Lectures 9 - 17

Type	Revision
Materials	https://loop.dcu.ie/course/view.php?id=57769
Reviewed	☐

Reliability of System - Lecture 8

Asystem consist of components which determine whether or not it will work. There are various types of configurations of the compinents in different systems.

- Series System → this is a system where the comonents are in series and they all have to work for the system to work. If one component fails, the entire system fails.
- Parallel System → this is a system that will fail only if they all fail.
- Series-Parallel System → this is a system where some of the comonents in series are replicated in parallel.

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1. Reliability of Series Systems

i.e.

1. A simple computer consists of a processor, a bus and a memory. The computer will work only if all three functioning correctly. The probability that the processor is functioning is 0.99, that the but is functioning is 0.95 and that the memory is functioning is 0.99.

NOTE: in this case the probavility wolod by like the probabilty of a word being right beside each other, something like _ _ _ ← our case.

Thus, the probabilities would end up being something like:

```
prob <- 0.99 * 0.95 * 0.99
```

2. A system consists of 5 components in series, each having a reliability of 0.97. What is the reliability of the system?

NOTE: remember the above example!

```
prob <- (0.97)^5 # this represents _ _ _ _ _ <- 5 comonents!
```

3. An electroning product contains 100 integrated circuits. The probability that any integrated circuit is defective os 0.001 and the integrated circuits are independent. The product operates only if all tha integrated circuits are operational (this means that this is a series system). What is the probability that the product is operational?

```
prob <- (0.99)^100 # we need the systems which succeed
                   # the answer is 0.9047921
```

Note that the answer's reliability is lower than that of an individual component. This is because an individual comonent will never be accurate enough to determine the reliability, regardless of how high it is. The components need to be backed up in parallel.

2. Reliability of a Parallel System

Systems with a parallel structure have build in redundancy. Components are backed up, and the system will work even if some of these cease to function.

Given a system of 5 comonents, it will work if at least one of the components works, thus

```
prob ← P(one component works) # be careful at the workding
```

Taking the complementary approach,

```
# P(one compoment works) = 1 - P(all comonents fail)
# Thus, given a system of 5 components, each having a probability of 0.97
# we have the following pribability that the system will function:

prob <- 1 - (1 - 0.97)^5
```

With parallelm systems, the law of diminishing returns operates with n comonents. The bigger the number of comonents the more accurate the reliability of the system.

We can plot this in R.

```
k <- 1:10 # this represents the individual components
p <- 0.97 # this represents the probability of each individual comonent
plot(k, 1-(1-p)^k, xlab="Number of components", ylab="Reliability") # this plots the reliability of each component

abline(1,0) # creates a linear line -- not important for us...
```

3. Reliability of Series-Parallel Systems

4. Consider a system with 5 kinds of component, with reliabilities:
    - Component 1: 0.95;
    - Component 2: 0.95;
    - Component 3: 0.70 (replicated 3 times) ← PARALLEL SYSTEM;
    - Component 4: 0.75 (replicated 2 times) ← PARALLEL SYSTEM;
    - Component 5: 0.90

NOTE: because of the low reliability of the third and fourth components, they are replicated; the system contains 3 of the third component and 2 of the fourth component.

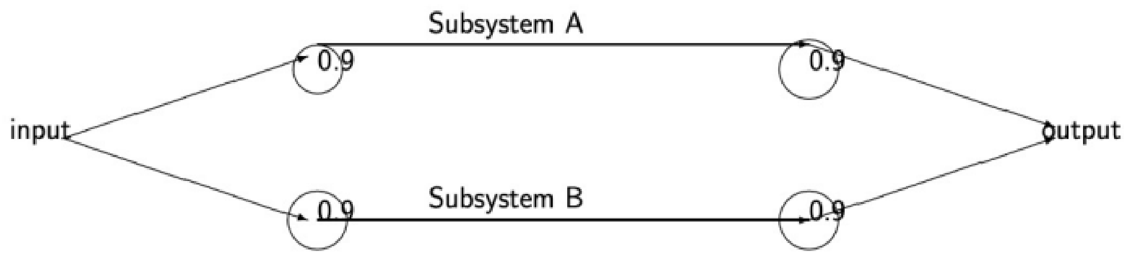
Thus, the following operation will take place.

```
#P(C1) * P(C2) * (1 - (1 - P(C3))^(no.comp3)) * (1 - (1 - P(C4))^(no.comp2)) * P(C5)

prob <- 0.95 * 0.95 * (1 - (1 - 0.70)^3) * (1 - (1 - 0.75)^2) * 0.90
```

