t) = \begin{cases} At^2 & 0 < t \leq 1 \\ 0 & \text{en otro caso} \end{cases}$ $T = 2$ $\omega_0 = \pi$

$$C_n = \frac{A}{2}\int_0^1 t^2 e^{-in\pi t}\,dt = \frac{A}{2}\left[\frac{1}{-in\pi}t^2 e^{-in\pi t}\Big|_0^1 + \frac{2}{in\pi}\int_0^1 t e^{-in\pi t}\,dt\right]$$

$$= -\frac{A}{2in\pi}\left[e^{-in\pi}\right] + \frac{A}{in\pi}\left[\frac{te^{-in\pi t}}{-in\pi} + \frac{e^{-in\pi t}}{n^2\pi^2}\right]_0^1 = -\frac{A}{2in\pi}e^{-in\pi} + \frac{A}{in\pi}\left[-\frac{e^{-in\pi}}{in\pi} + \frac{e^{-in\pi}}{n^2\pi^2}\right.$$

$$\left.-\left(\frac{1}{n^2\pi^2}\right)\right] = -\frac{A}{2in\pi}e^{-in\pi} + \frac{A}{n^2\pi^2}e^{-in\pi} + \frac{A}{in^3\pi^3}e^{-in\pi} - \frac{A}{in^3\pi^3} =$$

$$= e^{-in\pi}\left[\frac{A}{n^2\pi^2} - \frac{A}{2in\pi}\right] + \frac{A}{in^3\pi^3}\left[e^{-in\pi} - 1\right] = \frac{A(2in\pi - n^2\pi^2)}{2in^3\pi^3}\cos n\pi + \frac{A}{in^3\pi^3}\left[\cos n\pi - 1\right]$$

$$= \frac{A}{in^3\pi^3}\left[\frac{2in\pi - n^2\pi^2}{2}\cos n\pi + \cos n\pi - 1\right] = \frac{A}{in^3\pi^3}\left[\cos n\pi\left(\frac{2in\pi - n^2\pi^2}{2} + 1\right) - 1\right]$$

Problema 4.-

a). $f(t) = \begin{cases} -(t+2) & -2 < t \le -\frac{3}{2} \\ 3t+4 & -\frac{3}{2} < t \le 0 \\ -3t+4 & 0 < t \le \frac{3}{2} \\ (t-2) & \frac{3}{2} < t \le 2 \end{cases}$

$$F(w) = \int_{-\infty}^{\infty} f(t)e^{-iwt}dt = -\int_{-2}^{-3/2} te^{-iwt}dt - 2\int_{-2}^{-3/2} e^{-iwt}dt + 3\int_{-3/2}^{0} te^{-iwt}dt + 4\int_{-3/2}^{0} e^{-iwt}dt - 3\int_{0}^{3/2} te^{-iwt}dt + 4\int_{0}^{3/2} e^{-iwt}dt$$

$$+ \int_{3/2}^{2} te^{-iwt}dt - 2\int_{3/2}^{2} e^{-iwt}dt.$$

$$= -\left[\frac{te^{-iwt}}{-iw} + \frac{e^{-iwt}}{w^2}\right]_{-2}^{-3/2} - 2\left[-\frac{1}{iw}e^{-iwt}\right]_{-2}^{-\frac{3}{2}} + 3\left[\frac{te^{-iwt}}{-iw} + \frac{e^{-iwt}}{w^2}\right]_{-\frac{3}{2}}^{0} + 4\left[-\frac{1}{iw}e^{-iwt}\right]_{-\frac{3}{2}}^{0}$$

$$-3\left[\frac{te^{-iwt}}{-iw} + \frac{e^{-iwt}}{w^2}\right]_{0}^{\frac{3}{2}} + 4\left[-\frac{1}{iw}e^{-iwt}\right]_{0}^{\frac{3}{2}} + \left[\frac{te^{-iwt}}{-iw} + \frac{e^{-iwt}}{w^2}\right]_{\frac{3}{2}}^{2} - 2\left[-\frac{1}{iw}e^{-iwt}\right]_{\frac{3}{2}}^{2}$$

