AMATH 481/581 - Autumn 2022

Homework #1 presentation skill ex. 3

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As a basis, Figure 1 shows the approximation of the solutions to the Fitzhugh model with no interaction. Here we note that v_1 is significantly higher than v_2 and the same holds for w_1 and w_2.

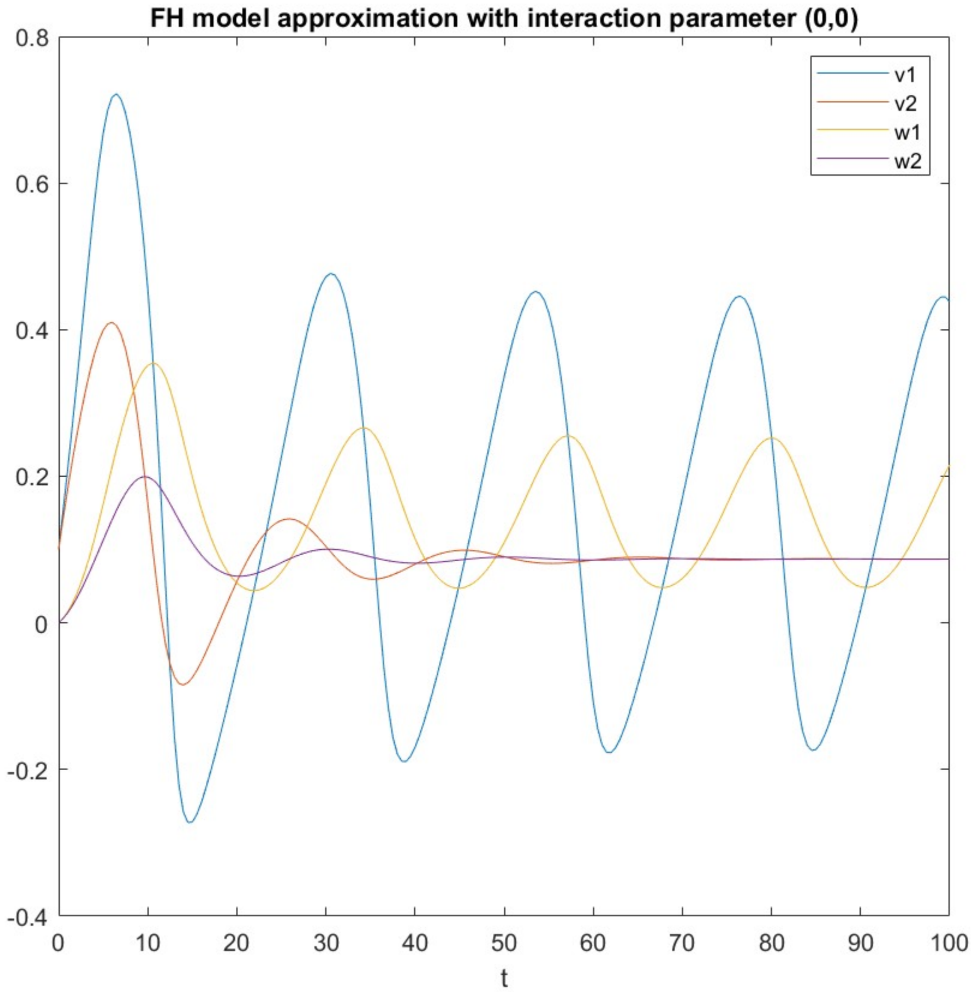

Figure 1: Numerical approximation of the Fitzhugh model using a variable-order implicit multi-step method with no interaction

Next, the approximations to the solutions to the Fitzhugh model with interaction can be seen. Note that the Fitzhugh model is given by,

$$
\begin{aligned}
\frac{dv_1}{dt} &= -v_1^3 + (1+a_1)v_1^2 - a_1 v_1 - w_1 + I + d_{12}v_2,\\
\frac{dw_1}{dt} &= bv_1 - cw_1,\\
\frac{dv_2}{dt} &= -v_2^3 + (1+a_2)v_2^2 - a_2 v_2 - w_2 + I + d_{21}v_1,\\
\frac{dw_2}{dt} &= bv_2 - cw_2.
\end{aligned}
$$

The interaction parameters are always given by (d_{12}, d_{21}). The differentials with respect to time should always be seen as how a certain function changes with respect to time. Since the interaction parameters are the only terms deciding the interaction between v_1 and v_2, these are detrimental. There is no explicit interaction between w_1 and w_2, but they of course change as v_1 and v_2 change. The numerical approximations can be seen in Figure 2.