Subsystems

The following diagram is a representation of a system that comprises two subsystems operating in parallel



The system operates if at least one of subsystem A operates, thus

```
prob <- P(one subsystem works) # or
prob <- 1 - P(both subsystems fail)
```

Now for a little exapmple:

```
P(subsystem A fails) = 1 - 0.9^2
P(subsystem B fails) = 1 - 0.9^2

# thus

prob <- 1 - (1 - 0.9^2)^2 # == 0.9639
                          # note that the final ^2 is due to the prob of A and B
                          # failing is the same!

# with k subsystems the reliability is:

prob <- 1 - (1-0.9^2)^k

# in R this would look something like:

k <- 1:10
plot(k, 1-(1-0.9)^k, xlab="Number of subsystems", ylab="Reliability")

abline(1, 0)
```

Further on

The reliability of a system, (the probability that it functions prorely) depends on:

- the reliability of each of its components;
- the type of system

Reliability with Series Systems → the problem with series systems is that reliability quickly decreases as the number of components increases.

Reliability woth Parallel Systems → the problem with parallel systems is that the low of diminishing returns operates. The rate of increase in reliability with each additional component decreases as the number of components increases.

Random Variables - Lecture 9

A random variable is a rule which assigns a numerical value to each possible outcome of an experiment; mapping from sample space S to the number line.

Random variables are discrete and continous.

Discrete Random Variables → its values can assume isolated points on the number line.

Continous Random Variables → its values can assume all points in a particular interval.

5. Let X be the number of heads. Then the values of X are 0, 1, or 2. The sample space consists of the following equally likely outcomes: {(T,T), (T, H), (H, H), (H, T)}.

   Thus P(X = 0) = 1/4, P(X = 1) = 2/4 and P(X = 2) = 1/4

NOTE: the p(0) + p(1) + p(2) = 1, where p(0) == p(2);

Probability Density/Mass Functions

$$\sum_{x} P(X = x) = 1$$

The above block states that all probablities add up to 1.

Probability Distributions

6. In order to obtain a model for hardware failures in computer system, the number of crashes occuring each week was observed over a period of one year. It was found that:
   - 0 failures occured in each of 9 weeks;
   - 1 failure occured in each of 14 weeks;
   - 2 failurs occured in each of 13 weeks;

- 3 failurs iccures in each of 9 weels;
- 4 failures occured in each of 4 weeks;
- 5 failures occured in each of 2 weeks;
- 6 failures occured in 1 week;
- TOTAL WEEKS: 52.

NOTE: we need to calculate the propabilities such that the total propabilities are 1.

```
weeks <- c(9,14,13,9,4,2,1);
prob <- weeks/sum(weeks)
round(prob, 2) # roundig the probabilities to 2 decimal places
# 0.17, 0.27, 0.25, 0.17, 0.08, 0.04, 0.02
```

Probability Mass

Given the previous example, this graphs the probabilities in R.

```
fail <- 0:6 # setting up the rows
plot(fail, prob, xlab="Number of Hardware Failures", type="H")
```

Probability Distribution

7. A particular Java assembler interface was used 2000 times, and the stack size was observed
    - 0 size: 100 times;
    - 1 size: 200 times;
    - 2 size: 500 times;
    - 3 size: 500 times;
    - 4 size: 400;
    - 5 size: 200;
    - 6 size: 80;
    - 7 size: 20;

