Dynamic Modelling II

Ali AlBeladi

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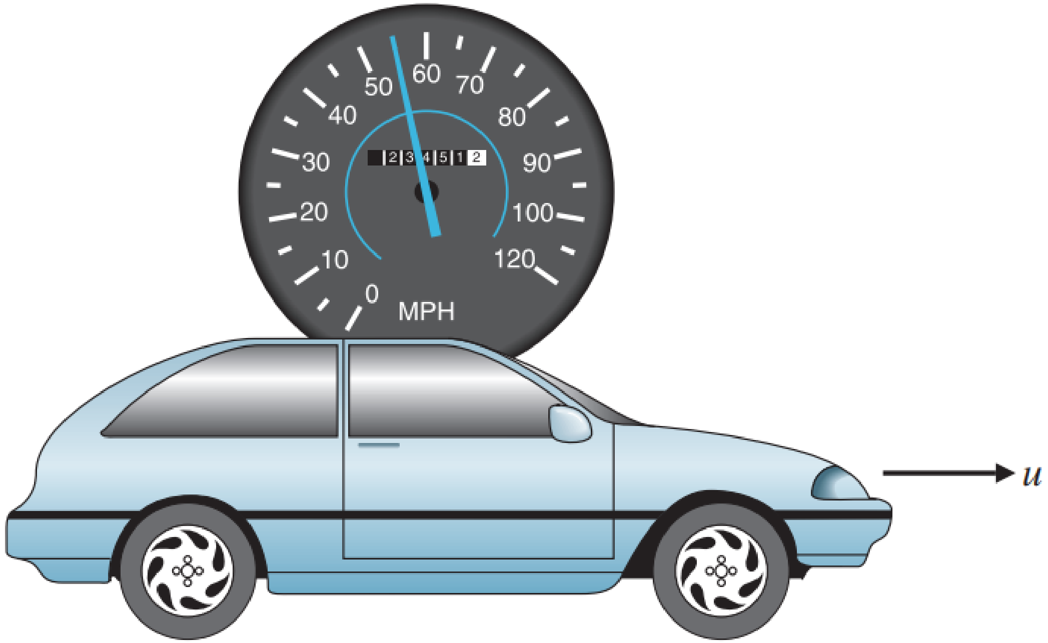
Objectives

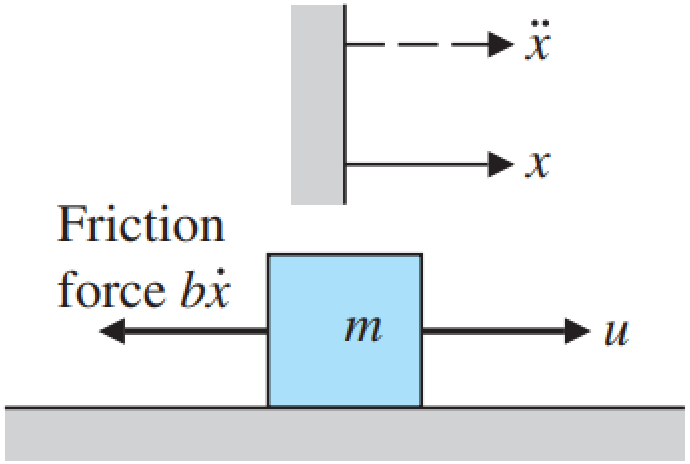
- Dynamic Model of Rotational Motion
- Dynamic Model of Electromechanical Systems

Dynamic Model of Translational Motion (Example)

A Simple System; Cruise Control Model

1. Write the equations of motion for the speed and forward motion of the car shown in Fig. 2.1, assuming the engine imparts a force u as shown. Take the Laplace transform of the resulting differential equation and find the transfer function between the input u and the output v.

