Code Homework 1

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[4]:
```
%matplotlib inline
```

[5]:
```
from pylab import *
from numpy.linalg import eig, solve
```

[6]:
```
def plot_pt(q, label, color='b'):
    plot([q[0]], [q[1]], 'o', color=color, label=label)
```

[176]:
```
def solve_Rp(A,ql,qr):
    """
    Solve the Riemann problem for a 2x2 matrix A and given left and right states.
    """

    print('Matrix A:\n', A)

    # compute eigenvalues and vectors:
    evals, R = eig(A)

    # reorder if necessary so evals[0] <= evals[1]
    jorder = argsort(evals)  # indices in order needed to sort
    evals = evals[jorder]
    R = R[:,jorder]

    print('Eigenvalues of A:', evals)
    print('Matrix R of eigenvectors:\n',R)

    # check that problem is hyperbolic:
    assert isreal(evals).all(), '*** Eigenvalues are not real'
    condR = cond(R)
    print('Condition number of R is %g' % condR)
    assert cond(R) < 1e12, '*** Matrix probably is not diagonalizable'

    dq = qr - ql
    alpha = solve(R, dq)
    qm = ql + alpha[0]*R[:,0]

    print('alpha = ', alpha)
    print('qm = ', qm)
```

```
figure(figsize=(6,5))
plot_pt(ql,'ql','r')
plot_pt(qr,'qr','g')
plot_pt(qm,'qm','b')
plot((ql[0],qm[0],qr[0]), (ql[1],qm[1],qr[1]), 'b')
legend()
grid(True)
xlabel('first component of q')
ylabel('second component of q')
title('phase plane solution of Riemann problem')

t1 = 1
t2 = 2
n = 10000
x = linspace(-1,5,n)
q1t1 = ones(n)
q1t2 = ones(n)
q2t1 = ones(n)
q2t2 = ones(n)
for index1 in range(len(x)):

    if x[index1]<t1*evals[0]:
        q1t1[index1] = ql[0]
        q2t1[index1] = ql[1]
    elif x[index1]>t1*evals[1]:
        q1t1[index1] = qr[0]
        q2t1[index1] = qr[1]
    else:
        q1t1[index1] = qm[0]
        q2t1[index1] = qm[1]

    if x[index1]<t2*evals[0]:
        q1t2[index1] = ql[0]
        q2t2[index1] = ql[1]
    elif x[index1]>t2*evals[1]:
        q1t2[index1] = qr[0]
        q2t2[index1] = qr[1]
    else:
        q1t2[index1] = qm[0]
        q2t2[index1] = qm[1]

figure(figsize=(6,5))
plot(x,q1t1, color="orange", label='q1')
plot(x,q2t1, 'b', label='q2')
legend()
xlabel('x')
```

```
    ylabel('q value')
    title('Value of q at time t=1')
    figure(figsize=(6,5))
    plot(x,q1t2, color="orange",label='q1')
    plot(x,q2t2, 'b',label='q2')
    legend()
    xlabel('x')
    ylabel('q value')
    title('Value of q at time t=2')


    return qm
```

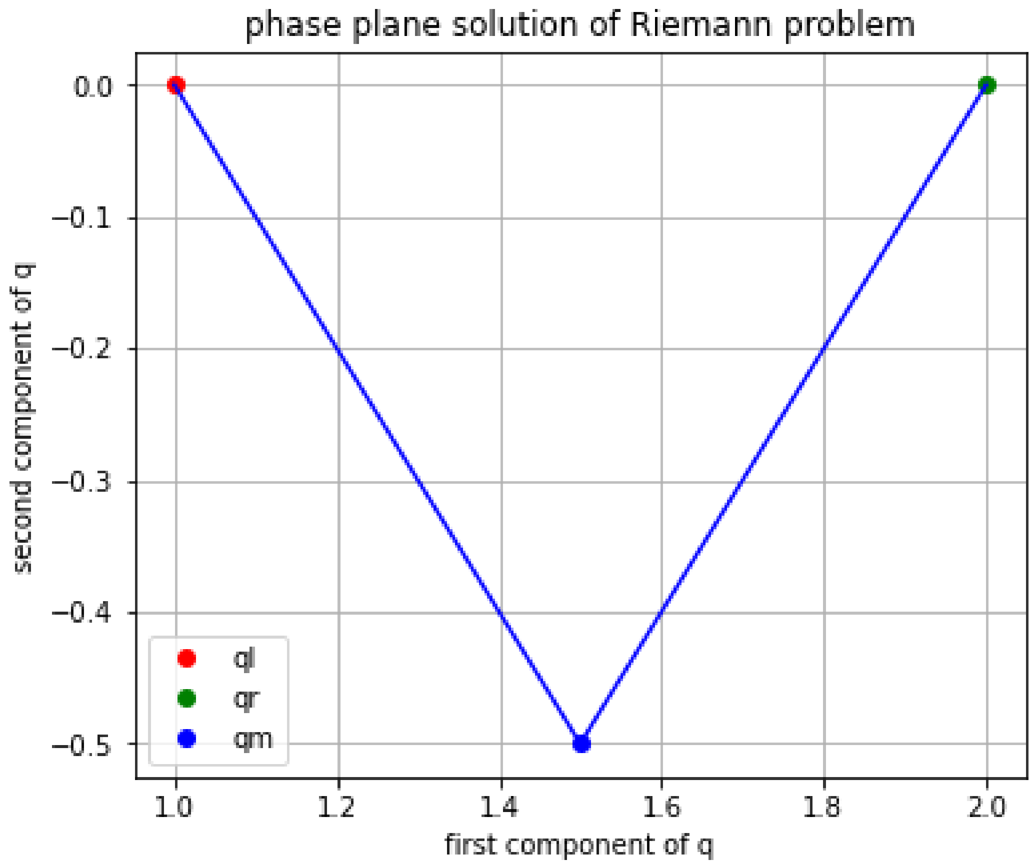
[240]:

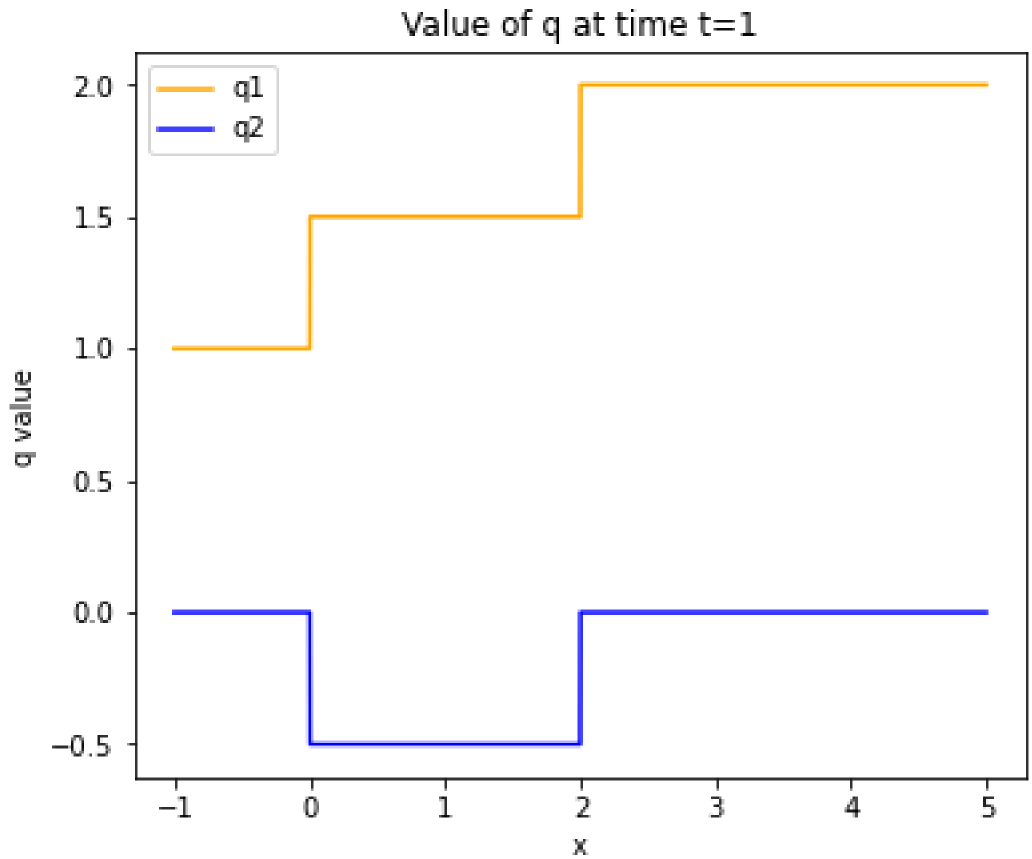
```
def xtplane(A,ql,qr):
    print('Matrix A:\n', A)

    # compute eigenvalues and vectors:
    evals, R = eig(A)

    # reorder if necessary so evals[0] <= evals[1]
    jorder = argsort(evals)   # indices in order needed to sort
    evals = evals[jorder]
    R = R[:,jorder]

    print('Eigenvalues of A:', evals)
    print('Matrix R of eigenvectors:\n',R)

    # check that problem is hyperbolic:
    assert isreal(evals).all(), '*** Eigenvalues are not real'
    condR = cond(R)
    print('Condition number of R is %g' % condR)
    assert cond(R) < 1e12, '*** Matrix probably is not diagonalizable'

    dq = qr - ql
    alpha = solve(R, dq)
    qm1 = ql + alpha[0]*R[:,0]
    qm2 = qm1 + alpha[1]*R[:,1]

    print('alpha = ', alpha)
    print('ql* = ', qm1, 'and qr* =', qm2)


    x = linspace(0,10,10)
    figure(figsize=(6,5))
    plot(evals[0]*x,x,'y',label = '$x=\lambda^1t$')
    plot(evals[1]*x,x,'b',label = '$x=\lambda^2t$')
    plot(evals[2]*x,x,'r',label = '$x=\lambda^3t$')
```

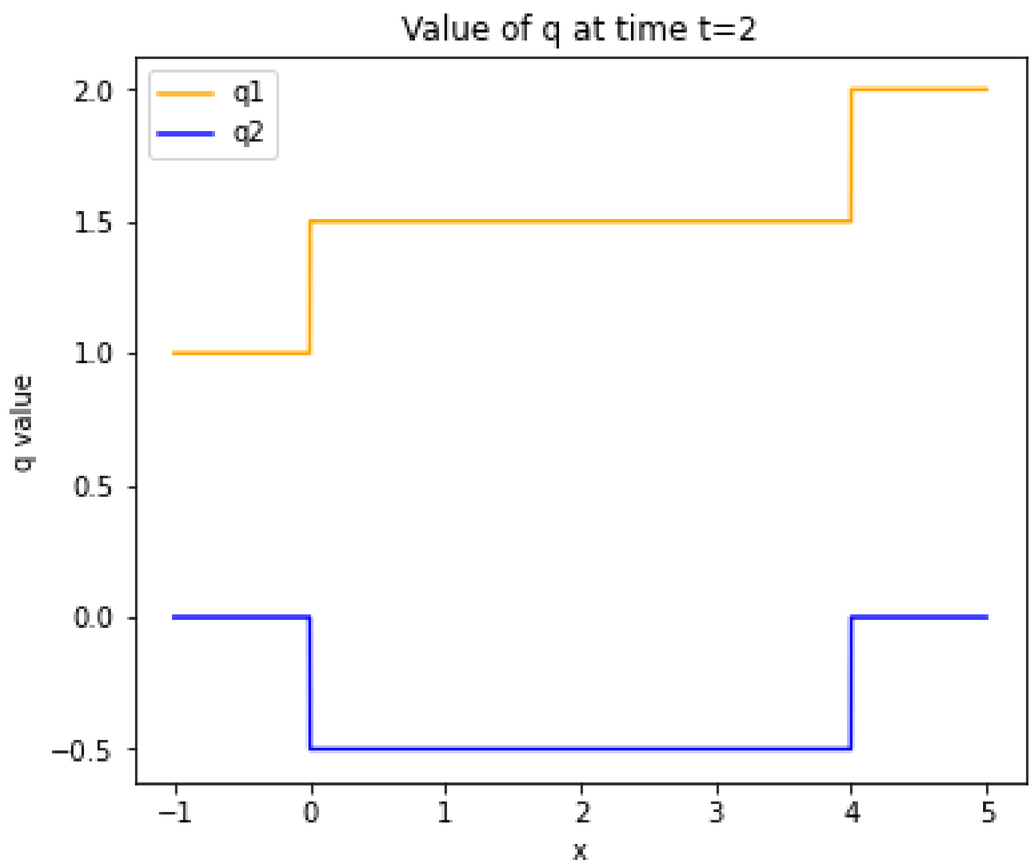
```
    legend()
    matplotlib.pyplot.text(-5, 1, '$q_l$')
    matplotlib.pyplot.text(3, 1, '$q_r$')
    matplotlib.pyplot.text(0, 4, '$q^*_l$')
    matplotlib.pyplot.text(5, 4, '$q^*_r$')
    plot(linspace(min(evals[0]*10,-1),max(evals[2]*10,0),10),zeros(10),'k')
    plt.axis('off')
    return
```