$$= \frac{1}{iw}\left[-\frac{3}{2}e^{\frac{3iw}{2}} + 2e^{2iw}\right] - \frac{1}{w^2}\left[e^{\frac{3iw}{2}} - e^{2iw}\right] + \frac{2}{iw}\left[e^{\frac{3iw}{2}} - e^{2iw}\right] - \frac{3}{iw}\left[\frac{3}{2}e^{\frac{3iw}{2}}\right]$$

$$+ \frac{3}{w^2}\left[1 - e^{\frac{3iw}{2}}\right] - \frac{4}{iw}\left[1 - e^{\frac{3iw}{2}}\right] + \frac{3}{iw}\left[\frac{3}{2}e^{-\frac{3iw}{2}}\right] - \frac{3}{w^2}\left[e^{-\frac{3iw}{2}} - 1\right] - \frac{4}{iw}\left[e^{-\frac{3iw}{2}} - 1\right]$$

$$- \frac{1}{iw}\left[2e^{-2iw} - \frac{3}{2}e^{-\frac{3iw}{2}}\right] + \frac{1}{w^2}\left[e^{-2iw} - e^{-\frac{3iw}{2}}\right] + \frac{2}{iw}\left[e^{-2iw} - e^{-\frac{3iw}{2}}\right]$$

$$= -\frac{3}{2iw}e^{\frac{3iw}{2}} + \cancel{\frac{2}{iw}e^{2iw}} - \frac{1}{w^2}e^{\frac{3iw}{2}} + \frac{1}{w^2}e^{2iw} + \frac{2}{iw}e^{\frac{3iw}{2}} - \cancel{\frac{2}{iw}e^{2iw}} - \frac{9}{2iw}e^{\frac{3iw}{2}} + \frac{3}{w^2} - \frac{3}{w^2}e^{\frac{3iw}{2}}$$

$$- \cancel{\frac{4}{iw}} + \frac{4}{iw}e^{\frac{3iw}{2}} + \frac{9}{2iw}e^{-\frac{3iw}{2}} - \frac{3}{w^2}e^{-\frac{3iw}{2}} + \frac{3}{w^2} - \cancel{\frac{4}{iw}}e^{\frac{3iw}{2}} + \frac{4}{iw} - \frac{2}{iw}e^{-2iw} + \frac{3}{2iw}e^{-\frac{3iw}{2}}$$

$$+ \frac{1}{w^2}e^{-2iw} - \frac{1}{w^2}e^{-\frac{3iw}{2}} + \frac{2}{iw}e^{-2iw} - \frac{2}{iw}e^{-\frac{3iw}{2}}$$

$$= -\frac{6}{iw}e^{\frac{3iw}{2}} - \frac{4}{w^2}e^{\frac{3iw}{2}} + \frac{1}{w^2}\left[e^{2iw} + e^{-2iw}\right] + \frac{2}{iw}\left[e^{\frac{3iw}{2}} - e^{-\frac{3iw}{2}}\right] + \frac{6}{w^2}$$

$$+ \frac{3}{2iw}\left[3e^{-\frac{3iw}{2}} + e^{-\frac{3iw}{2}}\right] - \frac{4}{w^2}\left[e^{-\frac{3iw}{2}} + e^{-\frac{3iw}{2}}\right]$$

$$= -\frac{6}{iw}e^{\frac{3iw}{2}} - \frac{4}{w^2}e^{\frac{3iw}{2}} + \frac{2}{w^2}\cos 2w + \frac{4}{w}\operatorname{Sen}\frac{3w}{2} + \frac{6}{w^2} + \frac{3}{2iw}\left[3e^{-\frac{3iw}{2}} + e^{-\frac{3iw}{2}}\right] \ldots (A)$$

b). $f(t)\begin{cases} A\cos w_1 t & T_1=2\pi \quad w_0=1 \Rightarrow A\cos t \quad -\frac{\pi}{2}<t\leq\frac{\pi}{2} \\ 0 \text{ en otro caso} \end{cases}$

$$F(w)=A\int_{-\frac{\pi}{2}}^{\pi/2}\cos t\, e^{-iwt}dt=\frac{e^{-iwt}}{\underset{-(w^2-1)}{-w^2+1}}((-iw)\cos t+\operatorname{sen}t)\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\left[\frac{iw}{w^2-1}e^{-iwt}\cos t-\frac{1}{w^2-1}e^{-iwt}\operatorname{sen}t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=-\frac{1}{w^2-1}\left[e^{-\frac{iw\pi}{2}}+e^{\frac{iw\pi}{2}}\right]=-\frac{2}{w^2-1}\cos\frac{w\pi}{2}$$

