$V, \mathbb{R}^n$

старый

$V, (\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n)$

$(\vec{f}_1, \vec{f}_2, \ldots, \vec{f}_n)$ новый

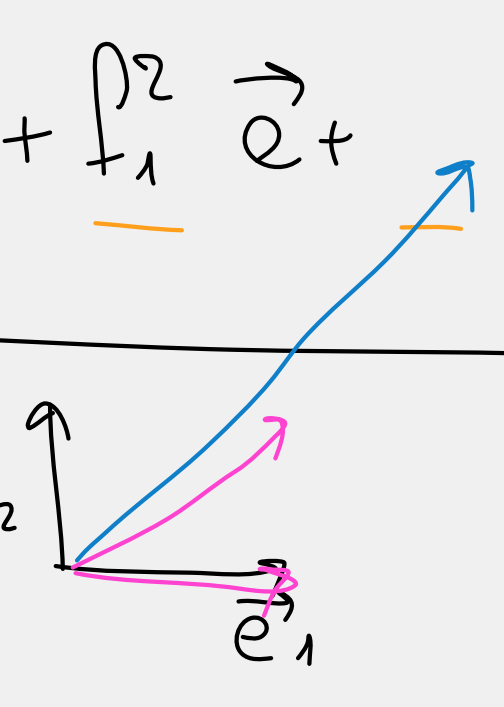
$A = \begin{pmatrix} f_1^1 & \\ f_1^2 & \\ & \\ f_1^n & \end{pmatrix}$

$f = eA$

$\vec{f}_1 = f_1^1 \vec{e}_1 + f_1^2 \vec{e}_2 +$

$$\overset{1\times n}{(\vec{f}_1, \vec{f}_2, \ldots \vec{f}_n)} = \overset{1\times n}{(\vec{e}_1, \vec{e}_2, \ldots \vec{e}_n)}\ \overset{n\times n}{A}$$

$\vec{e}_1, \vec{e}_2$



$\vec{x} \in V,\quad \vec{x} = x_1^e \vec{e}_1 + x_2^e \vec{e}_2 + \ldots + x_n^e \vec{e}_n$

$\vec{x} = x_1^f \vec{f}_1 + \ldots + x_n^f \vec{f}_n$

$\vec{f}_1 = 1\vec{e}_1 + 1\vec{e}_2$

$\vec{f}_2 = 1\vec{e}_1 + 0$

$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$\vec{x}$ коорд $(x_1^e, x_2^e \ldots x_n^e) = x^e$

$(x_1^f \ldots x_n^f) = x^f$

$AB \neq BA$

$x^f = x^e A^{-1}$

$x^e = (2,3)$

$A^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \end{array}\right)$$

$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$x^f = x^e A^{-1} = (2,3)\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} = (3, -1)$

$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

$(2, 3)(3\ \ -1)$

$x^f = (3, -1)$

$\vec{x} = 3\vec{f}_1 - \vec{f}_2 = 3(\vec{e}_1 + \vec{e}_2) - (\vec{e}_1) =$

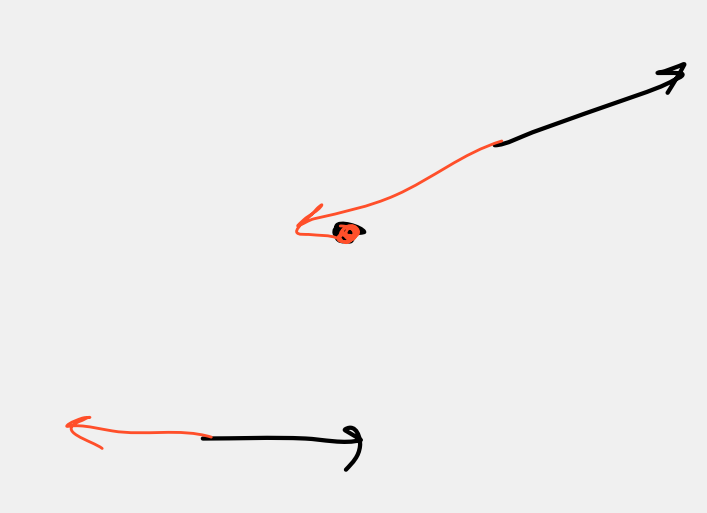
$= 2\vec{e}_1 + 3\vec{e}_2$

$\mathbb{R},\ f(x)\quad \mathbb{R} \to \mathbb{R}$

$V,\ A\quad V \to V$

A – матр $n \times n$ $\qquad Ax$ – столбец

x – Вектор – столбец



$A(x),\quad A(x+y) = A(x) + A(y)$

$f(x_0 + x_1) = f(x_0) + f(x_1)$

$f(x) = x$

$f(x) = x^2 \qquad 2x_0x_1$

$y = Ax,\quad A\ \vec{x} \to \vec{y}$

$\vec{x} = x_1\vec{e}_1 + x_2\vec{e}_2 +$

$(\vec{e}_1, \ldots \vec{e}_n)$

$A\vec{e}_1$

$A^e = \Big| \qquad \Big|$

$y = Ax$

$A^{-1}y = \underset{}{\overset{1}{(A^{-1}A)}}x$

~~$y A^{-1} = A x A^{-1}$~~

$(\vec{f}_1 \ldots \vec{f}_n)\qquad A^f$

$x = A^{-1}y$

C – матр перехода

$A^f = C^{-1} A^e C$

Собств зн и вект

$A\ \vec{x} \to \vec{y}\qquad A = \begin{pmatrix} n \times n \end{pmatrix}$ $\qquad$ фикс базис

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ a_{21} & a_{22} - \lambda & a_{23} \\ a_{31} & a_{32} & a_{33} - \lambda \end{vmatrix} = P(\lambda)$$

$\lambda_1, \lambda_2, \lambda_3,\qquad P(\lambda_1) = 0$

$|A - \lambda I|$

$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix},\quad \begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = (1-\lambda)(-\lambda) - 1 = \lambda^2 - \lambda - 1 = 0$

$\lambda = \frac{1 \pm \sqrt{5}}{2}$

$(\lambda - 1)^2(\lambda + 1) = 0$

$\lambda = 1$ кр 2

$\lambda = -1$ кр 1

$\lambda_1,\quad \tilde{A} = \begin{pmatrix} a_{11} - \lambda_1 \ldots & \\ & \end{pmatrix}$

$\tilde{A}x = 0$

$\lambda_1, v_1 \qquad Av_1 = \lambda_1 v_1$

$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$\overrightarrow{(1,0)}\ \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2$

$\lambda = 1$ кр 2

$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

↑ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ↑

$x_2 = 0 \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$0 = 0$

$A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 3 \end{pmatrix}.$$

$$\begin{vmatrix} 2-\lambda & -1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 1 & 3-\lambda \end{vmatrix} =$$

$= (2-\lambda)\big((1-\lambda)(3-\lambda) + 1\big) =$

$= (2-\lambda)(\lambda^2 - 4\lambda + 4) = 0$

$(2-\lambda)(\lambda - 2)^2 = 0$

$\lambda = 2$ кр 3

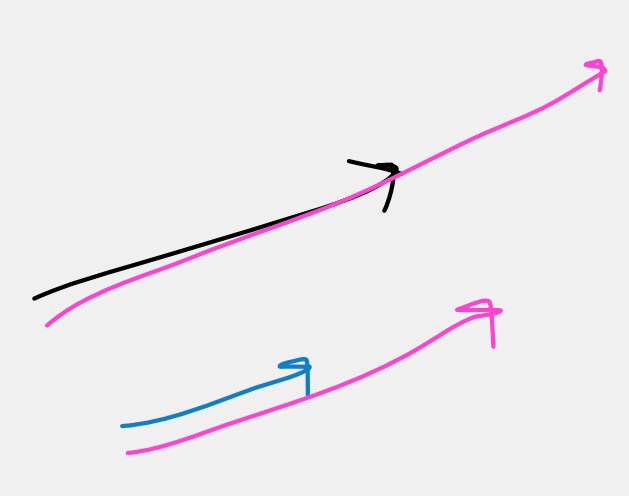
$\begin{pmatrix} 0 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

$-x_2 = 0$

$-x_2 - x_3 = 0$

$x_2 + x_3 = 0$

$-x_2 = 0 \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\ \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$



$A \times B = C$