AMath 574
Homework 4
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Due by 11:00pm on February 9, 2023

For submission instructions, see:

http://faculty.washington.edu/rjl/classes/am574w2023/homework4.html

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Problem #8.3 in the book

Consider the equation

$$q_t + \bar{u} q_x = aq, \qquad q(x,0) = \mathring{q}(x),$$

with solution $q(x,t) = e^{at}\mathring{q}(x - \bar{u}t)$.

(a) Show that the method

$$Q_i^{n+1} = Q_i^n - \frac{\bar{u}\Delta t}{\Delta x}(Q_i^n - Q_{i-1}^n) + \Delta t a Q_i^n$$

is first-order accurate for this equation by computing the local truncation error.

(b) Show that this method is Lax-Richtmyer-stable in the 1-norm provided $0 \leq \bar{u}\Delta t/\Delta x \leq 1$, by showing that a bound of the form

$$\|\mathcal{N}(E^n)\| \leq (1 + \alpha\Delta t)\|E^n\|$$

holds. Note that when $a > 0$ the numerical solution is growing exponentially in time (as is the true solution) but the method is stable and convergent at any fixed time.

(c) Show that this method is TVB. Is it TVD?

Solution:

(a) Say that

$$Q_i^{n+1} = \tilde{\mathcal{N}}(Q_i^n, Q_{i-1}^n) = \mathcal{N}(Q^n).$$

Local truncation error is given by

$$\|q(x, t + \Delta t) - \mathcal{N}(q(x,t), q(x - \Delta x, t))\|.$$

We now work this out using Taylor

$$\begin{aligned}
q(x, t+\Delta t) - \mathcal{N}(q(x,t), q(x-\Delta x, t)) &= q(x,t) + q_t(x,t)\Delta t - q(x,t) \\
&\quad + \frac{\bar{u}\Delta t}{\Delta x}(q(x,t) - q(x - \Delta x, t)) - \Delta t\, q(x,t) + \mathcal{O}(\Delta t^2) \\
&= q_t(x,t)\Delta t + \bar{u}\Delta t q_x(x,t) - \bar{u}\Delta t\Delta x q_{xx}/2 - \Delta t q(x,t) + \mathcal{O}(\Delta t^2, \Delta x^2) \\
&= \Delta t(q_t + \bar{u} q_x - aq) - \bar{u}\Delta t \Delta x q_{xx}/2 + \mathcal{O}(\Delta t, \Delta x^2) \\
&= -\bar{u}\Delta t\Delta x q_{xx}/2 + \mathcal{O}(\Delta t^2, \Delta x^2)
\end{aligned}$$

Therefore we see that it's first-order accurate.

(b) We have that $\mathcal{N}(E^n) = E_i^n - \frac{u\bar{\Delta}t}{\Delta x}(E_i^n - E_{i-1}^n) + \Delta t\ aE_i^n \leq E_{i-1}^n + \Delta t\ aE_i^n$. if $E_i^n - E_{i-1}^n < 0$ and $\mathcal{N}(E_i^n) = E_i^n + \Delta t a E_i^n$ otherwise. But it's both the same eventually. We thus have

$$\begin{aligned}\|\mathcal{N}(E^n)\|_1 &= \Delta x \sum_i |E_i^n - \frac{u\bar{\Delta}t}{\Delta x}(E_i^n - E_{i-1}^n) + \Delta t\ aE_i^n| \\ &\leq \Delta x \sum_i |E_{i-1}^n| + |a|\Delta t \Delta x \sum_i |E_i^n| = (1 + |a|\Delta t)\Delta x \sum_i |E_i^n| \\ &= (1 + |a|\Delta t)\|E^n\|_1\end{aligned}$$

In conclusion, it's Lax-Richtmyer-stable in the 1-norm provided $|\bar{u}\Delta t/\Delta x| \leq 1$.

(c) We start of by saying that a numerical method is total-variation bounded (TVB) if, for any data Q^0, with $TV(Q^0) < \infty$, and time T, there is a constant $R > 0$ and a value $\Delta t_0 > 0$ such that

$$TV(Q^n) \leq R$$

for all $n\Delta t \leq T$ when $\Delta t < \Delta t_0$.

