



My Dissertation

by

Johnny Elvis Bravo

*Dissertation presented for the degree of Doctor of Philosophy in Economics in the
Faculty of Economic and Management Sciences at Stellenbosch University*

Supervisor: Prof. F. Flintstone

Co-supervisors: Prof. H. Simpson
Prof. P. Griffin

April 2025

The financial assistance of the National Research Foundation (NRF) towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the author and are not necessarily to be attributed to the NRF.

Declaration

By submitting this dissertation electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Date: 22nd April 2025

Abstract

My Dissertation

J. E. Bravo

*Department of Economics,
Stellenbosch University,
Private Bag X1, Matieland 7602, South Africa.*

Dissertation: PhD (Economics)

April 2025

In this dissertation...

Our results show...

Uittreksel

My Proefskrif

(“My Dissertation”)

J. E. Bravo

*Departement van Ekonomie,
Stellenbosch Universiteit,
Privaatsak X1, Matieland 7602, Suid Afrika.*

Proefskrif: PhD (Ekonomie)

April 2025

In hierdie proefskrif...

Ons resultate dui aan...

Acknowledgements

I would like to express my sincere gratitude to the following people:

- My supervisor...
- My family...

Table of contents

Declaration	i
Abstract	ii
Uittreksel	iii
List of figures	vi
List of tables	vii
Nomenclature	viii
1 Chapter Heading	1
1.1 Section heading	1
1.1.1 Subsection heading	1
2 Code Chunks	2
3 Basics	3
3.1 Lists	3
3.2 Equations and symbols	3
3.3 Floats	4
3.4 Figures	5
3.5 Tables	6
3.6 Footnotes	6
3.7 Acronyms	7
3.8 Referencing, Citations, and Cross-Referencing	7
4 Conclusion	9
A Mathematics	10
B Experiments	12
References	13

List of figures

3.1	Water plants	5
3.2	Manufacturer fuel efficiency	5

List of tables

3.1	Caption centered above table	6
-----	--	---

Nomenclature

Variables

θ	elasticity of demand
$distance$	population-weighted distance [km]

Vectors

$\beta'w_{idt}$	set of gravity covariates
-----------------	---------------------------

Subscripts

i	importer
j	exporter
t	period

Acronyms

MNO	mobile network operator
OLS	ordinary least squares
PPML	Poisson Pseudo-Maximum Likelihood

Chapter 1

Chapter Heading

This is an R Markdown (.Rmd) document. Markdown is a simple formatting syntax for authoring HTML, PDF, and MS Word documents. For more details on using R Markdown see this link. When you click the **Knit** button (or Ctrl + Shift + K) a document will be generated based on the content of this document, as well as the underlying LaTeX template at resources/template.tex.

1.1 SECTION HEADING

Amet arcu condimentum lacus eget nisl penatibus ridiculus quisque mi. Fusce tempor volutpat velit accumsan montes blandit? Vestibulum netus scelerisque ultrices; mollis id lacus in, quis per. Et primis aliquet taciti et condimentum volutpat, scelerisque taciti auctor vitae sollicitudin tempor fusce

1.1.1 SUBSECTION HEADING

Elit ridiculus sagittis posuere malesuada dictumst mattis. Penatibus est arcu dis volutpat eu urna netus litora cursus aliquet purus massa. Dictumst vulputate tristique congue consequat habitasse rhoncus placerat per odio! Vehicula est arcu; inceptos accumsan venenatis taciti, luctus nulla vivamus fames ante donec habitasse volutpat

1.1.1.1 SUBSUBSECTION HEADING

Adipiscing porttitor porttitor commodo fermentum, dis tortor tortor leo convallis. Hac mauris tempus convallis interdum ligula. Lectus tempus cubilia ridiculus, varius vestibulum iaculis. Netus ridiculus porta ut eu pellentesque ad dis vel habitasse, interdum, metus erat

PARAGRAPH HEADING Elit enim eget, senectus non arcu congue eros neque posuere natoque. Morbi sociosqu facilisi viverra eu, posuere rhoncus sagittis vestibulum torquent molestie ultrices. Curae condimentum netus, erat dignissim himenaeos taciti odio euismod quam egestas venenatis risus. Nam tellus

