Dynamic Models

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Introduction to Control systems

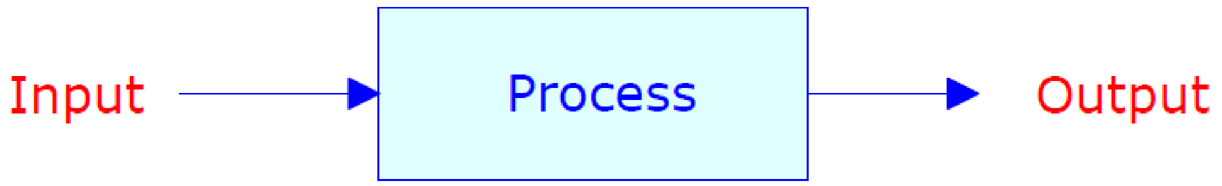
System

- Definition

A system is an interconnection of components forming a system configuration that will provide a desired system response.

Each component is described by a cause-effect relation.

Therefore a component or process to be controlled can be represented by a block



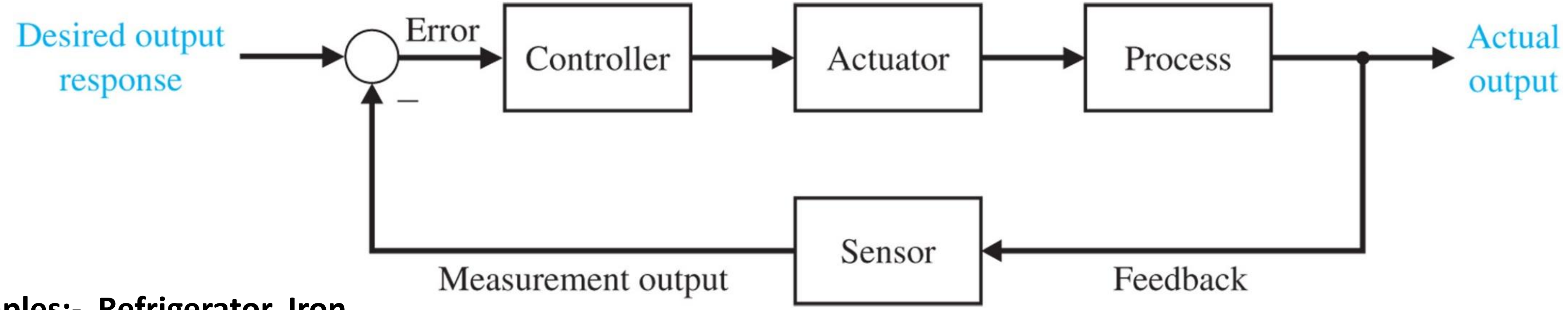
The input- output relation represents the cause-and-effect relationship of the process

Classification of Control systems

An **open-loop** control system utilizes an actuating device to control the process directly without feedback

