Quant Macro Final Project

Szymon Wieczorek

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1 Simple variant of Krusell-Smith Algorythm

In these exercise we were supposed to introduce simple variant of Krusell-Smith algorythm step by step. Each of 3 subpoints was constructed to help us in this case.

1.1 Restated proof of the Proposition 3

Statement

Under assumption of logarythmic utility and shock distribution such that they have positive support, mean of 1 and are independent, the equilibrium dynamics are given by the following formulas:
$k_{t+1} = \frac{1}{(1+g)(1+\lambda)} s(\tau)(1-\tau)(1-\alpha)\zeta_t k_t^{\alpha}$
$s(\tau) = \frac{\beta\Phi(\tau)}{1+\beta\Phi(\tau)} <= \frac{\beta}{1+\beta}$

Where $\Phi(\tau)$ is defined as:

$\Phi(\tau) = E_t[\frac{1}{1+\frac{1-\alpha}{\alpha(1+\lambda)\rho_{t+1}}(\lambda\eta_{i,2,t+1}+\tau(1+\lambda(1-\eta_{i,2,t+1})))}] <= 1$

Proof

Capital is made of assets created by savings of young households that come from the labor income. Since utility functions for all household is exactly the same (no individual heterogeneity), aggregate capital equation can be written in the form:
$K_{t+1} = a_{2,t+1} = s(\tau)(1-\tau)w_t = s(\tau)(1-\tau)(1-\alpha)\Upsilon_t\zeta_t k_t^{\alpha}$
Capital per unit of effective labor dynamics is:
$k_{t+1} = \frac{1}{(1+\lambda)(1+g)} s(\tau)(1-\tau)(1-\alpha)\zeta_t k_t^{\alpha}$
So the capital path has been proven. Now we can focus on consumption. After several transformations in original formula and implementation of formula for capital we get that:
$c_{2,t+1} = \Upsilon_{t+1}\zeta_{t+1}k_{t+1}^{\alpha}(\alpha(1+\lambda)\rho_{t+1}+(1-\alpha)(\lambda\eta_{i,2,t+1}+\tau(1+\lambda(1-\eta_{i,2,t+1}))))$
And since all non-saved income is consumed,
$c_{1,t} = (1-s(\tau))(1-\tau)(1-\alpha)\Upsilon_t k_t^{\alpha}\zeta_t$.
Now, we can use the Euler equation to relate consumption in both periods:

$1 = \beta E_t[\frac{c_{1,t}(1+r_{t+1})}{c_{i,2,t+1}}] = \beta E_t[\frac{(1-s(\tau))(1-\tau)(1-\alpha)k_t^{\alpha}\zeta_t k_{t+1}^{-1}\rho_{t+1}}{(1+g)(\alpha(1+\lambda)\rho_{t+1}+(1-\alpha)(\lambda\eta_{i,2,t+1}+\tau(1+\lambda(1-\eta_{i,2,t+1}))))}]$

Which after transformations becomes:

$1 = \frac{\beta(1-s(\tau))}{s(\tau)}\Phi(\tau)$, where $\Phi(\tau)$ has the same value as described in Harenberg and Ludwig (2015).

And after small transformation we get that:

$s(\tau) = \frac{\beta\Phi(\tau)}{1+\beta\Phi(\tau)}$,

which is exactly what we looked for.

1.2 FOD simulation

In this point we were supposed to simulate the capital path of the first-order equation of capital in logs:

$ln(k_{t+1}) = ln(\frac{1-\alpha}{1-\lambda} + ln(s(\tau)) + ln(1-\tau) + ln(\zeta_t) + \alpha ln(k_t)$,

using pre-defined values of parameters: g = 0, α = 0.3, λ = 0.5, $\beta = 0.99^{40}$ and $\tau = 0$.

Additionally, we assume that z^r are represented by such values of ζ_t, ρ_t, $\eta_{i,2,t}$ such that $k_t < k_{ss}$. Analogically, z^b are represented by such values of ζ_t, ρ_t, $\eta_{i,2,t}$ such that $k_t > k_{ss}$.

As it can be seen on the plot, the steady state value of capital in log is about -2.362.