[241]:
```
A = array([[1,1],[1,1]])
qleft = array([1,0])
qright = array([2,0])
qm = solve_Rp(A,qleft,qright)
```

```
Matrix A:
 [[1 1]
 [1 1]]
Eigenvalues of A: [0. 2.]
Matrix R of eigenvectors:
 [[-0.70710678  0.70710678]
 [ 0.70710678  0.70710678]]
Condition number of R is 1
alpha =  [-0.70710678  0.70710678]
qm =  [ 1.5 -0.5]
```

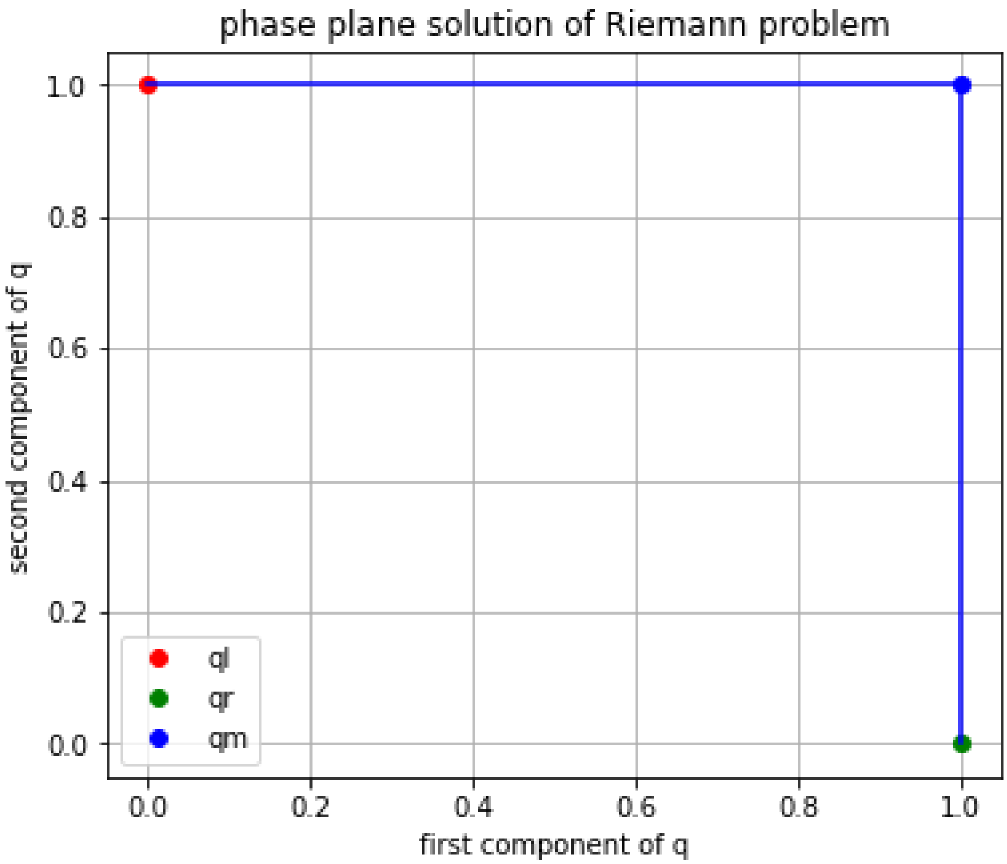

phase plane solution of Riemann problem
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
second component of q
ql
qr
qm
1.0
1.2
1.4
1.6
1.8
2.0
first component of q

