SATFD

Lab. 4

April 12, 2024

Exercise 4: Filters

Introduction

Frequency filters are systems that modify the frequency spectrum of signals passing through them, i.e. they remove certain components from the signal. In the time domain, the filtering operation corresponds to the convolution of the signal with the impulse response function, while in the frequency domain it corresponds to the multiplication of each component by a complex number. Examples of filtering used in ECG analysis include:

- Removal of electrical network noise from the ECG signal (50 Hz in Poland, 60 Hz in the USA).
- Removing high frequency noise from the ECG signal caused by muscle vibration.
- Removing the low frequency baseline from the ECG signal caused by breathing and body movement. The human heart rate can drop to 40 BPM, which is 0.67 Hz, so a cut-off frequency of 0.5 Hz can be assumed.

In terms of signal processing methods, digital filters can be divided into:

- Finite Impulse Response Filters - FIR (otherwise: moving average).
- Infinite Impulse Response filter - IIR (otherwise: autoregressive filter): Butterworth filter, Type I Chebyshev filter, Type II Chebyshev filter, Elliptic filter.
- Infinite impulse response filter - ARMA (otherwise: autoregressive moving average).

Characteristics of Filters

Each filter is characterized by its transfer function, which for a FIR filter is:

$$H(z) = b_0 + b_1 z^{-1} + \ldots + b_M z^{-M} \tag{1}$$

The transfer function of an IIR filter is:

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}} \tag{2}$$

where a and b are the filter coefficients.
The multiplication of signal components by complex numbers results in a phase shift, so that a given frequency appears at the output of the filter with a certain delay (phase delay). FIR filters have a linear phase characteristic in the passband, which ensures a constant group delay. For IIR filters, the relationship is nonlinear.

Realization of the task

Suggested environment: Matlab, Python.

Prepare scripts for the following tasks:

1. Generate a noisy signal, for example:

```
N=2000; A=5; f=5; fs=1000;
dt=1/fs; % sampling period
t=dt*(0:N-1); % sampling time vector
x=A*sin(2*pi*f*t) + A*randn(1, N); % noisy signal
plot(t,x); grid; title("Sygnal x(t)"); xlabel("Czas [s]");

```

   Filter the signal using FIR and IIR high-pass, low-pass, and band-pass filters. Present the signals and their spectra before and after filtering, and calculate the SNR (Signal-to-Noise ratio).

2. Generate a composition of two sine waves, for example:

```
N=2000; A=5; f1=5; f2=50; fs=1000;
dt=1/fs; % sampling period
t=dt*(0:N-1); % sampling time vector
y=A*sin(2*pi*f1*t) + 2*sin(10*pi*f2*t); % signal
plot(t,y); grid; title("Sygnal x(t)"); xlabel("Czas [s]");

```

   Filter the signal to obtain one sine wave after each filtration. Present the signals and their spectra before and after filtering.

3. Read the signal from the file ecg.mat. Choose the type of filtration to obtain the best signal, i.e., eliminate baseline wander and electrical network interference. Justify your choice. The sampling frequency of the signal is 500 Hz.

Comment on all obtained results.