

Quantitative Macroeconomics Homework 5

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1 Exercise 1. Factor Input Misallocation

1.1

We simulated 10,000,000 observations with the use of `numpy.random.multivariate_normal` function. In our exercise the mean of k is equal to 1. Hence we know that $E(k) = e^{mean-0.5 \cdot 1} = 1$, so the mean of $\ln(k)$ will be equal to -0.5 . Then we plot the joint density in logs:

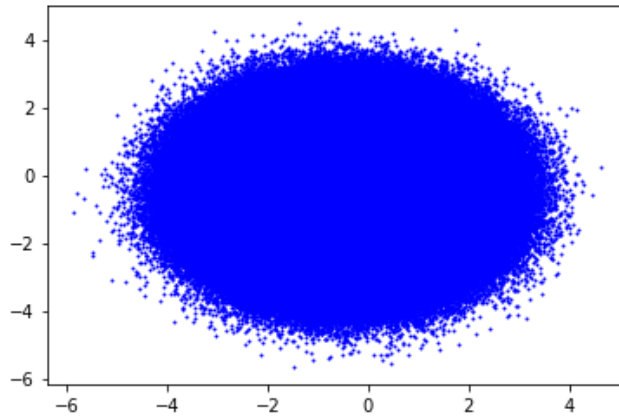


Figure 1: Joint density in logs

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And in levels:

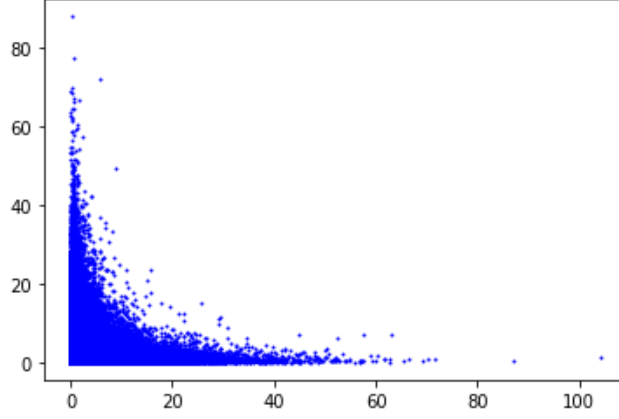


Figure 2: Joint density in levels

1.2

Now we compute firms output y_i for each of our observations. In our sample data the average $y_i = 0.787$ and the total output is $Y = 7,866M$.

1.3

We computed optimal capital as if the most productive firms receive the biggest amount of capital. Then the average output is $y_i = 1.00008$ and the total output is $Y = 10,001M$.

1.4

Now we compare the optimal allocations against the data. The sum of all k_i (K) is obviously the same, but capital is allocated differently.

1.5

Thanks to the reallocation we gave following output gains:

Total gain	Percentage gain	Gain per capita
2,134M	27.13%	0.213

Table 1: Production gain for zero correlation

1.6

1.6.1 For correlation equal to 0.5:

The plot with the joint density in logs:

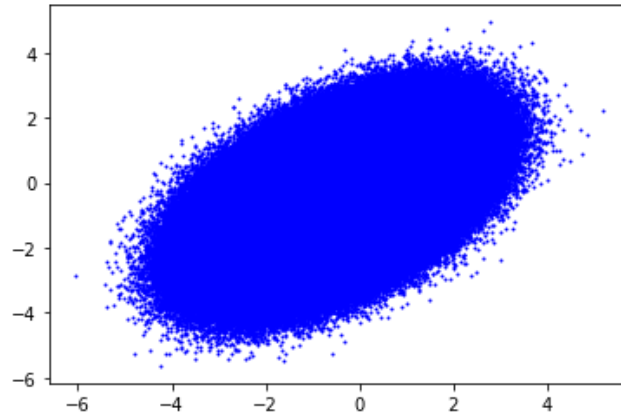


Figure 3: Joint density in logs for correlation=0.5

And in levels:

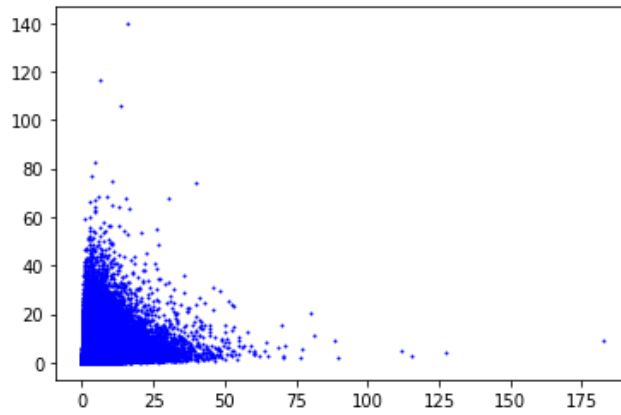


Figure 4: Joint density in levels for correlation=0.5

Firms total output is: 8,863M.
Firms total optimal output is: 9,993M.

And the production gains are:

Total gain	Percentage gain	Gain per capita
1,129M	12.74%	0.113

Table 2: Production gain for correlation=0.5

1.6.2 For correlation equal to -0.5:

The plot with the joint density in logs:

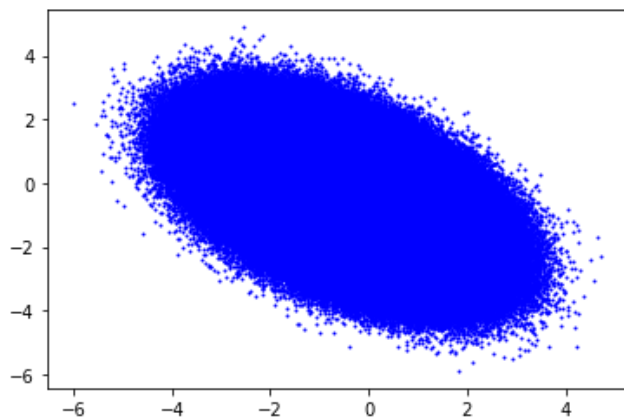


Figure 5: Joint density in logs for correlation=-0.5

And in levels:

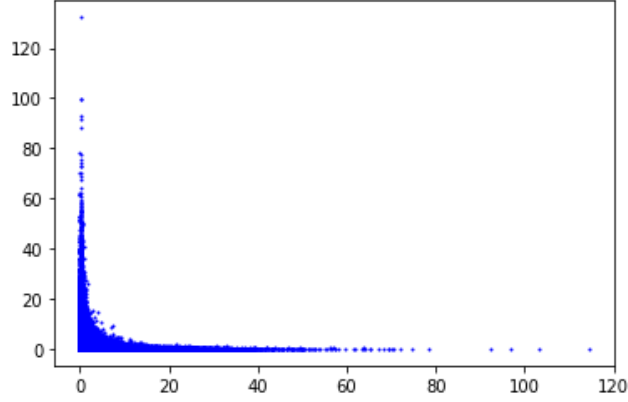


Figure 6: Joint density in levels for correlation= -0.5

Firms total output is: $6,977M$
Firms total optimal output is: $9,999M$

In this case, thanks to reallocation we gained almost 50% bigger output. Production gains are as follows:

Total gain	Percentage gain	Gain per capita
$3,022M$	43.32%	0.302

Table 3: Production gain for correlation= -0.5

2 Exercise 2: Higher Span of Control

The plots for each correlation levels are the same as in Exercise 1. The only thing that is changing is γ , now equal to 0.8.

2.1 Zero correlation

Firms total output is: $8,523M$.
Firms total optimal output is: $10.0M$.

And the production gains are:

Total gain	Percentage gain	Gain per capita
,1478M	17.35%	0.148

Table 4: Production gain - zero correlation

2.2 For correlation equal to 0.5

Firms total output is: 9,229M.

Firms total optimal output is: 9,998M.

In this case gains from reallocation are not that significant and the production are as follows:

Total gain	Percentage gain	Gain per capita
0,768M	8.33%	0.077

Table 5: Production gain for correlation=0.5

2.3 For correlation equal to -0.5

Firms total output is: 7,866M.

Firms total optimal output is: 9,999M.

And the production gains are:

Total gain	Percentage gain	Gain per capita
2,134M	27.12%	0.213

Table 6: Production gain for correlation=-0.5

3 Exercise 3: From Complete Distributions to Random Samples