c). $f(t)=\begin{cases} A & 4<t\leq 6. \end{cases}$

$$F(w)=A\int_4^6 e^{-iwt}dt=-\frac{A}{iw}\left(e^{-iwt}\right)_4^6=-\frac{A}{iw}\left[e^{-6iw}-e^{-4iw}\right].$$

d). $f(t)=\begin{cases} At^2 & 0<t\leq 1 \end{cases}$

$$F(w)=A\int_0^1 t^2e^{-iwt}dt=A\left[\frac{1}{-iw}t^2e^{-iwt}\Big|_0^1+\frac{2}{iw}\int_0^1 te^{-iwt}dt\right]$$

$$=-\frac{A}{iw}e^{-iw}+\frac{2A}{iw}\left[\frac{te^{-iwt}}{-iw}+\frac{e^{-iwt}}{w^2}\right]\Bigg|_0^1$$

$$=-\frac{A}{iw}e^{-iw}+\frac{2A}{w^2}e^{-iw}+\frac{2A}{iw^3}\left[e^{-iw}-1\right]=-\frac{A}{iw}e^{-iw}+\frac{2A}{w^2}e^{-iw}+\frac{2A}{iw^3}e^{-iw}+\frac{2A}{iw^3}$$

$$=e^{-iw}\left[-\frac{A}{iw}+\frac{2A}{w^2}+\frac{2A}{iw^3}\right]+\frac{2A}{iw^3}=e^{-iw}\left[\frac{-A(w^2-2iw-2)}{iw^3}\right]+\frac{2A}{iw^3}$$

$$=\frac{A}{iw^3}\left[2-(w^2-2iw-2)e^{-iw}\right].$$

e) $f(t)=\begin{cases}\frac{A}{2}(t+10) & -10<t\le -8\\ -\frac{A}{2}(t+6) & -8<t\le -6\end{cases}$

$$F(w_1)=\frac{A}{2}\int_{-10}^{-8} te^{-iwt}dt+5A\int_{-10}^{-8}e^{-iwt}dt=\frac{A}{2}\left[\frac{te^{-iwt}}{-iw}+\frac{e^{-iwt}}{w^2}\right]_{-10}^{-8}-\frac{5A}{iw}\left[e^{-iwt}\right]_{-10}^{-8}$$

$$=-\frac{A}{2iw}\left[-8e^{8iw}+10e^{10iw}\right]+\frac{A}{2w^2}\left[e^{8iw}-e^{10iw}\right]-\frac{5A}{iw}\left[e^{8iw}-e^{10iw}\right].$$

$$=\frac{4A}{iw}e^{8iw}+\frac{5A}{iw}e^{10iw}+\frac{A}{2w^2}e^{8iw}-\frac{A}{2w^2}e^{10iw}-\frac{5A}{iw}e^{8iw}+\frac{5A}{iw}e^{10iw}$$

$$=e^{8iw}\left[\frac{4A}{iw}+\frac{A}{2w^2}-\frac{5A}{iw}\right]+e^{10iw}\left[\frac{5A}{iw}-\frac{A}{2w^2}+\frac{5A}{iw}\right]$$

$$=e^{8iw}\left[\frac{A}{2w^2}-\frac{A}{iw}\right]+e^{10iw}\left[\frac{10A}{iw}-\frac{A}{2w^2}\right].$$

$$F(w_2)=-\frac{A}{2}\int_{-8}^{-6} te^{-iwt}dt-3A\int_{-8}^{-6}e^{-iwt}dt=-\frac{A}{2}\left[\frac{te^{-iwt}}{-iw}+\frac{e^{-iwt}}{w^2}\right]_{-8}^{-6}-\frac{3A}{iw}\left[e^{-iwt}\right]_{-8}^{-6}$$

$$=\frac{A}{2iw}\left[-6e^{6iw}+8e^{8iw}\right]-\frac{A}{2w^2}\left[e^{6iw}-e^{8iw}\right]-\frac{3A}{iw}\left(e^{6iw}-e^{8iw}\right)$$