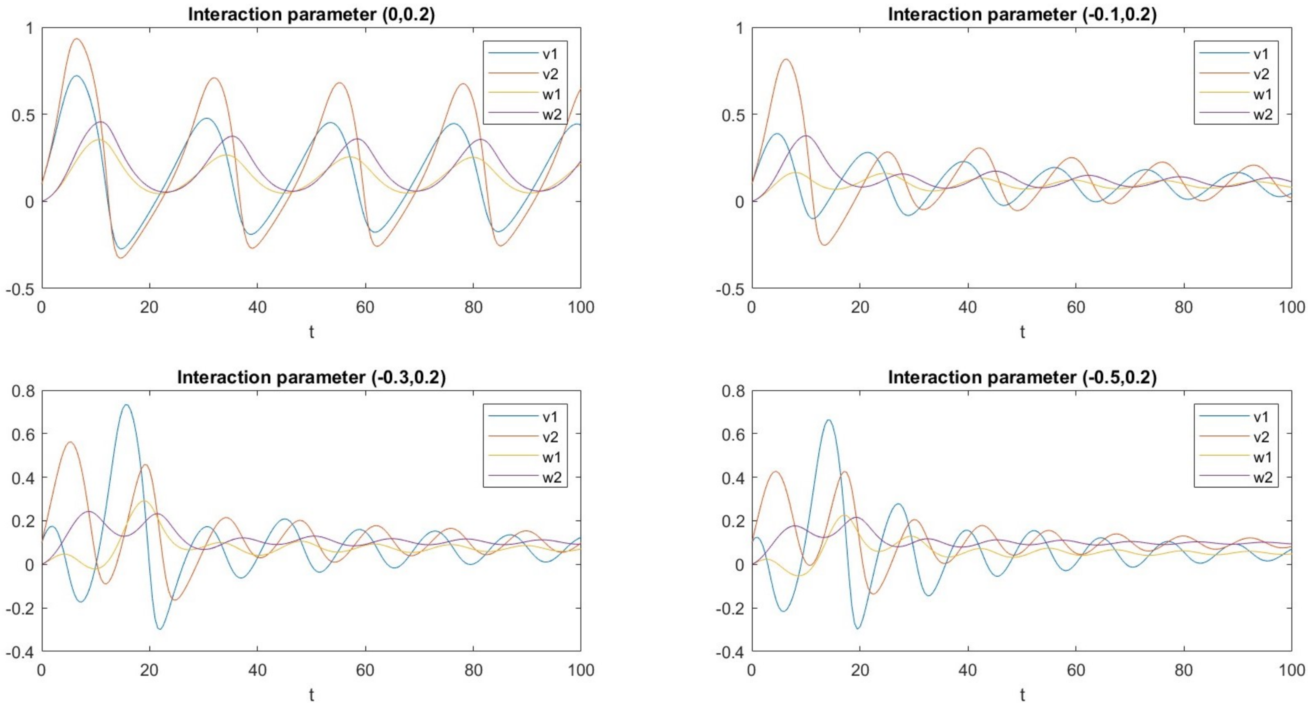

Figure 2: Multiple numerical approximations of the Fitzhugh model using a variable-order implicit multi-step method with different interactions

The first figure of Figure 2 shows the case where there is no interaction coming from v_2, since $d_{12} = 0$, but there is interaction from v_1 to v_2. This is clear as v_1 remains unchanged, and thus w_1 too, and v_2 seems to have grown in amplitude. This can be further explained by the Fitzhugh model since v_1 now adds to the change of v_2 when it is positive and subtracts from it when it is negative. At this moment both v_1 and v_2 seem to be positive and negative at the same moment, thus amplifying the amplitude.

The second figure shows interaction between both voltages. Here v_1 still adds (the same amount) to the change of v_2, however, v_2 now subtracts to the change of v_1. As a result, the functions no longer are periodic, due to an increase in v_1 causing an increase in v_2 causing an decrease in v_1 thus causing a difference in growth. It is no longer the case that v_1 and v_2 have the same sign. The last two statements were caused by the different signs of the interaction terms, this leads me to believe that all function would dampen out because of it and this can be clearly seen on the second figure. Further proof can be suggested if we plot the approximations of the solutions using only positive interaction parameters, as seen on the figure to Figure 3.

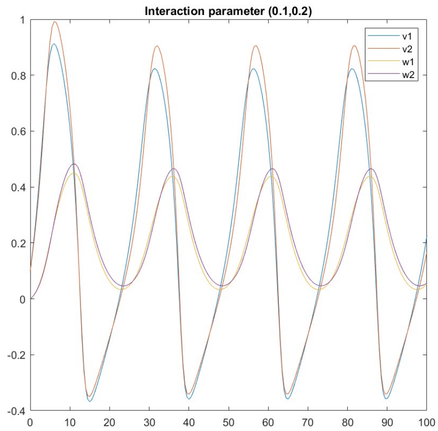

Figure 3: Numerical approximation of the Fitzhugh model using a variable-order implicit multi-step method with positive interaction

The last two figures of Figure 2 show what a change of magnitude in interaction parameter does to the solution. It seems to be a bit volatile which is to be expected from a rather complicated differential equation. At first sight the smoothing out of the curves seem to happen faster, however, v_1 seems to grow from the second figure to the third, but decreases from the third to the fourth. v_2 always decreases in magnitude, this can be explained by the accelerated time at which v_1 starts to decrease. It might be that, due to the decrease in v_2, v_1 temporarily increases, but this doesn't hold for the fourth figure. Once again, the solutions seem to be volatile when the interaction terms increase in amplitude.