Quantitative Macroeconomics
Part 2
Project 3

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1 Problem A

A function `lookup` was renamed to `lookup_custom` as we encountered the function overwrite error (some built in function had the same name).

1.1

Comparison between the explicit and the numerical solution for 2nd method (DC-Endogenous Grid Method).

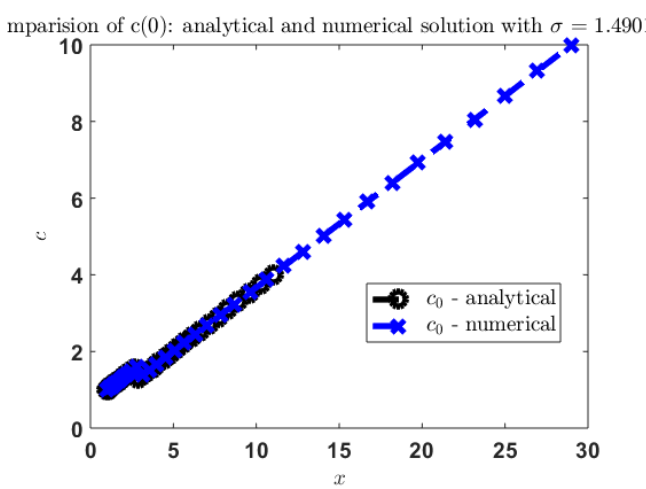


Figure 1: 1st vs 2nd method

*In collaboration with Sebastian A. Roy, Jakub Bławat, and Zuzanna Brzóska-Klimek

Comparison between the explicit and the numerical solution for 3rd method (Exogenous Grid Method).

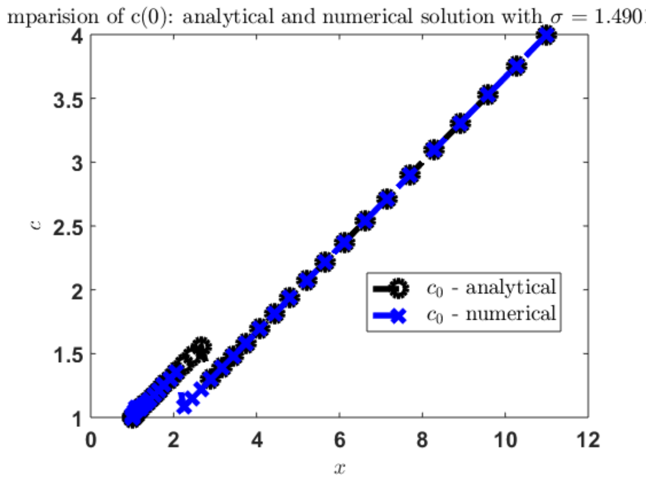


Figure 2: 1st vs 3rd method

All three methods are giving similar results. One of the things that can be noticed is that the cash-on-hand grid is longer for the 2nd method. Also 1st and 3rd methods have a 'jump' around x=2, although the 3rd method has it a bit earlier. After that they follow the same path.

1.2

The first method (opt.meth = 1) of solving our dynamic programming model is a standard global solution method. The model is solved by the implementation of the analytical solution shown in handout notes. The second method (opt.meth = 2) is a endogenous grid-point method (EGM) with discrete-continuous (DC) choice of variables, which should significantly speed up our solution and as we saw in subpoint (1) there is no significant loss of accuracy. This solution method is based on Iskhakov, Jorgensen, Rust and Schjerning (2017). The last one (opt.meth = 3) is the exogenous grid method where we use Brent's method of finding roots to solve Euler equations in order to find optimal consumption for each period which could be time-consuming.

1.3

We change the value of $param.nt$ from 2 to 1. After that the comparison between 1st and 2nd method looks like:

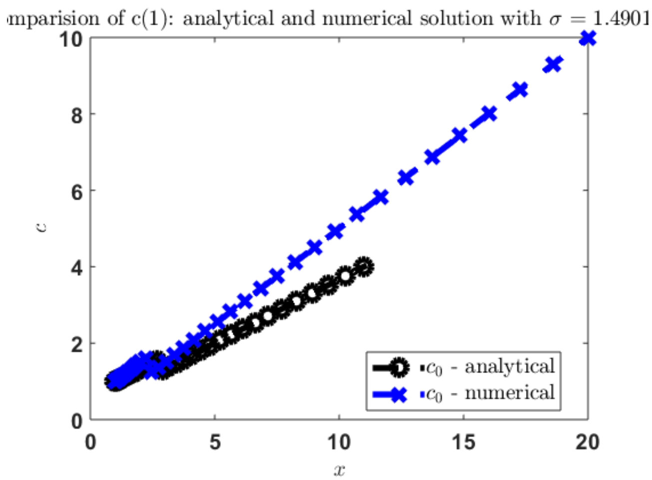


Figure 3: 1st vs 2nd method

Now the solutions are different and they do not follow the same path after the kink. They diverge the way that DC-EGM is increasing faster than the global solution method.

