# Turbofan aero-engine efficiency evaluation: An integrated approach using VSBM two-stage network DEA[*]

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## ABSTRACT

The application of Data Envelopment Analysis (DEA) has been wide, especially for the purpose of evaluating efficiency among similar production processes within enterprises belonging to particular industries. Although research pertinent to DEA has primarily focused on efficiency of production systems or corporate entities/organizations (e.g., terminals, hospitals, universities/schools, banks), fairly little attention has been given to efficiency evaluation among engineering systems featuring common configurations (e.g., automobiles, power plants). Furthermore, the limited previous literature involving efficiency evaluation of engineering systems has implemented DEA methodologies with limited discriminative power, i.e. there is a quite increased portion of efficient Decision Making Units (DMUs). In the current paper, a methodological framework deploying Variable intermediate measures Slacks-Based Measure (VSBM) Two-Stage Network DEA is implemented, in order to evaluate the efficiency of turbofan aero-engines, currently utilized by active-duty commercial and military aircraft. Apart from exploring the positive correlation of DEA efficiency with engineering efficiency, we also develop a methodology evaluating the features of near-future turbofan designs in terms of DEA efficiency, thus comprising a potential tool for efficiency assessment of any turbofan aero-engine being in the conceptual or preliminary design stage.

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## 1. Introduction

The development of effective and efficient production methods in several industrial sectors constitutes a major aspect of the effort conducted by industry professionals worldwide. In this framework, researchers have dealt with the production efficiency evaluation by deploying a great variety of methodologies. Data Envelopment Analysis (DEA) has been so far designated as a quite prominent methodology for evaluating efficiency of production systems. In essence, DEA enables the evaluation of relative efficiency of entities possessing similar features otherwise referred to as Decision Making Units (DMUs), with minimal prerequisites pertinent to relations among inputs and outputs within these units [23]. Although DEA has been introduced by Charnes et al. [7] for the purpose of evaluating non-profit organizations like schools, Liu et al. [37] claim

that relevant research deploying DEA methodology lies mainly in four sectors, namely the banking sector, the health care sector, the agricultural sector, and the transport sector.

The use of DEA in research pertinent to engineering systems is quite limited, while very few scholars and especially engineers have implemented efficiency measurement concepts for the purpose of assessing and improving performance of engineering products. In this context, it is vital to highlight the fact that in the framework of the conceptual or preliminary design stage of any engineering system/asset, engineers tend to implement a "stand-alone" approach (i.e. develop a design by merely adopting engineering principles) in order to optimize the particular system/asset, thus omitting to adopt a more holistic approach which encompasses benchmarking their design against any existing systems/assets of similar configuration [56].

The current paper primarily aims to evaluate the efficiency of turbofan aero-engines utilized by contemporary commercial and military aircraft implementing an advanced two-stage network DEA approach, thus further extending the previous research conducted by Bulla et al. [6]. The research contribution of the current paper is threefold, which is described as follows:

- Assess the efficiency of engineering systems/assets in an advanced DEA framework, subsequently conducting comparison with efficiency assessment within an engineering framework. While Bulla et al. [6] implement a single-stage DEA model, the current paper deploys a two-stage network DEA model in order to better represent the functional concept of turbofan aero-engines and concurrently possess enhanced discriminative power.
- Evaluate the DEA efficiency of commercial and military turbofan aero-engines with respect to technological progress over time and certain distinctive technological features, which are not investigated in the previous research of Bulla et al. [6].
- Create a fundamental framework for the development of methodology appropriate for benchmarking turbofan aero-engines being in the conceptual/preliminary design stage or entering service in the near future.

The structure of the rest of the paper is as follows: Section 2 describes the technological aspects of commercial and military turbofan aero-engines, concurrently highlighting contemporary design trends. Section 3 attempts to conduct a thorough literature review, with the first part dealing with previous research pertinent to engineering systems' efficiency evaluation with DEA, while the second part deals with previous research specifically investigating turbofan aero-engine performance optimization within an engineering framework. Section 4 has also two parts. In the first part description of the implemented DEA methodological approach is given, simultaneously providing a thorough justification of the two-stage network model. In the second part of Section 4, the turbofan aero-engine sample and the associated quantified features are presented, subsequently numerically assessing efficiency via the VSBM Two-Stage Network DEA methodology. In Section 5, post-hoc analysis of the efficiency results is conducted, thus designating DEA efficiency determinants and subsequently comparing them with the engineering efficiency determinants. Moreover, in Section 5 results of the post-hoc analysis are being discussed, taking into account the current and near-future technological limitations, concurrently setting the fundamentals for a method which could evaluate efficiency of any turbofan aero-engine design being in the conceptual/preliminary design stage or development stage. Finally, Section 6 makes a summary of the concluding remarks, while it additionally discusses some recommendations for future research.

## 2. Turbofan aero-engine description and performance measures

Turbofan aero-engines comprise the most common propulsion system for modern era commercial and military aircraft, alternatively referred to as "bypass" engines. Unlike turbojet engines, which do not divert any portion of suctioned air mass, turbofans divert suctioned air in two different streams, commonly quoted as "cold" stream (not passing through the high-pressure compressor, combustion chamber, and turbine) and "hot" stream (passing through the high-pressure compressor, combustion chamber, and turbine) [17,46]. In addition, it should be mentioned that the arrangement of high-pressure compressor (HPC), combustion chamber, and turbine is frequently quoted as "core engine" [46].

In terms of number of spools utilized by turbofan aero-engines on a global scale, the two-spool configuration is the most widespread among commercial turbofan aero-engines in service, as well as in a significant number of military turbofan aero-engines [46]. In this configuration, the fan, low-pressure compressor (LPC) and low-pressure turbine (LPT) are connected by a single shaft [alternatively referred to as low-pressure (LP) shaft], thus rotating with the same speed. Simultaneously, high-pressure compres-

sor (HPC) and high-pressure turbine (HPT), which are located immediately before and after the combustion chamber respectively, are also connected by a shaft [alternatively referred to as high-pressure (HP) shaft], which is concentric with the LP shaft. In Fig. 1 we can observe a two-spool, high-bypass ratio (BPR), unmixed flow commercial turbofan aero-engine, while in Fig. 2 we can observe a two-spool, low-BPR, mixed flow military turbofan aero-engine.

As a general rule, commercial turbofan aero-engines tend to adopt fairly high BPR (even reaching values of 7-8), while military turbofan aero-engines mostly adopt low BPR (usually values ranging from 0.3 to 0.5). Moreover, military turbofan aero-engines tend to add augmentor (alternatively referred to as "reheat" or "afterburner"), in which bypass and engine core flows are mixed and subsequently additional fuel is injected to create a supplemental combustion process, thus enhancing thrust [17,48].

As far as concerning the performance of turbofan aero-engines, thrust is the most widely used metric. Nevertheless, thrust alone is considered as inadequate to reflect the efficiency of any aero-engine, taking into account the fact that greater thrust can be achieved by simply making the aero-engine bigger, which is essentially deemed inefficient [39]. In this context, we shall refer to the main types of aero-engine efficiency, which are as follows [16,17,21]:

- Propulsive Efficiency: It comprises the efficiency of the conversion of kinetic energy of airflow passing through the engine into thrust power. It is considered an external efficiency measure, which is expressed by the following equation:

$$n_p = \frac{Thrust\ Power}{Power\ imparted\ to\ engine\ airflow} \tag{1}$$

For different types of propulsion systems, the propulsive efficiency is greatly differentiated with respect to flight airspeed, hence each propulsion system type is more efficient than other propulsion system types in certain airspeed ranges. As a general rule, higher propulsive efficiency can be attained when airflow velocity increase across the propulsion system is the smaller possible. A representative depiction of the aforementioned statements is displayed in Fig. 3, which clearly designates the superior propulsive efficiency of turbofan propulsion systems compared to turboprop and turbojet propulsion systems in the high subsonic/transonic flight speed region.

- • Thermal Efficiency: It comprises the ratio of the kinetic energy obtained by the airflow during the whole engine cycle (compression, combustion, expansion) divided by the thermal energy inherently existent in the fuel provided. It is deemed as an internal efficiency measure expressed by the following equation:

$$n_{th} = \frac{Power\ imparted\ to\ engine\ airflow}{Rate\ of\ energy\ supplied\ in\ the\ fuel} \tag{2}$$

- Overall Efficiency: It constitutes the portion of the thermal power provided by fuel combustion, which is converted into thrust power by the propulsion system. It is expressed as the product of propulsive efficiency multiplied by the thermal efficiency ($n_p \times n_{th}$). For all types of propulsion systems overall efficiency can be expressed by the equation:

$$n_o = \frac{T \times u}{\dot{m}_f \times Q_R} \tag{3}$$

Where (T) is the thrust, (u) is the aircraft speed, ($\dot{m}_f$) is the fuel flow, and ($Q_R$) the fuel calorific value. Given the fact that overall efficiency depends on aircraft speed and produced thrust (which balances the aircraft drag), it is deduced that overall efficiency is not a performance parameter solely dependent on the engine, but influenced by the propulsion system/airframe combination and the state of flight operating conditions.
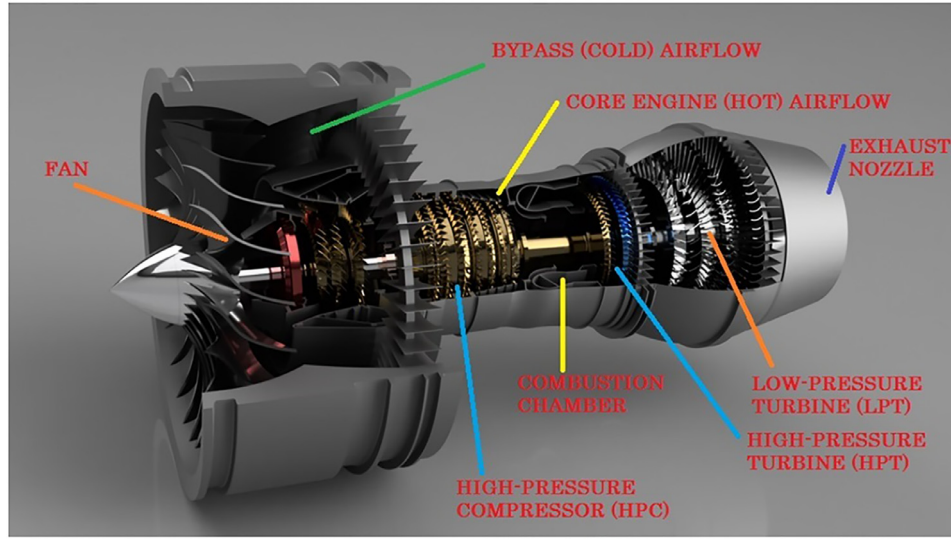
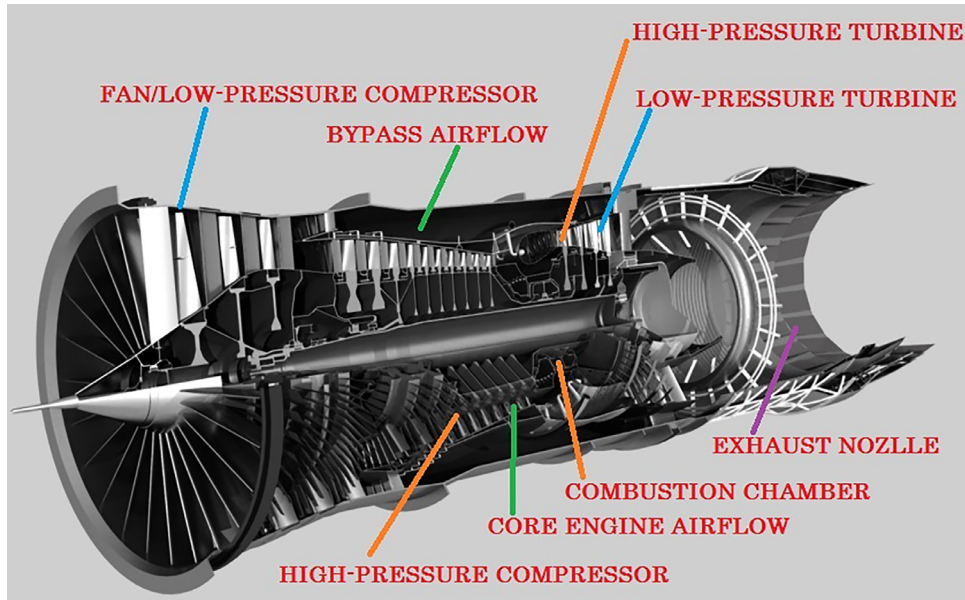Fig. 1. Two-spool, High-BPR, unmixed flow commercial turbofan aero-engine (Source: https://grabcad.com/library/pratt-whitney-turbofan-engine-1)



Fig. 2. Two-spool, low-BPR, mixed flow military turbofan aero-engine (Source: https://grabcad.com/library/low-bypass-turbofan-pratt-whitney-f100)

- **Specific Fuel Consumption**: For the case of turbofan propulsion system, the particular performance parameter is defined per unit of thrust force, otherwise referred to as *Thrust Specific Fuel Consumption* (TSFC), which is expressed by the fraction of fuel flow divided by the produced thrust i.e.
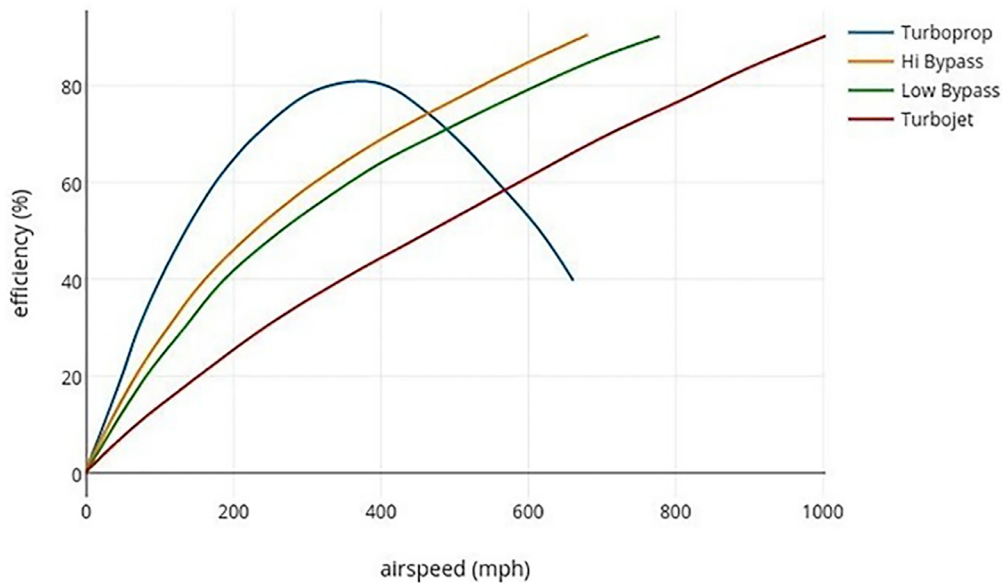
$$TSFC = \frac{\dot{m}_f}{T} \tag{4}$$

As far as commercial aviation is concerned, TSFC represents maybe the most important parameter of a propulsion system, as fuel expenditures constitute the biggest part of airline operating costs.