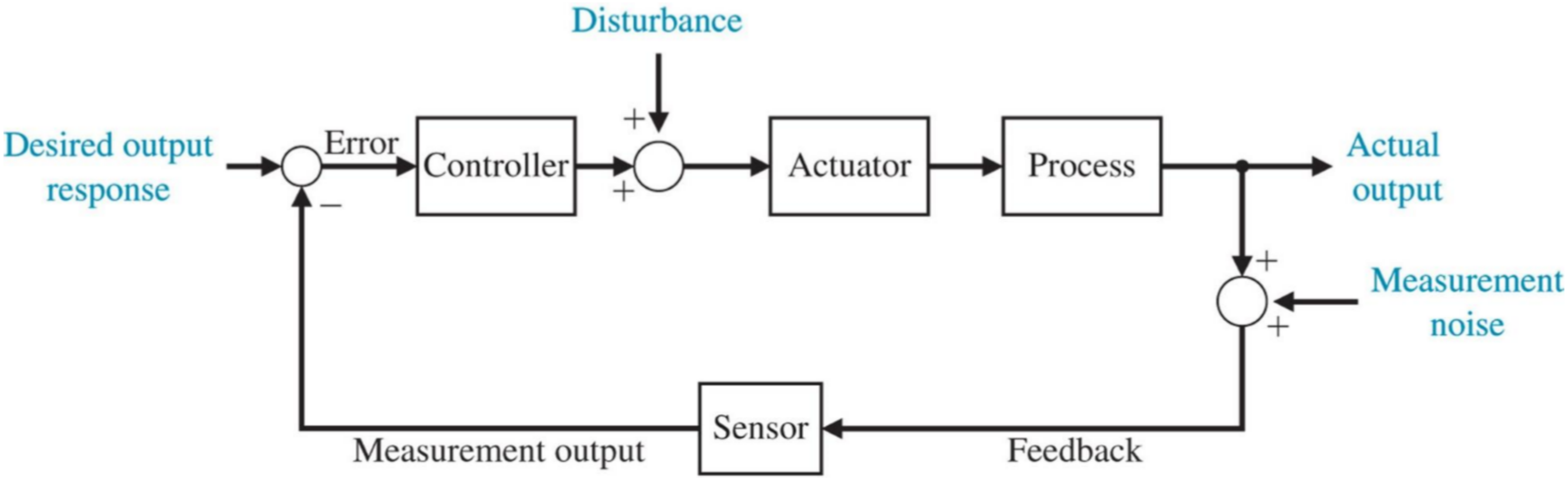
Examples:- Washing Machine, Toaster

A **closed-loop** control system uses a measurement of the out put and feedback of this signal to compare it with the desired output (reference or command)