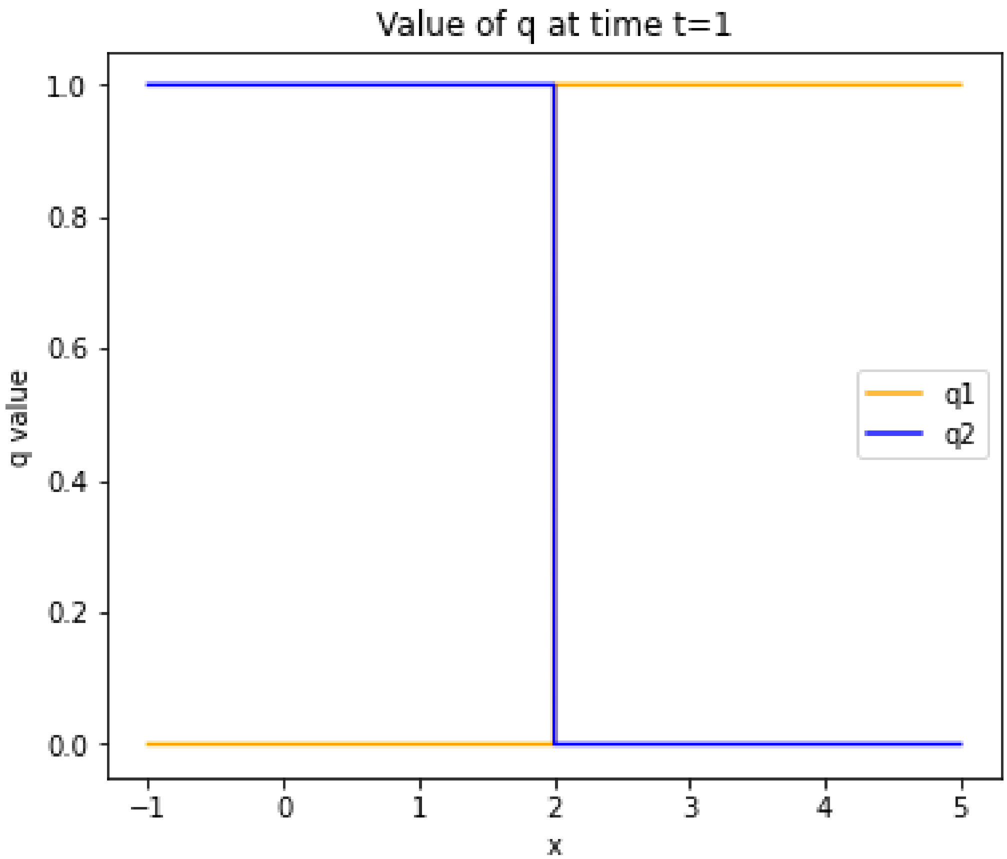

Value of q at time t=1
q1
q2
q value
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1
0
1
2
3
4
5

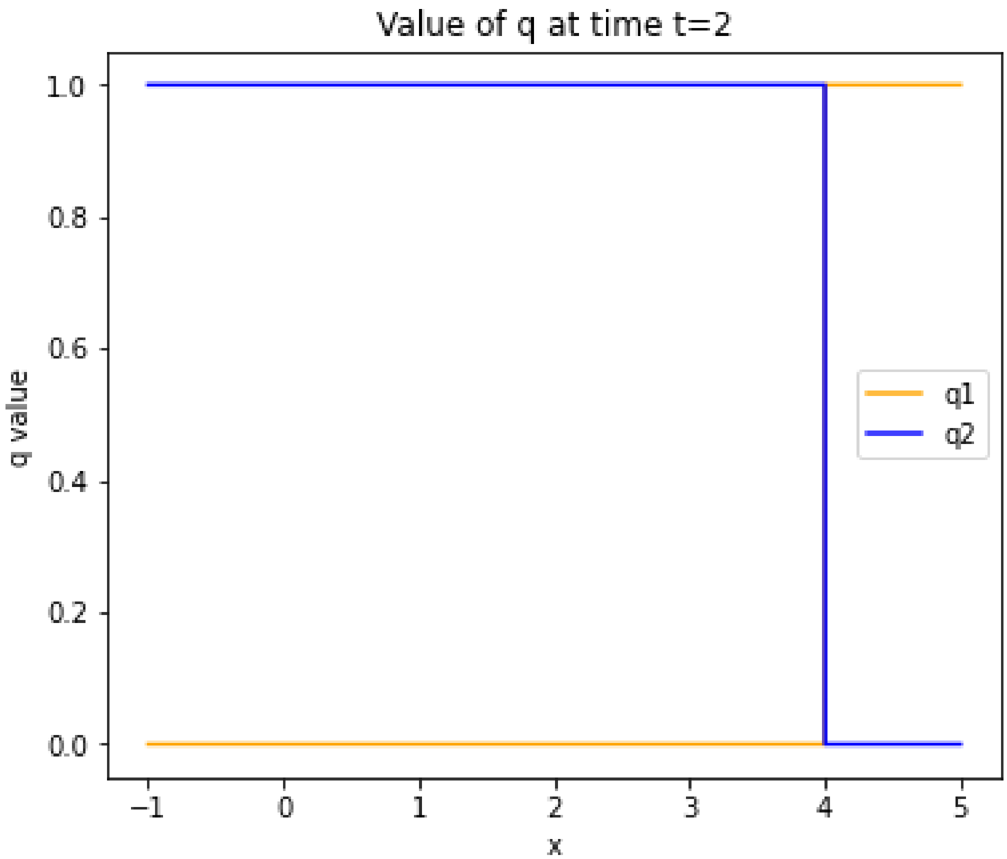


[228]:
```
A = array([[2,0],[0,2]])
qleft = array([0,1])
qright = array([1,0])
qm = solve_Rp(A,qleft,qright)
```

```
Matrix A:
 [[2 0]
 [0 2]]
Eigenvalues of A: [2. 2.]
Matrix R of eigenvectors:
 [[1. 0.]
 [0. 1.]]
Condition number of R is 1
alpha =  [ 1. -1.]
qm =  [1. 1.]
```