Let's calculate $TV(Q^{n+1})$ for when 1-norm stability holds, thus $(1 - \frac{\bar{u}\Delta x}{\Delta t} > 0)$:

$$\begin{aligned}TV(Q^{n+1}) &= \sum_i |Q_i^{n+1} - Q_{i-1}^{n+1}| \\ &= \sum_i |Q_i^n - Q_{i-1}^n - \frac{\bar{u}\Delta x}{\Delta t}(Q_i^n - Q_{i-1}^n) + \frac{\bar{u}\Delta x}{\Delta t}(Q_{i-1}^n - Q_{i-2}^n) + \Delta t\ a(Q_i^n - Q_{i-1}^n)| \\ &\leq (1 + \Delta t\ |a| - \frac{\bar{u}\Delta x}{\Delta t}) \sum_i |Q_i^n - Q_{i-1}^n| + \frac{\bar{u}\Delta x}{\Delta t} \sum_i |Q_i^n - Q_{i-1}^n| \\ &= (1 + \Delta t\ |a|)TV(Q^n) \\ &\leq (1 + \frac{\Delta t n\ |a|}{n})^n TV(Q^0)\end{aligned}$$

Since $TV(Q^0) < \infty$ we have that for n finite that this is definitely bounded. For $n \to \infty$ we have

$$\lim_{n\to\infty} TV(Q^{n+1}) \leq \lim_{n\to\infty}(1 + \frac{\Delta t n\ |a|}{n})^n TV(Q^0) \leq e^{|a|T} TV(Q^0).$$

Since $(1 + \frac{|a|T}{n})^n$ is always rising we have that

$$TV(Q^n) \leq e^{|a|T} TV(Q^0) < \infty,$$

for all n. Therefore it is TVB.

Looking at if it's TVD, even if we have this best, in my belief, case scenario where $|\bar{u}\Delta t/\Delta x| < 1$, we have that

$$TV(Q^{n+1}) \leq (1 + \Delta t\ |a|)TV(Q^n).$$

I cannot see a way to further better this inequality. Also, the bound would only get worse if $1 < \frac{\bar{u}\Delta x}{\Delta t}$.

In conclusion, the method is TVB but not TVD

Problem #8.5 in the book

Prove Harten's Theorem:
Consider a general method of the form

$$Q_i^{n+1} = Q_i^n - C_{i-1}^n(Q_i^n - Q_{i-1}^n) + D_i^n(Q_{i+1}^n - Q_i^n)$$

over one time step, where the coefficients C_{i-1}^n and D_i^n are arbitrary values (which in particular may depend on values of Q^n in some way, i.e., the method may be nonlinear).
Then

$$TV(Q^{n+1}) \leq TV(Q^n)$$

provided the following conditions are satisfied:

$$\begin{aligned} C_{i-1}^n &\geq 0 \forall i, \\ D_i^n &\geq 0 \forall i, \\ D_i^n + C_i^n &\leq 1 \forall i, \end{aligned}$$

Hint: Note that

$$\begin{aligned} Q_{i+1}^{n+1} - Q_i^{n+1} = &(1 - C_i^n - D_i^n)(Q_{i+1}^n - Q_i^n) + D_{i+1}^n(Q_{i+2}^n - Q_{i+1}^n) \\ &+ C_{i-1}^n(Q_i^n - Q_{i-1}^n) \end{aligned}$$

Sum $|Q_{i+1}^{n+1} - Q_i^{n+1}|$ over i and use the nonnegativity of each coefficient, as in the stability proof of Section 8.3.4.