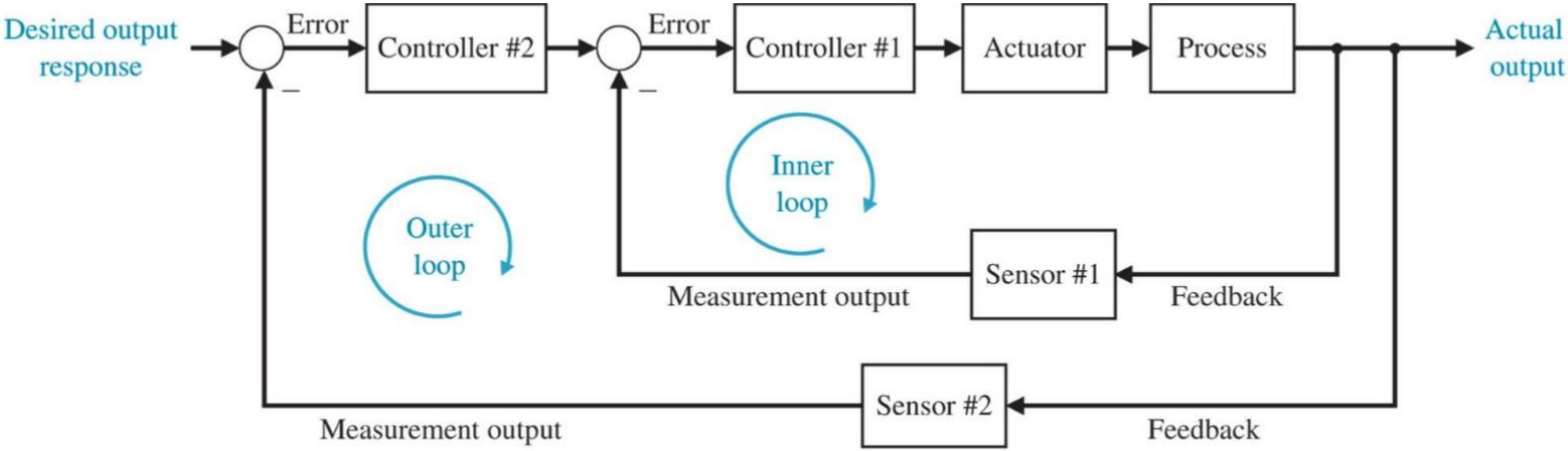


Examples:- Refrigerator, Iron

Closed-loop Feedback System With External Disturbances And Measurement Noise



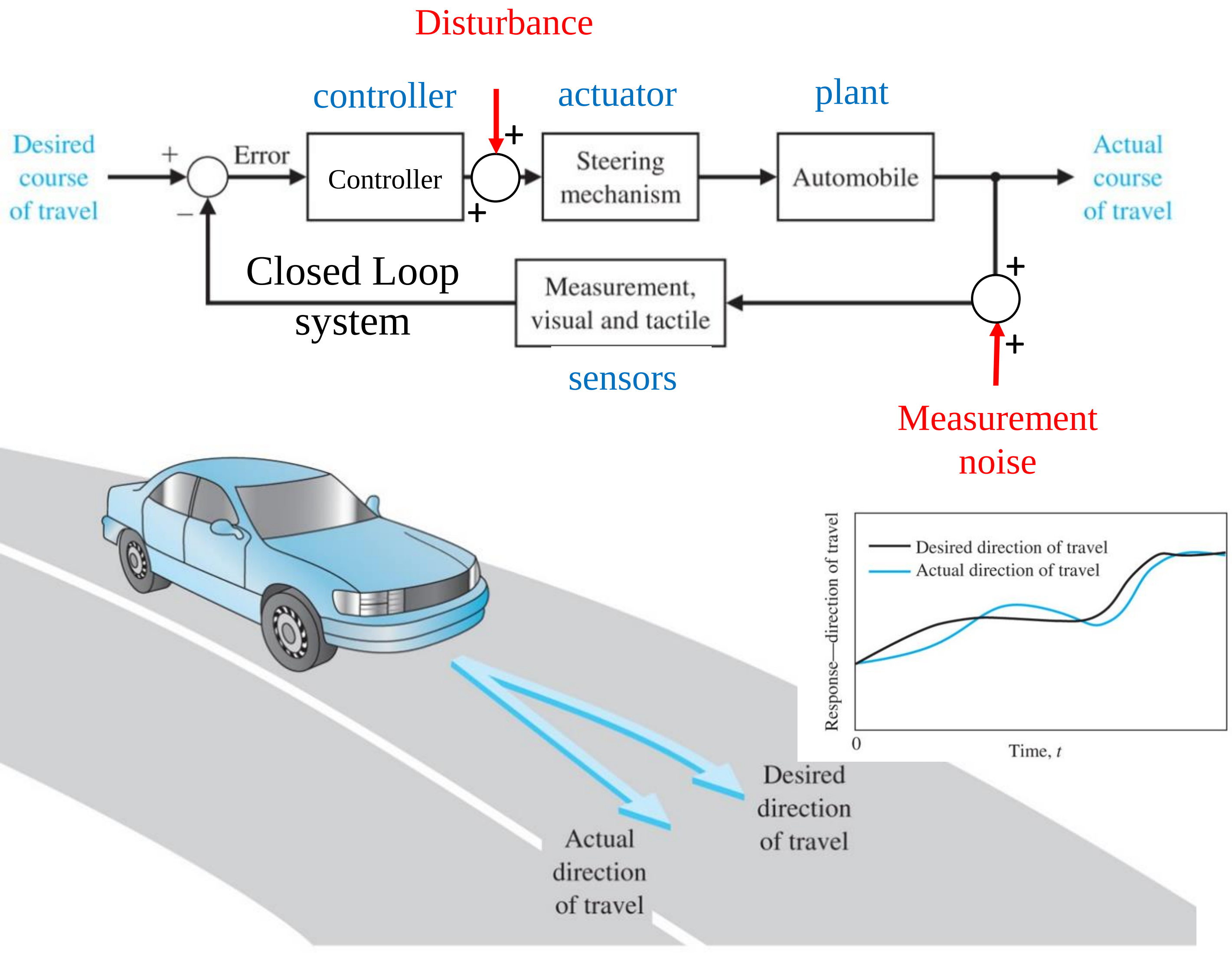
Multiloop Feedback System



Feedback Control

Some terminology:

- the plant is the system being controlled
- the sensors measure the quantity that is subject to control
- the actuators act on the plant
- the controller processes the sensor signals and drives the actuators
- the control law is the rule for mapping sensor signals to actuator signals



(b)

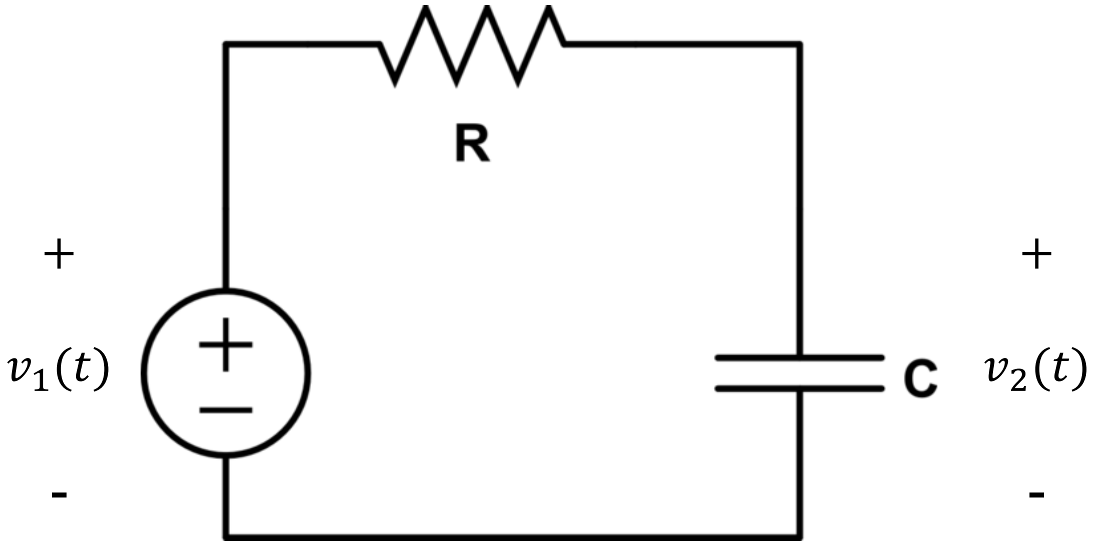
What is a Model?

- A model can be obtained using principles of the underlying physics or by testing a prototype of the device, measuring its response to inputs, and using the data to construct an analytical model.

- We will focus only on using physics

Modelling an Electric System

Consider the RC circuit shown here



Using KVL, we can obtain

$$v_1(t) = R\, i(t) + v_2(t)$$

We also have the relationship

$$v_2(t) = \frac{1}{C}\int_0^t i(\tau)\, d\tau$$

(assuming no initial charge)

