Exercise 4

Let $x_1, x_2, ..., x_m$ be a sequence of inputs where $x_i \in \{0,1\}^{2*m-1}$ and contains $2*i-1$ of $1's$ (with $i \in \{1, ...m\}$). We can show that we can shatter them by $\mathcal{H}$.
Let $ones(x)$ be a function which returns the number of ones in x and let $\phi_i(x)$ be a symmetric function which labels input x_i with 1 and is defined as follows: $\phi_i(x) = ((ones(x)) \mod i = 0)$. Then for any labeling we can find a symmetric function ψ_j (with $j \in \{1, ...2^m\}$) which is a composition of different $\phi_i(x)$'s by disjunction.
For instance, let the desired labeling of inputs be $(-, -, ...+, +, ..., -)$ where only inputs $x_{m/2}$ and $x_{m/2+1}$ are labeled with ´$+$´. The corresponding symmetric function would be $\psi(x) = \phi_{m/2}(x) \vee \phi_{m/2+1}(x)$. So a sequence of inputs of any size can be shattered by $\mathcal{H}$ and VC dimension of $\mathcal{H}$ is of ∞.