Chapter 2

Code Chunks

When you knit this file a document will be generated based on its contents as well as the output of any code chunks embedded within the file. You can embed an R code chunk, like the one `cars` below, with `Ctrl + Alt + I`. The chunk label for each chunk, e.g., `cars`, is assumed to be unique within the document.

```
summary(cars)
```

```
##           speed           dist
##  Min.      : 4.0    Min.      :  2.00
##  1st Qu.:12.0    1st Qu.: 26.00
##  Median :15.0    Median : 36.00
##  Mean   :15.4    Mean   : 42.98
##  3rd Qu.:19.0    3rd Qu.: 56.00
##  Max.   :25.0    Max.   :120.00
```

One of the most crucial code chunks in an R Markdown document is the `setup` code chunk placed at the beginning of the document (after the YAML header). It can be used to load packages and data, and configure the default behaviour of code chunks, used throughout the remainder of the document. Default chunk options are set globally using `knitr::opts_chunk$set` with arguments such as `echo`, `warning`, and `message`.¹ Learn more about code chunk options at [this link](#).

With `source`, you can load scripts of code stored elsewhere in your directory. In the example below, all custom functions stored in `code/functions.R` are loaded, e.g., the `hello_function` which prints the name stored in the YAML metadata. It may be worthwhile to run `source("code/functions.R")` in your `setup` chunk, making your own functions accessible throughout your `.Rmd` document.

```
source("code/functions.R")
hello_function()
```

```
## Hallo Johnny Elvis Bravo! This a sourced function.
```

¹For example, the code chunk option `echo` controls whether the code in an code chunk is printed in the final document. This is set globally to `echo = FALSE` in the `setup` chunk. However, in an individual code chunk, like `cars`, you can specify `echo = TRUE` if you want its code printed.

Chapter 3

Basics

3.1 LISTS

Itemized lists can be created using Markdown syntax like this:

- Item 1
- Item 2
- Item 3

Numbered lists can be created using Markdown syntax like this:

1. Item 1
2. Item 2
3. Item 3

These are equivalent to using the LaTeX environments `itemize` and `enumerate`.

3.2 EQUATIONS AND SYMBOLS

Equations are created with the help of the LaTeX package `amsmath`. An equation must read like part of the text, using a full stop to indicate the end of the sentence:

$$e^{i\theta} = \cos \theta + i \sin \theta. \quad (3.1)$$

End an equation with a comma if used in the middle of a sentence and start the subsequent text in lower case. For example, Euler's identity is

$$e^{i\pi} + 1 = 0, \quad (3.2)$$

where e is Euler's number, the base of natural logarithms. Here is another pair of equations, this time using `&` to horizontally align multiple lines on their equals signs:

$$a^2 + b^2 = c^2 \quad (3.3)$$

$$e^{i\pi} + 1 = 0. \quad (3.4)$$

A series of equations within the `subequations` LaTeX environment share a common label. In the `gather` LaTeX environment, equations are typeset sequentially. Using these

environments together produces, for example:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} [\rho u_j] = 0 \quad (3.5a)$$

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} [\rho u_i u_j + p \delta_{ij} - \tau_{ji}] = 0, \quad i = 1, 2, 3 \quad (3.5b)$$

$$\frac{\partial}{\partial t} (\rho e_0) + \frac{\partial}{\partial x_j} [\rho u_j e_0 + u_j p + q_j - u_i \tau_{ij}] = 0 \quad (3.5c)$$

Markdown syntax can also be used. An equation is created below with a pair of \$\$ (double \$). This is particularly useful to observe a visual preview of the equation in your .Rmd document.

$$f_X(x) = \left(\frac{\alpha}{\beta}\right) \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^\alpha}; \alpha, \beta, x > 0.$$

A mathematical expression within a line of text can be created with the use of a pair of \$ (single \$), like this $\sum_{i=2}^{\infty} \{\alpha_i^\beta\}$, also providing a visual preview of the expression.