Differentiate v_2

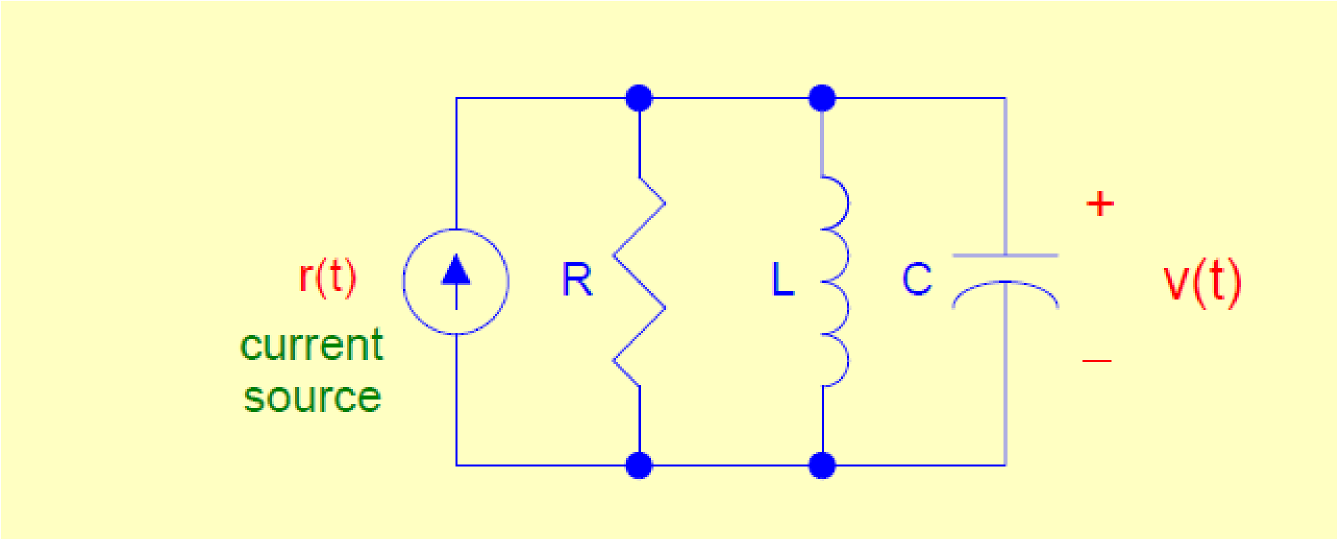
$$\dot{v}_2 = \frac{1}{C} i(t)$$

Substitute in $i(t)$

$$v_1(t) = RC\dot{v}_2 + v_2(t)$$

Modelling an Electric System

Consider the RLC circuit shown below.



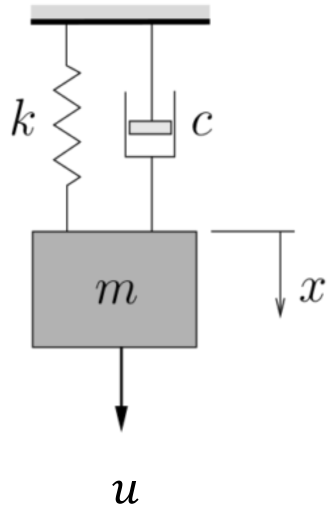
Using KCL, one obtains the following integro-differential equation,

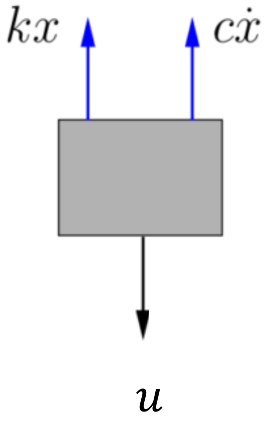
$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt = r(t)$$

Dynamics of Mechanical System

- The cornerstone for obtaining a mathematical model, or the dynamic equations, for any mechanical system is Newton's law

