CSCI 22012-Statistics for Decision Making

Assignment #01

Due on 10th May 2024, Time: 4.00 p.m.

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1. Given a random sample of $X_1, X_2, X_3, \dots, X_n$ size n from a geometric population . Use the method of moments to find an estimator for the parameter θ.

2. If $X_1, X_2, X_3, \dots, X_n$ constitute a random sample of size n from a population given by

$$g(x;\alpha) = \begin{cases} \frac{1}{\alpha}.e^{\frac{-(x-\delta)}{\alpha}}; & for\ x > \delta \\ 0 \quad ; & elsewhere \end{cases}$$

   Find an estimator for δ and α by using the method of moments.

3. Given a random sample of $X_1, X_2, X_3, \dots, X_n$ size n from a uniform distribution $\left(f(x;\alpha,\beta) = \frac{1}{\beta-\alpha}; \alpha < x < \beta\right)$, $\alpha = 0$, find an estimator for β by
   a) Method of moments,
   b) Method of maximum likelihood.

4. Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from a normal distribution with unknown mean μ and variance σ^2.
   a) Find maximum likelihood estimators of mean μ and variance σ^2.
   b) Suppose the weights of randomly selected American female college students are normally distributed with unknown mean μ and standard deviation σ. A random sample of 10 American female college students yielded the following weights (in pounds):

   115 122 130 127 149 160 152 138 149 180

   Using the given sample, find a maximum likelihood estimate of μ.

5. Let $Y_1, Y_2, Y_3, \dots, Y_n$ be random variables that follow a Weibull distribution,which has the density,

$$f(y;\alpha,\theta) = \frac{\alpha y^{\alpha-1}}{\theta^\alpha} e^{-\left(\frac{y}{\theta}\right)^\alpha}$$

   Suppose that α is known, but θ is unknown. Show that the MLE of θ is, $\left(\frac{1}{n}\sum_{i=1}^{n} y_i^\alpha\right)^{\frac{1}{\alpha}}$

6. Consider the distribution of serum cholesterol levels for all males in the US who are hypertensive and who smoke. This distribution has an unknown mean and a standard deviation of 46mg /100ml. Suppose we draw a random sample of 12 individuals from this population and find that the mean cholesterol level is 217mg/100ml. Find 95%,99%, and 90% confidence intervals for population mean μ.

7. a) Let be a random sample from the normal distribution with mean μ and variance σ^2 construct a 95% CI for μ.

   b)On the basis of the results obtained from a random sample of 100 men from a particular district, the 95% CI for the mean height of a man in the district is found to be $(177.22cm, 179.18cm)$. Find the values of the sample mean and the population standard deviation of the normal distribution from which the sample was drawn. Also find the 98% for the mean height.

8. A sample of 500 nursing applications included 60 men. Find the 90% confidence interval of the true proportion of men who applied to the nursing program.

9. A survey of 200,000 boat owners found that 12% of the pleasure boats were named *Serenity*. Find the 95% confidence interval of the proportion of boats named *Serenity*.

10. A social experiment conducted in 1962 involved $n = 123$ three and four-year-old children from poverty-level families in Ypsilanti and Michigan. The children were randomly assigned either to (1) a treatment group receiving two years of preschool instruction, or to (2) a control group receiving no preschool instruction. The participants were followed into their adult years. Here is a summary of the data:

	Arrested for some crime	
	Yes	No
Control	32	30
Treatment	19	42

Find a 95% confidence interval for $p_1 - p_2$, the difference in the two population proportions.