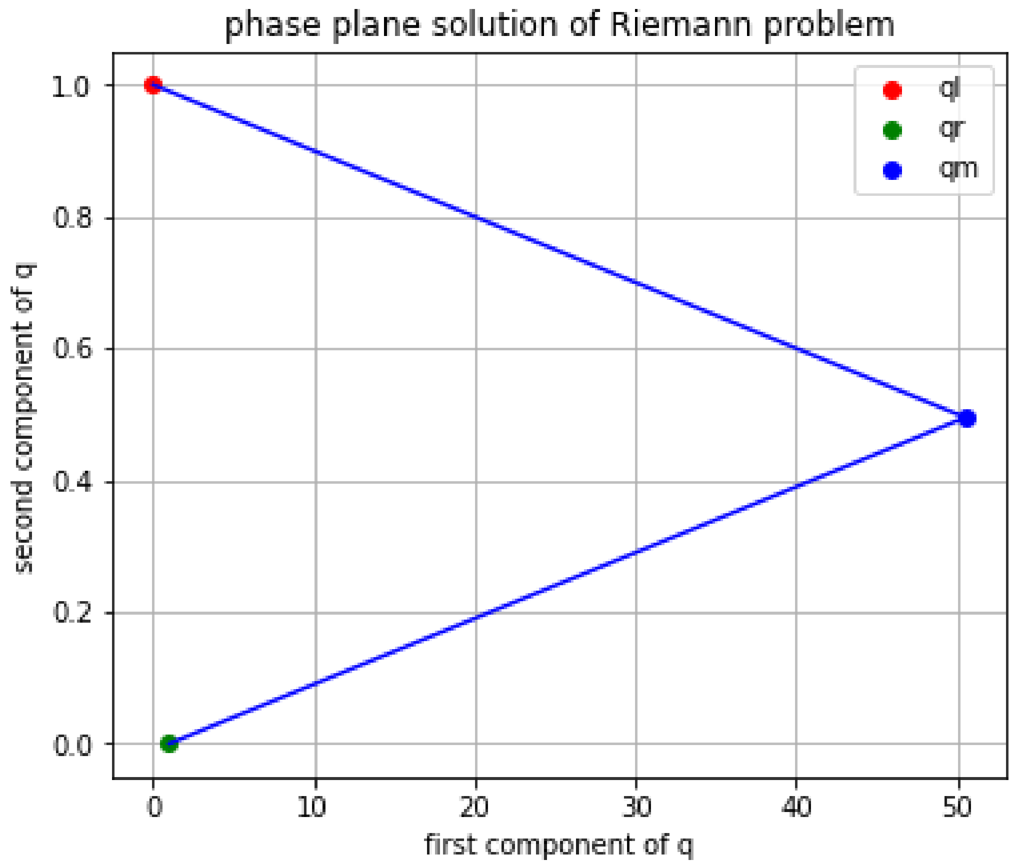

phase plane solution of Riemann problem
1.0
0.8
0.6
0.4
0.2
0.0
second component of q
ql
qr
qm
0.0
0.2
0.4
0.6
0.8
1.0
first component of q