Comparison between the explicit and the numerical solution for 3rd method (Exogenous Grid Method).

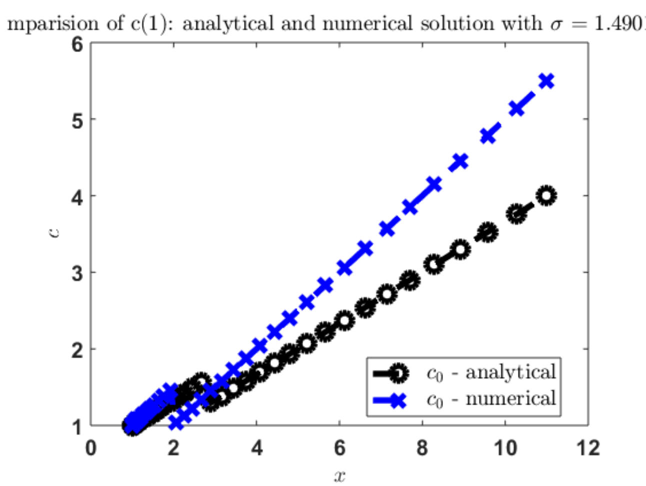


Figure 4: 1st vs 3rd method

Again the methods are not following the same path after the kink and again the 1st method is the one that increases slower. What is more the 3rd method again struggles to deal with the kink as it jumps to the new path earlier (in x) than the 1st method.

1.4

As shown in the table below, the second method is way faster than the others. Implementation of discrete-continuous endogenous grid method is visibly improving the speed of computation without significant loss of accuracy. The first and the last method used comparable amount of time.

Method	time elapsed [s]
1st method	109
2nd method	7
3rd method	115

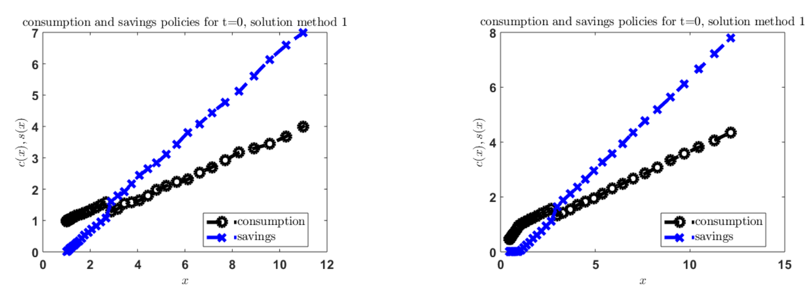
Table 1: Running times of the three alternative methods.

1.5

The third method is using Brent's algorithm to find the root of the Euler equation, however this method has a problem when dealing with kinks accurately. It can not catch the exact moment when the budget constraint binds.

1.6

The consumption and savings policy functions after we increase the variance of income shocks to 0.01 for the first method are shown on the plots below.



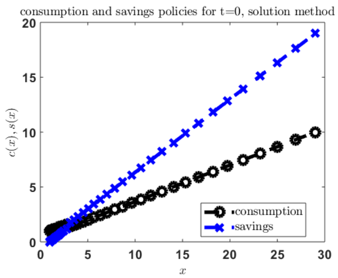
(a) For sig2e=sqrt(eps)

(b) For sig2e=0.01

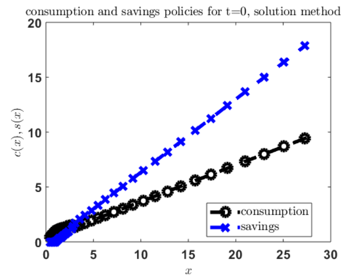
Figure 5: Method 1

After the increase of the income shock the shape of the policy functions for small values of cash-on-hand have changed. Now we can say that two kinks are visible, one the same as before, and one for even smaller values of x (around x=1).