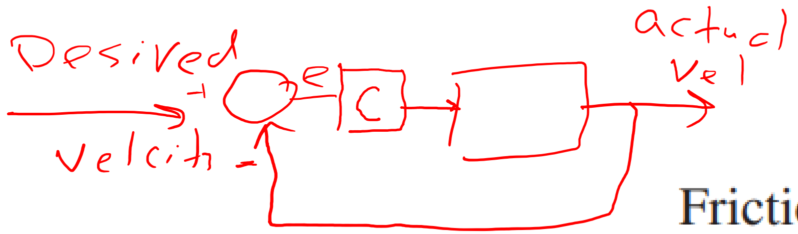


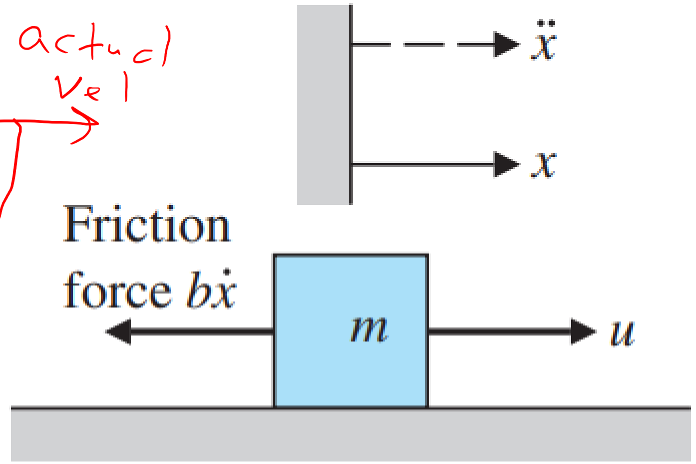


Dynamic Model of Translational Motion (Example)

$$u - b\dot{x} = m\ddot{x},$$

$$\ddot{x} + \frac{b}{m}\dot{x} = \frac{u}{m}.$$





For the case of the automotive cruise control where the variable of interest is the speed, v $(=\dot{x})$, the equation of motion becomes

$$\dot{v} + \frac{b}{m}v = \frac{u}{m}. \tag{2.4}$$

$$\frac{V_o}{U_o} = \frac{\frac{1}{m}}{s + \frac{b}{m}}.$$

Dynamic Model of Rotational Motion

Application of Newton's law to one-dimensional rotational systems

$$M = I\alpha$$

where

$M :=$ the sum of all external moments about the center of mass of a body, N $\cdot$ m,

$I :=$ the body's mass moment of inertia about its center of mass, kg·m2,

$\alpha :=$ the angular acceleration of the body, rad/sec2.

Rotational Motion: Satellite Attitude Control Model

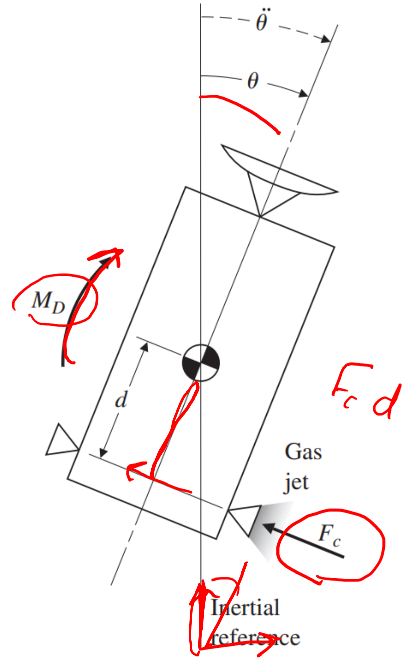
Satellites usually require attitude control so antennas, sensors, and solar panels are properly oriented.

Let us find the equations of rotational motion for one axis of this system.

θ ≔ the satellite orientation measured with respect to an inertial reference—that is, a reference that has no angular acceleration.

F_c ≔ control force from the gas jet

M_D ≔ small disturbance moments on the satellite



$U = I \Theta s^2$ $\qquad \frac{\Theta}{U} = \frac{1}{Is^2}$

Rotational equation of motion:

$\frac{\Theta}{u} = \frac{1}{dIs^2} \Leftarrow ud = I\Theta s^2$

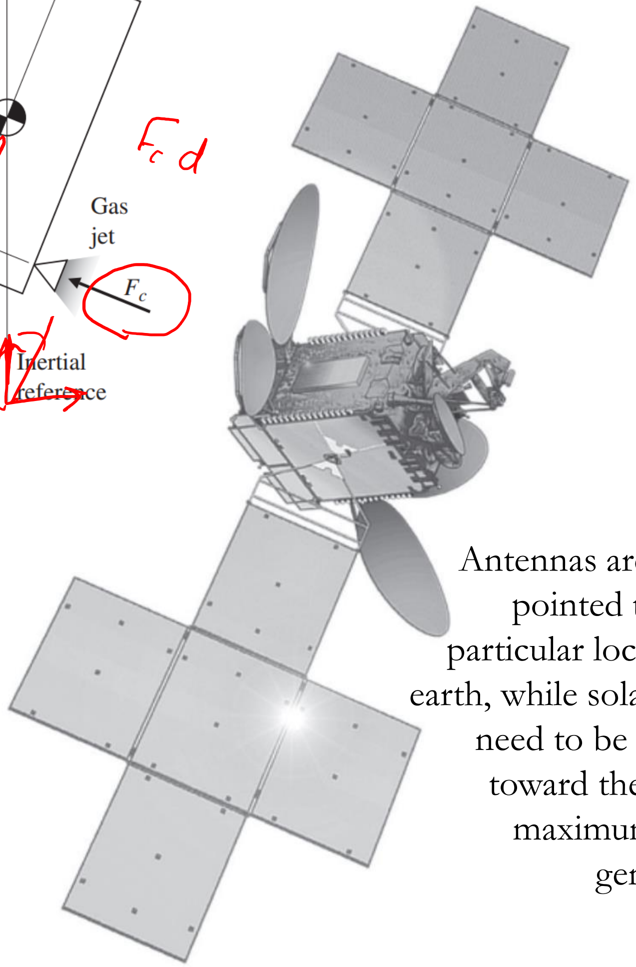
$$\overbrace{F_c d + M_D}^{u} = I\ddot{\theta}$$

Transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{1}{Is^2}$$

double-integrator plant

$u \rightarrow \boxed{\frac{1}{I}} \rightarrow \ddot{\Theta} \rightarrow \underbrace{\boxed{\boxed{\int} \rightarrow \boxed{\int}}}_{\frac{1}{s^2}} \rightarrow \Theta$

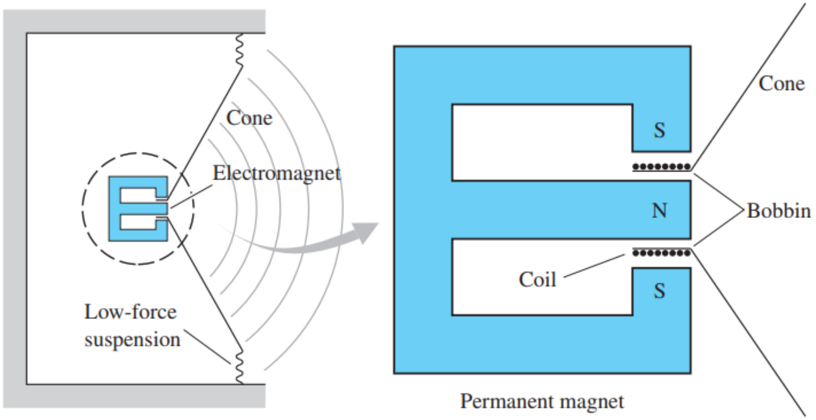


Antennas are usually pointed toward a particular location on earth, while solar panels need to be oriented toward the sun for maximum power generation.

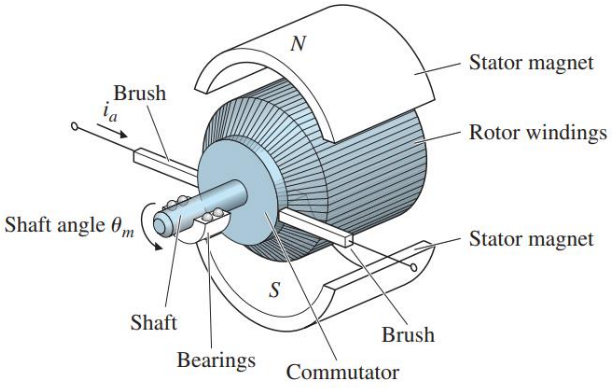
Dynamics of Electromechanical Systems

Examples of electromechanical systems

- Loudspeakers

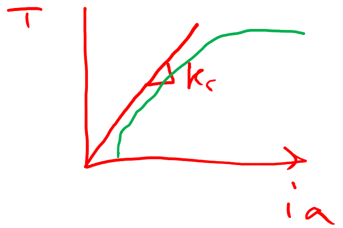


- Motors



Armature-Current Controlled DC Motor

In an armature-current controlled DC motor, the field current is fixed and the motor speed is controlled with the armature current:



$$T = K_a i_a$$

We assume a viscous model for friction

$$T_{fric} = b\dot{\theta}$$

A "back EMF“ is induced by the rotation of the armature windings in a magnetic field:

$$v_e = K_e\dot{\theta}$$

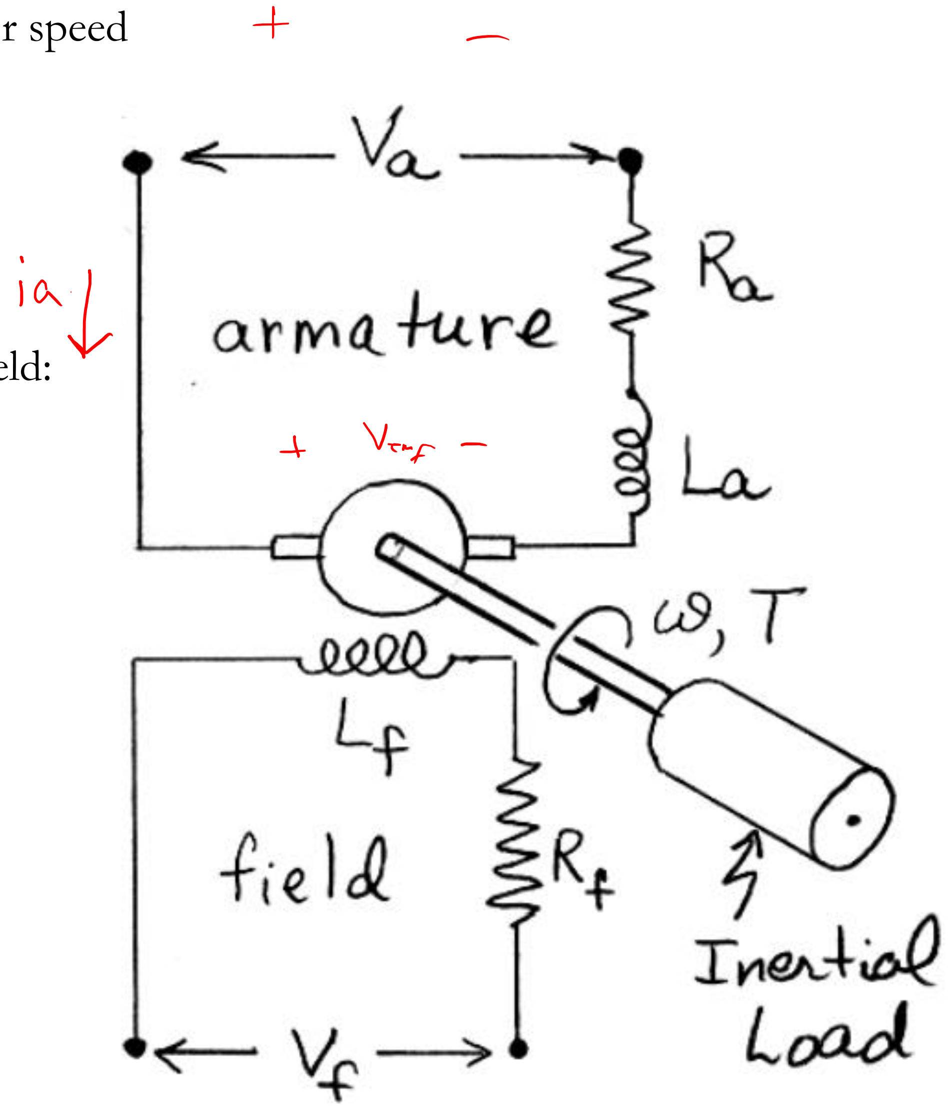
From KVL:

$$v_a = R_a i_a + L_a \frac{d}{dt} i_a + v_e$$

$$= R_a i_a + L_a \frac{d}{dt} i_a + K_e\dot{\theta}$$

From Newton’s second law (rotational):

$$T - T_{fric} = J\ddot{\theta} \quad \Longrightarrow \quad K_a i_a - b\dot{\theta} = J\ddot{\theta}$$



Armature-Current Controlled DC Motor

- Take Laplace the Laplace transform:

$$v_a = R_a i_a + L_a \frac{d}{dt} i_a + K_e \dot{\theta}$$

$$K_a i_a - b\dot{\theta} = J\ddot{\theta}$$

$$\Rightarrow$$

$$V_a = R_a I_a + L_a s I_a + K_e s\Theta$$

$$K_a I_a - bs\Theta = Js^2\Theta$$

$$\boxed{\frac{s\Theta}{V_a} = \frac{K_a}{(sL_a + R_a)(b + Js) + K_a K_e}}$$

Field-Current Controlled DC Motor

In an filed-current controlled DC motor, the armature current is fixed and the motor speed is controlled with the field current:

$$T = K_f i_f$$

We assume a viscous model for friction

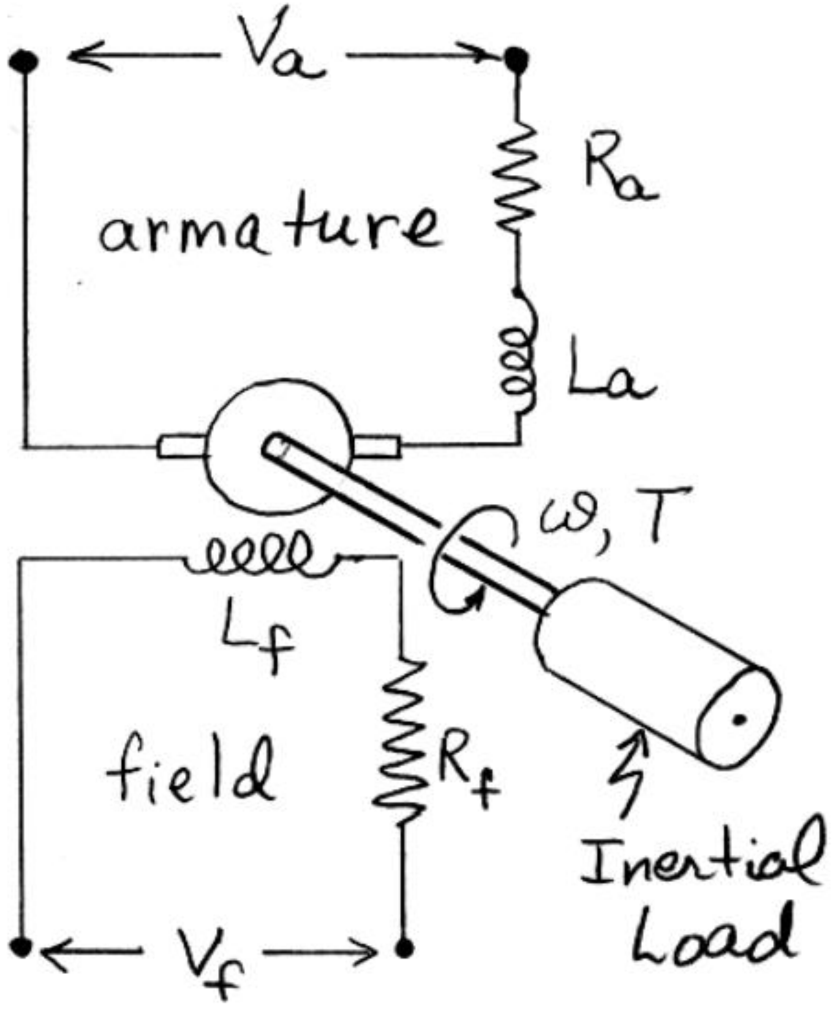
$$T_{fric} = b\dot{\theta}$$

From KVL:

$$v_f = R_f i_f + L_f \frac{d}{dt} i_f$$

From Newton's second law (rotational):

$$T - T_{fric} = J\ddot{\theta} \implies K_f i_f - b\dot{\theta} = J\ddot{\theta}$$



Field-Current Controlled DC Motor

- Take Laplace the Laplace transform:

$$v_f = R_f i_f + L_f \frac{d}{dt} i_f$$

$$K_f i_f - b\dot{\theta} = J\ddot{\theta}$$

$$\Rightarrow$$

$$V_f = R_f I_f + L_f s I_f$$

$$K_f I_f - bs\Theta = Js^2\Theta$$

$$\boxed{\frac{s\Theta}{V_f} = \frac{K_f}{\left(sL_f + R_f\right)(Js + b)}}$$