Solution: As a reminder:

$$TV(Q) = \sum_{i \in \mathbb{N}} |Q_i - Q_{i-1}| = \sum_{i \in \mathbb{N}} |Q_{i+1} - Q_i|.$$

Looking at the hint, this turns out to be rather easy. Since the conditions are satisfied, we know that, because of the last condition:

$$1 - (C_i^n + D_i^n) \geq 0, \quad \forall i.$$

Therefore, also because of the first two conditions:

$$\begin{aligned} TV(Q^{n+1}) &= \sum_{i \in \mathbb{N}} |Q_{i+1}^{n+1} - Q_i^{n+1}| \\ &\leq \sum_{i \in \mathbb{N}} (1 - C_i^n - D_i^n)|Q_{i+1}^n - Q_i^n| + \sum_{i \in \mathbb{N}} D_{i+1}^n |Q_{i+2} - Q_{i+1}^n| + \sum_{i \in \mathbb{N}} C_{i-1}^n |Q_i^n - Q_{i-1}^n| \\ &= \sum_{i \in \mathbb{N}} (1 - C_i^n - D_i^n)|Q_{i+1}^n - Q_i^n| + \sum_{i \in \mathbb{N}} D_i^n |Q_{i+1} - Q_i^n| + \sum_{i \in \mathbb{N}} C_i^n |Q_{i+1}^n - Q_i^n| \\ &= \sum_{i \in \mathbb{N}} |Q_{i+1}^n - Q_i^n| \\ &= TV(Q^n) \end{aligned}$$

In conclusion

$$TV(Q^{n+1}) \leq TV(Q^n),$$

and thus, if the conditions are satisfied, this general method is TVD.

Problem #8.6 in the book

Use the method of Section 8.3.4. to show that the method,

$$\begin{aligned} Q_i^{n+1} &= Q_i^n - (Q_i^n - Q_{i-1}^n) - \left(\frac{\bar{u}\Delta t - \Delta x}{\Delta x}\right)(Q_{i-1}^n - Q_{i-2}^n) \\ &= Q_{i-1}^n - \left(\frac{\bar{u}\Delta t}{\Delta x} - 1\right)(Q_{i-1}^n - Q_{i-2}^n) \end{aligned}$$

is stable in the 1-norm for $\Delta x \leq \bar{u}\Delta t \leq 2\Delta x$.

Solution: Since $\Delta x \leq \bar{u}\Delta t \leq 2\Delta x$, we have that $0 \leq \dfrac{\bar{u}\Delta t}{\Delta x} - 1 \leq 1$. It's positive and it's less than or equal to one. Therefore, we have that

$$\begin{aligned} \|Q^{n+1}\|_1 &= \Delta x \sum_{i\in\mathbb{N}} |Q_i^{n+1}| \\ &= \Delta x \sum_{i\in\mathbb{N}} |Q_{i-1}^n - \left(\frac{\bar{u}\Delta t}{\Delta x} - 1\right)(Q_{i-1}^n - Q_{i-2}^n)| \\ &\leq \Delta x \sum_{i\in\mathbb{N}} |Q_{i-1}^n| + \left(\frac{\bar{u}\Delta t}{\Delta x} - 1\right)|Q_{i-1}^n| + \Delta x \sum_{i\in\mathbb{N}} |Q_{i-2}^n| \\ &= \bar{u}\Delta t \sum_{i\in\mathbb{N}} |Q_i^n| + \Delta x \sum_{i\in\mathbb{N}} |Q_i^n| \\ &\overset{\Delta t \to 0}{=} \|Q^n\|_1. \end{aligned}$$

In conclusion, for $\Delta t \to 0$ (and thus also $\Delta x \to 0$ such that the other characteristics hold), we have that

$$\|Q^{n+1}\|_1 \leq \|Q^n\|_1,$$

and is thus stable in the one norm.

Problem #11.1 in the book

Show that in solving the scalar conservation law $q_t + f(q)_x = 0$ with smooth initial data $q(x, 0)$, the time at which the solution "breaks" is given by

$$T_b = \frac{-1}{\min_x[f''(q(x,0))q_x(x,0)]}$$

if this is positive. If this is negative, then characteristics never cross. **Hint:** Use $q(x,t) = q(\xi(x,t),0)$ from

$$q(x,t) = \mathring{q}(\xi),$$

differentiate this with respect to x, and determine where q_x becomes infinite. To compute ξ_x, differentiate the equation

$$x = \xi + f'(\mathring{q}(\xi))t,$$

with respect to x.