Symbols representing values of properties should be printed in italics, but SI units and names of functions (e.g. sin, cos and tan) should not be printed in italics. There should be a small hard space between a number and its unit. This can be achieved with, e.g., `\qty{120}{km}` which produces 120 km. Use the `siunitx` package to typeset numbers, angles and quantities with units:

```
\num{1.23e3}    → 1.23×103
\ang{30}        → 30°
\qty{20}{N.m}   → 20 N·m
```

3.3 FLOATS

Floating environments in LaTeX prevent figures and tables from being split across pages. Their placement can be controlled by the following placement specifiers:

- **h**: Here at approximately the same point in the source text.
- **t**: Top of the page.
- **b**: Bottom of the page.
- **p**: A special page for floats only.
- **!**: Override LaTeX's internal parameters.
- **H**: Precisely at this location in the source (with the float LaTeX package).

Specifiers can also be combined, e.g., `!b` forces placement at the bottom of a page. You may also considering making use of the LaTeX package `placeins` by specifying the `\FloatBarrier` command twice to create barriers that figures cannot escape.

3.4 FIGURES

A figure's placement can be set in its particular code chunk's options, as in the `waterplants` chunk with `fig.pos = "h"`. Other chunk options for figures include, among others, `fig.cap`, `fig.height`, `fig.width` and `out.width`. The default behavior for figures arising from all code chunks is set in the setup chunk with `knitr::opts_chunk$set`. Existing figures or images can be included with `knitr::include_graphics`, which can import PDF, PNG, or JPG files like Figure 3.1.

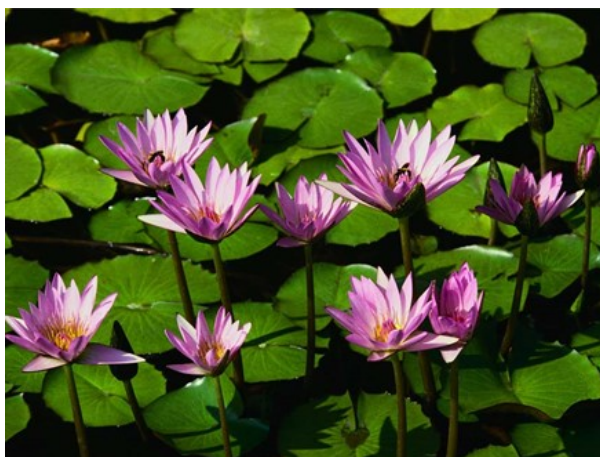


Figure 3.1 Water plants

Plots can also be created using, e.g., `ggplot` from the `tidyverse`, as shown below. The resulting plot, Figure 3.2, is embedded in the document.