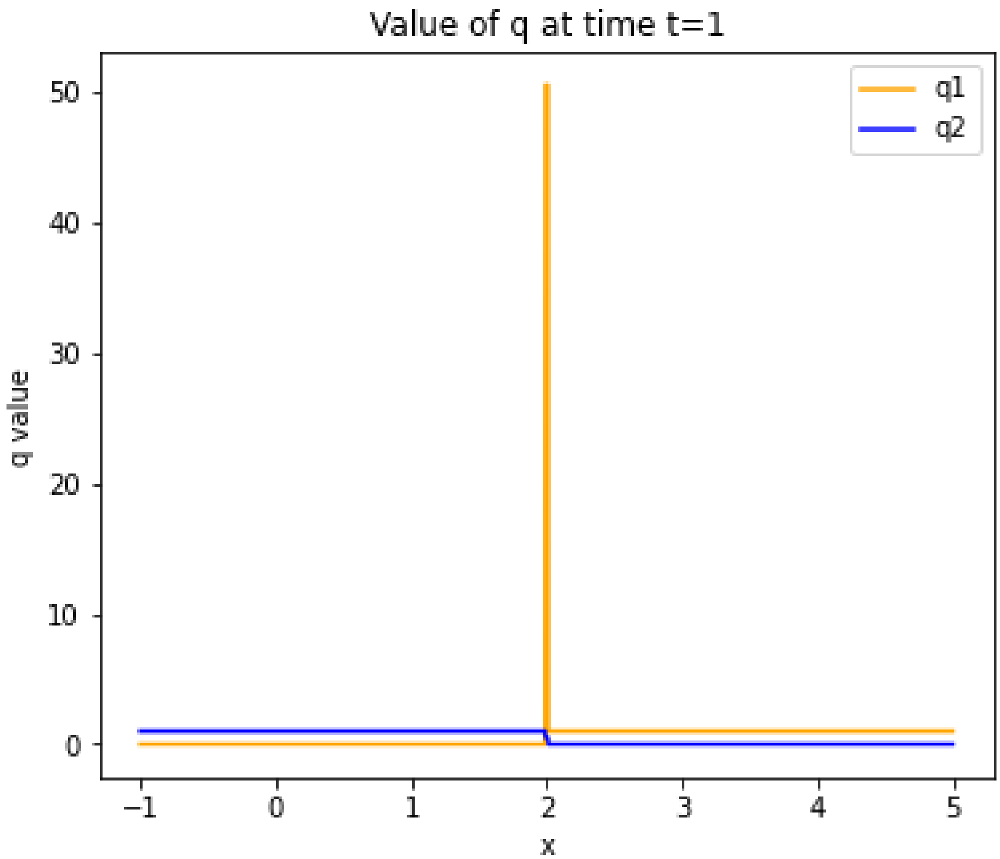

Value of q at time t=1
1.0
0.8
0.6
0.4
0.2
0.0
q value
−1
0
1
2
3
4
5
x
q1
q2



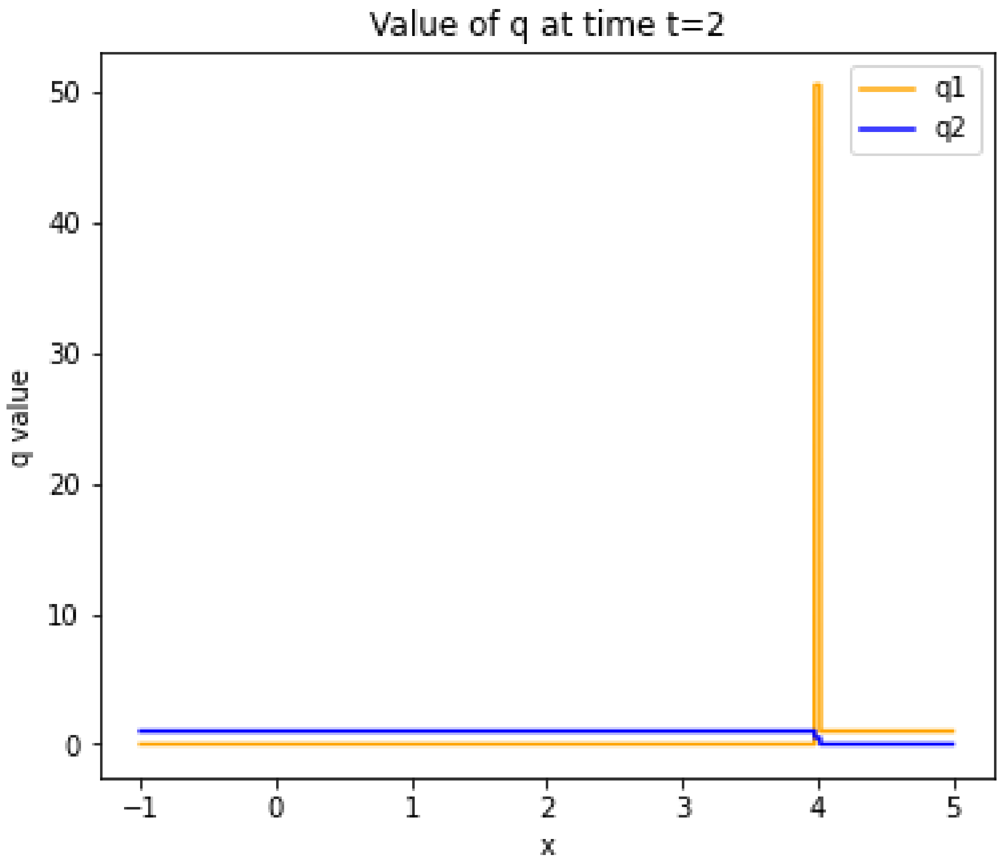
[229]:
```
A = array([[2,1],[.0001,2]])
qleft = array([0,1])
qright = array([1,0])
qm = solve_Rp(A,qleft,qright)
```

```
Matrix A:
 [[2.e+00 1.e+00]
 [1.e-04 2.e+00]]
Eigenvalues of A: [1.99 2.01]
Matrix R of eigenvectors:
 [[-0.99995    0.99995  ]
 [ 0.0099995  0.0099995]]
Condition number of R is 100
alpha =  [-50.50252494 -49.50247494]
qm =  [50.5    0.495]
```


phase plane solution of Riemann problem
ql
qr
qm
1.0
0.8
0.6
0.4
0.2
0.0
second component of q
0
10
20
30
40
50
first component of q