Solution: First we calculate q_x :

$$q_x(x,t) = \mathring{q}'(\xi)\xi_x.$$

For this we need ξ_x, which is

$$1 = \xi_x + tf''(\mathring{q}(\xi))\mathring{q}'(\xi)\xi_x \Rightarrow \xi_x = \frac{1}{1 + tf''(\mathring{q}(\xi))\mathring{q}'(\xi)}.$$

Thus

$$q_x(x,t) = \frac{\mathring{q}'(\xi)}{1 + tf''(\mathring{q}(\xi))\mathring{q}'(\xi)}.$$

The solution "breaks", or isn't smooth anymore, once q_x no longer exists or is infinity. This is clearly when the denominator is 0, thus

$$1 + tf''(\mathring{q}(\xi))\mathring{q}'(\xi) = 0 \Rightarrow t = \frac{-1}{f''(\mathring{q}(\xi))\mathring{q}'(\xi)}.$$

The solutions "breaks" once even at one x point this is true. Since we want to know when it starts breaking (thus the minimum) and that $q(x,t) = q(\xi, 0)$, we can use that to get back to an x on the initial condition. This then gives us

$$T_b = \frac{-1}{\min_x[f''(q(x,0))q_x(x,0)]}$$

Problem #11.3 in the book

For a general smooth scalar flux functions $f(q)$, show by Taylor expansion of

$$s = \frac{f(q_r) - f(q_l)}{q_r - q_l},$$

that the shock speed is approximately the average of the characteristic speed on each side,

$$s = \frac{1}{2}[f'(q_l) + f'(q_r)] + \mathcal{O}(|q_r - q_l|^2).$$

Solution: Let's Taylor approximate s first by approximating around q_r:

$$s = \frac{f(q_r) - f(q_l)}{q_r - q_l} = \frac{f(q_r) - f(q_r) - f'(q_r)(q_l - q_r) - f''(q_r)(q_l - q_r)^2/2}{q_r - q_l} + \mathcal{O}((q_l - q_r)^2).$$

Afterwards we approximate s by approximating around q_l:

$$s = \frac{f(q_r) - f(q_l)}{q_r - q_l} = \frac{f(q_l) + f'(q_l)(q_r - q_l) + f''(q_l)(q_r - q_l)^2/2 - f(q_l)}{q_r - q_l} + \mathcal{O}((q_l - q_r)^2).$$

summing these gives us

$$2s = f'(q_r) + f'(q_l) + \frac{1}{2}(q_r - q_l)(f''(q_l) - f''(q_r)) + \mathcal{O}((q_l - q_r)^2).$$

Let's approximate $f''(q_r)$ around q_l:

$$\begin{aligned}
s &= \frac{1}{2}[f'(q_r) + f'(q_l)] + \frac{1}{4}(q_r - q_l)(f''(q_l) - f''(q_l) - (q_r - q_l)f'''(q_l) + \mathcal{O}((q_r - q_l)^2)) + \mathcal{O}((q_l - q_r)^2) \\
&= \frac{1}{2}[f'(q_r) + f'(q_l)] - \frac{1}{4}f'''(q_l)(q_l - q_r)^2 + \mathcal{O}((q_l - q_r)^2) \\
&= \frac{1}{2}[f'(q_r) + f'(q_l)] + \mathcal{O}((q_l - q_r)^2)
\end{aligned}$$

In conclusion:

$$s = \frac{1}{2}[f'(q_l) + f'(q_r)] + \mathcal{O}((q_l - q_r)^2)$$

The exercises below require determining the exact solution to a scalar conservation law with particular initial data. In each case you should sketch the solution at several instants in time as well as the characteristic structure and shock-wave locations in the x–t plane.
You may wish to solve the problem numerically by modifying the CLAWPACK codes for this chapter in order to gain intuition for how the solution behaves and to check your formulas.