NOTE: please take note of the previous examples (6).

```
size <- c(100, 200, 500, 500, 400, 200, 80, 20);
prob <- size/sum(size)
round(prob, 2)
# 0.05, 0.10, 0.25, 0.25, 0.20, 0.10, 0.04, 0.01
```

Probability Density Function

```
stacksize <- 0:7
prob <- c(0.05, 0.10, 0.25, 0.25, 0.20, 0.10, 0.04, 0.01)
plot(stacksize, probability, xlab="Stack Size", type="h")
```

Cumulative Distribution Function

Up until now er have worked with P(X=x), but now instead, we might be interested in the probability for a group of outcomes of the sample space.

F(x) = P(X ≤ x)

Thus, the new values will be something like (based on example 6):

- We take the first value and divide by the total sum of the probavbilities (in this case 52);
- With the next value, we take the previous and current and add them together and tgen divide it by the total sum of the probabilities, and so on.
- The last value shouold be 1.

These are tho imortant concepts:

- Probability densitiy function (PDF);
- Cumulative Distribution Function (CDF);

The Mean of a Sample

8. Salaries of 6 recent computer graduates (000s euro): 20.3, 14.9,. 18.9, 21.7, 16.3, 17.7.

   The average is:

$$\bar{x} = \frac{20.3 + 14.9 + 18.9 + 21.7 + 16.3 + 17.7}{6} = 18.3$$

Generally, if x1, x2, … xn, in a seample of size n…

Then the average is:

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$$

Or it may be written as:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Summarising Random Variables (mean)

9. One humdred applicants for a certain degree had the following age distribution.

   Age | No. of Applicants

- 18: 9
- 19: 40
- 20: 18
- 21: 18
- 22: 8
- 23: 4
- 24: 3

$$\bar{x} = \frac{18 * 9 + 19 * 40 + 20 * 18 + 21 * 18 + 22 * 8 + 23 * 4 + 24 * 4}{100} = \sum xp(x)$$

The mean of a discrete random variable is defined as the wighted average of all possible values. The weights are the probabilities of respective values of the random variable.

$$E(X) = \sum_x xp(x)$$

Summarising Random Variables (Variances)

The variance (s^2 or V((X)) of a discrete random variable:

> The Variance is defined as the wighted average of the squared differences betweem each possible outcome and its mean; the wights being the probability of the respective outcome.

10. Salariaes of 6 recent computer science graduates (000's euro): 20.3, 14.9, 18.9, 21.7, 16.3, 17.7.

    Recall that the mean of x = 18.3

    Thus , the calculation of V(x) or s^2:

$$s^2 = \frac{[(20.3 - 18.3)^2 + (18.9 - 18.3)^2 + ...]}{5}$$

Generally, of $x_1, x_2, ...x_n$ is a sample size n:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \sqrt{6.368} = 2.523$$

The above formula can be rewriteen as:

$$V(X) = \sum_x (x - \bar{x})^2 p(x)$$

NOTE: that p(x) is $\frac{1}{prob}$, which is then multiplied by $(x - \bar{x})^2$.

Or equivalently:

$$V(X) = E(X - (E(X)))^2$$

The Variance of a descrete random variable

The variance is often deonoted by: $\delta_x^2 = E(X - \bar{x})^2$

The standard deviatiob is denoted by: $\delta_x = \sqrt{E(X - \bar{x})^2}$

```
# This is a worked example in R based in the above formulae's:

x <- 0:5 # range
prob <- c(.237, .396, .264, .088, .014)
expectation <- sum(x * prob)
variance <- sum((x - expectation)^2 * prob)
```

Propreties of Expectations

11. The number of hardware failures X of a computer in a week of operation, which has the following probabilities:

    0 → .17

    1 → .27

    2 → .35

    3 → .17

    4 → .08

    5 → .04

    6 → .02

    Loss = 10X + 200 (this formula is given by the exercies)

    Estimate:

    - the expected weekly loss;
    - the variance of the expected weekly loss

    We know that:

    $E(10X+200)=\sum_x xp(x)$
    $V(10X+200)=\sum_x (x-\bar{x})^2 p(x)$

    Thus, based on the previous computations we get:

    - E(X) = 1.904
    - V(X) = 2.0869

Binomial Distribution — Lecture 10

$$P(X=x)=\binom{n}{x}p^x q^{n-q}$$

n = no. of trials;

p = prob of success

q = 1 - p = prob of failure

X = no of successes in n trials

---

12. The probability that 3 ready temrinals un 5 terminals

$$P(X=3) = \binom{5}{3}(0.95)^3(0.05)^{5-3}$$

```
# The above example can be caluclated as follows in R

x <- 0:5
dbinom(x, size=5, prob=0.95)

# Or tossing a coin 10 times?
x <- 0:10
round(dbinom(x, 10, .5), 4) # the 4 is how many digits after the number
```

Binomial Probability Density Functions

```
# The number of ready terminals (5) where the probavility of
# a terminal being ready is 0.95;

x <- 0:5
plot(x, dbinom(x, size=5, prob=0.95), xlab="Number of Ready Terminals",
ylab= "P(X=x)", type="h", main="Ready Terminals, n=5, p=0.95")
```

Binomial Cumulative Distribution Function

$$P(X \leq x) = P(X=0) + P(X=1) + ... + P(X=x)$$
$$= q^n + \binom{n}{1} p^1 q^{n-1} + ... + \binom{n}{x} p^x q^{n-x}$$

NOTE: LOOK IN LECTURE 10 FOR EXAMPLES ON THIS!!!

```
# In R, with the same example:
# P(X <= 3):
pbinom(3, 5, .95)

# P(X > 3):
1 - pbinom(3, 5, 0.95)

# NOW onto the example,

x <- 0:5
plot(x, pbinom(x, size = 5, prob=0.95), xlab="Number of Ready Terminals",
ylab="P(X<=x)", ylim=c(0, 1), type="s", main="n=5, p=.95")
```

 To calculate the mean of a binomial distribution we need “np”, where n is the nr of successes and p is the probability of success.

Variance of a binomial distribution

$$\delta^2 = V(X) = \sum_x (x - np)^2 \binom{n}{x} p^x q^{n-x} = npq$$

NOTE: rember that np is the mean!!

```
# some R functions for the binomial distribution

# P(X = 4) with n = 20 and p = .2(the pdf)
dbinom(x = 4, size = 20, prob = .2)

# P(x <= 4) with n = 20 and p = .2 (the cdf)
pbinom(x = 4, size = 20, prob = .2)

# Choose k so that P(X <= k) >= 0.95 (inverse if the cdf)
qbinom(.95, size = 20, prob = .2)
```

Geometric Distribution — Lecture 10

PDF of a Geometric Random Variable

$$P(X = x) = q^{x-1}p,\\ where\\ x = 1, 2, 3, ...$$

Probability will happen on the Xth trial.

```
# Calculating PDF of a Geometric Random Variable in R
# Producs produced by a machine has a 3% defective rate

# P(X = 5) = P(1st 4 non-defectove)P(5th defective) = 0.94^4 * 0.03

dgeom(x=4, prob=0.03) # note that x in the number of failures before the first success
```

CDF of a Geometric random variable

$$P(X = x) = 1 - q^x$$

```
# Calculating CDF of a Geometric random variable of a Geometric Variable in R
# What is the probability that the first defective occurs in the first five inspectons?

# P(X <= 5) = 1 - P(first 5 non-defective) = 1 - 0.97^5

pgeom(x=4, prob=0.03)
```

Probability will happen before the Xth trial.

The Quantile Function

A production line whoch has 20% defective ratem what is the minimum numer of inspections, that would be necessary so that the probability of observing a defectove is more than 75%?

```
qgeom(.75, .2) # returns the number of truals before the first defective that
               # has a probabulity of .75
```