$$\mathbf{F} = m\mathbf{a}$$

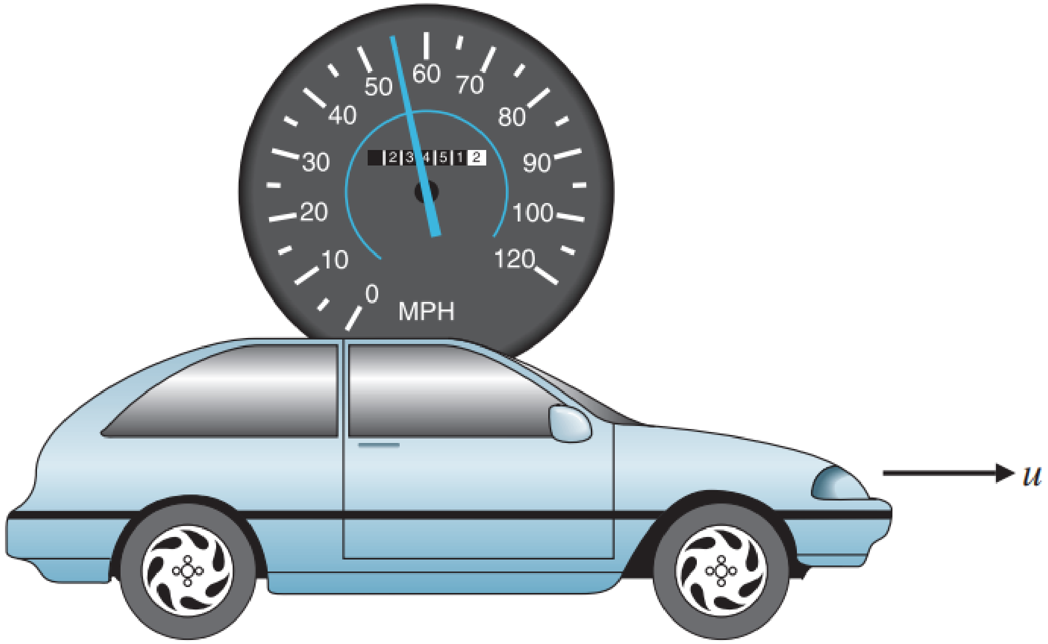


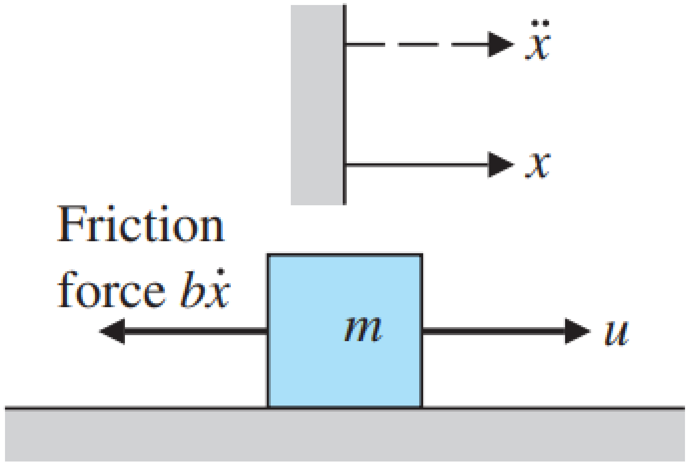


- $m\ddot{x} = -kx - c\dot{x} + u$

A Simple System; Cruise Control Model

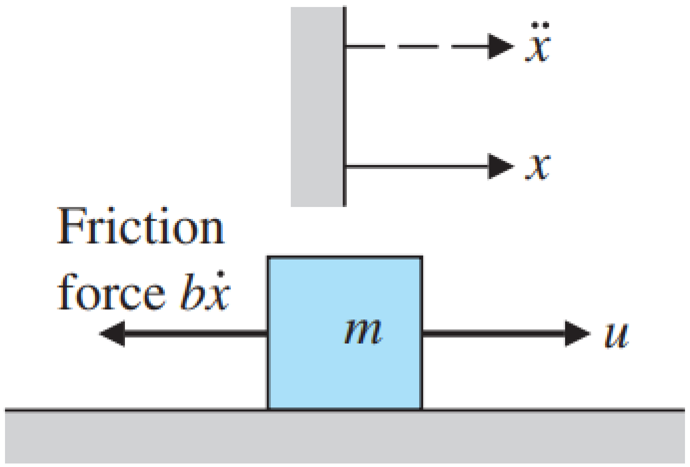
1. Write the equations of motion for the speed and forward motion of the car shown in Fig. 2.1, assuming the engine imparts a force u as shown. Take the Laplace transform of the resulting differential equation and find the transfer function between the input u and the output v.





$$u - b\dot{x} = m\ddot{x},$$

$$\ddot{x} + \frac{b}{m}\dot{x} = \frac{u}{m}.$$



For the case of the automotive cruise control where the variable of interest is the speed, v $(=\dot{x})$, the equation of motion becomes

$$\dot{v} + \frac{b}{m}v = \frac{u}{m}. \tag{2.4}$$

$$\frac{V_o}{U_o} = \frac{\frac{1}{m}}{s + \frac{b}{m}}.$$