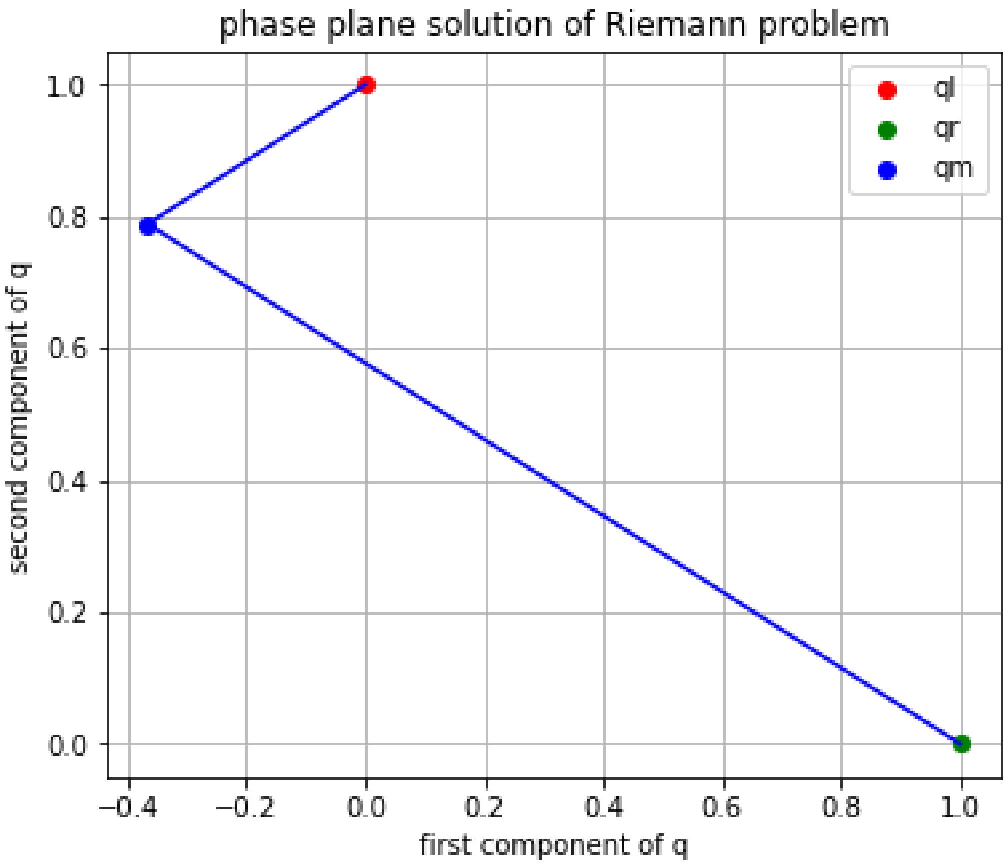


Value of q at time t=1
q1
q2
q value
x
50
40
30
20
10
0
−1
0
1
2
3
4
5

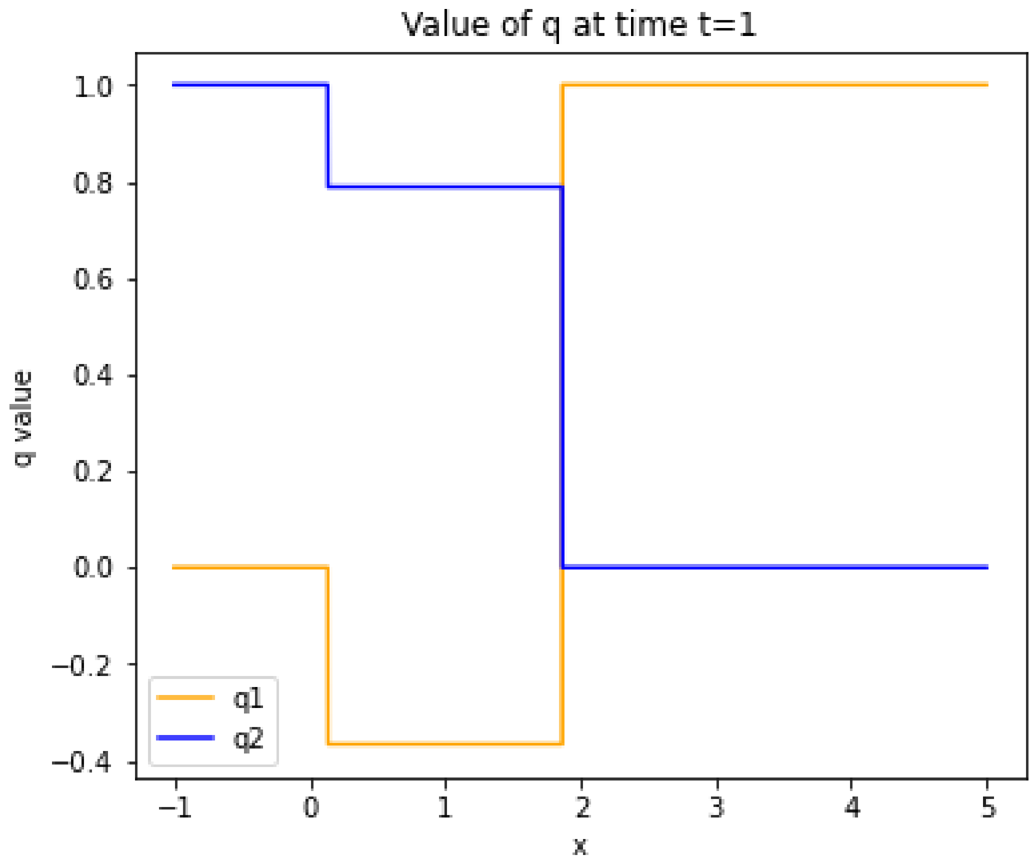


[230]:
```
A = array([[1,-1.5],[-.5,1]])
qleft = array([0,1])
qright = array([1,0])
qm = solve_Rp(A,qleft,qright)
```

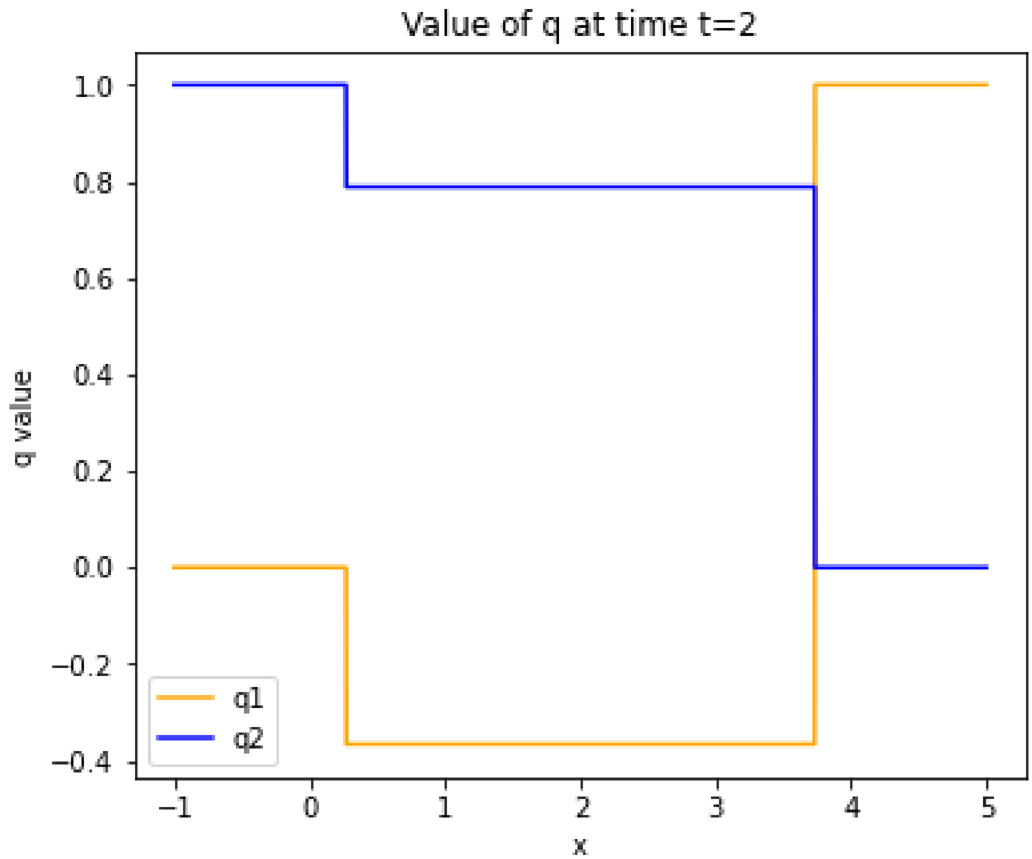
```
Matrix A:
 [[ 1.  -1.5]
 [-0.5  1. ]]
Eigenvalues of A: [0.1339746 1.8660254]
Matrix R of eigenvectors:
 [[ 0.8660254  0.8660254]
 [ 0.5       -0.5      ]]
Condition number of R is 1.73205
alpha =  [-0.42264973  1.57735027]
qm =  [-0.3660254   0.78867513]
```



phase plane solution of Riemann problem
ql
qr
qm
second component of q
first component of q
1.0
0.8
0.6
0.4
0.2
0.0
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