- **Take-Off/Specific thrust**: Take-off thrust comprises a major determinant of propulsion system performance, as it represents its ability to propel an aircraft during take-off run and render it airborne. In certain instances, the parameter of specific thrust is implemented as it designates the capability of the propulsion system to produce thrust per unit of mass of inducted air,

thus expressed as the fraction of the produced thrust divided by the inducted air mass flow ($\frac{T}{\dot{m}m_{air}}$). It is important to stress the fact that optimum TSFC is feasible through lower values of specific thrust, thus comprising a vital aspect associated with reduced fuel consumption in high-BPR turbofan propulsion systems like the ones utilized by commercial aircraft [48].

The above measures are used to evaluate performance and efficiency solely within an engineering context. Though, it is evident that overall efficiency depends on flight speed. Concurrently, flight speed affects the aircraft drag and subsequently the required thrust in order to maintain level flight [42]. Due to the fact that every individual aircraft type has different drag, it is deduced that even the same engine in different aircraft should produce different thrust performance in order to maintain level flight. As a result, aero-engine comparison is inherently very subjective and inaccurate if it is assumed that aero-engines under comparison are installed on aircraft.

**Fig. 3.** Propulsive efficiency versus airspeed for different propulsion system types (Source: https://aviation.stackexchange.com/questions/48221/is-a-turboprop-or-a-turbofan-more-eco-friendly)

On the other hand, take-off thrust represents thrust in static conditions, i.e. conditions where there is zero speed, thus being independent whether the aero-engine is installed on aircraft or not. However, although comparison of aero-engines in static conditions would be less subjective, it ignores the important parameter of engine weight. Hence, the aforementioned drawbacks which render aero-engine comparison vague when using engineering approach, designate the need to establish a benchmarking concept sufficient enough to perform efficiency evaluation in a more integrated manner.

## 3. Literature review

### 3.1. Engineering systems efficiency evaluation with DEA

As previously stressed, application of DEA for the purpose of evaluating the comparative efficiency of engineering systems is fairly scarce in the research conducted so far. Adoption of an engineering approach for the purpose of developing an engineering product usually, encompasses optimization of sub-systems and/or components. In certain cases, the engineering approach ignores certain aspects which can have an impact on overall performance. Consequently, engineering design and development phases preceding the production of any engineering asset should implement methods to evaluate engineering systems' performance in a broader manner [57]. In the framework of the current paper, an extensive literature search has been conducted in order to pinpoint previous research related to engineering systems' evaluation using DEA. The results of the search confirmed the scarcity and subsequently the gap in scholarly research, pertinent to DEA implementation for benchmarking engineering assets. As observed in Table 1 of the current paper, 20 instances of research relevant to engineering asset benchmarking deploying DEA methodology were found. All research efforts incorporate single-stage basic DEA models, except for the research of Zhao et al. [60] which incorporates multi-stage network DEA.

The earliest piece of relevant research is the one by Färe et al. [20] which evaluates the efficiency of 22 steam-electric utility plants in the Western United States for the time period 1977-1979. Subsequent research conducted by Doyle and Green

[15] benchmarks 37 computer printers. Like Färe et al. [20], Golany et al. [24] conduct efficiency assessment of power plants belonging to the Israel Electric Corporation for the time period between September 1981 and November 1987. The research efforts of Hjalmarsson and Odeck [29] and Odeck [43] are deemed quite unique, with the former evaluating the efficiency of heavy construction trucks belonging to the Norwegian public roads administration, while the latter evaluates the efficiency of rock-blasting units also belonging to the Norwegian public roads administration. The research of Lo Storto [38] also involves vehicle efficiency evaluation, namely efficiency evaluation of 29 Italian market passenger cars. In a similar framework to Lo Storto [38], Papahristodoulou [44] assesses the comparative efficiency of 121 passenger cars being on sale for the year 1996.

Electrical power-generating units' efficiency evaluation is revisited by Chitkara [9], thus comparing the performance of 37 coal-based generating units belonging to National Thermal Power Corporation of India for the period from 1991 to 1995. Sarkis and Talluri [49] conduct a quite novel efficiency assessment of Flexible Manufacturing Systems (FMS) incorporating cardinal and ordinal data, thus employing an evolved form of the methodology introduced by Cook et al. [10]. Subsequently, Braglia and Petroni [4] evaluate the efficiency of industrial robots by implementing a sophisticated single-stage model embedding restricted weights and sensitivity analysis with varying weight threshold values, while they concurrently compare the obtained results with the results obtained by implementing the methodology of Sarkis and Talluri [49].

Prominent research effort regarding the efficiency evaluation of engineering systems is the one conducted by Bulla et al. [6], which evaluates the efficiency of 29 turbofan jet engines used on commercial aircraft. The prominence of the particular research lies in the fact that DEA efficiency is compared with engineering efficiency, thus attempting to assess whether those types of efficiencies are correlated. The research of Sun [52] is also of interest, evaluating the efficiency of 21 Computer Numerical Control (CNC) machines.

The evaluation of power-generating units is repeatedly conducted in the research of Cook and Zhu [11] along with goal-programming. In a similar manner, the subsequent research of

**Table 1**
Previous research summary of engineering systems' Benchmarking using DEA

| Author(s), Year | Engineering System Type | Number of DMUs | DEA Model Type | DEA Model Inputs | DEA Model Outputs |
|---|---|---|---|---|---|
| Färe et al. [20] | Steam-electric Utility Plants | 22 | CRS & VRS (Input-oriented) | Labor, Fuel Consumed, Installed Generating Capacity | Kilowatt-Hours |
| Doyle and Green [15] | Computer Printers | 37 | CRS (Input-oriented) | Acquisition Cost | Throughput, Print Quality, Reliability, Disruption through Noise, Delay through Occupancy |
| Golany et al. [24] | Powerplants | 87 | CRS & VRS (Input-oriented) | Installed Capacity, Fuel Consumed, Manpower | Generated Power, Operational Availability, Deviation from Operational Parameters, Sulphur Dioxide Emissions |
| Hjalmarsson and Odeck [29] | Heavy Construction Trucks | 72 | VRS (Input & Output-oriented) | Driver Wages, Fuel Consumed, Tire Costs, Maintenance Costs | Total Transport Distance, Effective Hours in Production |
| Odeck [43] | Rock-Blasting Units | 170 | VRS (Input & Output-oriented) | Capital (transport & machines), Material Costs, Labor | Physical Volume Produced |
| Lo Storto [38] | Cars (Italian Market) | 29 | CRS (Input-oriented) | Acquisition Price, Fuel Consumption | Max Speed, Mass, Engine Capacity, Max Torque, Specific Power, Specific Torque, Acceleration, Pick-Up, Noise, Braking, Safety, Quality |
| Papahristodoulou [44] | Cars (1996 Models) | 121 | CRS, VRS, NIRS (Input-oriented) | 5 inputs (not specified) | 9 outputs (not specified) |
| Chitkara [9] | Power-Generating Units | 37 | CRS (Output-oriented) | Specific Coal Consumption, Specific Oil Consumption, Auxiliary, Vintage, Part Load, Outage, Availability, Capacity, Coal Quality | Power Generated |
| Sarkis and Talluri [49] | Flexible Manufacturing Systems | 12 | Single-stage capable of incorporating cardinal and ordinal data | Capital & Operating Costs, Floor Space | Improvements in Work-in-Process (ordinal), Percentage of Tardy Jobs, Yield (Throughput minus scrap & rework) |
| Braglia and Petroni [4] | Industrial Robots | 12 | CRS (Input-oriented) along with Weight Restrictions, Sensitivity Analysis with Varying Weight Restriction Thresholds, and Cross-Efficiency | Price | 10 Operational Performance Parameters |
| Bulla et al. [6] | Turbofan Jet Engines (Commercial Aircraft) | 29 | CRS (Input-oriented) | Fuel Consumption, Engine Weight, Drag | Airflow, Cruise Thrust |
| Sun [52] | Computer Numerical Machines (CNC) | 21 | VRS (Input-oriented) | Acquisition Cost | Spindle Speed Range, Number of Tool Capacity, X-Axis Traverse Rate, Z-Axis Traverse Rate, Maximum Machine Turning Diameter, Maximum Machine Turning Length |
| Cook and Zhu [11] | Power-Generating Units | 40 | CRS (Input-oriented) along with Goal Programming | Labor & Materials Total Expenditures, Total Occupied Hours | Equivalent Full-Capacity Operating Hours, Number of Forced & Sudden Outages, Number of Forced Deratings due to Equipment Failure |
| Meenakumari et al. [41] | Power-Generating Units | 29 | CRS & VRS (Input-oriented) | Installed Capacity, Distribution Line Length, Power Losses | Energy Supplied, Number of Consumers (as profit) |
| Karsak [34] | Flexible Manufacturing Systems | 15 | Single-Stage capable of incorporating crisp, ordinal, and fuzzy data | Capital & Operating Cost, Required Floor Space, Work-In-Process (fuzzy) | Product Flexibility (ordinal), Quality Improvement (ordinal), Lead Time Reduction (fuzzy) |
| Lee and Lee [36] | Government Office Buildings | 47 | CRS & VRS (Input-oriented) | Floor Area, Occupants Number, Climate-Adjusted Energy Consumption | Total Energy Consumption |

**Table 1** (*continued*)

| Author(s), Year | Engineering System Type | Number of DMUs | DEA Model Type | DEA Model Inputs | DEA Model Outputs |
|---|---|---|---|---|---|
| Zhao et al. [60] | Downtown Space Reservation System | 28 (as scenarios) | Radial Network DEA & Slacks-Based Network DEA (multistage) | Operation Cost, Fuel Cost, Travel Time | Vehicle Miles Travelled (also used as interstage input), Average Speed (also used as interstage input), Revenue, Person Miles Traveled |
| Voltes-Dorta et al. [58] | Automobiles sold in Spanish market during year 2004-2010 | 281 | VRS (Input-oriented) & Malmquist Index | Vehicle Mass, Fuel Consumption | Engine Power, Vehicle Volume, Range |
| Hampf and Krüger [28] | Automobiles sold in German market for year 2010 | 3961 | CRS & VRS (Ouput-oriented) along with Frontier Separation (VRS) and Efficiency Scores' Bootstrappin | | |
| Lozano et al. [40] | Reconfigurable Machine Tools | 14 (each producing 7 part types) | MMF (Multiple Modes of Functioning) DEA | Modules/tools Usage, Labor, Energy Consumption | Number of Units Produced of Each Type |

Meenakumari et al. [41] also deals with efficiency evaluation of 29 state-owned electric utilities in India. Efficiency evaluation of Flexible Manufacturing Systems (FMS) is revisited by Karsak [34] utilizing single-stage DEA model, albeit taking into account crisp, ordinal, and fuzzy data.

The research of Lee and Lee [36] is also deemed quite unique, as it attempts to evaluate the energy efficiency of 47 government buildings in Taiwan. In addition, the research of Zhao et al. [60] comprises a quite notable effort regarding engineering systems' evaluation, thus assessing the efficiency of a novel Downtown Space Reservation System (DSRS) for 28 scenarios, with each scenario representing an individual Decision Making Unit (DMU).

Automobile efficiency is also evaluated by Voltes-Dorta et al. [58], thus implementing VRS (Variable Returns to Scales) methodology along with Malmquist index for car models sold in Spain during the years 2004 to 2010. In a similar vein, Hampf and Krüger [28] evaluate automobiles sold in the German market, concurrently taking into account $CO_2$ emissions. Finally, Lozano et al. [40] apply their own-developed novel MMF (Multiple Modes of Functioning) DEA methodology, in order to assess technical, cost, and allocative efficiency of 14 Reconfigurable Machine Tools (RMTs) producing 7 different part types.

Table 1 designates that previous research dealing with engineering systems' evaluation using DEA, has mainly implemented single-stage CRS (Constant Returns to Scale) or VRS (Variable Returns to Scales) models. However, the fairly recent research of Zhao et al. [60] implements a multistage network DEA model, thus incorporating the functional relationships among the major segments of the engineering system. Consequently, the implemented DEA model represents the internal processes and their interdependencies within DMUs in a more accurate manner, compared to the single-stage basic DEA models where the conversion process of inputs to outputs essentially comprises a "black box".