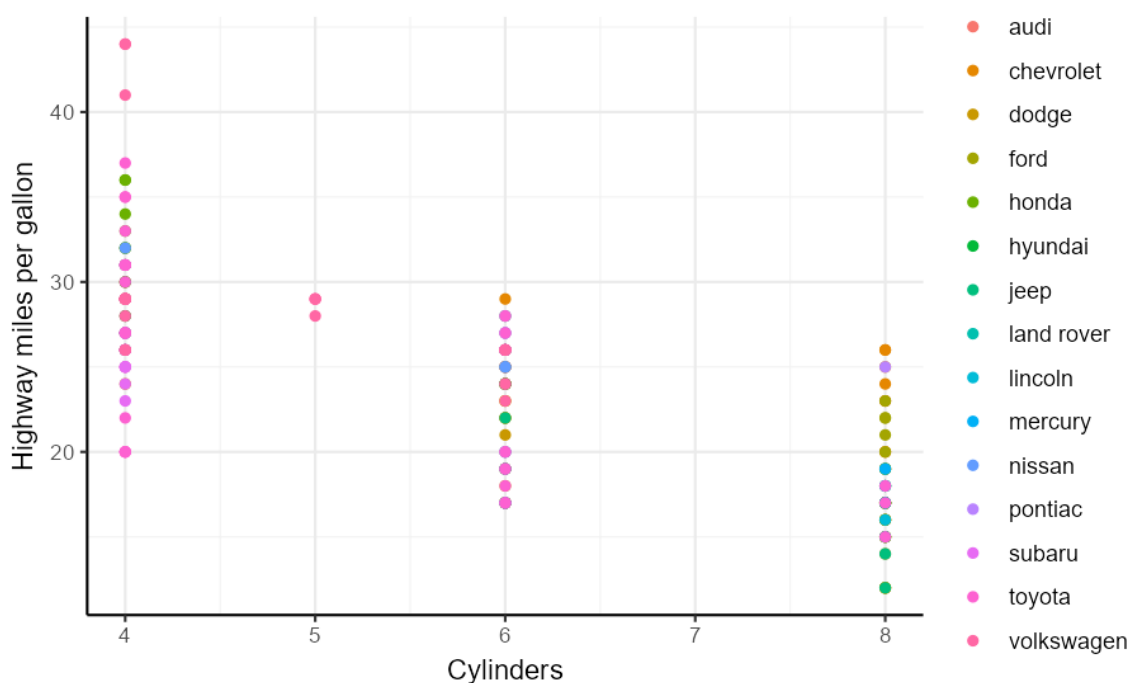


Figure 3.2 Manufacturer fuel efficiency

3.5 TABLES

To create tables, I recommend using the `kbl` and `kable_styling` functions from the `kableExtra` R package. Table 3.1 is created with the code chunk below.

Table 3.1 Caption centered above table

	mpg	cyl	disp	hp
Mazda RX4	21.0	6	160	110
Mazda RX4 Wag	21.0	6	160	110
Datsun 710	22.8	4	108	93
Hornet 4 Drive	21.4	6	258	110
Hornet Sportabout	18.7	8	360	175
Valiant	18.1	6	225	105

My footnote: This is my table footnote.

The first four columns of the `mtcars` data frame are used as input for `kbl`, which generates a basic table. The function includes several useful arguments:

- `digits = 2`: Sets the number of decimal places.
- `booktabs = TRUE`: Uses booktabs for a more appealing formatting.
- `linesep = ""`: Removes default line separators.
- `centering = TRUE`: Centres the table.
- `position = "H"`: Positions the table precisely with the float specifier.
- `escape = TRUE`: FALSE allows the use of special LaTeX characters.
- `table.envir = "table"`: Sets the LaTeX environment.
- `caption = "Caption centered above table"`: Adds a caption above the table.

In turn, `kable_styling` enhances the appearance of the table by setting, e.g., the table's font size. Additionally, a footnote is added using `footnote`, with arguments that set the title, content and layout of the footnote.

3.6 FOOTNOTES

Footnotes provide additional information without cluttering the main text and are inserted using `\footnote{}` in LaTeX or `^[]` in Markdown. See this example.¹ Footnotes can also be managed with `\footnotemark` and `\footnotetext` for more control over their placement and numbering.

- `\footnotemark` adds a superscript number at the text location, and can be used in LaTeX when you need to refer to the same footnote multiple times or to place

¹This is an example of a footnote.

the marker manually. In Markdown, the equivalent is using `[^1]` to reference a footnote.

- `\footnotetext` is used to input the footnote’s text and can be placed anywhere in the text to match the corresponding `\footnotemark`. In Markdown, define the footnote content at the document’s end with `[^1]: Text of the footnote`.

3.7 ACRONYMS

Using the acronym LaTeX package, acronyms are defined in the `acronym` environment of the `matter/nomenclature.Rmd` file, making them available throughout your document. Acronyms are defined with `\acro{key}[short]{long}` where `short` the abbreviated form, and `long` the full form, e.g., `\acro{arpu}{ARPU}{average revenue per unit}`. In most cases, use `\ac{key}` to insert an acronym. It will automatically expand to its full form the first time and to its short form on subsequent references. Here are some variations on the use of `\ac{key}`.