For the second method:



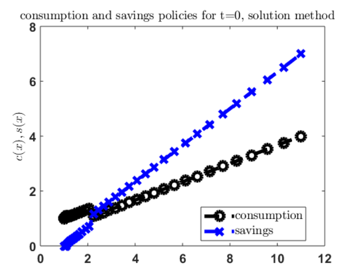
(a) For sig2e=sqrt(eps)



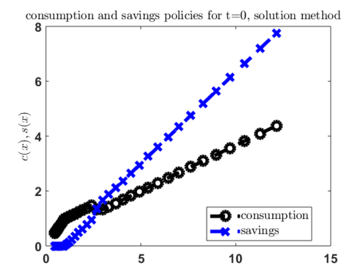
(b) For sig2e=0.01

Figure 6: Method 2

There is no significant difference in this case.
And finally for 3rd method:



(a) For sig2e=sqrt(eps)



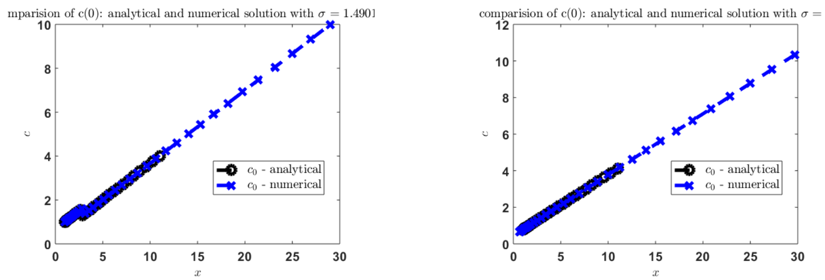
(b) For sig2e=0.01

Figure 7: Method 3

Same as in the case of the first method, the policy for small values of x is now different.

1.7

When we change the taste shock to 1.0 we can see that it smoothed out kinks in consumption policy functions. Comparison between the situation before the change and after for the 1st and 2nd method (the plot on the left side is from subpoint (1)):

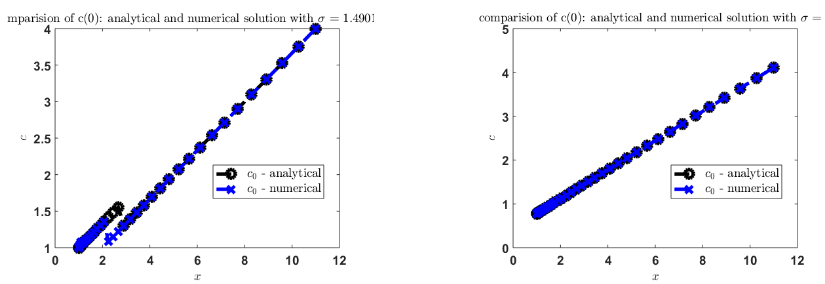


(a) For σ=sqrt(eps)

(b) For σ=1.0

Figure 8: Before and after the change of taste shock. 1st vs 2nd

When we compare the 1st and the 3rd method we can also notice that the 'jump' has been reduced. The increase of the taste shock is making the 1st and the 3rd method basically identical in terms of solution, as shown on the graph below:



(a) For σ=sqrt(eps)

(b) For σ=1.0

Figure 9: Before and after the change of taste shock. 1st vs 3rd

2 Problem B

2.1 Problem 1

2.1.1

MATLAB does not take into consideration the order of the code in terms of defining functions, so the sections of the code wouldn't be described in the order in which they are written in the code.

Let's split our code into 5 sections:

- Section 1 - Defining usage of survival risk, Markov chain and deterministic approach in the model; solution and plots
- Section 2 - Model calibration, defining shocks

- Section 3 - Defining HH's optimization problem, backward iteration
- Section 4 - Defining extrapolation and interpolation, defining aggregation and cross-sectional measure, distribution of newborns, transfers, incomes and lifetime cycle process
- Section 5 - Defining Markov chain, grid and utility functions

1. Section 5 defines utility function and its derivative. The utility funtion is $u(c_t) = log(c_t)$ if $\theta = 1$ and $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$ otherwise. Then we define the Markov chain parameters that will be used to determine the income shocks in Section 2.
2. Section 4 defines the aggregation and cross-sectional measure. First, we define the distribution of assets among the agents. Then we define income and cash-on-hand for newborn households. Further onwards, we define the transfers that are supposed to represent cash transfers from parents to their children. Next, we define general equilibrium in aggregate measures and life-cycle profiles for assets, consumption, labor and retirement.
3. Section 3 of the MATLAB code defines the household's problem and brings a solution to it. First, we define grid size and income grid. Then we define income and cash on hand as well as final period values of consumption and utility function. The following step is backward iteration of income, consumption and utility function.
4. Section 2 defines all the parameters used in defined functions and shocks based on the Markov chain proces.
5. Finally, Section 1 defines the use of survival risk, Markov chain and deterministic approach in the model. Additionally, Section 1 contains 13 plots corresponding to consumption, distribution of assets, income, assets, consumption, savings and value function.