$$=-\frac{3A}{iw}e^{6iw}+\frac{4A}{iw}e^{8iw}-\frac{A}{2w^2}e^{6iw}+\frac{A}{2w^2}e^{8iw}-\frac{3A}{iw}e^{6iw}+\frac{3A}{iw}e^{8iw}$$

$$=e^{6iw}\left[-\frac{3A}{iw}-\frac{A}{2w^2}-\frac{3A}{iw}\right]+e^{8iw}\left[\frac{4A}{iw}+\frac{A}{2w^2}+\frac{3A}{iw}\right]$$

$$=e^{6iw}\left[-\frac{6A}{iw}-\frac{A}{2w^2}\right]+e^{8iw}\left[\frac{7A}{iw}+\frac{A}{2w^2}\right]$$

$$F(w_1)+F(w_2)=e^{8iw}\left[\frac{A}{2w^2}-\frac{A}{iw}+\frac{7A}{iw}+\frac{A}{2w^2}\right]+e^{10iw}\left[\frac{10A}{iw}-\frac{A}{2w^2}\right]+e^{6iw}\left[-\frac{6A}{iw}-\frac{A}{2w^2}\right]$$

$$=e^{8iw}\left[\frac{A}{w^2}+\frac{6A}{iw}\right]+e^{10iw}\left[\frac{10A}{iw}-\frac{A}{2w^2}\right]+e^{6iw}\left[-\frac{6A}{iw}-\frac{A}{2w^2}\right].$$

$$=\frac{A}{w^2}e^{8iw}+\frac{6A}{iw}e^{8iw}+\frac{10A}{iw}e^{10iw}-\frac{A}{2w^2}e^{10iw}-\frac{6A}{iw}e^{6iw}-\frac{A}{2w^2}e^{6iw}.$$

$$=\frac{A}{w^2}\left[e^{8iw}-\frac{1}{2}e^{10iw}-\frac{1}{2}e^{6iw}\right]+\frac{A}{iw}\left[6e^{8iw}+10e^{10iw}-6e^{6iw}\right]$$

$$=\frac{A}{w^2}\left[e^{8iw}-\frac{1}{2}e^{10iw}-\frac{1}{2}e^{6iw}\right]+\frac{A}{iw}\left[6e^{8iw}+10e^{10iw}-6e^{6iw}\right]$$

f) $f(t)=\begin{cases} 1 & 0<t\leq 1 \quad y \quad 4<t\leq 5 \\ 2 & 1<t\leq 2 \quad y \quad 3<t\leq 4 \\ 3 & 2<t\leq 3. \end{cases}$

$$F(\omega)=\int_0^1 e^{-i\omega t}dt+\int_4^5 e^{-i\omega t}dt+2\int_1^2 e^{-i\omega t}dt+2\int_3^4 e^{-i\omega t}dt+3\int_2^3 e^{-i\omega t}dt.$$

$$=-\frac{1}{i\omega}\left[e^{-i\omega}-1\right]-\frac{1}{i\omega}\left[e^{-5i\omega}-e^{-4i\omega}\right]-\frac{2}{i\omega}\left[e^{-2i\omega}-e^{-i\omega}\right]-\frac{2}{i\omega}\left[e^{-4i\omega}-e^{-3i\omega}\right]-\frac{3}{i\omega}\left[e^{-3i\omega}-e^{-2i\omega}\right]$$

$$=-\frac{1}{i\omega}e^{-i\omega}+\frac{1}{i\omega}-\frac{1}{i\omega}e^{-5i\omega}+\frac{1}{i\omega}e^{-4i\omega}-\frac{2}{i\omega}e^{-2i\omega}+\frac{2}{i\omega}e^{-i\omega}-\frac{2}{i\omega}e^{-4i\omega}+\frac{2}{i\omega}e^{-3i\omega}-\frac{3}{i\omega}e^{-3i\omega}+\frac{3}{i\omega}e^{-2i\omega}$$

$$=\frac{1}{i\omega}+\frac{1}{i\omega}e^{-i\omega}+\frac{1}{i\omega}e^{-2i\omega}-\frac{1}{i\omega}e^{-3i\omega}-\frac{1}{i\omega}e^{-4i\omega}-\frac{1}{i\omega}e^{-5i\omega}.$$

$$=\frac{1}{i\omega}\left[1+e^{-i\omega}+e^{-2i\omega}-e^{-3i\omega}-e^{-4i\omega}-e^{-5i\omega}\right].$$