- `\ac{ppml}` produces Poisson Pseudo-Maximum Likelihood (PPML) the first time, and PPML the second time.
- `\acf{ols}` uses the long form ordinary least squares (OLS) every time.
- `\acs{ols}` uses the short form OLS every time.
- Plural forms can also be automatically managed by adding an `p` to the command, e.g., `\acp{mno}` for mobile network operators (MNOs).

Notably, the acronym package is set up with the `printonlyused` option, implying that the list of acronyms in your final document (printed in the “Nomenclature” chapter) will only include those used at least once in the document.

3.8 REFERENCING, CITATIONS, AND CROSS-REFERENCING

The bibliography file is stored at `matter/mybib.bib`, formatted in `bibtex` with unique keys for each entry. The `natbib` package manages citations effectively, supporting a range of styles:

- `\citet{key}` for textual citations (e.g., Anderson et al. (2016)). `\citet*{key}` expands to list all authors.
- `\citep{key}` for parenthetical citations (e.g., (Anderson et al., 2016)), with `\citep*{key}` for a full author list.
- `\citeauthor{key}` and `\citeyear{key}` (`\citeyearpar{key}`) cite the author(s) or year (in parentheses), respectively, e.g., Anderson and Van Wincoop, 2003, and (2003).

Markdown offers similar functionality using `@key` for textual citations and `[@key]` for parenthetical citations. The `-@key` form cites just the year.

For bibliography management, I recommend using Zotero to automatically export collections of bibliography entries to `matter/mybib.bib`. See **Better BibTeX, Citations in R**

Markdown’s Visual Mode, and **ZotFile** for additional tools making citation and reference management easier.

Cross-referencing is made possible with LaTeX package `hyperref`, and streamlined with the `cleveref` package, configured with `\usepackage[capitalize,noabbrev]{cleveref}` to capitalize and use full names (e.g., Figure, Equation) in references:

- `\label{key}` marks a target location.
 - Tables and figures are automatically labelled in R Markdown, using code chunk names with an appropriate prefix as keys, e.g., `fig:fancyplot` and `tab:tab1`.
 - Headings are also automatically labelled, taking their text (in lower-case, spaces swapped with hyphens) as keys, e.g. the “Equations and symbols” section is labelled as `equations-and-symbols`.
- `\ref{key}` retrieves a basic reference.
 - `\ref{fig:fancyplot}` produces 3.2
 - `\ref{tab:tab1}` produces 3.1
 - `\ref{eq:Euler1}` produces 3.1
 - `\ref{equations-and-symbols}` produces 3.2.
- `\eqref{key}` provides a parenthesized number for equations
 - `\eqref{eq:Euler1}` produces (3.1).
- `\cref{key}` offers context-dependent referencing, adapting to the type of the referenced object.
 - `\cref{fig:fancyplot}` produces Figure 3.2
 - `\cref{tab:tab1}` produces Table 3.1
 - `\cref{eq:Euler1}` produces Equation (3.1)
 - `\cref{equations-and-symbols}` produces Section 3.2.
- `\notag` is used to suppress the numbering of an equation when it is unnecessary, ensuring that un-referenced equations remain unnumbered.

With both citations and cross-references, dynamic links are created, aiding document navigation.

Chapter 4

Conclusion

Lorem aenean pulvinar litora cum proin lacus habitant primis. Proin vehicula augue nulla massa nostra augue nec porttitor blandit aenean conubia?

Dolor class penatibus eu fringilla phasellus neque dis nullam eleifend rhoncus posuere enim feugiat. Vulputate mauris enim morbi – diam tortor, quisque curabitur in erat! Nisi curae – mus nascetur habitasse porta interdum? Ad magna ridiculus fermentum duis nostra at fringilla et vestibulum enim, mattis proin eleifend. Na eleifend.

Adipiscing porttitor quam nascetur libero montes posuere penatibus sodales. Neque hendrerit quam eros urna, purus primis velit fermentum nisi mollis nec vitae. Torquent euismod potenti mus ullamcorper elementum etiam eu porttitor? Aliquam primis lacinia enim leo, erat urna dignissim dictumst: tellus pretium. Non egestas scelerisque mi lectus elementum litora a fermentum torquent? Himenaeos convallis nascetur purus tellus vivamus tristique risus taciti.

Dolor turpis blandit convallis nunc inceptos! Neque pulvinar elementum fringilla vivamus dis condimentum est. Maecenas ad taciti cras mauris lobortis vivamus litora aliquet. Habitant a posuere blandit, proin imperdiet ante sodales fringilla!

Adipiscing fusce placerat nibh. Scelerisque mauris nulla in nulla augue varius natoque malesuada! Ultricies mollis consequat vulputate quis torquent viverra, orci fringilla malesuada pretium – sociis praesent himenaeos mollis orci natoque venenatis sodales aliquam!

Adipiscing et proin tellus nam commodo commodo class justo neque litora libero? Habitasse vitae eget mauris lectus in porttitor elementum primis. At congue inceptos, purus, enim taciti, dui cum cubilia dictumst volutpat sollicitudin. Auctor habitant lectus, dictumst mus purus ad erat consequat. Sociis lobortis id molestie magna massa: morbi scelerisque semper. Mus scelerisque consequat auctor semper?

Appendix A

Mathematics

An inversion formula: Let $g : \mathbb{R}^+ \rightarrow \mathbb{R}$ be bounded and right continuous, and let $\varphi(\alpha) := \int_0^\infty e^{-\alpha t} g(t) dt$ denote its Laplace transform. Then, for every $t > 0$,

$$g(t) = \lim_{\varepsilon \rightarrow 0} \lim_{\lambda \rightarrow \infty} \varepsilon^{-1} \sum_{\lambda t < k \leq (\lambda + \varepsilon)t} \frac{(-1)^k}{k!} \lambda^k \varphi^{(k)}(\lambda). \quad (\text{A.1})$$

Solutions of systems of ODEs: Let $\mathbf{v}(\mathbf{x}, \alpha)$ denote a parametrized vector field ($\mathbf{x} \in U$, $\alpha \in A$) where U is a domain in \mathbb{R}^n and the parameter space A is a domain in \mathbb{R}^m . We assume that \mathbf{v} is C^k -differentiable as a function of (\mathbf{x}, α) , where $k \geq 2$. Consider a system of differential equations in U :

$$\dot{\mathbf{x}} = \mathbf{v}(\mathbf{x}, \alpha), \quad \mathbf{x} \in U \quad (\text{A.2})$$

Fix an initial point \mathbf{p}_0 in the interior of U , and assume $\mathbf{v}(\mathbf{p}_0, \alpha_0) \neq \mathbf{0}$. Then, for sufficiently small t , $|\mathbf{p} - \mathbf{p}_0|$ and $|\alpha - \alpha_0|$, the system (A.2) has a unique solution $\mathbf{x}_\alpha(t)$ satisfying the initial condition $\mathbf{x}_\alpha(0) = \mathbf{p}$, and that solution depends differentiably (of class C^k) on t , \mathbf{p} and α .

Stirling's formula:

$$\Gamma(z) \sim e^{-z} z^{z-1/2} \sqrt{2\pi} \left[1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} + \dots \right], \quad z \rightarrow \infty \text{ in } |\arg z| < \pi. \quad (\text{A.3})$$

Bézier curves: Given z_1, z_2, z_3, z_4 in \mathbb{C} , define the Bézier curve with control points z_1, z_2, z_3, z_4 by

$$z(t) := (1-t)^3 z_1 + 3(1-t)^2 t z_2 + 3(1-t)t^2 z_3 + t^3 z_4, \quad 0 \leq t \leq 1.$$

Because $(1-t)^3 + 3(1-t)^2 t + 3(1-t)t^2 + t^3 = (1-t+t)^3 = 1$ and all summands are positive for $0 \leq t \leq 1$, $z(t)$ is a convex combination of the four points z_k , hence the curve defined by $z(t)$ lies in their convex hull. As t varies from 0 to 1, the curve moves from z_1 to z_4 with initial direction $z_2 - z_1$ and final direction $z_4 - z_3$.

Maxwell's equations:

$$\begin{aligned} \mathbf{B}' &= -c \nabla \times \mathbf{E} \\ \mathbf{E}' &= c \nabla \times \mathbf{B} - 4\pi \mathbf{J}. \end{aligned}$$

Residue theorem: Let f be analytic in the region G except for the isolated singularities a_1, a_2, \dots, a_m . If γ is a closed rectifiable curve in G which does not pass through any of the points a_k and if $\gamma \approx 0$ in G , then

$$\frac{1}{2\pi i} \int_{\gamma} f = \sum_{k=1}^m n(\gamma; a_k) \operatorname{Res}(f; a_k).$$

Maximum modulus principle: Let G be a bounded open set in \mathbb{C} and suppose that f is a continuous function on \bar{G} which is analytic in G . Then

$$\max\{|f(z)| : z \in \bar{G}\} = \max\{|f(z)| : z \in \partial G\}.$$

Jacobi's identity: Define the *theta function* ϑ by

$$\vartheta(t) = \sum_{n=-\infty}^{\infty} \exp(-\pi n^2 t), \quad t > 0.$$

Then

$$\vartheta(t) = t^{-1/2} \vartheta(1/t).$$

Appendix B

Experiments

Elit magna rutrum sed tortor. Ultricies praesent dictum tempor quisque purus fermentum, maecenas posuere tempus ad aliquet dis. Accumsan erat taciti porta rutrum ultricies sollicitudin dis. Aenean cum ante accumsan libero auctor sagittis orci sapien! Ultrices ultricies posuere, metus per nulla torquent tellus viverra inceptos odio; quis; nisi quam dapibus.

Lorem magnis enim at aliquet eu nibh. Mollis maecenas varius facilisis nibh vehicula, egestas libero senectus risus. Curae suspendisse netus mi lacinia habitasse nulla est himenaeos, dis aliquet cubilia. Per aliquam mattis interdum a nibh auctor? Tellus ante nam fames maecenas pharetra orci, proin quis pharetra, nisi facilisi lacus mus consequat inceptos viverra neque praesent.

Lorem platea porta cursus, commodo a eros suspendisse? Maecenas est dictumst ridiculus eros ligula mollis; sed facilisi! Lectus pharetra nulla eros pharetra montes, augue inceptos, pretium – taciti augue fringilla. Parturient volutpat condimentum egestas ultricies viverra: vulputate urna malesuada nostra vitae vehicula facilisi? Suspendisse fames consequat hendrerit nam penatibus. Justo conubia habitasse fusce mattis leo molestie placerat – commodo lectus imperdiet.

Lorem eget metus, enim sapien sagittis donec, primis et euismod eu. Placerat conubia condimentum; sem aliquam nec lacinia nisl turpis sem suscipit venenatis. Nisl euismod, non platea nascetur eleifend vehicula nam. Per orci sollicitudin – ornare quis nisl enim viverra magna. Platea aenean lobortis purus enim vulputate nec porttitor bibendum, primis: consequat gravida luctus nostra.

Dolor justo integer nascetur quis nostra scelerisque imperdiet sollicitudin habitasse aenean inceptos torquent. Libero luctus eu: arcu at ultricies nisi, commodo pulvinar penatibus bibendum? Aenean est montes – risus mattis varius fermentum dui cursus scelerisque? Praesent lacinia luctus nostra.

Amet egestas class sapien vivamus, mauris pharetra feugiat porta lacinia! Ligula inceptos ridiculus donec tellus velit erat dictum proin ridiculus fames turpis? Tristique felis, nullam nisi iaculis velit parturient convallis? Semper imperdiet suspendisse purus leo iaculis. Dapibus euismod placerat, varius tristique eros magnis, vestibulum egestas consequat blandit dis placerat

References

- James E Anderson and Eric Van Wincoop. Gravity with Gravitas: A Solution to the Border Puzzle. *American Economic Review*, 93(1):170–192, 2003. doi: 10.1257/000282803321455214.
- James E. Anderson, Mykyta Vesselovsky, and Yoto V. Yotov. Gravity with scale effects. *Journal of International Economics*, 100:174–193, 2016. doi: 10.1016/j.jinteco.2016.03.003.