2.1.2

The solution presented in ***func_hh*** differs from Exogenous Grid Method (EXOGM). In EXOGM, the grid is created on cash on hand, while in ***func_hh*** the grid is created on savings. In ***func_hh*** we calculate cash on hand and assets based on savings and consumption, while in EXOGM savings and assets are calculated with respect to cash on hand and consumption.

Advantages of working with savings grid:

- The method avoids the need to numerically solve the nonlinear Euler equation for the optimal continuous choice at each grid point in the state space.
- The method handles borrowing constraints in an efficient manner.
- Therefore, the method is faster and more accurate.

Disadvantages of working with savings grid:

- Interpolation of assets is problematic, the assets grid is not necessarily equally spaced and monotone
- Standard Endogenous Grid Model doesn't work well with 3 or more dimensions in state space.

2.1.3

In this part of the Problem 1. we are supposed to introduce exogenous grid for cash-on-hand x and compare the running time for both methods.

We tried to do this part, but our code has a mistake making it impossible to run properly.

2.1.4

TT is an income transfer function between generations and ***Phi*** represents cross-sectional distribution of assets conditional by age and shock. These functions assure that wealth is transfered from older households to the younger ones and also that wealth is transfered among the whole population.

The modification of the code proposed boosted the speed of calculations Original solution of ***towards_olg*** takes between 7 and 8 seconds, while after modification the computation time is able to get below 7 seconds.

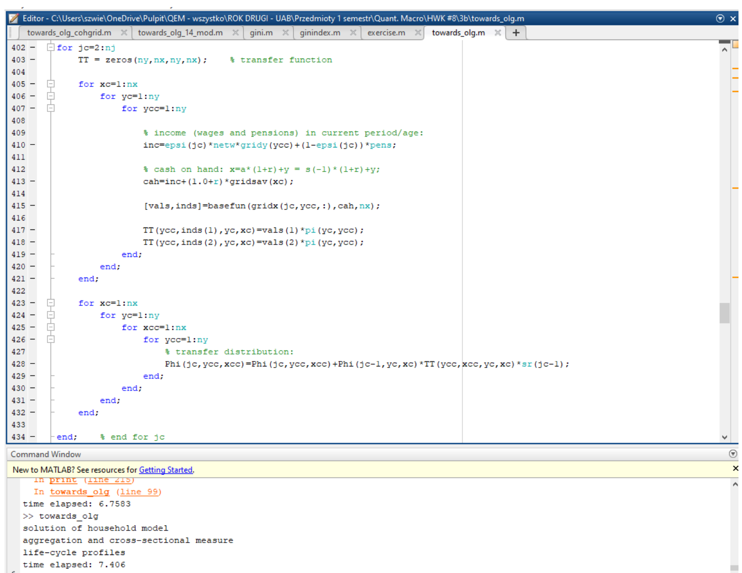


Figure 10: Original loop with computation time

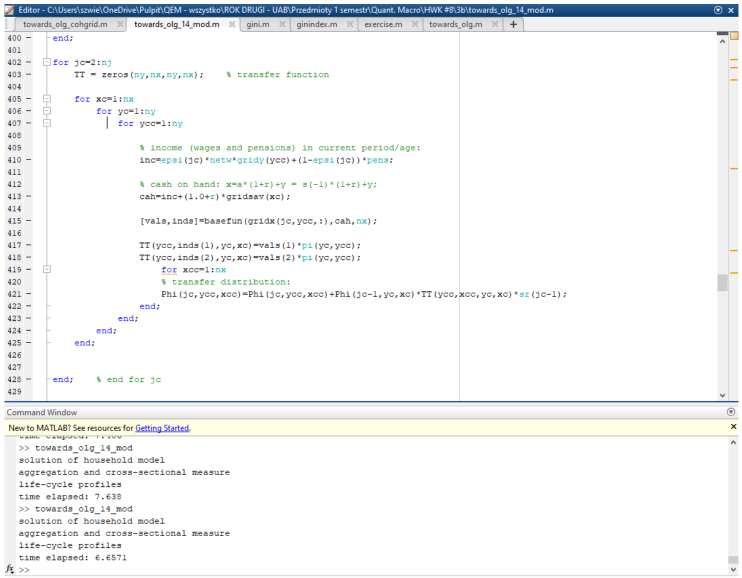
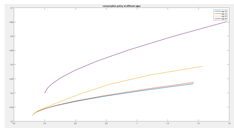


Figure 11: Modified loop with computation time

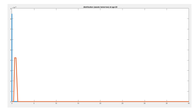
2.1.5

In this part we solve the model with alternative settings corresponding to ***opt_det*** as well as ***opt_nosr***.

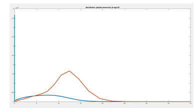
The original solution to the model is displayed on the figures below:



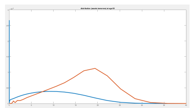
(a) Consumption at different ages



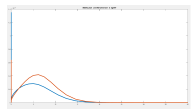
(b) Assets tomorrow distribution at age 20



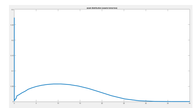
(c) Assets tomorrow distribution at age 40



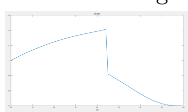
(d) Assets tomorrow distribution at age 60



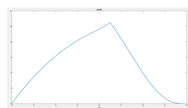
(e) Assets tomorrow distribution at age 80



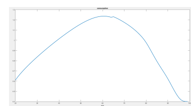
(f) Assets distribution



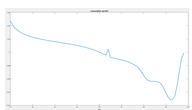
(g) Income of each individual



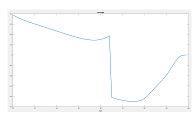
(h) Assets of each individual



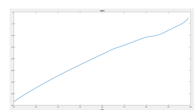
(i) Consumption of each individual



(j) Consumption dynamics of each individual

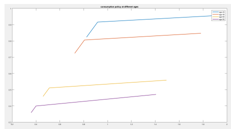


(k) Savings of each individual

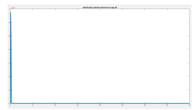


(l) Household's value function

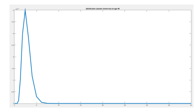
1. Here ***opt_det*** is set as True and ***opt_nosr*** is also set as True. The modification causes households to have exactly the same consumption in each period, which has a huge impact on the life-cycle values of variables and on asset distribution.



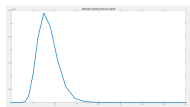
(a) Consumption at different ages



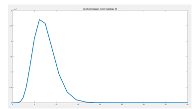
(b) Assets tomorrow distribution at age 20



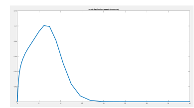
(c) Assets tomorrow distribution at age 40



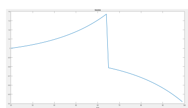
(d) Assets tomorrow distribution at age 60



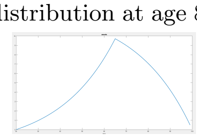
(e) Assets tomorrow distribution at age 80



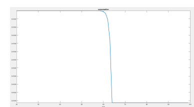
(f) Assets distribution



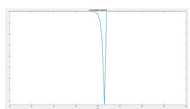
(g) Income of each individual



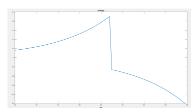
(h) Assets of each individual



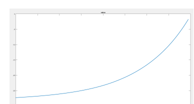
(i) Consumption of each individual



(j) Consumption dynamics of each individual

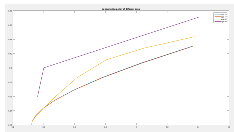


(k) Savings of each individual

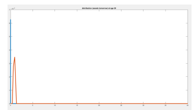


(l) Household's value function

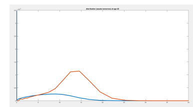
2. Here ***opt_det*** is set as False and ***opt_nosr*** is also set as True. In this case the consumption in the first periods is small and as the time goes by it increases. What is more, when household retires, its consumption does not change.



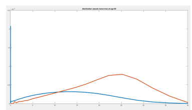
(a) Consumption at different ages



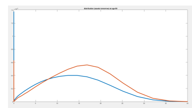
(b) Assets tomorrow distribution at age 20



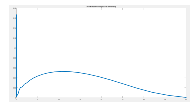
(c) Assets tomorrow distribution at age 40



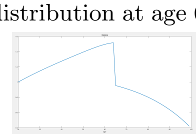
(d) Assets tomorrow distribution at age 60



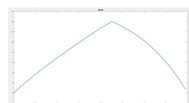
(e) Assets tomorrow distribution at age 80



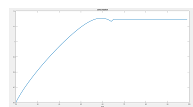
(f) Assets distribution



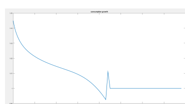
(g) Income of each individual



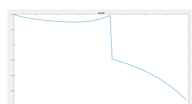
(h) Assets of each individual



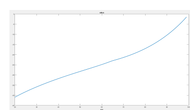
(i) Consumption of each individual



(j) Consumption dynamics of each individual

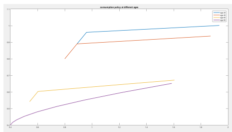


(k) Savings of each individual

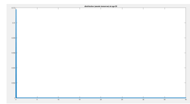


(l) Household's value function

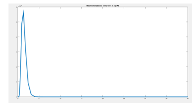
3. Here ***opt_det*** is set as True and ***opt_nosr*** is also set as False. In this case households consume the most in the first periods and transitions to lower level of consumption as the time goes by.



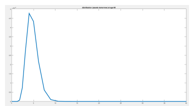
(a) Consumption at different ages



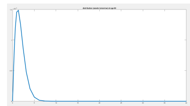
(b) Assets tomorrow distribution at age 20



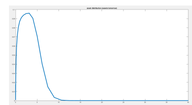
(c) Assets tomorrow distribution at age 40



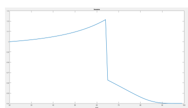
(d) Assets tomorrow distribution at age 60



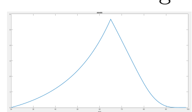
(e) Assets tomorrow distribution at age 80



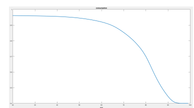
(f) Assets distribution



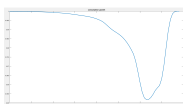
(g) Income of each individual



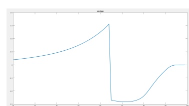
(h) Assets of each individual



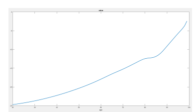
(i) Consumption of each individual



(j) Consumption dynamics of each individual



(k) Savings of each individual



(l) Household's value function

An increase in θ creates a kink in value function at the retirement age, below this age the values are significantly lower and in case of decreased θ the whole value function is steeper and takes more extreme values.

Modification in which r is significantly greater than ρ leads to a scenario in which younger households consume less and older households consume more (consumption dynamics are steeper). The income increases quicker for working households, they accumulate more assets (distribution of assets in time is moved more to later periods). A modification of r and ρ in the other side would lead to opposite results.

2.2 Problem 2

2.2.1

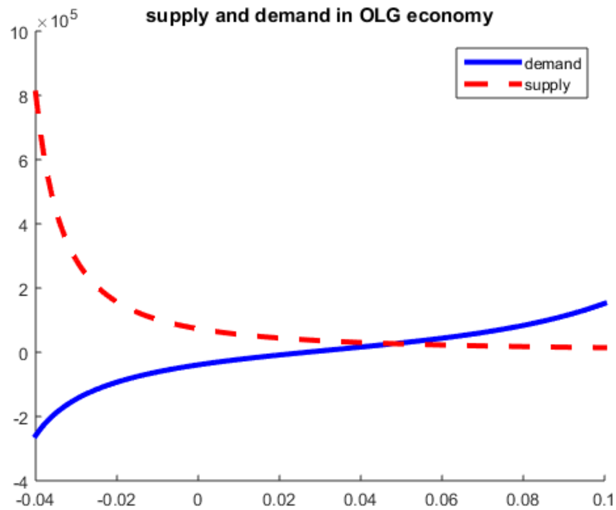


Figure 16: Asset market without pension system

Actually, general equilibrium solution has been provided in code `exercise`.

2.2.2

OLG economy can be inefficient, because retired agents need to consume while their only income comes from asset returns; therefore, efficiency requires that return on capital is sufficiently high. If it is too low, retired agents make sub-optimal choices as they are desperate to consume.

More formally one could state a theorem that if gross interest rate (R) is smaller than combined growth rates of population (n) and productivity (γ), then OLG economy is inefficient:

$$R < n\gamma \tag{1}$$

Obviously, a pension scheme might improve efficiency of the OLG economy regardless of the interest rate - that is why we are asked to make sure that interets rate is high enough to make economy efficient even with no pension system.