Problem #11.5 in the book

Determine the exact solution to Burgers' equation for $t > 0$ with initial data

$$\mathring{u}(x) = \begin{cases} 2 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the rarefaction wave catches up to the shock at some time T_c. For $t > T_c$ determine the location of the shock by two different approaches:

(a) Let $x_s(t)$ represent the shock location at time t. Determine and solve an ODE for $x_s(t)$ by using the Rankine-Hugoniot jump condition

$$s = \frac{f(q_r) - f(q_l)}{q_r - q_l},$$

which must hold across the shock at each time.

(b) For $t > T_c$ the exact solution is triangular-shaped. Use conservation to determine $x_s(t)$ based on the area of this triangle. Sketch the corresponding "over-turned" solution, and illustrate the equal-area rule

Solution: Let's start off by saying that the initial shock location is at $x = 1$ ($x_s(0) = 1$) for both ways of solving it. We also have the Burgers' equation:

$$u_t + (\frac{1}{2}u^2)_x = 0,$$

with flux $f(u) = \frac{1}{2}u^2$

(a) First note that There is a rarefaction wave at $x = 0$ and shock at $x = 1$. At first this shock has speed

$$x_s'(t) = s(t) = \frac{2+0}{2} = 1.$$

Thus $x_s = t$. The right side of the rarefaction fan travels at speed 2, because we have the burger's equation ($f'(u) = u$) and $\mathring{u}(0) = 2$. since the two ends of the discontinuity are 1 apart, the rarefaction wave catches up to the shock at $t = 1$. At $t = 1$ we have that the shock is at $x_s = 2$. We thus have initial condition $x_s(1) = 2$. Now that our rarefactionwave and shockwave have collided, we have a new q_l. We want to calculate this, we do this by knowing that inside the rarefaction wave we have that

$$f'(\tilde{u}(x/t)) = x/t \Rightarrow \tilde{u}(x/t) = x/t.$$

Therefore, our $q_l = x/t$ at the new discontinuity, this now gives us the new Rankine-Hugoniot jump conditions:

$$x_s'(t) = s(t) = \frac{x/t}{2}.$$

Solving this ODE with Wolfram Mathematica, gives

$$x_s(t) = c_1\sqrt{t},$$

using the initial condition $x_s(1) = 2$, we get that

$$x_s(t) = 2\sqrt{t}.$$

In conclusion: The location of the shock at time t can be found at

$$x_s(t) = 2\sqrt{t}, \quad \text{for } t > 1$$

(b) We will always have that the are needs to be 2 under the curve. The formula for the area of a triangle is given by $\frac{u(x_s)x_s}{2}$, since our triangle has one fixed point $(0, 0)$ and it's a perpendicular triangle. We thus always want

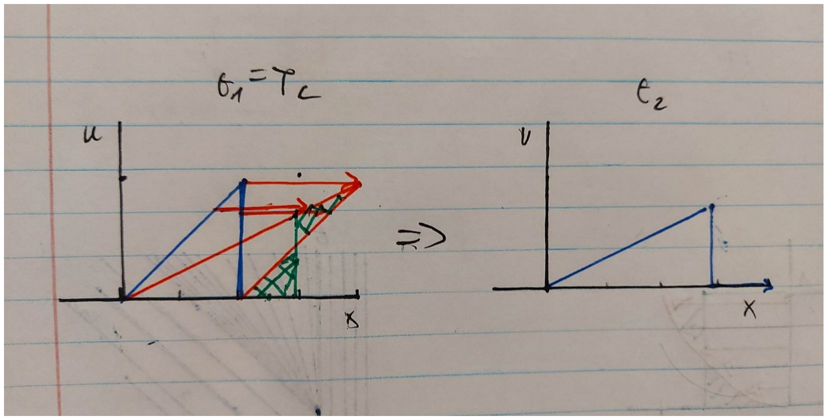
$$u(x_s(t))x_s(t) = 4.$$

We also must have that this $(x_s(t), u(x_s(t)))$ must be on the line caused by the rarefaction wave, which is known to be x_s/t. Thus $x_s/t = u(x_s(t))$. Therefore

$$\begin{cases} u(x_s(t))x_s(t) & = 4 \\ x_s/t & = u(x_s(t)) \end{cases} \Rightarrow \begin{cases} x_s^2(t)/t & = 4 \\ x_s/t & = u(x_s(t)) \end{cases} \Rightarrow \begin{cases} x_s(t) & = 2\sqrt{t} \\ u(x_s(t)) & = 2\sqrt{t}/t \end{cases}.$$

This gives us the solution we expected and also the height of the functions at the shock.