In the current paper, a two-stage network DEA model is implemented, in order to better reflect the functions within DMUs (i.e. turbofan aero-engines) and subsequently obtain an improved outlook regarding the relative performance of DMUs under evaluation. Compared to the DEA model implemented in the previous research of Bulla et al. [6], the two-stage network DEA model deployed within the framework of current research seeks to attain the following advantages:

- Represent the turbofan aero-engine operation in a more accurate manner, when compared to its actual operation principles. More specifically, the energy which is exogenously provided by the fuel is transformed into another form of energy

i.e. of a compressed air mass (aero-engine "cold" section). The compressed air which has certain flow and pressure magnitude is subsequently entered into the combustion chamber and converted into thrust within the so-called "hot" section of the aero-engine. Hence, our model concurrently assesses the efficiency of the "cold" section and the "hot" section of the turbofan aero-engine. However, as in Bulla et al. [6] aero-engine weight is also considered as an exogenous input, which is explained by the actual advantage that is gained by an aero-engine which adopts weight-saving technologies.

- Improved discriminative power, which usually comprises a major drawback of the single-stage basic DEA models. As a matter of fact, the DEA model of Bulla et al. [6] results in 8 out of 29 DMUs being efficient (or 27.58 percent of the sample), which is in any case deemed quite increased. Also, lack of ranking methods among efficient DMUs renders post-hoc statistical analysis quite subjective, thus significantly affecting conclusions.

### 3.2. Turbofan aero-engine performance optimization implementing engineering approach

Although the research of Bulla et al. [6] is the only effort to date evaluating turbofan aero-engine efficiency both implementing DEA and engineering approaches, research dealing with turbofan aero-engine comparative efficiency evaluation exclusively implementing engineering approach has not really developed. Instead, researchers of the gas turbine engineering scientific field primarily tend to optimize various turbofan aero-engine components or integrated configurations taking into account thermodynamic aspects and features of the aircraft that shall utilize the prospective aero-engine. Consequently, they lack a coherent outlook regarding the performance of their design, compared to already in-service turbofan aero-engine designs.

For example, Guha [26] develops a systematic methodology to optimize commercial aircraft turbofan aero-engines from a thermodynamic perspective on an individual major component basis and subsequently on an integrated turbofan aero-engine basis. Thus, Guha [26] determines optimum fan pressure ratio, optimum exhaust stream speed, and optimum BPR for given specific thrust. Additionally, optimum specific thrust with respect to certain BPR is determined, along with optimum specific thrust optimization with respect to direct engineering cost. On the other hand, Guha [27] attempts to exclusively optimize a single major component, thus defining the optimum fan pressure ratio for turbofan aero-engines incorporating separate or mixed exhaust streams.

Following a similar pattern to Guha [27], the research of Von der Bank et al. [59] determines the optimum design features for compressors incorporated in ultra-high-pressure-ratio turbofan aero-engines, from structural, aerodynamic, and thermodynamic perspectives. Though, the research of El-Sayed at al. [18] determines the optimum thermodynamic features on an individual major component and integrated engine basis, for the purpose of maximizing efficiency and absolute performance. Similarly, the research of Dik et al. [14] performs thermodynamic analysis for the purpose of determining optimum pressure ratio, jet velocity ratio, and temperatures in order to subsequently achieve minimum specific fuel consumption (SFC).

Nevertheless, we should also refer to the research effort of Zhu et al. [61] who deploy a mathematical approach in order to evaluate aero-engine efficiency for particular aircraft types. More specifically, Zhu et al. [61] utilize fuzzy mathematical theory in order to designate the most suitable aero-engine for diverse aircraft types (commercial, military) by taking into account certain features like thrust, BPR, thrust-to-weight ratio, fuel consumption rate, and total engine life.

## 4. Turbofan aero-engine efficiency evaluation

### 4.1. Network DEA model formulation

Before proceeding with the DEA model that shall be implemented for the purpose of benchmarking in-service commercial and military aircraft turbofan aero-engines, a brief description of the turbofan aero-engine operation shall be given. More specifically, a brief architectural and functional description of a two-spool commercial turbofan aero-engine shall be given, in order to justify the structure of the developed network DEA model, along with the implemented inputs, intermediate inputs/outputs, and final outputs of the model.

In essence, air is initially fed to the turbofan engine through the fan, which compresses the air. Subsequently, the major portion of the suctioned air, i.e. the bypass (cold) airflow is expanded to produce thrust (otherwise referred to as "cold stream thrust"), while a significantly smaller portion (depending on the BPR) is routed into the compressor section (also referred to as "core airflow") to be further compressed. After exiting the compressor section, core airflow is driven into the combustion section, where compressed air is mixed with fuel and combustion takes place by ignition. The combustion gases are then routed into the turbine section, thus providing kinetic energy in order to rotate the multistage high- and low-pressure turbine (HPT and LPT). Finally, the combustion gases are expanded to the atmosphere through an exhaust nozzle, producing additional thrust. Due to mechanical connection of the HPT with the compressor section and of the LPT with the fan unit section, more air is suctioned by the fan and the aforementioned sequence is rendered continuous [17]. Considering the previous turbofan engine operation description, it is evident that both cold stream and hot stream thrust are essentially produced by the HPT and LPT, which are part of the so-called "core engine" [50].

Another aspect that is seriously taken into consideration by companies designing and manufacturing turbofan aero-engines and propulsion systems in general, is the engine weight. As a matter of fact, weight-saving practices are of paramount importance in the aviation industry sector. Given the maximum take-off weight (MTOW) of an aircraft, its payload-carrying capability can be significantly enhanced if the empty weight (or structural weight) is reduced [22]. Consequently, reducing the propulsion system weight is a desirable practice by designers and manufacturers, thus dictating the integrated approach of concurrently optimizing performance, mechanical robustness and reduced weight [39]. In this framework, thrust-to-weight ratio has been established as a key
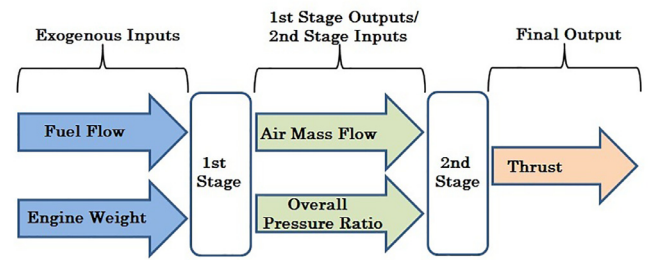


**Fig. 4.** Network DEA functional model for turbofan aero-engine (Source: Own elaboration)

design parameter among propulsion systems designers and manufacturers [18].

Taking into account the aforementioned operational, design and manufacturing aspects, we construct the functional model depicted in Fig. 4, in order to subsequently implement it for the purpose of benchmarking commercial and military turbofan engines.

We next proceed with the justification of the implemented exogenous inputs, intermediate measures (1st stage outputs, 2nd stage inputs), and final output of the proposed network DEA model, which is reported as follows:

#### 4.1.1. Exogenous inputs
- *Fuel flow*: Fuel is stored in the aircraft tanks, thus pressurized and subsequently fed to the engines through pumps. Depending on the thrust setting, fuel flow is properly regulated in order to sustain the desired speed, climb rate etc. It is considered an exogenous input, assuming that it is not an element pre-existing in the engine. In the current paper, fuel flow is recorded in the same way as in aviation, i.e. as pounds per hour (lb/hr).
- *Engine weight*: As previously stated, weight is an important aspect for propulsion system designers and manufacturers. In fact, aero-engine weight is essentially the sum of surrounding structure weight plus the weight of the rotating modules and the accessories. Irrespective of the thermodynamic efficiency and thrust of a turbofan engine, increased weight negatively affects the payload-carrying capability of the aircraft and subsequently the marketability of the engine [19]. Hence, weight is regarded as an exogenous input, assuming that excess weight is translated into excess resource consumption in the form of materials and energy, along with the fact that engine weight results from manufacturing processes, which do not take place within the aero-engine. The units utilized for this input are recorded as pounds (lbs).

#### 4.1.2. Intermediate measures
- *Air mass flow*: In order for a turbofan engine to produce thrust, sufficient air mass should be suctioned, regardless of the portion that shall be routed through the fan and the core. Hence, air mass flow comprises a representative measure of an aero-engine's potential to provide thrust. We consider air mass flow as an intermediate measure, taking into account that in the turbofan engine self-sustained continuous operation the power consumed to induct air into the engine comes from the turbine section (HPT and LPT). The units for this measure are pounds per second (lb/sec), which are the most frequently used by scientific community dealing with propulsion systems.
- *Overall pressure ratio (OPR)*: It literally comprises the ratio of the pressure of the compressed air after exiting the HPC (before entering the combustion chamber), to the pressure of the ambient air (before entering the fan). In a similar manner to air mass flow, OPR also represents an aero-engine's potential to produce

thrust, taking into account that higher pressure exhaust gases possess higher energy, consequently leading to higher energy extraction by the HPT and LPT. OPR was selected instead of fan pressure ratio and HPC pressure ratio, due to the fact that both fan and HPC are driven by the LPT and HPT respectively, thus rendering the adoption of OPR more proper for our study. Moreover, it should be stated that OPR has no units, as it comprises a ratio.

### 4.1.3. Final outputs

- *Thrust*: It comprises the final desirable product of a turbofan aero-engine. Produced thrust propels the aircraft during ground and flight operations, i.e. taxi, take-off, climb, cruise, descend, and landing. Regarding the cruise phase, alternatively referred to as *design point*, engine designers assign increased gravity to efficient engine operation during this phase, taking into account that it consists the biggest portion of an engine's operational life [25]. Regarding the utilized units, thrust is usually expressed in pounds (lbs).

Furthermore, it is essential to refer to a similar previous research effort dealing with turbofan aero-engine efficiency evaluation using DEA, namely the one conducted by Bulla et al. [6]. In the particular research effort, fuel flow, engine weight, air mass flow, and thrust (i.e. cruise thrust) are also implemented using the very same units. However, the current paper deploys two-stage network DEA instead of single-stage CRS DEA, while all inputs, intermediate measures, and output refer to static conditions at sea level instead of cruise conditions. It should be stressed that the rationale behind opting for static sea level conditions is described as follows:

- Engine data sources mostly contain figures for static conditions at sea level, otherwise referred to as take-off at sea level (T-O @ SL). Especially for the case of air mass flow, OPR, and thrust, reported figures almost solely refer to T-O @ SL conditions.
- Figures referring to cruise conditions and especially thrust, depend on a vast degree to the type of aircraft that the engine is installed. In particular, the type of aircraft defines the resulting drag and consequently the required thrust at any flight speed. Hence, it is deemed more appropriate to evaluate DEA efficiency of turbofan engines in T-O @ SL conditions, where all figures are independent of the engine/aircraft combination. The above statement is further augmented by formula (3) of the current paper, which denotes that overall efficiency i.e. the combination of propulsive and thermal efficiency does not solely depend on the engine features, but it directly depends on speed (u) and thrust (T) balancing aircraft drag, thus confirming its high degree of correlation with the aircraft/engine combination [17].

After extensive study regarding Network DEA models that have been developed for evaluating efficiency of multistage systems and taking into account the special features of turbofan aero-engines, the current paper implements the *variable intermediate measures slacks-based measure* (VSBM) model developed by Chen et al. [8]. More specifically, the variable returns to scale (VRS) approach of the VSBM model is adopted for the purpose of evaluating commercial and military turbofan aero-engines.

The justification for implementing the VRS approach of the VSBM model developed by Chen et al. [8] is reported as follows:

- It comprises a non-radial model, thus excluding the assumption of proportional changes in inputs and outputs, dealing directly with slacks [54,55]. This property fits well in engineering systems' evaluation, having in mind that processes within these systems are not necessarily linear.
- It is non-oriented, i.e. the model is structured to both minimize the inputs and maximize the outputs [55]. Hence, it is rendered

ideal for engineering systems' evaluation, where designers apply optimization techniques which simultaneously reduce utilized resources and maximize performance.

- The VRS approach aims to calculate pure technical efficiency as claimed by Ramanathan [45], which is by definition the efficiency of a production process in converting inputs into outputs, independent of the operations' scale and the associated prices and costs [2]. Consequently, the specific type of efficiency is the exact type of efficiency incorporated within engineering systems, where resource consumption and subsequent performance output are dealt with absolutely technical terms and conditions.
- The particular model enables the simultaneous assessment of stage efficiency and overall system efficiency, while it additionally makes feasible to compute the frontier projections for inputs, intermediate measures, and outputs. Hence, the researcher can locate where inefficiencies are occurring and quantify the excess inputs and the deficit of outputs of each evaluated DMU (i.e. engineering system). However, as cited by Tone [53] the excess in inputs and deficit in outputs that are expressed by slacks, may not always be feasible in actual situations, thus designating the need to further investigate to which degree each evaluated DMU is able to improve its relative efficiency.

From the above justification, it is evident that current research comprises a major leap compared to the early research effort of Bulla et al. [6] dealing with the very same topic, i.e. turbofan aero-engine efficiency evaluation deploying DEA. In specific, the adoption of the VRS VSBM DEA model of Chen et al. [8] along with the network DEA functional model depicted in Figure 4, is expected to enhance discriminative power and subsequently the objectivity of the undertaken efficiency evaluation. The additional features of VSBM model, namely the dual orientation (input & output), stage efficiency evaluation, and frontier projection, certainly comprise differentiating factors over the CCR input-oriented DEA model adopted by Bulla et al. [6], which assumes proportional changes of outputs with respect to inputs. Moreover, it should be stressed that the steady-state continuous operation of turbofan aero-engines does not encompass multiple time periods with carry-overs, thus rendering incompatible in our case the deployment of DEA models dealing with DMUs incorporating multiple divisions and multiple time periods as those adopted by Kao [33] and Kou et al. [35].