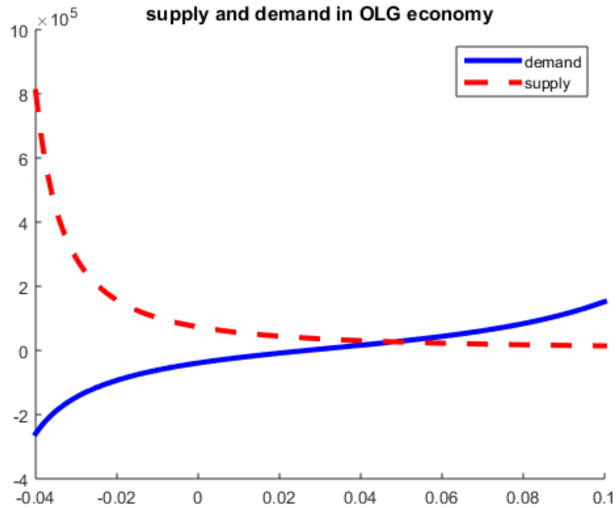


Figure 17: Asset market without pension system

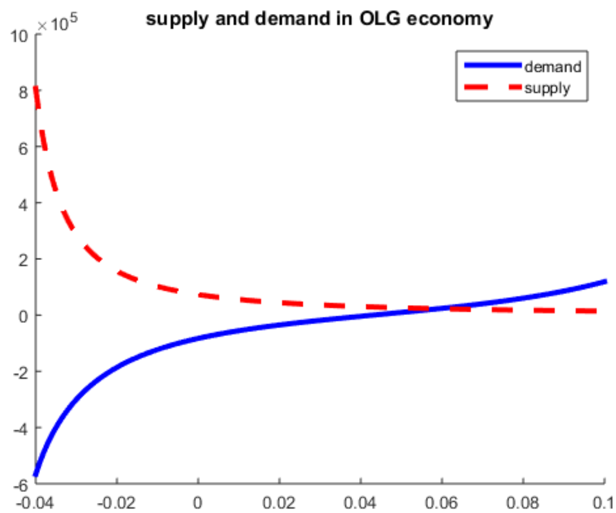


Figure 18: Asset market with pension system and 60% replacement rate

Variable	No pension (`rr`=0%)	With pension (`rr`=60%)
Interest rate	0.048	0.059
Contribution rate	0	0.175
Capital-output ratio	3.363	3.025

Table 2: OLG equilibrium with and without pension system - comparison

2.2.3

2.2.4

Pension system is one of the most interesting features that might be analysed with OLG model. Figures (17) and (18) compare asset demand and supply curves across cases without a pension system and with one. We can see that the supply curve remains unchanged while demand differs significantly. More specifically, in the early stages of life agents exhibit lower demand for assets under a pension regime than without one. This can be tracted back to the fact that with pension system present, young agents do not need to save that much as they would without pension system.

This strikes a chord also with calculated parameters of the model. As we can see in Table (2), presence of a pension system lowers capital-to-output ratio - again, because, agents do not need to save that much.

2.2.5

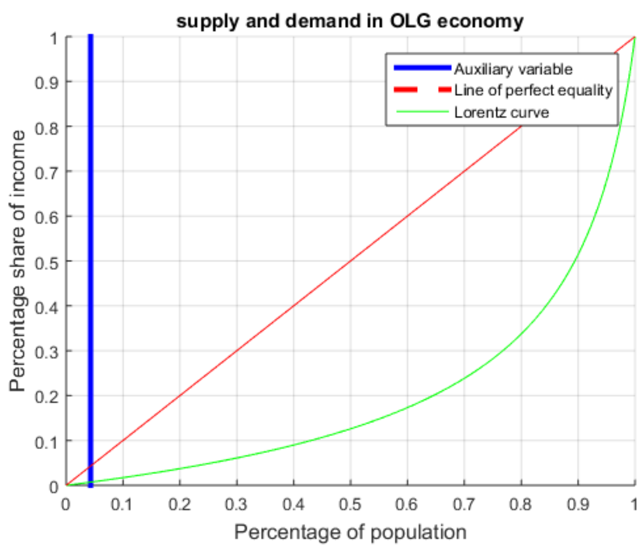


Figure 19: Lorentz curve

The `ginindex` code calculates Gini coefficient at 0.5957. Graphical representation can be found in Figure (19), which depicts Lorentz curve. Hence ratio of the area under green curve and red line is precisely 0.5957.

2.3 Problem 3

3 Problem C

3.1 Krueger, D. and A. Ludwig (2016). Progressive taxation versus education subsidies in general equilibrium

Krueger and Ludwig focused in their work on optimal mix of income taxation and tertiary education subsidies both in terms of steady state and dynamic transition. The general conclusions are such that optimal steady state, optimal transition path and also *status quo* differ from each other.

This Overlapping Generations model with heterogeneous households closely simulates real economy. First of all it reflects demographics thanks to multiperiod life cycle, where new households spend their early life supported by parents till age j_a. Then they become fully independent households, which at that moment need to decide, whether or not attain the collage. Later these agents have their own children at the age j_f and retire at j_r. Every year after that they die with probability $1-_t$ until maximum age J.