g) $f(t)=\begin{cases} 4 & -8<t\leq -7 \\ 4+2\cos 100t & -7<t\leq -2 \\ 4 & -2<t<-1 \\ 4t & -1<t<1 \\ -4 & 1<t<2 \\ -4-2\cos 100t & 2<t<7 \\ -4 & 7<t<8. \end{cases}$

$$F(\omega_1)=4\int_{-8}^{-7} e^{-i\omega t}dt+4\int_{-2}^{-1} e^{-i\omega t}dt-4\int_1^2 e^{-i\omega t}dt-4\int_7^8 e^{-i\omega t}dt.$$

$$=-\frac{4}{i\omega}\left[e^{7i\omega}-e^{8i\omega}\right]-\frac{4}{i\omega}\left[e^{i\omega}-e^{2i\omega}\right]+\frac{4}{i\omega}\left[e^{-2i\omega}-e^{-i\omega}\right]+\frac{4}{i\omega}\left[e^{-8i\omega}-e^{-7i\omega}\right]$$

$$=-\frac{4}{i\omega}e^{7i\omega}+\frac{4}{i\omega}e^{8i\omega}-\frac{4}{i\omega}e^{i\omega}+\frac{4}{i\omega}e^{2i\omega}+\frac{4}{i\omega}e^{-2i\omega}-\frac{4}{i\omega}e^{-i\omega}+\frac{4}{i\omega}e^{-8i\omega}-\frac{4}{i\omega}e^{-7i\omega}$$

$$=-\frac{4}{i\omega}\left[e^{7i\omega}+e^{-7i\omega}\right]+\frac{4}{i\omega}\left[e^{8i\omega}+e^{-8i\omega}\right]-\frac{4}{i\omega}\left[e^{i\omega}+e^{-i\omega}\right]+\frac{4}{i\omega}\left[e^{2i\omega}+e^{-2i\omega}\right]$$

$$=-\frac{8}{i\omega}\cos 7\omega+\frac{8}{i\omega}\cos 8\omega-\frac{8}{i\omega}\cos\omega+\frac{8}{i\omega}\cos 2\omega$$

$$=-\frac{8}{i\omega}\left[\cos 7\omega-\cos 8\omega+\cos\omega-\cos 2\omega\right].$$

$$F(w_2) = 4\int_{-1}^{1} t e^{-iwt}\,dt = 4\left[\frac{t e^{-iwt}}{-iw} + \frac{e^{-iwt}}{w^2}\right]_{-1}^{1} = \frac{-4}{iw}\left[e^{-iw} + e^{iw}\right] + \frac{4}{w^2}\left[e^{-iw} - e^{iw}\right]$$

$$= \frac{4}{iw}\left[e^{iw} - e^{-iw}\right] - \frac{4}{w^2}\left[e^{iw} - e^{-iw}\right] = \frac{8}{w}\,\text{Sen}\,w - \frac{8i}{w^2}\,\text{Sen}\,w$$

$$= \text{Sen}\,w\left[\frac{8}{w} - \frac{8i}{w^2}\right]$$

$$F(w_3) = 4\int_{-7}^{-2} e^{-iwt}\,dt + 2\int_{-7}^{-2} \cos 100t\, e^{-iwt}\,dt - 4\int_{2}^{7} e^{-iwt}\,dt - 2\int_{2}^{7} \cos 100t\, e^{-iwt}\,dt.$$

$$= -\frac{4}{iw}\left[e^{2iw} - e^{7iw}\right] + 2\left[\frac{e^{-iwt}}{-w^2+100^2}\left(-iw\cos 100t + 100\,\text{sen}\,100t\right)\right]_{-7}^{-2}$$

$$+ \frac{4}{iw}\left[e^{-7iw} - e^{-2iw}\right] - 2\left[\frac{e^{-iwt}}{-w^2+100^2}\left(-iw\cos 100t + 100\,\text{sen}\,100t\right)\right]_{2}^{7}$$

$$= \frac{-4}{iw}e^{2iw} + \frac{4}{iw}e^{7iw} + \frac{2}{100^2-w^2}\Big[e^{2iw}\left(-iw\cos(-200) + 100\,\text{sen}(-200)\right)$$