In accordance with Chen et al. [8], envelopment form of their VSBM model is deployed for the purpose of evaluating overall system efficiency and frontier projections, while the multiplier form VSBM model is deployed to obtain efficiency decomposition, i.e. to determine efficiency of each individual stage. The envelopment form of the deployed VRS VSBM DEA methodology of Chen et al. [8] has the following formulation:

$$\min \ e_o = \frac{1 - \frac{1}{m} \sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s} \sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j^1 x_{ij} = x_{io} - s_i^-, \quad i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j^1 z_{dj} = \widetilde{z_{do}}, \quad d = 1, 2, \ldots, D,$$

$$\sum_{j=1}^{n} \lambda_j^2 z_{dj} = \widetilde{z_{do}}, \quad d = 1, 2, \ldots, D,$$

$$\sum_{j=1}^{n} \lambda_j^2 y_{rj} = y_{ro} + s_r^+, \quad r = 1, 2, \ldots, s,$$

$$\sum_{j=1}^{n} \lambda_j^1 = \sum_{j=1}^{n} \lambda_j^2 = 1,$$

$$s_i^-, \; s_r^+ \geq 0, \quad i = 1, 2, \ldots, m; \quad r = 1, 2, \ldots, s, \qquad (5)$$

$$\lambda_j^1, \; \lambda_j^2 \geq 0, \; j = 1, 2, \ldots, n$$

Where:

- ($e_o$) is the overall system efficiency
- ($n$) is the number of DMUs under evaluation
- ($m$) is the number of exogenous inputs of each DMU
- ($D$) is the number of intermediate measures of each DMU
- ($s$) is the number of final outputs of each DMU
- ($x_{ij}$) is the i-th input of the j-th DMU
- ($x_{io}$) is the i-th input of the DMU$_o$ under evaluation
- ($s_i^-$) is the slack of the i-th input of the DMU$_o$ under evaluation
- ($z_{dj}$) is the d-th intermediate measure of the j-th DMU
- ($\tilde{z}_{do}$)　is the d-th variable intermediate measure of the DMU$_o$ under evaluation
- ($y_{rj}$) is the r-th final output of the j-th DMU
- ($y_{ro}$) is the r-th final output of the DMU$_o$ under evaluation
- ($s_r^+$) is the slack of the r-th final output of the DMU$_o$ under evaluation
- ($\lambda_j^1$) and ($\lambda_j^2$) are the intensity variables of the first and second stage respectively

### 4.2. Turbofan aero-engine sample and efficiency evaluation results

The turbofan engines comprising the sample under evaluation are all in-service (except for GE9X) either installed on commercial (passenger) aircraft or military aircraft. More specifically, the sample consists of 34 commercial turbofan engines and 21 military turbofan engines. As stressed in Section 2, commercial and military turbofan aero-engines are architecturally the same, except for the addition of the augmentor (alternatively referred to as "afterburner") in military engines. However, in our paper the function of the augmentor is fundamentally excluded, thus solely adopting unaugmented ("dry") thrust for all military turbofan aero-engines existent in our sample. In addition, military turbofan aero-engines usually adopt low BPR (bypass ratio), due to the fact that are mainly installed inside the aircraft fuselage. On the other hand, commercial turbofan aero-engines tend to adopt high BPR (usually greater than 5), thus exclusively installed underwing or adjacent to the vertical tail.

As previously stated, the data collected for the whole sample refer to take-off static conditions, for the purpose of ensuring a common benchmarking base and concurrently ensuring data availability. Primary source of data is Jane's Aero Engines 2018-2019 [13], while alternative sources of a quite high portion of the acquired data are the following:

- ICAO Aircraft Engine Emissions Databank, Version 25a [30]
- AIR International Magazine [1] (for GE9X engine)
- El-Sayed et al. [18]
- Farokhi [21]
- Roux [47]
- Jet Engine Specification Database (http://www.jet-engine.net/) [32]

It should be clarified that with respect to fuel flow data, these are either obtained by ICAO [30] by the figure relevant to fuel flow at take-off conditions or by multiplying the thrust specific fuel consumption (TSFC) by thrust. Also, for the purpose of benchmarking an engine that shall enter service in the near future, the General Electric GE9X was added. The specific engine is going to enter

service soon with the Boeing 777X commercial aircraft. Some data figures are obtained from Jane's Aero Engines [13], while others are obtained from AIR International [1]. Especially for the fuel flow figure, its value is obtained based on the claim cited by AIR International [1] and Jane's Aero Engines 2018-2019 [13] that it achieves 10 percent lower specific fuel consumption than the General Electric GE90-115B used in Boeing 777-300ER aircraft. The data for exogenous inputs, intermediate measures, and output of the DEA model represented by Fig. 4, are presented in Table 2. In addition, data pertinent to the year each engine entered service, type of aircraft each engine is installed on, and bypass ratio were also collected, thus being presented in Table 3. Also, Table 3 contains data pertinent to TSFC and thrust-to-weight ratio, for the purpose of acquiring a more detailed overview of each engine.

After obtaining the data contained in Table 2, we incorporate them in the VRS VSBM network DEA model, in order to evaluate the efficiency of each engine of the sample. The "envelopment form" model is used to determine overall efficiency and frontier projections, while the "multiplicative form" model is used to determine the first and second stage individual efficiencies. The gained results regarding overall efficiency, stage efficiency, and frontier projections are contained in Table 4.

Discriminative power of the implemented VRS VSBM network DEA can be considered as very satisfactory, regarding both overall and stage efficiency. In particular, as far as overall efficiency is concerned, no DMU out of 55 is overall efficient (i.e. overall efficiency equal to unity). Also, the discriminative power for both the first and second stage is also satisfactory, as four DMUs are efficient regarding the first stage and five DMUs are efficient regarding the second stage (7.27 and 9.09 percent of total DMUs respectively). The attained efficient DMU portion is significantly low and consequently the achieved discriminative power is at any case acceptable, as stipulated by Avkiran [2] who considers that acceptable discriminative power is achieved when the portion of efficient DMUs is less than one third of the total number of DMUs.

The most efficient engine of the sample is General Electric GE90-115B1, which is solely used by the Boeing 777-300ER. The second most efficient engine is the General Electric GE90-110B1, comprising along with GE90-115B1 the engine options for Boeing 777-200LR and the sole engine option for 777F (freighter version of Boeing 777). It should be mentioned that GE90-110B1 is considered a decreased thrust variant of the GE90-115B1, with both engines comprising an evolutionary improvement of the General Electric GE90-94B, which also belongs to the current engine sample. In particular, major improvements over GE90-94B comprise the larger fan with toughened composite fan blades, along with design and material enhancements in HPT and LPT blade aerofoils [13].

In addition, it is important to stress that if the GE90-110B1 engine is excluded from the sample, GE90-115B1 is rendered overall efficient. By observing the stage efficiencies in Table 4, it is observed that GE90-110B1 has 1$^{st}$ stage efficiency equal to one while GE90-115B1 is not efficient (efficiency score equal to 0.936370). With respect to 2$^{nd}$ stage efficiency, the situation is reversed with the GE90-115B1 obtaining unity efficiency score and GE90-110B1 achieving efficiency score equal to 0.84959. Hence, it is evident that the GE90-110B1 and GE90-115B1 influence each other in both stages, which can be explained by the respective lambda values obtained by VSBM "envelopment form" model.

As far as frontier projections are concerned, it is evident from Table 4 that they significantly deviate compared to actual values of inputs, intermediate measures, and final output. However, deviations are relatively smaller for the case of overall pressure ratio and thrust. Especially regarding thrust, there are 29 instances (mainly concerning commercial aircraft turbofan aero-engines) where actual engine thrust coincides with

**Table 2**
Sample turbofan aero-engines

| Manufacturer | Model | 1st Stage Inputs | | 1st Stage Outputs/2nd Stage Inputs | | Final Output |
|---|---|---|---|---|---|---|
| | | Weight (lbs) | Fuel Flow (lbs/hour) | Overall Pressure Ratio | Air Mass Flow (lbs/hour) | Thrust (lbs) |
| **Commercial** | | | | | | |
| Avco Lycoming | ALF502R-6 | 1,376 | 2,814 | 13.8 | 192 | 6,700 |
| CFE Company | CFE738-1-1B | 1,325 | 2,201 | 23.0 | 210 | 5,918 |
| CFM International | CFM56-3C | 4,301 | 9,150 | 30.6 | 710 | 23,500 |
| CFM International | CFM56-5B3/P | 5,250 | 11,340 | 34.4 | 968 | 33,300 |
| CFM International | CFM56-7B26 | 5,300 | 9,681 | 32.7 | 783 | 26,300 |
| Engine Alliance | GP7270 | 13,416 | 20,910 | 38.6 | 3,000 | 70,000 |
| General Electric | CF34-80C1 | 2,403 | 4,791 | 28.0 | 441 | 12,670 |
| General Electric | CF6-80C2B6F | 9,790 | 20,141 | 31.4 | 1,759 | 60,030 |
| General Electric | CF6-80E1A3 | 11,225 | 23,701 | 34.8 | 1,926 | 69,800 |
| General Electric | GE90-110B1 | 19,316 | 32,456 | 42.0 | 3,618 | 110,100 |
| General Electric | GE90-115B1 | 19,316 | 37,189 | 42.0 | 3,618 | 115,540 |
| General Electric | GE90-85B | 17,250 | 25,232 | 36.9 | 3,120 | 84,700 |
| General Electric | GE90-94B | 17,250 | 27,864 | 39.6 | 3,234 | 93,700 |
| General Electric | GE9X | 20,000 | 29,560 | 60.0 | 3,850 | 102,000 |
| General Electric | GEnx-1B70 | 19,856 | 13,540 | 44.3 | 2,545 | 69,800 |
| Honeywell | TFE731-60 | 988 | 2,025 | 22.0 | 187 | 5,000 |
| IAE | V2533-A5 | 5,200 | 11,307 | 33.4 | 858 | 33,000 |
| Ivchenko Progress | D-18T | 9,001 | 17,823 | 25.0 | 1,687 | 51,660 |
| Pratt & Whitney | PW2040 | 7,295 | 13,964 | 29.9 | 1,340 | 41,700 |
| Pratt & Whitney | PW4056 | 9,420 | 19,419 | 28.4 | 1,705 | 56,750 |
| Pratt & Whitney | PW4084 | 14,920 | 27,048 | 34.2 | 2,400 | 84,600 |
| Pratt & Whitney | PW4090 | 15,740 | 31,131 | 38.6 | 2,720 | 91,790 |
| Pratt & Whitney | PW4098 | 16,260 | 35,456 | 42.8 | 2,850 | 99,040 |
| Pratt & Whitney | PW4462 | 9,420 | 22,663 | 32.3 | 1,800 | 62,000 |
| Pratt & Whitney | PW6122A | 5,041 | 8,464 | 26.6 | 660 | 22,100 |
| Rolls-Royce | AE3007A3 | 1,586 | 2,841 | 21.0 | 240 | 7,040 |
| Rolls-Royce | BR715-C1-30 | 4,597 | 7,803 | 32.0 | 625 | 21,000 |
| Rolls-Royce | RB211-535C | 7,294 | 14,274 | 21.1 | 1,142 | 37,400 |
| Rolls-Royce | Tay 620-15 | 3,185 | 6,930 | 15.8 | 410 | 13,850 |
| Rolls-Royce | Trent 1000C | 11,924 | 19,887 | 47.7 | 2,659 | 73,000 |
| Rolls-Royce | Trent 556 | 10,660 | 17,762 | 36.3 | 1,939 | 56,000 |
| Rolls-Royce | Trent 700 | 10,467 | 25,374 | 35.5 | 2,030 | 71,100 |
| Rolls-Royce | Trent 895 | 13,100 | 31,956 | 41.6 | 2,664 | 95,000 |
| Rolls-Royce | Trent 970 | 14,190 | 20,617 | 38.5 | 2,655 | 70,000 |
| **Military** | | | | | | |
| Aviadvigatel | D-30F6 | 5,326 | 15,080 | 21.2 | 331 | 20,944 |
| Eurojet | EJ200 | 2,180 | 9,983 | 26.1 | 170 | 13,490 |
| General Electric | F101-GE-102 | 4,460 | 9,554 | 26.8 | 352 | 17,000 |
| General Electric | F110-GE-100 | 3,920 | 12,972 | 30.4 | 269.8 | 17,530 |
| General Electric | F110-GE-129 | 3,980 | 10,880 | 30.7 | 270 | 17,000 |
| General Electric | F110-GE-132 | 4,150 | 12,224 | 33.3 | 275.6 | 19,100 |
| General Electric | F118-GE-100 | 3,200 | 12,730 | 35.1 | 287 | 19,000 |
| General Electric | F404-GE-400 | 2,195 | 9,010 | 26.0 | 146 | 10,600 |
| General Electric | F414-GE-400 | 2,470 | 10,683 | 30.0 | 172 | 14,756 |
| Klimov | RD-33 | 2,683 | 8,569 | 21.7 | 169.8 | 11,128 |
| Lyul'ka Saturn Inc | AL-31F | 3,373 | 11,525 | 23.5 | 247 | 17,305 |
| Lyul'ka Saturn Inc | AL-37FU | 3,660 | 12,687 | 25.0 | 264 | 18,740 |
| Pratt & Whitney | F100-PW-220E | 3,245 | 10,651 | 25.0 | 228 | 14,590 |
| Pratt & Whitney | F100-PW-229 | 3,795 | 12,923 | 32.4 | 248 | 17,800 |
| Pratt & Whitney | F119-PW-100 | 3,900 | 16,470 | 35.0 | 305 | 27,000 |
| Pratt & Whitney | TF33-P-103 | 3,900 | 8,585 | 13.0 | 460 | 17,000 |
| Rolls-Royce | F402-RR-408 | 4,260 | 18,088 | 16.3 | 461 | 23,800 |
| SNECMA | M88-2 | 1,978 | 8,760 | 24.5 | 143.3 | 10,950 |
| SNECMA | M88-3 | 2,013 | 10,665 | 26.6 | 158.1 | 13,500 |
| Turbo-Union | RB199 Mk105 | 2,185 | 6,273 | 24.5 | 166 | 9,650 |
| Volvo Aero Corp. | RM12 | 2,326 | 10,198 | 27.5 | 152 | 12,140 |

the frontier projection calculated from the VSBM Network DEA model.