The decision on tertiary education depends on innate abilities e, initial idiosyncratic labor productivity , parent's education $_p$ and income shock γ_p, as well as *in vivo* transfers b between generations. More over government subsidize collage attainment by θ and there is available students-only debt up to fraction ϕ of overall expenses along studying period. Skilled and unskilled labor are substitutes no matter the age with elasticity of substitution $\frac{1}{1-\rho}$.

Government levies taxes on households to finance its expenditures including tertiary education subsidies. We assume that these are the income taxes, that are optimized in order to meet budget constraint while choosing subsidiary level. They characterise by linear parameter τ_l and deduction level d. The lower the latter one, the less progressive taxes are.

The market is incomplete as there are borrowing constraints and also it is impossible to insure against idiosyncratic income shock nor death. The model is calibrated to US economy.

According to results, the status quo subsidies on tertiary education are way too low and tax progressivity is too high, no matter if we are considering only steady state social welfare maximization or dynamic transition. Thanks to optimal policy we would be able to decrease collage premium wage and thus inequalities among society.

Optimal parameters in steady state are $\tau_l = 26.03\%$, $d = 10\%$ and $\theta = 200\%$. However, as authors notice, changing policy to one maximized only for steady state, would result in recession periods. Thus they carried out the similar maximization for the whole transition path. The optimal parameter then are $\tau_l = 21.9\%$, $d = 6\%$ and $\theta = 150\%$. Thus the taxes are even less progressive, but the collage subsidies are smaller. Authors thus also concludes, that education subsidies and progressive labor income taxes are imperfect substitutes.

In the end authors argue, that slow introduction of new subsidy - labor income policy matters, in terms of social welfare gains, only for steady state policy and not for dynamic transition one.

3.2 Paper of Abbott, Gallipoli, Meghir, Violante (2019)

The paper examined the equilibrium effects of college aid policies. Financial aid for college students works as an investment in human capital, working as a vehicle to boost aggregate productivity as well as social mobility. In the US there is a system of financial loans and grants for college students that is intended to to advance college attainment.

In order to check the effects of the system and look for possibility of upgrading it, authors created an life cycle, heterogeneous agent model set in overlapping generations framework. The model includes education, labor supply, and endogenous savings. Parents decide upon their savings having in mind both altruism and a paternalistic preference for children's education. Agents are heterogeneous in terms of both the returns to education and the psychic costs of schooling, which depend on both cognitive and noncognitive ability.

According to the results, current US system of financial aid improves long-run social welfare by 6 percent and accounts for more than 4 percent of GDP. Every additional dollar of government grants crowds out 25–50 cents of parental inter vivos transfers on average, and a $1,000 reduction in tuition fees lowers the annual earnings of college students by roughly $100 on average.

3.3 De Nardi, M. (2004). Wealth Inequality and Intergenerational Links.

The paper's aim is to find the reason for high inequality of wealth distribution.

The paper focuses on reasons of high inequality of wealth distribution, which is learnt to be the mix of productivity inheritance and bequest motives in utility function, which lead to higher wealth accumulation through the generations. The model was calibrated to both US and Swedish economy as robustness check. It worked well for both countries.

The Overlapping Generation Model was used here one period lasting 5 years and agents appearing at their age 20 (t=5), procreating a period later and retiring at age 60 (t=10). Only after retirement agents face positive probability of death and the maximum length of life equals 90 (T=14). All the assets owned by household at the moment of death is passed to young generation. The experiments taken accounted for both equal bequest to all other households and specifically to one's children. The latter specification appeared to be more in line with observed data. The intuition for why this happens is that the introduction of a nonhomothetic bequest motive makes bequests a luxury good. It is good to notice that in that specification there is no *inter vivo* transfers nor its equivalents. This assumption is probable to not hold in real life.

There was taken also the experiment whether or not inheritance of productivity may lead to observed wealth inequalities. The answer for that question is positive. Thus in general more productive households accumulate more assets before they retire, then consume relatively less on retirement for luxury bequest good and after their death, the outstanding assets are passed to their more productive children. Thus the capital accumulation raises through generations.

Apart from non-presence of *inter vivo* transfers, there are some other assumption, that might be violated. These are the exogeneity of fertility and death probability. As data shows it is probable that rich household would have less children and enjoy good health longer. The potential problem might be also the fact that in reality rich households may face higher rates of returns than poor ones.