In blue we have the solution at the collision of rarefaction wave and shock, this goes to the red solution before making it a function, green then displays the equal area rule where we have to add the area back equal to what we are subtracting:



It's not unreasonable that the height of the triangle will go down. This since we are most likely cutting off the last section of the triangle which gives it height, plus the left point of the triangle is fixed at $(0,0)$ while the right hand side is moving to the right. The conservation property that we want to keep must thus reduce the height of the triangle. Note that this height is equal to $u(x_s) = 4/x_s$.

Problem #11.8 in the book

Consider the scalar conservation law $u_t + (e^u)_x = 0$. Determine the exact solution with the following sets of initial data:

(a)
$$\mathring{u}(x) = \begin{cases} 1 & \text{if } x < 0, \\ 0 & \text{if } x > 0. \end{cases}$$

(b)
$$\mathring{u}(x) = \begin{cases} 0 & \text{if } x < 0, \\ 1 & \text{if } x > 0. \end{cases}$$

(c)
$$\mathring{u}(x) = \begin{cases} 2 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Hint: Use the approach outlined in the previous exercise (a).

Solution:

(a) It is clear that we have a shock since $q_l > q_r$. Here the jump speed is rather easy to calculate, we calculate the Rankine-Hugoniot jump condition:

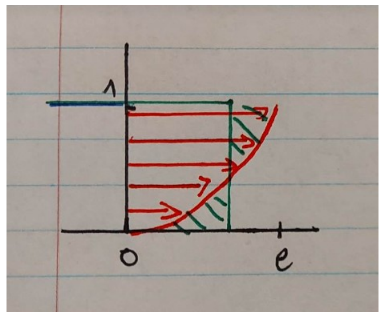
$$x_s'(t) = \frac{f(q_r) - q(q_l)}{q_r - q_l} = e - 1,$$

with x_s the point of shock. This value seems reasonable, if we draw this case out and use the a handwavy approach to the method where the areas need to cancel out:

Therefore we have that $x_s(t) = (e-1)t$ is the x value where the jump happens at time t. Thus, the solution here is

$$u(x,t) = \begin{cases} 1 & \text{if } x < (e-1)t, \\ 0 & \text{if } x > (e-1)t. \end{cases}$$

An attempt at drawing this gives (blue is solution at last time, green is equal-area rule and red is what it should go to if we're not dealing with math and physics):



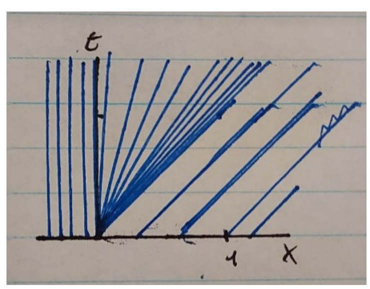
(b) Here we have a rarefaction wave ($q_l < q_r$). For this we want to find

$$f'(\tilde{u}(x/t)) = x/t \Rightarrow e^{\tilde{u}(x/t)} = x/t \Rightarrow \tilde{u}(x/t) = \ln(x/t).$$

Note that we only look at positive time and the rarefaction happens after $x > 0$. This thus exists. We now have that

$$u(x/t) = \begin{cases} 0 & \text{for } x/t \leq 1 \\ \ln(x/t) & \text{for } 1 \leq x/t \leq e \\ 1 & \text{for } e \leq x/t \end{cases} \Rightarrow u(x,t) = \begin{cases} 0 & \text{for } x \leq t \\ \ln(x) - \ln(t) & \text{for } t \leq x \leq et \\ 1 & \text{for } et \leq x \end{cases}.$$