$$- e^{7iw}\left(-iw\cos(-700) + 100\,\text{sen}(-700)\right)\Big] + \frac{4}{iw}e^{-7iw} - \frac{4}{iw}e^{-2iw} - \frac{2}{100^2-w^2}\Big[e^{-7iw}\left(-iw\cos 700 + 100\,\text{sen}\,700\right)$$

$$- e^{-2iw}\left(-iw\cos 200 + 100\,\text{sen}\,200\right)\Big]$$

$$= -\frac{4}{iw}\left[e^{2iw} + e^{-2iw}\right] + \frac{4}{iw}\left[e^{7iw} + e^{-7iw}\right] + \frac{2}{100^2-w^2}\Big[-iwe^{2iw}\cos 200 - 100e^{2iw}\,\text{sen}\,200 + iwe^{7iw}\cos 700$$

$$+ 100e^{7iw}\,\text{sen}\,700\Big] - \frac{2}{100^2-w^2}\Big[-iwe^{-7iw}\cos 700 + 100e^{-7iw}\,\text{sen}\,700 + iwe^{-7iw}\cos 200 - 100e^{-2iw}\,\text{Sen}\,200\Big]$$

$$= -\frac{8}{iw}\,\text{Sen}\,2w + \frac{8}{iw}\,\text{Sen}\,7w - \frac{2iw}{100^2-w^2}e^{2iw}\cos 200 - \frac{200}{100^2-w^2}e^{2iw}\,\text{sen}\,200 + \frac{2iw}{100^2-w^2}e^{7iw}\cos 700 + \frac{200}{100^2-w^2}e^{7iw}\,\text{sen}\,700.$$

$$+ \frac{2iw}{100^2-w^2}e^{-7wi}\cos 700 - \frac{200}{100^2-w^2}e^{-7iw}\,\text{sen}\,700 - \frac{2iw}{100^2-w^2}e^{-2iw}\cos 200 + \frac{200}{100^2-w^2}e^{-2iw}\,\text{sen}\,200.$$

$$= -\frac{8}{iw}\left[\text{sen}\,2w - \text{Sen}\,7w\right] - \frac{2iw\cos 200}{100^2-w^2}\left[e^{2iw} + e^{-2iw}\right] - \frac{200\,\text{sen}\,200}{100^2-w^2}\left[e^{2iw} - e^{-2iw}\right]$$

$$+ \frac{2iw\cos 700}{100^2-w^2}\left[e^{7iw} + e^{-7iw}\right] + \frac{200\,\text{Sen}\,700}{100^2-w^2}\left[e^{7iw} - e^{-7iw}\right]$$

$$= -\frac{8}{iw}\left[sen 2w - sen 7w\right] - \frac{4iw\cos 200 \cos 2w}{100^2 - w^2} - \frac{400i\, sen 200\, sen 2w}{100^2 - w^2}$$

$$+ \frac{4iw\cos 700 \cos 7w}{100^2 - w^2} + \frac{400i\, sen 700\, sen 7w}{100^2 - w^2}$$

$$= -\frac{8}{iw}\left[sen 2w - sen 7w\right] - \frac{4i}{100^2 - w^2}\left[w\cos 200 \cos 2w + 100\, sen 200\, sen 2w - w\cos 700 \cos 7w\right.$$

$$\left. - 100\, sen 700\, sen 7w\right].$$

$$F(w) = F(w_1) + F(w_2) + F(w_3) =$$

$$= -\frac{8}{iw}\left[\cos 7w - \cos 8w + \cos w - \cos 2w\right] + sen w\left[\frac{8}{w} - \frac{8i}{w^2}\right] - \frac{8}{iw}\left[sen 2w - sen 7w\right]$$

$$- \frac{4i}{100^2 - w^2}\left[w\cos 200 \cos 2w + 100\, sen 200\, sen 2w - w\cos 700 \cos 7w - 100\, sen 700\, sen 7w\right]$$

h) $f(t) \begin{cases} A\cos w_1 t & -\frac{\pi}{2} < t < \frac{\pi}{2} \Rightarrow A\cos t \quad -\frac{\pi}{2} < t \leq \frac{\pi}{2} \\ T_1 = 2\pi \quad w_1 = 1 \end{cases}$