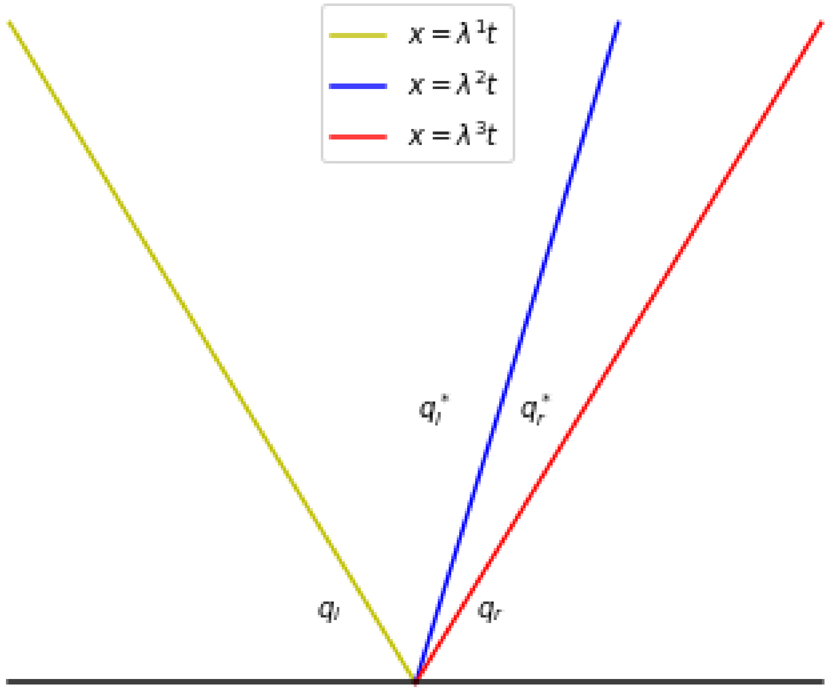


Value of q at time t=1
q value
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−1
0
1
2
3
4
5
x
q1
q2



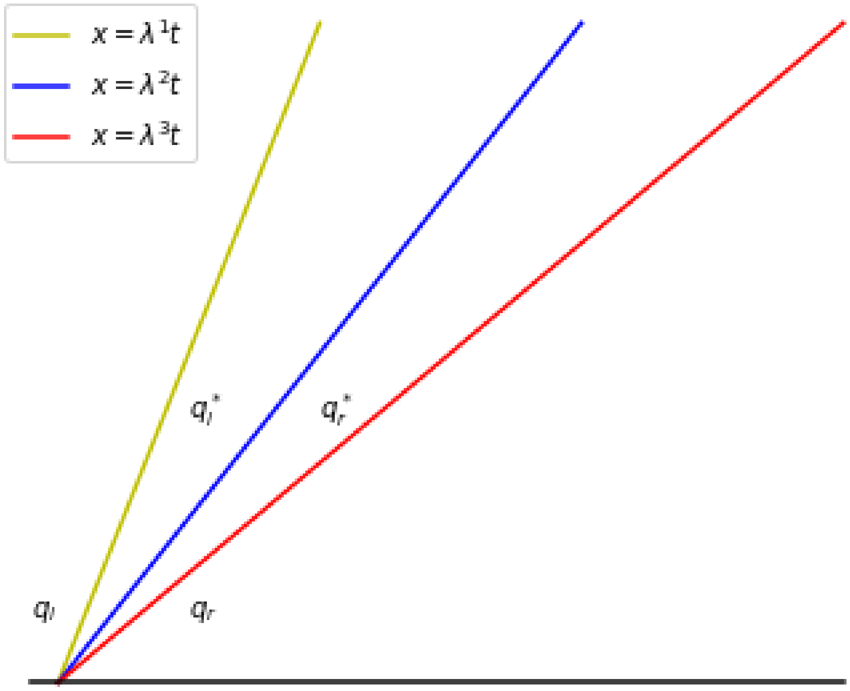
[242]:
```
A = array([[0,0,4],[0,1,0],[1,0,0]])
qleft = array([1,2,0])
qright = array([1,5,1])
xtplane(A,qleft,qright)
```

```
Matrix A:
 [[0 0 4]
 [0 1 0]
 [1 0 0]]
Eigenvalues of A: [-2.  1.  2.]
Matrix R of eigenvectors:
 [[-0.89442719  0.          0.89442719]
 [ 0.          1.          0.        ]
 [ 0.4472136   0.          0.4472136 ]]
Condition number of R is 2
alpha =  [1.11803399 3.         1.11803399]
ql* =  [0.  2.  0.5] and qr* = [0.  5.  0.5]
```



[234]:
```
A = array([[1,0,2],[0,2,0],[0,0,3]])
qleft = array([1,1,1])
qright = array([3,3,3])
xtplane(A,qleft,qright)
```

```
Matrix A:
 [[1 0 2]
 [0 2 0]
 [0 0 3]]
Eigenvalues of A: [1. 2. 3.]
Matrix R of eigenvectors:
 [[1.         0.         0.70710678]
 [0.         1.         0.        ]
 [0.         0.         0.70710678]]
Condition number of R is 2.41421
alpha =  [0.         2.         2.82842712]
ql* =  [1. 1. 1.] and qr* = [1. 3. 1.]
```



$x = \lambda^1 t$
$x = \lambda^2 t$
$x = \lambda^3 t$
q_l^*
q_r^*
q_l
q_r

[ ]: