CH5115

① $f(y|x) = cy^2 \ , \ -2 \le y \le 2$

$$\int_{-2}^{2} cy^2 dy = 1 = c\,\frac{y^3}{3}\Big|_{-2}^{2} = 1$$

$$\Rightarrow \frac{c(8+8)}{3} = 1$$

$$\Rightarrow \boxed{c = \frac{3}{16}}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \ , \ -\infty \le x \le \infty .$$

$$E(Y) = \int_{-\infty}^{\infty} E(Y|X=x)\, f_X(x)\, dx$$

$$E(Y|X=x) = \int_{-2}^{2} y\, cy^2 dy = \int_{-2}^{2} cy^3 dy$$

$$= \frac{cy^4}{4}\Big|_{-2}^{2} = \frac{c}{4}(2^4 - (-2)^4)$$

$$= 0$$

$$\Rightarrow \boxed{E(Y) = 0} = \text{mean}.$$

$$\text{var}(Y) = E(Y^2) - \cancel{(E(Y))^2}^{\,0}$$

; $f_{Y|X}(x) = \frac{f_{X,Y}(x,y)}{f_X}$

$\Rightarrow f_{X,Y} = cy^2\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\right)$

$-2 \le y \le 2$

$-\infty \le x \le \infty$

$$= \int_{-\infty}^{\infty} E(Y^2|X=x)\, f_X(x)\, dx$$

$$E(Y^2|X=x) = \int y^2 cy^2 dy$$

$$= \frac{cy^5}{5}\Big|_{-2}^{2} = \frac{3}{\cancel{16}}\left(\frac{1}{5}\right)(\cancel{64}^{\,4}) = \frac{12}{5}.$$

$$\Rightarrow \text{var}(Y) = \int_{-\infty}^{\infty} \frac{12}{5} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{12}{5\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$= \frac{12}{5\sqrt{2\pi}} \quad (\sqrt{2\pi}) \quad = \frac{12}{5}.$$

$$\boxed{\begin{aligned} var(Y) &= \frac{12}{5} \\ E(Y) &= 0 \end{aligned}}$$

② $\Sigma_X = \begin{bmatrix} 2 & 1.2 \\ 1.2 & 3 \end{bmatrix}$

$$\sigma_x^2 = 2 \quad \Rightarrow \quad \sigma_x = \sqrt{2} \quad | \quad E(X^2) - (E(X))^2 = 2$$

$$\sigma_y = \sqrt{3} \quad | \quad E(Y^2) - (E(Y))^2 = 3$$

$$\sigma_{xy} = 1.2 \quad | \quad E(XY) - E(X)E(Y) = 1.2$$

$\cancel{E(var(X))} =$

$$cov(X, Y) = E(cov(X, Y|Z)) + cov(E(X|Z), E(Y|Z))$$

$$= E(cov(X, Y|X)) + cov(\underbrace{E(X|X)}_{= X}, E(Y|X))$$

$$= E(cov(X, Y|X)) + cov(X, E(Y|X))$$

$cov(X, c)$
$= E(Xc) - E$