НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
УНИВЕРСИТЕТ

Numerical Study of the Current-Voltage Characteristics of Mesoscopic Superconductors

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HSE MIEM

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- Zero electrical resistance
- Magnetic field screening (Meissner effect)

- Qubit
- Single-photon detector
- SQUID
- MRI
- Maglev

Ginzburg-Landau theory

Ψ — *order parameter*, a complex-valued function. Wavefunction of charge carriers (Cooper pairs).

$\Psi = 0$ — normal (non-superconducting) state

$\Psi \neq 0$ — superconducting state

Ginzburg-Landau theory

GL equations. General form

$$-\frac{\mathcal{K}}{D}\left(\partial_t + i\frac{2e}{\hbar}\varphi\right)\Psi = a(1 - T/T_c)\Psi + b|\Psi|^2\Psi - \mathcal{K}\vec{D}^2\Psi$$

$$\frac{\sigma}{c}\left(\vec{\nabla}\varphi + \frac{1}{c}\partial_t\vec{A}\right) = i\mathcal{K}\frac{2e}{\hbar c}\left(\Psi\vec{D}^*\Psi^* - \Psi^*\vec{D}\Psi\right) - \frac{1}{4\pi}\vec{\nabla}\times\vec{\nabla}\times\vec{A}$$

My work
The idea

- GL equations
- Skew coordinates

- Write GL equations in skew coordinates
- Discretize the equations
- Run simulation
- Discuss results

- Gauge invariance
- Thin film limit
- Dimensionless units

$$\partial_t \Psi = \kappa \left(\vec{\nabla} - i\vec{A} \right)^2 \Psi + \left(1 - |\Psi|^2 \right) \Psi$$

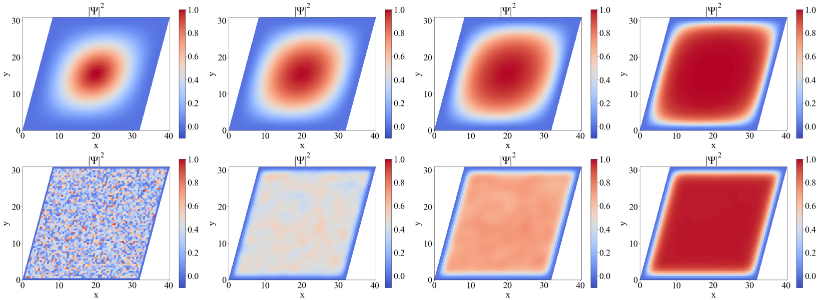

Figure: Plots of $|\Psi|^2$, for $\phi = \pi/12$. The snapshots in each row are taken at times $t = 0$, 5, 10, 25, respectively.

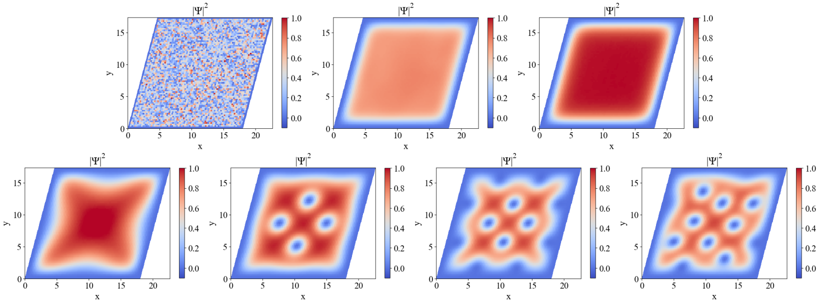

Figure: Plots of $|\Psi|^2$, for $\phi = \pi/12$. The snapshots in each row are taken at times $t = 0$, 10, 20 at the first row and $t = 200$, 300, 400, 500 at the second row.

Results

Non-zero magnetic field, moderate current

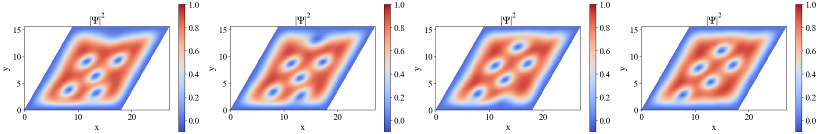

Figure: Plots of $|\Psi|^2$, for $\phi = \pi/6$. The snapshots are taken at times $t = 2300, 2500, 2700, 2900$.

Results
High current

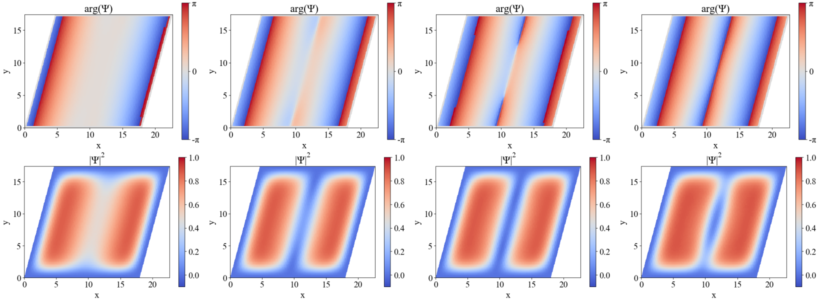

Figure: Plots of phase of Ψ (first row), $|\Psi|^2$ (second row). The snapshots in each row are taken at times $t = 680$, 780, 800, 820, respectively.

In the work *Weyl gauge* is used. Scalar potential of electric field is zero.

$$\varphi = 0$$

Ginzbug-Landau parameter is different compared to bulk superconductor

$$\kappa = \frac{\lambda^2}{d\xi}$$

Technical details
Dimensionless units

Order Parameter	$\Psi_0 = \sqrt{\frac{\lvert a\rvert(1-T/T_c)}{b}}$
Distances	$\xi(T) = \sqrt{\frac{\mathcal{K}}{\lvert a\rvert(1-T/T_c)}}$
Time	$t_0 = \frac{\xi^2}{D}$
Vector Potential	$A_0 = \frac{\Phi_0}{2\pi\xi}$
Scalar potential	A_0
Current	$\frac{A_0}{c\sigma_n t_0}$

To preserve gauge auxiliary variables are used

$$\mathcal{W}^x(x',y') = \exp\left[-i\int_0^{x'} A_{x'}(X,y')dX\right]$$

$$\mathcal{W}^y(x',y') = \exp\left[-i\int_0^{y'} A_{y'}(x',Y)dY\right]$$