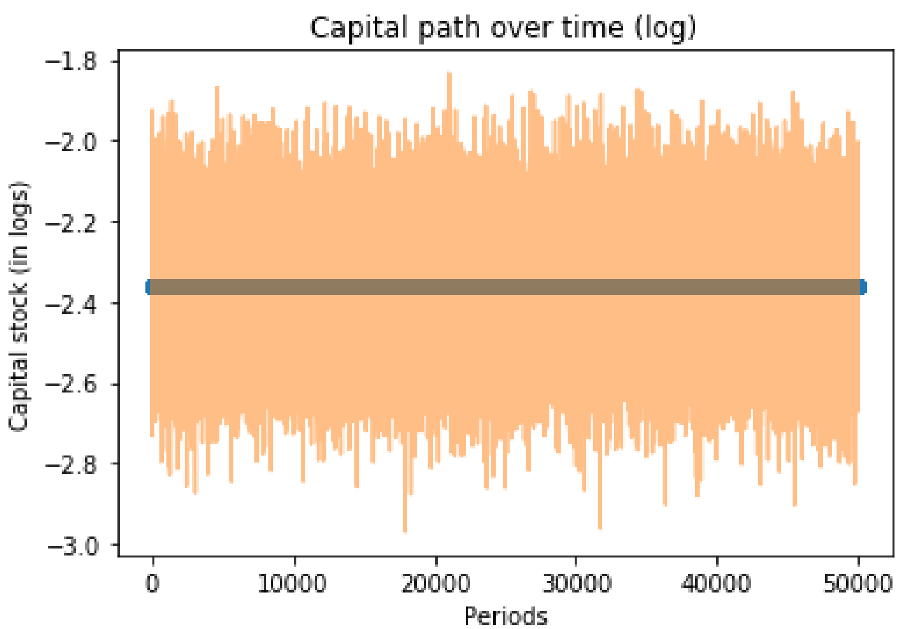

Figure 1: Simulation of path of capital with g = 0 in 50k periods

The value of capital stock between periods varies between -3 and -1.8 in log values, depending on the values of ζ_t.

The next step was to implement Gaussian quadrature method to calculate the capital path. ... (Gaussian quadrature)

1.3 Implementation of Krusell-Smith Algorythm

1.3.1 Theoretical values of coefficients of capital equation

According to the egquations in paper written by Harenberg and Ludwig (2015), the theoretical values of coefficients ψ_i were calculated, according to the equation below:

$ln(k_{t+1}) = \psi_0(z) + \psi_1(z)ln(k_t)$

And the equations in **1.2** as well as equations concerning savings from Harenberg and Ludwig(2015) imply that:

$ln(k_{t+1}) = ln(\frac{1-\alpha}{1-\lambda}) + ln(s(\tau)) + ln(1-\tau) + ln(\zeta_t) + \alpha ln(k_t)$, where
$s(\tau) = \frac{\beta\Phi(\tau)}{1+\beta\Phi(\tau)}$, and
$\Phi(\tau) = E_t[\frac{1}{1+\frac{0.7}{0.3\lambda\rho_{t+1}}(\lambda\eta_{i,2,t+1}+\tau(1+\lambda(1-\eta_{i,2,t+1})))}]$

After using the fact that $\tau = 0$, $\lambda = 0.5$, $\alpha = 0.3$ and $\beta = 0.99^{40}$:
$\Phi(0) = E_t[\frac{1}{1+\frac{0.7}{0.15\rho_{t+1}}(0.5\eta_{i,2,t+1})}]$,
$s(0) = \frac{0.99^{40}\Phi(0)}{1+0.99^{40}\Phi(0)}$,
$ln(k_{t+1}) = ln(\frac{0.7}{0.5}) + ln(s(0)) + ln(1) + ln(\zeta_t) + 0.3ln(k_t)$

Inserting $\Phi(0)$ and $s(0)$ into the capital function as well as using the properties of logarythmic function yields:

$$ln(k_{t+1}) = ln(1.4\zeta_t\frac{0.99^{40}E_t[\frac{1}{1+\frac{0.7}{0.15\rho_{t+1}}(0.5\eta_{i,2,t+1})}]}{1+0.99^{40}E_t[\frac{1}{1+\frac{0.7}{0.15\rho_{t+1}}(0.5\eta_{i,2,t+1})}]}) + 0.3ln(k_t)$$

And since expected value of all the shock terms is 1:

$$ln(k_{t+1}) = ln(\frac{0.99^{40}*0.15*1.4\zeta_t}{0.99^{40}*0.15+0.5}) + 0.3ln(k_t)$$

Therefore:

$\psi_0 = ln(\frac{0.99^{40}*0.15*1.4\zeta_t}{0.99^{40}*0.15+0.5}) = ln(\zeta_t) - 1.452$
$\psi_1 = 0.3$
(is this value correct?)

1.3.2 Carrying out the algorythm

In this part we implement the algorythm using our Python code.
PART I

Here (calculation of savings) ...
PART II
(simulation of economy) ...
PART III
(self-updating algorythm) ...

1.3.3 Comparison of numerical and analytical solutions

('steady' state of capital) calculated earlier vs ...

1.3.4 Changed τ and calculated expected utility

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2 Complex variant of Krusell-Smith Algorythm

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