Also, it should be noted that nearly all military aircraft turbofan engines (except for F119-PW-100 and F402-RR-408) have identical frontier projections for all inputs, intermediate measures, and final output. More specifically:

- Fuel flow frontier projections lie in the range between 3,014.7 and 3,034.4 lbs/hour
- Engine weight frontier projections lie in the vicinity between 1,593.8 and 1,605.9 lbs
- Overall pressure ratio frontier projections take certain distinctive values of either 23.4 or 23.5.

- Air mass flow frontier projections lie in the vicinity between 323.9 and 326.7 lbs/sec.
- Thrust frontier projections lie within the value range between 21,757 lbs and 21,951 lbs.

As depicted in Table 5, the frontier projections designate that in order for a military turbofan aero-engine to become efficient, the optimal thrust specific fuel consumption (TSFC) should be approximately 0.1383 lb$_f$/lb•hr, while the optimal thrust-to-weight ratio (T/W ratio) should take an approximate value of 13.65. Consulting Table 3 of the current paper, it can be concluded that the aforementioned optimal TSFC and T/W ratio values vastly differ from the actual values, taking into account that typical actual

**Table 3**
Aero-engine additional data.

| Model | Entry into Service (Year) | Aircraft Type(s) | Thrust-Specific Fuel Consumption (lb$_f$/lb•hr) | Bypass Ratio | Thrust-to-Weight Ratio |
|---|---|---|---|---|---|
| Commercial | | | | | |
| ALF502R-6 | 1984 | BAe 146-200/300 | 0.42 | 5.6 | 4.87 |
| CFE738-1-1B | 1993 | Dassault Falcon 2000 | 0.372 | 5.3 | 4.47 |
| CFM56-3C | 1986 | Boeing 737-400 | 0.389 | 5.0 | 5.46 |
| CFM56-5B3/P | 1994 | Airbus A321-200 | 0.34 | 5.4 | 6.34 |
| CFM56-7B26 | 2001 | Boeing 737-900ER | 0.368 | 5.1 | 4.96 |
| GP7270 | 2005 | Airbus A380-800 | 0.299 | 8.7 | 5.70 |
| CF34-80C1 | 1995 | Bombardier CRJ700 | 0.378 | 4.8 | 5.27 |
| CF6-80C2B6F | 1985 | Boeing 767-300ER | 0.336 | 5.1 | 6.13 |
| CF6-80E1A3 | 2001 | Airbus A330-200 | 0.34 | 5.3 | 6.22 |
| GE90-110B1 | 2004 | Boeing 777-200LR | 0.311 | 8.9 | 5.70 |
| GE90-115B1 | 2004 | Boeing 777-300ER | 0.322 | 8.9 | 5.98 |
| GE90-85B | 1995 | Boeing 777-200 | 0.298 | 8.3 | 4.91 |
| GE90-94B | 2000 | Boeing 777-300 | 0.2974 | 8.4 | 5.43 |
| GE9X | Beyond 2020 | Boeing 777-8X/-9X | 0.2898* | 10.3 | 5.10 |
| GEnx-1B70 | 2014 | Boeing 787-9 | 0.2845 | 9.1 | 5.16 |
| TFE731-60 | 1995 | Dassault Falcon 900 | 0.405 | 3.9 | 5.06 |
| V2533-A5 | 1995 | Airbus A321-200 | 0.342 | 4.5 | 6.35 |
| D-18T | 1993 | Antonov An-124-100 | 0.345 | 5.6 | 5.74 |
| PW2040 | 1987 | Boeing 757-200/757F | 0.335 | 5.5 | 5.72 |
| PW4056 | 1997 | Boeing 747-400 | 0.342 | 4.9 | 6.02 |
| PW4084 | 1995 | Boeing 777-200 | 0.32 | 6.4 | 5.67 |
| PW4090 | 1997 | Boeing 777-300 | 0.339 | 6.3 | 5.83 |
| PW4098 | 1999 | Boeing 777-300 | 0.358 | 5.8 | 6.09 |
| PW4462 | 1993 | MD-11 | 0.366 | 4.8 | 6.58 |
| PW6122A | 2007 | Airbus A318-100 | 0.383 | 5.0 | 4.38 |
| AE3007A3 | 1999 | Embraer ERJ-135 | 0.403 | 4.8 | 4.44 |
| BR715-C1-30 | 1999 | Boeing 717-200 | 0.372 | 4.55 | 4.57 |
| RB211-535C | 1983 | Boeing 757-200 | 0.382 | 4.4 | 5.13 |
| Tay 620-15 | 1987 | Fokker 70 | 0.5 | 3.04 | 4.35 |
| Trent 1000C | 2014 | Boeing 787-9 | 0.273 | 11.0 | 6.12 |
| Trent 556 | 2002 | Airbus A340-500/600 | 0.317 | 7.6 | 5.25 |
| Trent 700 | 1995 | Airbus A330-200 | 0.357 | 5.0 | 6.79 |
| Trent 895 | 2000 | Boeing 777-200ER | 0.336 | 5.8 | 7.25 |
| Trent 970 | 2007 | Airbus A380-800 | 0.295 | 8.7 | 4.93 |
| Military | | | | | |
| D-30F6 | 1980 | MiG-31 | 0.52 | 0.52 | 3.93 |
| EJ200 | 2003 | EF2000 | 0.74 | 0.4 | 6.19 |
| F101-GE-102 | 1985 | B-1B | 0.562 | 2.1 | 3.81 |
| F110-GE-100 | 1986 | F-16C/D | 0.74 | 0.76 | 4.47 |
| F110-GE-129 | 1992 | F-16C/D & F-15 | 0.64 | 0.76 | 4.27 |
| F110-GE-132 | 2003 | F-16E/F | 0.64 | 0.68 | 4.60 |
| F118-GE-100 | 1993 | B-2A | 0.67 | 0.76 | 5.94 |
| F404-GE-400 | 1981 | F/A-18A/B/C/D | 0.85 | 0.27 | 4.83 |
| F414-GE-400 | 2001 | F/A-18E/F | 0.724 | 0.27 | 5.97 |
| RD-33 | 1981 | MiG-29 | 0.77 | 0.55 | 4.15 |
| AL-31F | 1985 | Su-27 | 0.666 | 0.57 | 5.13 |
| AL-37FU | 1995 | Su-30MK | 0.677 | 0.65 | 5.12 |
| F100-PW-220E | 1985 | F-16 & F-15 | 0.73 | 0.6 | 4.50 |
| F100-PW-229 | 1991 | F-16 & F-15 | 0.726 | 0.36 | 4.69 |
| F119-PW-100 | 2005 | F-22A | 0.61 | 0.45 | 6.92 |
| TF33-P-103 | 1962 | B-52H | 0.505 | 1.36 | 4.36 |
| F402-RR-408 | 1986 | AV-8B | 0.76 | 1.2 | 5.59 |
| M88-2 | 1999 | Rafale | 0.8 | 0.3 | 5.54 |
| M88-3 | 2005 | Rafale | 0.79 | 0.3 | 6.71 |
| RB199 Mk105 | 1999 | Tornado ECR | 0.65 | 1.1 | 4.42 |
| RM12 | 1986 | JAS39 | 0.84 | 0.31 | 5.22 |

* Assuming 10 percent improvement over GE90-115B1 [1,13]

TSFC values for military turbofan aero-engines are within the 0.65 - 0.75 lb$_f$/lb•hr range, while typical actual thrust-to-weight ratios are within the 4.5 - 6.0 range.

By observing the column of Table 5 named "Air Mass Flow Change Percentage", it can be deduced that frontier projections regarding air mass flow for military aircraft turbofan aero-engines designate the requirement for substantially increasing the air mass flow for the majority of these engines (except for D-30F6, F101-GE-102, TF33-P-103, and F402-RR-408). The aforementioned frontier projection pattern for air mass flow coincides with the engineering approach which is associated with propulsive efficiency. In particular, military turbofan aero-engines usually propel a rel-

atively small air mass to a high velocity (compared to commercial turbofan aero-engines), thus resulting in a fairly low propulsive efficiency within the subsonic speed region. In general, propulsion systems' theory suggests that higher propulsive efficiency is typically obtained when a large mass flow is given a slightly small speed increase. Hence, it is evident that the VSBM Network DEA efficiency score pattern follows the propulsive efficiency pattern for the subsonic speed region (see Figure 3 of the current paper), which is confirmed by the relatively low efficiency scores (mean value equal to 28.66 %) of military turbofan aero engines, when compared with the efficiency scores of the commercial turbofan aero-engines (mean value equal to 56.49 %).

**Table 4**
Overall efficiency, stage efficiencies, and frontier projections.

| Model | Overall Efficiency | Stage Efficiencies | | Frontier Projections* | | | | |
| | | 1st Stage | 2nd Stage | Fuel Flow | Weight | Overall Pressure Ratio | Air Mass Flow | Thrust |
|---|---|---|---|---|---|---|---|---|
| **Commercial** | | | | | | | | |
| ALF502R-6 | 0.35014 | 0.45837 | 0.89177 | 2,657.4 | 1,376.0 | 22.9 | 274.7 | 18,601 |
| CFE738-1-1B | 0.38181 | 0.94221 | 0.43960 | 2,201.5 | 1,101.9 | 22.2 | 212.8 | 14,194 |
| CFM56-3C | 0.38278 | 0.73280 | 0.64998 | 3,275.1 | 1,753.4 | 23.8 | 360.0 | 23,500 |
| CFM56-5B3/P | 0.52680 | 0.80122 | 0.72558 | 5,380.1 | 3,042.1 | 26.8 | 651.3 | 33,300 |
| CFM56-7B26 | 0.39114 | 0.71387 | 0.67726 | 3,792.5 | 2,070.1 | 24.5 | 431.6 | 26,300 |
| GP7270 | 0.63064 | 0.98121 | 0.64944 | 13,696.4 | 8,133.9 | 38.8 | 1,802.3 | 70,000 |
| CF34-80C1 | 0.37565 | 0.89518 | 0.48047 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,951 |
| CF6-80C2B6F | 0.62867 | 0.70239 | 0.92628 | 11,438.0 | 6,751.8 | 35.5 | 1,489.8 | 60,030 |
| CF6-80E1A3 | 0.64905 | 0.67122 | 0.97783 | 13,651.1 | 8,106.1 | 38.7 | 1,796.0 | 69,800 |
| GE90-110B1 | 0.84959 | 1 | 0.84959 | 27,009.4 | 16,747.1 | 42.3 | 3,364.8 | 110,100 |
| GE90-115B1 | 0.93637 | 0.93637 | 1 | 32,456.3 | 19,316.0 | 42.0 | 3,618 | 115,540 |
| GE90-85B | 0.63619 | 0.82903 | 0.80716 | 16,953.7 | 10,363.4 | 40.6 | 2,307.0 | 84,700 |
| GE90-94B | 0.68004 | 0.82145 | 0.85859 | 18,935.8 | 11,739.5 | 41.5 | 2,619.0 | 93,700 |
| GE9X | 0.69864 | 1 | 0.69864 | 21,280.6 | 13,559.7 | 41.7 | 2,989.6 | 102,000 |
| GEnx-1B70 | 0.64309 | 0.89209 | 0.75100 | 13,651.9 | 8,106.9 | 38.7 | 1,796.1 | 69,800 |
| TFE731-60 | 0.40219 | 1 | 0.40219 | 2,025 | 988.0 | 22.0 | 187.0 | 12,424 |
| V2533-A5 | 0.52327 | 0.72595 | 0.79732 | 5,312.7 | 3,000.8 | 26.7 | 641.9 | 33,000 |
| D-18T | 0.57809 | 0.71313 | 0.86496 | 9,544.1 | 5,591.5 | 32.8 | 1,227.5 | 51,660 |
| PW2040 | 0.54910 | 0.75917 | 0.78992 | 7,283.0 | 4,207.1 | 29.6 | 914.6 | 41,700 |
| PW4056 | 0.60947 | 0.69364 | 0.91583 | 10,698.9 | 6,298.9 | 34.5 | 1,387.3 | 56,750 |
| PW4084 | 0.65962 | 0.67099 | 0.98863 | 16,930.5 | 10,347.8 | 40.6 | 2,303.4 | 84,600 |
| PW4090 | 0.66097 | 0.69562 | 0.96535 | 18,514.0 | 11,446.6 | 41.3 | 2,552.7 | 91,790 |
| PW4098 | 0.67973 | 0.68122 | 0.99852 | 20,401.1 | 12,783.7 | 41.7 | 2,854.1 | 99,040 |
| PW4462 | 0.63498 | 0.70084 | 0.93414 | 11,883.2 | 7,023.7 | 36.2 | 1,551.3 | 62,000 |
| PW6122A | 0.34131 | 0.62403 | 0.71728 | 3,057.6 | 1,620.4 | 23.5 | 329.9 | 22,100 |
| AE3007A3 | 0.33705 | 0.74335 | 0.59371 | 2,841.0 | 1,524.9 | 22.7 | 308.5 | 20,479 |
| BR715-C1-30 | 0.35312 | 0.69952 | 0.65360 | 3,033.5 | 1,605.5 | 23.5 | 326.6 | 21,941 |
| RB211-535C | 0.46843 | 0.59399 | 0.87445 | 6,309.4 | 3,611.3 | 28.2 | 779.9 | 37,400 |
| Tay 620-15 | 0.29720 | 0.47487 | 0.82233 | 3,026.5 | 1,604.0 | 23.4 | 326.2 | 21,888 |
| Trent 1000C | 0.72058 | 1 | 0.72058 | 14,368.3 | 8,569.4 | 39.5 | 1,900.8 | 73,000 |
| Trent 556 | 0.58662 | 0.80296 | 0.78366 | 10,523.9 | 6,191.5 | 34.2 | 1,363.2 | 56,000 |
| Trent 700 | 0.67064 | 0.71373 | 0.95690 | 13,948.9 | 8,288.8 | 39.1 | 1,837.2 | 71,100 |
| Trent 895 | 0.75641 | 0.75641 | 1 | 19,222.3 | 11,938.2 | 41.6 | 2,664.0 | 95,000 |
| Trent 970 | 0.61876 | 0.86287 | 0.75589 | 13,697.9 | 8,134.7 | 38.8 | 1,802.5 | 70,000 |
| **Military** | | | | | | | | |
| D-30F6 | 0.23984 | 0.23984 | 1 | 3,034.1 | 1,605.8 | 23.5 | 326.6 | 21,948 |
| EJ200 | 0.31976 | 0.35895 | 0.96081 | 3,014.7 | 1,593.8 | 23.4 | 323.9 | 21,757 |
| F101-GE-102 | 0.26241 | 0.38635 | 0.87606 | 3,034.1 | 1,606.0 | 23.5 | 326.7 | 21,951 |
| F110-GE-100 | 0.25699 | 0.33296 | 0.92403 | 3,033.5 | 1,605.4 | 23.5 | 326.6 | 21,942 |
| F110-GE-129 | 0.26424 | 0.35593 | 0.90831 | 3,032.9 | 1,605.1 | 23.5 | 326.5 | 21,937 |
| F110-GE-132 | 0.27635 | 0.35552 | 0.92084 | 3,034.4 | 1,605.9 | 23.5 | 326.7 | 21,950 |
| F118-GE-100 | 0.32035 | 0.44028 | 0.88007 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,950 |
| F404-GE-400 | 0.25797 | 0.33736 | 0.92061 | 3,034.1 | 1,606.1 | 23.4 | 326.7 | 21,947 |
| F414-GE-400 | 0.31400 | 0.36923 | 0.94477 | 3,034.2 | 1,605.9 | 23.5 | 326.7 | 21,950 |
| RD-33 | 0.24148 | 0.27945 | 0.96203 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,951 |
| AL-31F | 0.29145 | 0.30036 | 0.99109 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,951 |
| AL-37FU | 0.28939 | 0.30141 | 0.98798 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,950 |
| F100-PW-220E | 0.25915 | 0.31360 | 0.94555 | 3,033.5 | 1,605.9 | 23.5 | 326.6 | 21,939 |
| F100-PW-229 | 0.26678 | 0.33941 | 0.92737 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,950 |
| F119-PW-100 | 0.39777 | 0.39777 | 1 | 3,951.7 | 2,167.5 | 24.8 | 453.6 | 27,000 |
| TF33-P-103 | 0.29632 | 0.40430 | 0.89202 | 3,030.7 | 1,603.7 | 23.4 | 326.0 | 21,894 |
| F402-RR-408 | 0.30095 | 0.30095 | 1 | 3,321.6 | 1,781.8 | 23.9 | 366.4 | 23,800 |
| M88-2 | 0.28890 | 0.34019 | 0.94871 | 3,034.2 | 1,605.9 | 23.5 | 326.7 | 21,939 |
| M88-3 | 0.33281 | 0.36358 | 0.96923 | 3,033.4 | 1,605.3 | 23.5 | 326.5 | 21,939 |
| RB199 Mk105 | 0.26789 | 0.39447 | 0.87342 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,951 |
| RM12 | 0.27320 | 0.33668 | 0.93653 | 3,034.3 | 1,605.9 | 23.5 | 326.7 | 21,951 |

* Underlined numerical values designate frontier projections equal to actual values

## 5. Post-hoc analysis and discussion

### 5.1. Non-parametric post-hoc analysis

The theoretical distribution of DEA efficiency is generally deemed as unknown, consequently designating the need to implement non-parametric statistical tests, fundamentally dealing with cases where observations are considered as statistically independent [5,12]. In order to perform the necessary statistical tests in the framework of the current paper, the IBM SPSS Version 22 software is used.

Initially, it is investigated whether statistically significant mean efficiency differences exist between commercial aircraft and military aircraft aero-engines. After consulting Table 3 of the current paper, it is deduced that there is a group of 34 commercial aircraft aero-engines and a group of 21 military aero-engines. The mean DEA efficiency of those groups is as follows:

- Mean efficiency of commercial aircraft aero-engines = 56.49% ($n_1 = 34$)
- Mean efficiency of military aircraft aero-engines = 28.66% ($n_2 = 21$)

**Table 5**
DEa optimal values & value change percentages.

| Model | DEA Optimal TSFC | DEA Optimal Thrust-to-Weight | Weight Change Percentage | Air Mass Flow Change Percentage | Thrust Change Percentage |
|---|---|---|---|---|---|
| <u>Commercial</u> | | | | | |
| ALF502R-6 | 0.1429 | 13.52 | 0% | +43.07% | +177.62% |
| CFE738-1-1B | 0.1551 | 12.88 | -16.84% | +1.32% | +139.84% |
| CFM56-3C | 0.1394 | 13.40 | -59.23% | -49.29% | 0% |
| CFM56-5B3/P | 0.1616 | 10.95 | -42.05% | -32.72% | 0% |
| CFM56-7B26 | 0.1442 | 12.70 | -60.94% | -44.88% | 0% |
| GP7270 | 0.1957 | 8.61 | -39.37% | -39.92% | 0% |
| CF34-80C1 | 0.1382 | 13.67 | -33.17% | -25.91% | +73.25% |
| CF6-80C2B6F | 0.1905 | 8.89 | -31.03% | -15.30% | 0% |
| CF6-80E1A3 | 0.1956 | 8.61 | -27.79% | -6.75% | 0% |
| GE90-110B1 | 0.2453 | 6.57 | -13.30% | -7.00% | 0% |
| GE90-115B1 | 0.2809 | <u>5.98</u> | 0.00% | 0.00% | 0% |
| GE90-85B | 0.2002 | 8.17 | -39.92% | -26.06% | 0% |
| GE90-94B | 0.2021 | 7.98 | -31.950% | -19.02% | 0% |
| GE9X | 0.2086 | 7.52 | -32.20% | -22.35% | 0% |
| GEnx-1B70 | 0.1956 | 8.61 | -40.13% | -29.43% | 0% |
| TFE731-60 | 0.1630 | 12.58 | 0% | 0% | 148.48% |
| V2533-A5 | 0.1610 | 11.00 | -42.29% | -25.19% | 0% |
| D-18T | 0.1847 | 9.24 | -37.88% | -27.24% | 0% |
| PW2040 | 0.1747 | 9.91 | -42.33% | -31.74% | 0% |
| PW4056 | 0.1885 | 9.01 | -33.13% | -18.63% | 0% |
| PW4084 | 0.2001 | 8.18 | -30.64% | -4.02% | 0% |
| PW4090 | 0.2017 | 8.02 | -27.28% | -6.15% | 0% |
| PW4098 | 0.2060 | 7.75 | -21.38% | +0.14% | 0% |
| PW4462 | 0.1917 | 8.83 | -25.44% | -13.82% | 0% |
| PW6122A | 0.1384 | 13.64 | -67.86% | -50.01% | 0% |
| AE3007A3 | 0.1387 | 13.43 | -3.85% | +28.54% | +190.89% |
| BR715-C1-30 | 0.1383 | 13.67 | -65.08% | -47.75% | +4.48% |
| RB211-535C | 0.1687 | 10.36 | -50.49% | -31.71% | 0% |
| Tay 620-15 | 0.1383 | 13.65 | -49.64% | -20.44% | +58.04% |
| Trent 1000C | 0.1968 | 8.52 | -28.13% | -28.51% | 0% |
| Trent 556 | 0.1879 | 9.04 | -41.92% | -29.70% | 0% |
| Trent 700 | 0.1962 | 8.58 | -20.81% | -9.50% | 0% |
| Trent 895 | 0.2023 | 7.96 | -8.87% | 0% | 0% |
| Trent 970 | 0.1957 | 8.61 | -42.67% | -32.11% | 0% |
| <u>Military</u> | | | | | |
| D-30F6 | 0.1382 | 13.67 | -69.85% | -1.31% | +4.79% |
| EJ200 | 0.1386 | 13.65 | -26.89% | +90.54% | +61.28% |
| F101-GE-102 | 0.1382 | 13.67 | -63.99% | -7.19% | +29.12% |
| F110-GE-100 | 0.1382 | 13.67 | -59.05% | +21.04% | +25.17% |
| F110-GE-129 | 0.1382 | 13.67 | -59.67% | +20.92% | +29.04% |
| F110-GE-132 | 0.1383 | 13.67 | -61.30% | +18.53% | +14.92% |
| F118-GE-100 | 0.1382 | 13.67 | -49.81% | +13.82% | +15.53% |
| F404-GE-400 | 0.1382 | 13.66 | -26.83% | +123.76% | +107.50% |
| F414-GE-400 | 0.1382 | 13.67 | -34.98% | +89.92% | +48.75% |
| RD-33 | 0.1382 | 13.67 | -40.14% | +92.39% | +97.26% |
| AL-31F | 0.1382 | 13.67 | -52.39% | +32.26% | +26.85% |
| AL-37FU | 0.1382 | 13.67 | -56.12% | +23.74% | +17.13% |
| F100-PW-220E | 0.1383 | 13.67 | -50.53% | +43.23% | +50.37% |
| F100-PW-229 | 0.1382 | 13.67 | -57.68% | +31.72% | +23.32% |
| F119-PW-100 | 0.1464 | 12.46 | -44.42% | +48.73% | 0% |
| TF33-P-103 | 0.1384 | 13.65 | -58.88% | -29.13% | +28.79% |
| F402-RR-408 | 0.1396 | 13.36 | -58.17% | -20.51% | 0% |
| M88-2 | 0.1382 | 13.67 | -18.81% | +127.97% | +100.46% |
| M88-3 | 0.1383 | 13.67 | -20.25% | +106.53% | +62.51% |
| RB199 Mk105 | 0.1382 | 13.67 | -26.50% | +96.79% | +127.47% |
| RM12 | 0.1382 | 13.67 | -30.96% | +114.92% | +80.81% |

For the purpose of assessing whether the mean efficiency difference between the two aforementioned groups is statistically significant, the Mann-Whitney rank-sum test is deployed. The Mann-Whitney non-parametric statistical test evaluates the hypothesis whether two groups belong to the same population or they differ in a significant grade. It is important to stress that the validity of Mann-Whitney rank-sum test is enhanced for the case where at least 10 observations are contained into the investigated groups [5,12]. In addition, the hypothesis shall be examined at 0.05 significance level.

The hypothesis investigated in order to evaluate whether the mean efficiency differentiation between commercial and military aircraft aero-engines is statistically significant, is stated as follows:

$H_1$: Commercial and military aircraft aero-engine have identical mean efficiency.

The Mann-Whitney rank-sum test provides a Z-value equal to -5.942 and a p-value far less than 0.01 (see Table 6 for further details). Hence, hypothesis $H_1$ is rejected at 0.05 significance level, consequently designating that the mean efficiency difference between commercial and military aero-engines is statistically significant.

Observing the numerical values regarding BPR in Table 3, it is evident that the great majority of military aircraft aero-engines incorporate a significantly low BPR (usually less than 1) compared to commercial aircraft aero-engines. Hence, the obtained result for hypothesis $H_1$ coincides with the superior propulsive efficiency

**Table 6**
Mann-Whitney statistical test for hypothesis $H_1$.

| Group | N | Mean Rank | Sum of Ranks |
|---|---|---|---|
| 1 | 34 | 38.09 | 1,295.00 |
| 2 | 21 | 11.67 | 245.00 |
| Total | 55 | | |
| Mann-Whitney U | 14.000 | | |
| Wilcoxon W | 245.00 | | |
| Z | -5.942 | | |
| p-value | $2.8135 \cdot 10^{-9}$ | | |

of high-BPR over low-BPR turbofan aero-engines in the subsonic speed region, which is qualitatively expressed in Fig. 3 of the current paper. Subsequently, it is deduced that the resulting efficiency scores of the VSBM Network DEA model follow the same pattern as the propulsive efficiency dictated by the propulsion systems' fundamental theory (as also noted in Section 4.2 of the current paper).

Another turbofan aero-engine feature that shall be investigated how it is associated with VSBM Network DEA efficiency is the Entry Into Service (EIS) year. The scope of investigating how EIS year is associated with DEA efficiency, is to evaluate whether technological progress achieved by aero-engine manufacturers has resulted into significant efficiency enhancement. Taking into account the data in Table 3, we divide the sample into three groups. In particular, the first group consists of turbofan aero-engines with EIS year in the 1980s decade and earlier, the second group consists of turbofan aero-engines with EIS year in the 1990s decade, while the third group consists of turbofan aero-engines with EIS year spanning from 2000 to near-future i.e. 2020.

The mean DEA efficiency scores for each of the formed group were calculated as follows:

- Mean efficiency of turbofan aero-engine group with EIS year in the 1980s decade and earlier = 33.48% ($n_1 = 16$).
- Mean efficiency of turbofan aero-engine group with EIS year in the 1990s decade = 46.32% ($n_2 = 21$).
- Mean efficiency of turbofan aero-engine group with EIS year in the 2000s decade and later = 56.35% ($n_3 = 18$).

In order to assess whether there are statistically significant differences regarding the mean DEA efficiency scores among the aforementioned turbofan aero-engine groups, we shall deploy the Games-Howell statistical test. The specific statistical test has been designated as the most appropriate for the purpose of conducting pairwise comparisons among multiple groups, given that the samples' size and associated variance is not equal. As stated by Shingala and Rajyaguru [51], Games-Howell statistical test possesses the best capability with respect to pairwise comparisons among multiple groups. In a similar concept with the previous Mann-Whitney rank-sum test, any pairwise comparison with significance level (p-value) less than or equal to 0.05 shall be considered as statistically significant. The results of the conducted Games-Howell statistical tests are shown in Table 7.

By observing the pairwise comparison results provided by the Games-Howell statistical test, it is deduced that there are statistically significant differences between the first group (i.e. turbofan aero-engines which entered service in 1980s decade and earlier) and both other groups (i.e. turbofan aero-engines which entered service in 1990s and beyond). Hence, it can be stated that a major technological leap has been achieved from 1990s decade and onwards. However, it would be potentially of great interest to form a group consisting of turbofan aero-engines with EIS year from 2010 to date, in order to better assess the efficiency difference compared to the groups consisting of turbofan aero-engines having EIS date within the 1990s and 2000s decades.

In addition, we attempt to evaluate the effect of BPR in DEA efficiency, in order to confirm whether the DEA approach coincides with the engineering approach. More specifically, we divide the sample into the following groups with respect to BPR [19] (along with their respective mean DEA efficiency):

- Low-BPR engines, i.e. engines featuring BPR $\leq 2$ with mean DEA efficiency score equal to 28.78% ($n_1 = 20$).
- Medium-BPR engines, i.e. engines featuring $2 < $ BPR $\leq 5$ with mean DEA efficiency score equal to 43.53% ($n_2 = 13$).
- High-BPR engines, i.e. engines featuring $5 < $ BPR $\leq 8$ with mean DEA efficiency score equal to 56.91% ($n_3 = 13$).
- Ultrahigh-BPR engines, i.e. engines featuring BPR $> 8$ with mean DEA efficiency score equal to 71.27 % ($n_4 = 9$).

In a similar way to the previous efficiency differences' evaluation with respect to EIS year, we conduct the Games-Howell statistical test, taking into account that it is considered the best-performing in terms of pairwise comparison. Moreover, although the population of the last group can be deemed as quite small, Shingala and Rajyaguru [51] state that Games-Howell statistical test can be deployed for samples with population greater than five. The results of the Games-Howell statistical test evaluating whether there are statistically significant mean efficiency differences among the aforementioned aero-engine groups are shown in Table 8.

The results obtained by performing pairwise comparisons via the Games-Howell statistical test can be characterized as quite interesting. More specifically, it is clearly evident that the mean efficiency differences are nearly all statistically significant. Subsequently, it is deduced by observing the p-values of Table 8 that the statistical significance is increasing as the gap in terms of BPR range increases. In particular, with respect to the ultrahigh-BPR engine group the p-values of the pairwise comparisons between the first and the second group (0.000007 and 0.000202 respectively) are significantly lower to the p-value when compared to the third group (0.047898).

The results of the aforementioned Games-Howell statistical test designate the positive association of BPR and turbofan aero-engine DEA efficiency. The particular positive association is fully aligned with the results of previous research using the engineering approach. A typical example is the research of Guha [26] which clearly demonstrates the positive effect of ultrahigh BPR (usually between 8 and 10) on specific fuel consumption, namely reduced specific fuel consumption. Also, it is of paramount importance to stress that in accordance with El-Sayed [16] high BPR is positively associated with increased propulsive efficiency and reduced specific fuel consumption, especially for BPR within 8 to 10 value range. Consequently, the aforementioned engineering research conclusions constitute a strong indication of the validity of VSBM Network DEA methodology, as an alternative approach to evaluate turbofan aero-engine efficiency without incorporating complex engineering calculations.

## 5.2. Regression analysis

Apart from non-parametric statistical analysis, regression comprises a quite popular method for assessing the effect of various exogenous factors on DEA efficiency. The regression methodologies implemented by researchers dealing with DEA efficiency post-hoc analysis are diverse and in certain cases pretty sophisticated (e.g., truncated regression).

In the current paper, we adopt the following regression methodologies:

- Generalized Linear Model (GLM) equipped with the quasi-binomial family and logit link function. The GLM is applied to separately test the null hypothesis that DMU efficiency is independent of the DMU grouping in terms of BPR and EIS.

**Table 7**

Games-Howell test for pairwise comparison of turbofan aero-engine groups based on entry into service (EIS) year.

| (I) Turbofan Aero-engine Group | (J) Turbofan Aero-engine Group | Mean Difference (I-J) | p-value | 95% Confidence Interval | |
|---|---|---|---|---|---|
| | | | | Lower Bound | Upper Bound |
| 1 | 2 | -0.1284417 | 0.020659* | -0.2397012 | -0.0171821 |
| | 3 | -0.2287411 | 0.000964* | -0.3673748 | -0.0901074 |
| 2 | 1 | 0.1284417 | 0.020659* | 0.0171821 | 0.2397012 |
| | 3 | -0.1002994 | 0.223214 | -0.2458241 | 0.0452252 |
| 3 | 1 | 0.2287411 | 0.000964* | 0.0901074 | 0.3673748 |
| | 2 | 0.1002994 | 0.223214 | -0.0452252 | 0.2458241 |

* Statistically significant at 0.05 significance level

**Table 8**

Games-Howell test for pairwise comparison of turbofan aero-engine groups based on bypass ratio (BPR).

| (I) Turbofan Aero-engine Group | (J) Turbofan Aero-engine Group | Mean Difference (I-J) | p-value | 95% Confidence Interval | |
|---|---|---|---|---|---|
| | | | | Lower Bound | Upper Bound |
| 1 | 2 | -0.1474897 | 0.008954* | -0.2591569 | -0.0358226 |
| | 3 | -0.2813090 | 0.000013* | -0.3865049 | -0.1761130 |
| | 4 | -0.4248761 | 0.000007* | -0.5424199 | -0.3073322 |
| 2 | 1 | 0.1474897 | 0.008954* | 0.0358226 | 0.2591569 |
| | 3 | -0.1338192 | 0.067098 | -0.2747984 | 0.0071599 |
| | 4 | -0.2773863 | 0.000202* | -0.4238899 | -0.1308828 |
| 3 | 1 | 0.2813090 | 0.000013* | 0.1761130 | 0.3865049 |
| | 2 | 0.1338192 | 0.067098 | -0.0071599 | 0.2747984 |
| | 4 | -0.1435671 | 0.047898* | -0.2860546 | -0.0010796 |
| 4 | 1 | 0.4248761 | 0.000007* | 0.3073322 | 0.5424199 |
| | 2 | 0.2773863 | 0.000202* | 0.1308828 | 0.4238899 |
| | 3 | 0.1435671 | 0.047898* | 0.0010796 | 0.2860546 |

* Statistically significant at 0.05 significance level

**Table 9**

Regression analysis results.

| Model | Explanatory Variable | Intercept (p-value) | Slope (p-value) | Linearity Assumption (p-value) | Null Deviance Reduction | Adjusted Coefficient of Determination |
|---|---|---|---|---|---|---|
| GLM | BPR-based Groups | $-1.492(2.95 \bullet 10^{-15})$ | $0.598(4.44 \bullet 10^{-15})$ | Not Applicable | 69.93% | Not Applicable |
| GLM | EIS-based Groups | $-1.125(6.44 \bullet 10^{-5})$ | $0.468(2.12 \bullet 10^{-4})$ | Not Applicable | 22.88% | Not Applicable |
| LM | BPR | $0.251(4.13 \bullet 10^{-16})$ | $0.051 (< 2 \bullet 10^{-16})$ | 0.844 | Not Applicable | 72.57% |
| LM | EIS | $-17.868(1.76 \bullet 10^{-4})$ | $0.009 (1.25 \bullet 10^{-4})$ | 0.502 | Not Applicable | 23.01% |

- Standard Linear Model (LM), which is particularly deployed separately on raw BPR and EIS data for the purpose of testing the null hypothesis that DMU efficiency is independent of BPR and EIS respectively. For the particular model, it should be stressed that before proceeding with the aforementioned LM methodologies, an assessment for the linear model assumptions has been conducted by using the global test.

After applying the GLM and LM regression methodologies on the resulting DEA efficiency scores with respect to BPR and EIS data, it is highly evident that both BPR and EIS are positively correlated with increased DEA efficiency. The regression analysis output is summarized in Table 9, while Figs. 5 and 6 display the boxplots and scatterplots corresponding to prediction models regarding BPR and EIS respectively.

More specifically, GLM assessing the four BPR-based engine groups (as previously determined in sub-Section 5.1) designates the remarkable reduced null deviance reduction value of 69.93 percent, while the LM for assessing DEA efficiency with respect to BPR features a relatively increased adjusted coefficient of determination value of 72.57 percent. However, EIS demonstrates fairly low explanatory potential when regressed against DEA efficiency. In particular, for the case of LM assessing DEA efficiency with respect to EIS, although the adjusted coefficient of determination gets a fairly low value of 23.01 percent, EIS is found to be significantly correlated with DEA efficiency at the 5 percent significance level. In addition, it is vital to stress that for LM the assumption of linearity is not rejected at the 5 percent significance level, which is clearly designated by the p-value of 0.844 for DEA efficiency against BPR and the p-value of 0.502 for DEA efficiency against EIS.

### 5.3. Discussion

By observing the numerical values in Tables 4 and 5 regarding the frontier projections and the associated optimal values/change percentage respectively, it is notably evident that for the vast majority of the engines/DMUs the frontier projections do not correspond to figures achievable in the near future. Especially for the case of TSFC, although progress in aerodynamics, thermodynamics, and combustion technology has contributed to significantly reduce TSFC, the TSFC figures contained in Table 5 are certainly unrealistic.

With respect to future TSFC reduction due to technological progress, Birch [3] stresses that by year 2020 an annual reduction of 0.5% in TSFC per year shall be feasible, taking into account the current and near-future technological constraints. Hence, considering the lowest TSFC values in Table 3 (Trent 1000C and GE9X) and the claim of Birch [3], it can be deduced that the TSFC threshold for commercial turbofan aero-engines at the end of the next decade (i.e. year 2030) is predicted to lie within the 0.255 – 0.275 value range. Consequently, the optimal TSFC values in Table 5 which result from the frontier projections can be characterized as technologically infeasible.

Also, frontier projections designate extensive weight reduction for the majority of the engine sample, which is observable in

**Table 10**
Revised efficiency & corresponding exogenous inputs/final output values.

| Model | Maximum Attained Revised Efficiency | Efficiency Improvement (% of original value) | Corresponding Fuel Flow* (% of actual value) | Corresponding Weight* (% of actual value) | Corresponding Thrust* (% of actual value) |
|---|---|---|---|---|---|
| Commercial |
| ALF502R-6 | 0.39147 | 11.8 % | 95.00 % | 95.00 % | 105.00 % |
| CFE738-1-1B | 0.42629 | 11.6 % | 95.05 % | 95.18 % | 105.00 % |
| CFM56-3C | 0.42709 | 11.6 % | 95.00 % | 95.00 % | 105.00 % |
| CFM56-5B3/P | 0.59520 | 13.0 % | 95.00 % | 95.00 % | 105.00 % |
| CFM56-7B26 | 0.44605 | 14.0 % | 95.00 % | 95.00 % | 105.00 % |
| GP7270 | 0.68187 | 8.1 % | 95.00 % | 95.00 % | 105.00 % |
| CF34-80C1 | 0.41520 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| CF6-80C2B6F | 0.70193 | 11.6 % | 95.00 % | 95.00 % | 105.00 % |
| CF6-80E1A3 | 0.71736 | 10.5 % | 95.00 % | 95.00 % | 104.81 % |
| GE90-110B1 | 1 | 17.7 % | 95.90 % | 96.22 % | 104.96 % |
| GE90-115B1 | 1 | 6.8 % | 98.46 % | 95.56 % | 104.09 % |
| GE90-85B | 0.70894 | 11.4 % | 95.00 % | 95.00 % | 105.00 % |
| GE90-94B | 0.76654 | 12.7 % | 95.00 % | 95.00 % | 105.00 % |
| GE9X | 0.82276 | 17.7 % | 95.00 % | 95.00 % | 105.00 % |
| GEnx-1B70 | 0.71737 | 11.5 % | 95.00 % | 95.00 % | 105.00 % |
| TFE731-60 | 0.42251 | 5.0 % | 97.48 % | 96.82 % | 104.99 % |
| V2533-A5 | 0.59139 | 13.0 % | 95.00 % | 95.00 % | 105.00 % |
| D-18T | 0.64676 | 11.8 % | 95.00 % | 95.00 % | 105.00 % |
| PW2040 | 0.61668 | 12.3 % | 95.00 % | 95.00 % | 105.00 % |
| PW4056 | 0.68098 | 11.7 % | 95.00 % | 95.00 % | 105.00 % |
| PW4084 | 0.71014 | 7.6 % | 95.00 % | 95.00 % | 105.00 % |
| PW4090 | 0.73222 | 10.8 % | 95.00 % | 95.00 % | 104.85 % |
| PW4098 | 0.71754 | 5.4 % | 95.00 % | 95.00 % | 104.48 % |
| PW4462 | 0.70875 | 11.6 % | 95.00 % | 95.00 % | 105.00 % |
| PW6122A | 0.38092 | 11.6 % | 95.00 % | 95.00 % | 105.00 % |
| AE3007A3 | 0.37586 | 11.5 % | 95.00 % | 95.00 % | 105.00 % |
| BR715-C1-30 | 0.39064 | 10.6 % | 95.00 % | 95.00 % | 105.00 % |
| RB211-535C | 0.52744 | 12.6 % | 95.00 % | 95.00 % | 105.00 % |
| Tay 620-15 | 0.32849 | 10.4 % | 95.00 % | 95.00 % | 105.00 % |
| Trent 1000C | 0.77153 | 7.1 % | 95.00 % | 95.00 % | 105.00 % |
| Trent 556 | 0.65549 | 11.7 % | 95.00 % | 95.00 % | 105.00 % |
| Trent 700 | 0.74924 | 11.7 % | 95.00 % | 95.00 % | 105.00 % |
| Trent 895 | 0.79622 | 5.3 % | 95.00 % | 95.00 % | 104.60 % |
| Trent 970 | 0.69028 | 11.5 % | 95.00 % | 95.00 % | 105.00 % |
| Military |
| D-30F6 | 0.25493 | 6.3 % | 95.00 % | 95.00 % | 105.00 % |
| EJ200 | 0.35342 | 10.3 % | 95.00 % | 95.00 % | 105.00 % |
| F101-GE-102 | 0.29004 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F110-GE-100 | 0.28404 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F110-GE-129 | 0.29206 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F110-GE-132 | 0.30544 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F118-GE-100 | 0.35408 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F404-GE-400 | 0.28512 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F414-GE-400 | 0.34705 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| RD-33 | 0.26690 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| AL-31F | 0.31796 | 9.1 % | 95.00 % | 95.00 % | 105.00 % |
| AL-37FU | 0.31856 | 10.1 % | 95.00 % | 95.00 % | 105.00 % |
| F100-PW-220E | 0.28643 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F100-PW-229 | 0.29486 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| F119-PW-100 | 0.43897 | 10.3 % | 95.00 % | 95.00 % | 105.00 % |
| TF33-P-103 | 0.32751 | 10.4 % | 95.00 % | 95.00 % | 105.00 % |
| F402-RR-408 | 0.32738 | 8.8 % | 95.00 % | 95.00 % | 105.00 % |
| M88-2 | 0.31931 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| M88-3 | 0.36784 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| RB199 Mk105 | 0.29609 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |
| RM12 | 0.30196 | 10.5 % | 95.00 % | 95.00 % | 105.00 % |

* Numerical values in bold designate improvement below the 5 percent threshold

"Weight Change Percentage" column of Table 5. Simultaneously, frontier projections for several engines stipulate for significant thrust increase, which in some cases are well above 50% or even 100%. Similarly to TSFC, it is highly doubtful whether near-future technology shall render the weight and thrust figures stipulated by frontier projections achievable. Although the frontier projections' viability is not stressed in the research effort of Chen et al. [8], the earlier research of Tone [53] expresses the need to impose bounds on the slacks in order to render reduction of inputs and increase of outputs to lie within a technically feasible spectrum. Hence, in the framework of evaluating efficiency of production or engineering systems, the developed methodologies implementing slacks-based measures should additionally investigate whether the resulting slacks represent technically feasible frontier projections.