$$F(w) = A\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\, e^{-iwt}\,dt = A\left[\frac{e^{-iwt}}{-w^2+1}\left(-iw\cos t + \operatorname{sen} t\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= -\frac{A}{w^2-1}\left[-iw\,e^{-iwt}\cos t + e^{-iwt}\operatorname{sen} t\right]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\frac{A}{w^2-1}\left\{e^{-\frac{\pi i w}{2}} - \left(-e^{\frac{\pi i w}{2}}\right)\right\}.$$

$$= -\frac{A}{w^2-1}\left[e^{\frac{\pi w}{2}i} + e^{-\frac{\pi w}{2}i}\right] = -\frac{2A}{w^2-1}\cos\frac{\pi w}{2}$$

i) $f(t) = \begin{cases} \frac{3A}{5}\left(t+\frac{5}{3}\right) & -\frac{5}{3} < t \leq 0. \\ -\frac{3A}{5}\left(t-\frac{5}{3}\right) & 0 < t \leq \frac{5}{3}. \end{cases}$

$$F(w) = \frac{3A}{5}\int_{-\frac{5}{3}}^{0}\left(t+\frac{5}{3}\right)e^{-iwt}\,dt - \frac{3A}{5}\int_{0}^{5/3}\left(t-\frac{5}{3}\right)e^{-iwt}\,dt.$$

$$= \frac{3A}{5}\left[\int_{-\frac{5}{3}}^{0}\left(t+\frac{5}{3}\right)e^{-iwt}\,dt - \int_{0}^{5/3}\left(t-\frac{5}{3}\right)e^{-iwt}\,dt.\right.$$

$u = t+\frac{5}{3} \qquad u_1 = t-\frac{5}{3} \qquad dv = e^{-iwt}\,dt$

$du = dt. \qquad du_1 = dt. \qquad v = -\frac{1}{iw}e^{-iwt}.$

$$= \frac{3A}{5}\left\{-\frac{\left(t+\frac{5}{3}\right)}{iw}e^{-iwt}\Big|_{-\frac{5}{3}}^{0} + \frac{1}{iw}\int_{-\frac{5}{3}}^{0} e^{-iwt}\,dt - \left[-\frac{\left(t-\frac{5}{3}\right)}{iw}e^{-iwt}\Big|_{0}^{\frac{5}{3}} + \frac{1}{iw}\int_{0}^{\frac{5}{3}} e^{-iwt}\,dt\right]\right\}$$

$$= \frac{3A}{5}\left\{-\left[\frac{\frac{5}{3}}{iw}\right] + \frac{1}{w^2}e^{-iwt}\Big|_{-\frac{5}{3}}^{0} + \left[\frac{+\frac{5}{3}}{iw}\right] - \frac{1}{w^2}e^{-iwt}\Big|_{0}^{\frac{5}{3}}\right\}$$

$$= \frac{3A}{5}\left\{\cancel{-\frac{5}{3iw}} + \frac{1}{w^2}\left[1 - e^{\frac{5w}{3}i}\right] + \cancel{\frac{5}{3iw}} - \frac{1}{w^2}\left[e^{-\frac{5w}{3}i} - 1\right]\right\}.$$

$$= \frac{3A}{5}\left\{\frac{1}{w^2} - \frac{1}{w^2}e^{\frac{5w}{3}i} - \frac{1}{w^2}e^{-\frac{5wi}{3}} + \frac{1}{w^2}\right\} = \frac{3A}{5}\left[\frac{2}{w^2} - \frac{1}{w^2}\left[e^{\frac{5w}{3}i} + e^{-\frac{5wi}{3}}\right]\right]$$

$$= \frac{3A}{5}\left[\frac{2}{w^2} - \frac{2}{w^2}\cos\frac{5w}{3}\right] = \frac{3A}{5}\left[\frac{2}{w^2}\left(1-\cos\frac{5w}{3}\right)\right] = \frac{6A}{5w^2}\left[2\operatorname{sen}^2\frac{5w}{3}\right] = \frac{12A}{5w^2}\operatorname{sen}^2\frac{5w}{3}$$

$$= \frac{12 \cdot 12A \ \operatorname{Sen}\frac{5w}{3} \ \operatorname{Sen}\frac{5w}{3} \left(\frac{1}{3}\right)\left(\frac{5}{3}\right)}{\left(\frac{5w}{3}\right) \cdot \left(\frac{w5}{3}\right) \cdot} = \frac{20}{3} Sa^2 \frac{5w}{3}$$