Q-2

(1) $f(y;\theta) = \begin{cases} \theta y^{\theta-1} & , 0 < y < 1 \\ 0 & \text{o.t.} \end{cases}$

N obs

$$L(\theta; y) = \ln\left(\prod_{i=1}^{N} \theta\, y[i]^{\theta-1}\right)$$

$$= \ln(\theta\, y[1]^{\theta-1}) + \ln(\theta\, y[2]^{\theta-1}) + \dots$$

$$= \sum_{K=1}^{N} \ln(\theta\, y[K]^{\theta-1})$$

$$\cancel{\frac{dL}{d\theta} = \left(\frac{1}{\theta y[1]^{\theta-1}} + \frac{1}{\theta y[2]^{\theta-1}} + \dots + \frac{1}{\theta y[n]^{\theta-1}}\right) \cdot y[K]}$$

$$\cancel{\frac{dL}{d\theta} = \frac{\sum_{K=1}^{N} y[K]^{1-\theta}}{\theta}}$$

$$\cancel{\frac{dL}{d\theta} = \frac{1}{\theta y[1]^{\theta-1}}\, y[1]^{(\theta-1)} + \frac{1}{\theta y[2]^{\theta-1}}\, y[2]^{\theta-1} + \dots + \frac{1}{\theta y[N]^{\theta-1}}\, y[N]^{\theta-1}}$$

$$\cancel{\frac{dL}{d\theta} = \frac{N}{\theta}}$$

$$\frac{dL}{d\theta} = \frac{1}{\theta y[1]^{\theta-1}}\left[y[1]^{\theta-1} + \theta\, y[1]^{\theta-1} \ln y[1]\right]$$

$$+ \frac{1}{\theta y[2]^{\theta-1}}\left[y[2]^{\theta-1} + \theta\, y[2]^{\theta-1} \ln y[2]\right]$$

$$+ \dots$$

$$= \frac{1}{\theta}\left[1 + \theta \ln y[1]\right] + \frac{1}{\theta}\left[1 + \theta \ln y[2]\right] + \dots$$

$$= \frac{1}{\theta}\left[N + \theta \sum_{K=1}^{N} \ln y[K]\right]$$

MLE

$$N = -\theta \sum_{k=1}^{N} \ln y[k]$$

$$N = -\theta \ln \left(\prod_{k=1}^{N} y[k] \right)$$

$$\Rightarrow \boxed{\theta = \frac{-N}{\ln [y[1]\, y[2] \ldots y[N]]}}$$

FI

$$\frac{\partial^2 L}{\partial \theta^2} = \frac{\partial}{\partial \theta} \left[\frac{1}{\theta} \left[N + \theta \ln [y[1]\, y[2] \ldots y[k]] \right] \right]$$

$$= \frac{\partial}{\partial \theta} \left[\frac{N}{\theta} + \ln (\quad) \right]$$

$$= \frac{-N}{\theta^2} + 0$$

$$I(\theta) = -E\left(\frac{\partial^2 L}{\partial \theta^2}\right) = E\left(\frac{N}{\theta^2}\right) = \frac{N}{\theta^2}$$

over all realizations.

(2) ACF ; $\rho[\pm 1] = 0$
$\downarrow$
Correlation $\rho[\pm 2] = 0.6$

$\rho[\pm l] = ?$ $|l| \geq 3$

AR(2)

$$e[k] \to \boxed{H} \to v[k]$$

$$v[k] = H(q^{-1})\, e[k]$$

$$= \frac{1}{1 + d_1 q^{-1} + d_2 q^{-2}}\, e[k]$$

$$\Rightarrow \quad v[k] + d_1 v[k-1] + d_2 v[k-2] = e[k] \quad \to (*)$$

$$\Rightarrow \quad v[k] = -d_1 v[k-1] - d_2 v[k-2] + e[k]$$

$$\sigma_{vv}[l] = E(v[k]\, v[k-l]) \qquad // E(v[k]) = 0$$

$$= E((-d_1 v[k-1] - d_2 v[k-2] + e[k])\, v[k-l])$$

$$= -d_1 \sigma_{vv}[l-1] - d_2 \sigma_{vv}[l-2] + \sigma_{ev}[l]$$

$$\sigma_{ev}[l] = \begin{cases} \sigma_e^2 & l = 0 \\ 0 & l > 0 \end{cases}$$

$$\sigma_{vv}[0] = -d_1 \sigma_{vv}[1] - d_2 \sigma_{vv}[2] + \sigma_e^2$$

$$\sigma_{vv}[1] = -d_1 \sigma_{vv}[0] - d_2 \sigma_{vv}[1] + 0$$

$$\sigma_{vv}[2] = -d_1 \sigma_{vv}[1] - d_2 \sigma_{vv}[0] + 0$$

$$\Rightarrow \quad \sigma_{vv}[0] + d_1 \sigma_{vv}[1] + d_2 \sigma_{vv}[2] = \sigma_e^2 \quad \to (1)$$

$$\Rightarrow \quad \sigma_{vv}[1] + d_1 \sigma_{vv}[0] + d_2 \sigma_{vv}[1] = 0 \quad \to (2)$$

$$\Rightarrow \quad \sigma_{vv}[2] + d_1 \sigma_{vv}[1] + d_2 \sigma_{vv}[0] = 0 \quad \to (3)$$

~~① ⇒ $1 + d_1 s[1] + d_2 s[2] = \frac{\sigma_e^2}{\sigma_{vv}[0]}$~~

① ⇒ $1 + d_1 \cancel{[s(1)]}^{0} + d_2 s[2] = \frac{\sigma_e^2}{\sigma_{vv}[0]}$

② ⇒ $\cancel{s[1]} + d_1 s[0] + d_2 \cancel{s[1]} = 0$

③ ⇒ $s[2] + d_1 \cancel{s[1]} + d_2 s[0] = 0$

$s[0] = 1$

$s[1] = 0$

$s[2] = 0.6$

$1 + 0.6 d_2 = \frac{\sigma_e^2}{\sigma_{vv}[0]}$

$d_1 = 0$

$0.6 + d_2 = 0$

$\boxed{d_1 = 0}$

$\boxed{d_2 = -0.6}$

$0.64 = \frac{\sigma_e^2}{\sigma_{vv}[0]}$

AR(2)

$$\boxed{v[k] = \frac{1}{1 - 0.6q^{-2}} e[k]}$$