Commercial and military aircraft take-off field requirements are directly associated with the take-off thrust-to-weight ratio, i.e. the total engine take-off thrust divided by the aircraft take-off weight. For the case of Boeing 777-8X/9X, although it has the same MTOW of 775,000 lbs as its predecessor 777-200LR/300ER [31], the reduced take-off thrust of GE9X engine is definitely a compromise in terms of required take-off field distance, compared to the GE90-110B1 and GE90-115B1. As a matter of fact, the take-off thrust-to-weight ratio for the 777-300ER/GE90-115B1 at MTOW is 0.2982, while the take-off thrust-to-weight ratio for
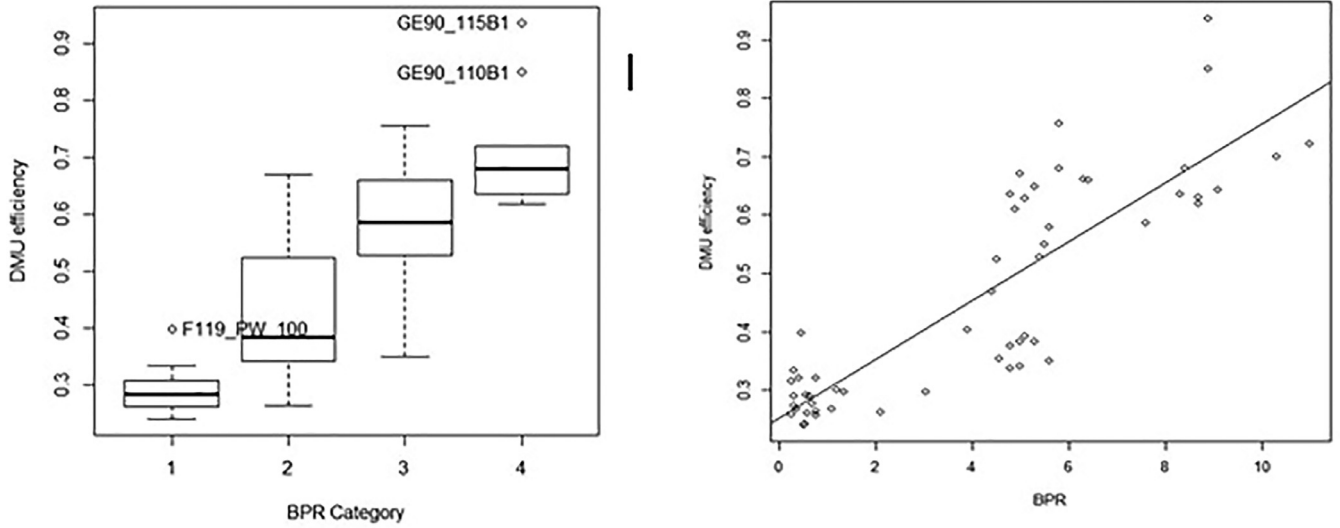
**Fig. 5.** Boxplot and Scatterplot of DEA efficiency versus BPR (Source: Own elaboration)
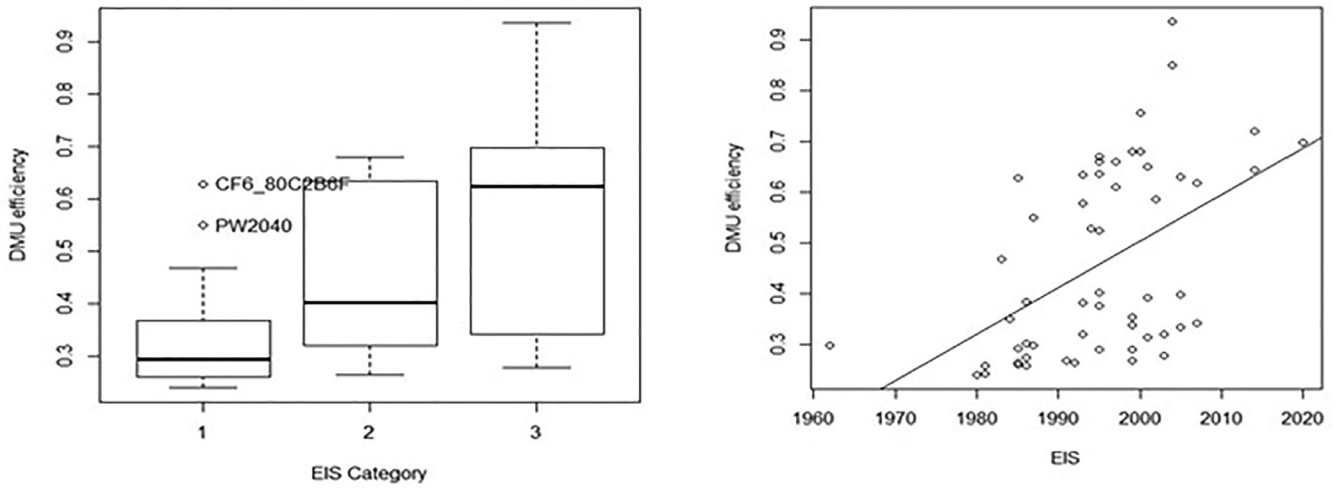


**Fig. 6.** Boxplot and scatterplot of DEA Efficiency versus EIS (Source: Own elaboration)

the 777-8X/GE9X(102,000 lbs version) at MTOW is 0.2632, which is certainly a quite notable reduction. Besides the reduced take-off thrust, the 3.5 percent more weight of GE9X compared to GE90-115B1 poses an additional feature, which negatively affects the DEA efficiency of the GE9X engine. Although the effort of the aero-engine manufacturers is concentrated on weight reduction, GE9X is not fulfilling the prospect of a lighter aero-engine, consequently leading to a payload reduction compared to GE90-115B1, taking into consideration that Boeing 777-300ER and 777-8X/-9X feature exactly the same MTOW.

Based on the previous two paragraphs, it is evident that incorporating weight as an exogenous input renders the current efficiency evaluation more objective. Existing research pertinent to performance optimization of turbofan aero-engines via an engineering approach mainly incorporates aerodynamic and thermodynamic aspects, thus omitting including weight considerations in the optimization process. Hence, turbofan aero-engine designers should include weight estimation techniques like the one developed by Lolis [39], after completing the aerodynamic and thermodynamic calculations and before finalizing their conceptual/preliminary designs.

For the purpose of assessing to which degree current sample turbofan engines could improve their efficiency given the technological boundaries, we subsequently impose additional constraints to the slacks of the VRS VSBM Network DEA formulation, albeit in an alternative manner to the one dictated by Tone [53]. In accordance with Birch [3], where a 5 percent decrease in TSFC is technologically feasible till the end of the next decade, we numerically approximate the optimal efficiency within a feasible range of exogenous inputs and final output.

More specifically, for each $DMU_o$ under consideration we assume fuel flow ($X_{1o}$), engine weight ($X_{2o}$), and take-off thrust ($Y_{1o}$) are uniformly distributed on $[0.95x_{1o}, x_{1o}] \times [0.95x_{2o}, x_{2o}] \times [y_{1o}, 1.05y_{1o}]$. Subsequently, we use simulation to generate $10^6$ sets of pseudo-random variables that follow the aforementioned multivariate uniform distribution. Through an iterative implementation of the multiplier form model, we obtain a range of technologically feasible $DMU_o$ efficiencies. The set that maximizes the $DMU_o$ efficiency is considered optimal. The VRS VSBM Network DEA has been duly implemented and executed in the Wolfram Mathematica v10.3 computation environment. The obtained results are reported in Table 9.

By observing the numerical values within Table 9, it is evident that only GE90-110B1 and GE90-115B1 can attain ideal efficiency by simultaneously improving the exogenous inputs and final output within the aforementioned technologically feasible ranges. In addition, it is worth mentioning that for both the above engines the ideal efficiency is achieved by simultaneously improving the exogenous inputs and final output by less than 5 percent, thus designating the feasibility of such attempt by resorting to fairly minor enhancements. Another notable example is TFE731-60, where maximum attained revised efficiency, which is lower than unity, is achieved by simultaneously improving exogenous inputs and final output by less than 5 percent.

## 6. Concluding remarks

The current paper attempts to evaluate turbofan aero-engine efficiency by deploying an advanced benchmarking methodology, namely the VRS VSBM Two-Stage Network DEA. The implemented approach comprises an alternative pathway to the engineering approach, which is predominantly of a thermodynamic texture. In any case, the engineering approach is by definition assuming that the aero-engine design process has the following main features:

- It takes place in isolation, i.e. there are no subsequent comparisons with similar designs.
- It excludes certain traits which would negatively affect the usability or the marketability. For example, an oversized/overweight aero-engine would affect usability, while an aero-engine with high emissions (pollutant/noise) is highly unlikely to be marketable.

Vital part of the implemented benchmarking approach comprises the selection of the appropriate exogenous inputs, intermediate measures, and final output. In addition, adherence to the functional concept of the turbofan aero-engine has rendered the adopted Network DEA model sufficiently representative, while incorporating the engine weight as an input enhances the objectivity of the benchmarking results.

Another novelty of the conducted research lies in the fact that without resorting to extremely complex engineering calculations, it has been rendered feasible to adequately assess the relative efficiency of in-service and near-future turbofan aero-engines. As previously reported, the resulting efficiency scores follow a pattern similar to the propulsive efficiency pattern implementing an engineering approach (see Fig. 3). Moreover, with respect to the GE9X turbofan aero-engine which is planned to enter service soon aboard the Boeing 777-8X/-9X, the current paper designates the additional effort that should be undertaken from General Electric regarding weight reduction and take-off thrust enhancement.

Additionally, the concurrent deployment of non-parametric statistics and regression analysis for the purpose of conducting efficiency scores' post-hoc analysis is another novel feature of the present research effort. Consequently, the convergence of both post-hoc analysis methods corroborates the positive association of BPR with increased DEA efficiency, which is also the case regarding the engineering approach. Moreover, post-hoc analysis results designate the advances in turbofan aero-engine technology by confirming the positive association of EIS with increased efficiency. However, lack of data pertinent to new-generation turbofan aero-engines like Rolls-Royce Trent XWB, Pratt & Whitney GTF (1100G/1500G/1700G Series), and CFM LEAP rendered infeasible to gain a more accurate outlook of the turbofan aero-engine evolution over time. Hence, it comprises a challenge for future research to evaluate the particular aero-engines within a similar benchmarking framework, thus acquiring a more coherent view regarding the association of technological leaps with enhanced efficiency.

From a purely technical standpoint, the present paper introduces a methodological framework that enables turbofan aero-engine designers to evaluate conceptual/preliminary designs against existent in-service designs. In particular, after defining the turbofan aero-engine features deploying engineering calculations, benchmarking the resulting overall configuration against existing designs with similar configurations can comprise a complementary tool in the process of design optimization. Consequently, adopting the aforementioned design evaluation framework could substantially reduce developmental effort along with minimizing the need to reconsider design features. As a matter of fact, non-oriented VRS VSBM Network DEA has a clear advantage over alternative benchmarking methodologies taking into account that:

- It does not assign weights to the inputs, intermediate measures, and final outputs in a predetermined manner (e.g., Analytic Hierarchy Process).
- It simultaneously attempts to reduce inputs and increase outputs in a non-linear way, similarly to the engineering approach.
- It designates the areas requiring improvement, thus avoiding unnecessary modifications to the design features.

Finally, it is worth mentioning that the devised simulation method for defining the optimum DEA efficiency within a certain improvement threshold of the incorporated exogenous inputs and final outputs, further augments the potential for implementing the herewith integrated approach for objectively assessing both existing relative efficiency and optimum efficiency enhancement within technological limitations. Hence, the particular simulation method could comprise a tool for turbofan aero-engine designers and manufacturing enterprises, in order to evaluate whether it is preferable to invest in using an existing engine as design baseline or develop a "white-paper" design.

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## Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.omega.2019.102167.

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