Homework 2

Due: Feb, 6, midnight

Problem 1. Consider the dynamics for the mass-spring system, as depicted in Figure 1.

Here, k is the spring constant and ρ is the friction coefficient (yes, the mass m in the figure does not touch the floor, but *assume it does!*).

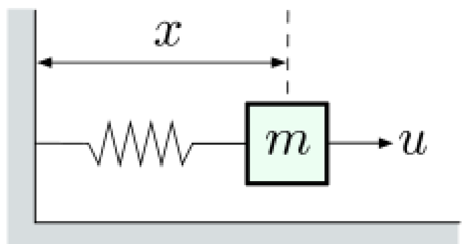


Figure 1: The mass-spring system.

(a) (5 points) Derive the equations of motion for the system, assuming the displacement of the mass is $x(t)$ and the input force is $u(t)$.
(b) (10 points) From the equations of motion, derive the transfer function $H(s) = \frac{X(s)}{U(s)}$, where $X(s)$ and $U(s)$ are the Laplace transforms of $x(t)$ and $u(t)$, respectively.
(c) (10 points) Express the natural frequency ω_n and the damping ratio ζ in terms of k, ρ, and m.

Problem 2. Consider the transfer function:

$$H(S) = \frac{25}{s^2 + 6s + 25}$$

(a) (5 points) Draw a block diagram for $H(s)$ using integrator, summation, and gain blocks.
(b) (5 points) Suppose you are given the following time-domain specs: rise time $t_r \leq 0.6$ and settling time $t_s \leq 1.6$. (Here we're considering settling time to within 5% of the steady-state value.) Plot the admissible pole locations in the s-plane corresponding to these two specs. Does this system satisfy these specs?
(c) (5 points) Repeat the previous problem for the specs: rise time $t_r \leq 0.6$, settling time $t_s \leq 1.6$, and magnitude $M_p \leq 1/e^2$. Plot the admissible pole locations; does this system satisfy these specs?
(d) (5 points) Draw a block diagram for $(s+1)H(s)$ using integrator, summation, and gain blocks.

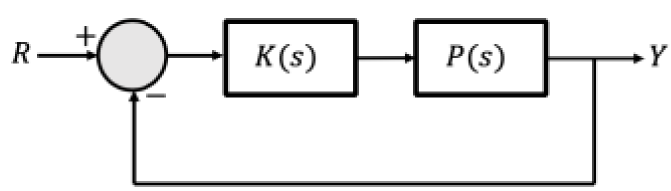


Figure 2: A diagram of a unity feedback system.

Problem 3. (15 points) Consider the unity feedback system in Figure 2. Let the plant's transfer function be given by:

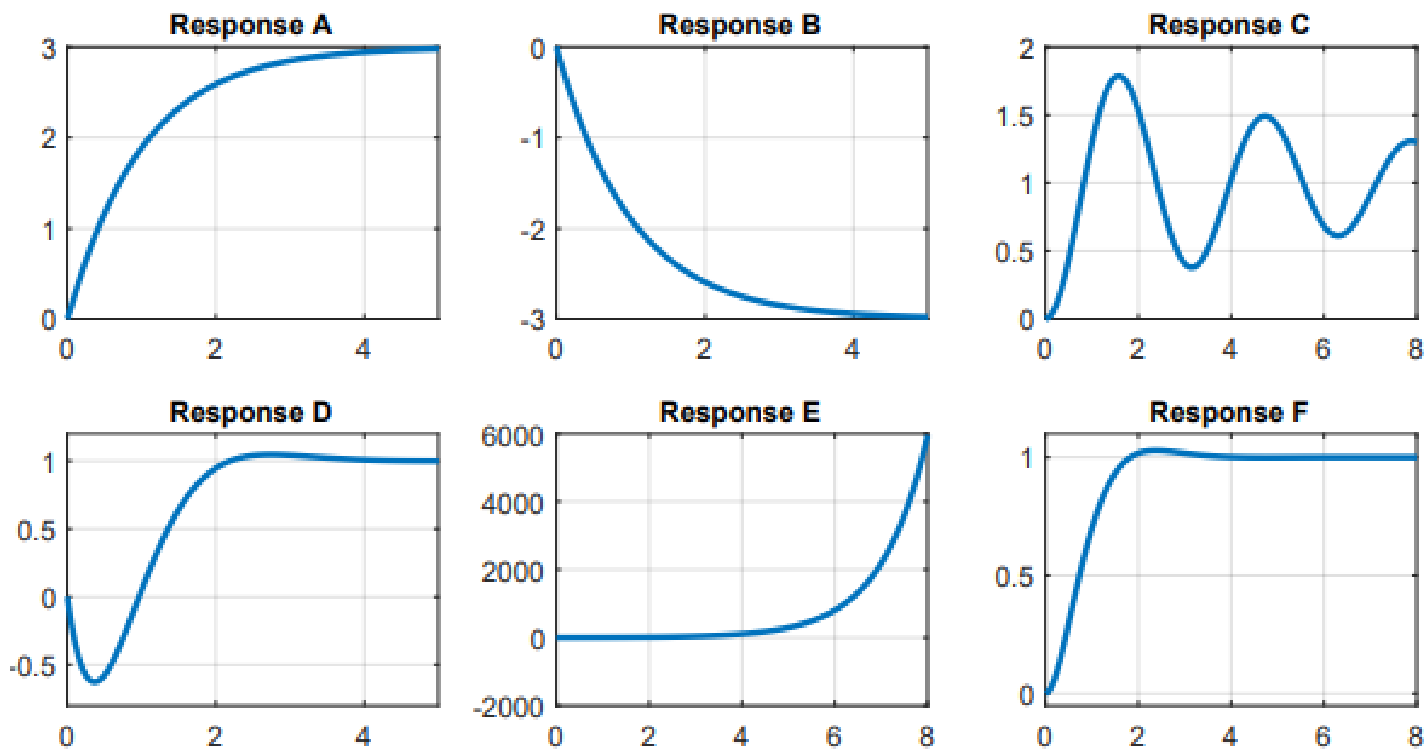
$$P(s) = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

Suppose our controller is given by $K(s) = 4$. What is the transfer function from R to Y? Use the Routh-Hurwitz criterion to determine whether this model is stable or not.

Problem 4. (30 points) Consider the six transfer functions given below. For each $G_i(s), i = 1, 2, \ldots 6$, specify the following **in turn**: (a) Poles, (b) Zeros (if any), (c) Stable or unstable, and (d) Steady-state gain **before proceeding to the next**. Use these answers to match each of the six transfer functions with one of the unit step responses in the figure below. All responses were generated with zero initial conditions.

$$G_1(s) = \frac{-4s+4}{s^2+3s+4} \qquad G_2(s) = \frac{4}{s^2+0.3s+4} \qquad G_3(s) = \frac{4}{s^2+3s+4}$$

$$G_4(s) = \frac{-s+3}{s-1} \qquad G_5(s) = \frac{300}{s^2+101s+100} \qquad G_6(s) = \frac{-3}{s+1}$$



Problem 5. (15 points) Without a computer, determine whether or not the following polynomials have any RHP roots:

(a) $s^6 + 2s^5 + 3s^4 + s^3 + s^2 - 3s + 5$
(b) $s^4 + 10s^3 + 10s^2 + 20s + 1$
(c) $s^4 + 10s^3 + 10s^2 + 1$