An attempt at drawing this gives:



(c) Here we once again have that the rarefaction will catch up to the shock. We have that the speed of the right side of the rarefaction fan, before interaction, will have a max speed of e^2 and the shock, according to the Rankine-Hugoniot jump condition will have

$$s = \frac{e^2-1}{2} \Rightarrow x_s = \frac{e^2-1}{2}t.$$

Thus they will collide when

$$e^2 t = 1 + \frac{e^2 - 1}{2} t \Rightarrow t = \frac{2}{e^2 + 1}.$$

This cross will happen when $x = \frac{2e^2}{e^2+1}$. Note that we added 1 to the shock since it starts at $x = 1$. Therefore we have initial condition $x_s(\frac{2}{e^2+1}) = \frac{2e^2}{e^2+1}$. After this we have a new q_l. For this we will need to look at what happens in the rarefaction wave (as seen before):

$$\tilde{u} = \ln(x/t).$$

Therefore we have that

$$x_s'(t) = s(t) = \frac{x_s/t - 1}{\ln(x_s/t)}.$$

Therefore we can find x_s by solving the ODE

$$\begin{cases} x_s'(t) = \dfrac{x_s/t - 1}{\ln(x_s/t)} \\ x_s\left(\dfrac{2}{e^2+1}\right) = \dfrac{2e^2}{e^2+1} \end{cases}$$

Programming Problem.

The notebook `$AM574/notebooks/advection_highres.ipynb` illustrates an implementation of Lax-Wendroff and some high-resolution limiters method for advection, using the wave propagation form that can be generalized to other problems. A rendered version can be viewed at http://faculty.washington.edu/rjl/classes/am574w2023/_static/advection_highres.html

The notebook `$AM574/homework/hw4/scalar_highres.ipynb` is a similar notebook for a more general scalar conservation law, but so far it only implements the first order Godunov method (with or without the entropy fix for transonic rarefaction waves). A rendered version can be viewed at http://faculty.washington.edu/rjl/classes/am574w2023/_static/scalar_highres.html

Following the approach used in the advection notebook, update the scalar notebook to include Lax-Wendroff and the minmod limiter methods.

Also add the MC limiter as another option.

Test your code on the examples in the notebook, many of which do not produce plots currently. Note that the animations at the end of the notebook are the ones that I used in lecture FVMHP14 for Monday 2/6/23. The static plots produced earlier are the same examples but at fixed times (in case you have problems with the animations). So you might want to rewatch the end of that video to see what plots are expected in each case.

As an additional test, try modifying the code in the notebook to solve the scalar problem from #11.5 in the book with $f(q) = \exp(q)$, and for initial data

$$q(x,0) = \begin{cases} 0 & \text{if } x < 0.5 \\ 5 & \text{if } x \geq 0.5 \end{cases}$$

on the domain $0 \leq x \leq 1$ up to time $t = 0.03$. Check that the numerical solution with the minmod method agrees well with the Osher solution for this case by plotting them together. Use the same grid with 50 cells but note that you will need to choose an appropriate time step for the method to be stable.

Also note that this problem has no sonic point, so you should set `efix = False` and then `qsonic` could be set to anything.

Solution: My code is submitted alongside this document. In there we have the problems that were ran during the 14th video, they seems correct. I also ran the scalar problem of 11.5. with the minmod function. When $\Delta t > .034\Delta x$ we see that the approximation does not fit the curve well, it even goes down and then up. At $\Delta t \approx .034\Delta x$ it's pretty much just a shockwave and when $\Delta t < .034\Delta x$ it actually start to approximate the solution. For big Δt, we see that it starts by following the curving and afterwards it has a jump. We get a good approximation when $\Delta t = 0.005\Delta x$, it does need to compute a lot and the not smooth starting jump has been smoothed out, which is to be expected. It won't show on the html, therefore we show $\Delta t = 0.01\Delta x$, which still has a jump.