DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Master's Thesis in Quantum Computing

Quantum and Classical Generative Modeling for Quantum States Preparation

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Quantenbasierte und klassische generative Modellierung zur Erzeugung von Quantenzuständen

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I confirm that this master's thesis in quantum computing is my own work and I have documented all sources and material used.

Munich, 15.06.2021 Wiktor Jurasz

Acknowledgments

Abstract

Kurzfassung

Contents

1. Introduction

1.1. Problem Statement

Generative Modeling aims to learn a conditional probability $P(X|Z = z)$, where X is some observable variable and Z is a target variable. With knowledge of this conditional probability, it is possible to generate new observations $\bar{x} \in X$. In general case, one would not try to obtain the probability $P(X|Z)$ exactly, but learn an approximation. To do so a set of samples $x \in X$ is necessary to train a generator function $G : Z \to X$ which given a target variable $z \in Z$ generates new observation $x \in X$.

In the generative framework, the variable X is a multidimensional vector, in particular it can be used to describe an arbitrary quantum state. With this setup, given a finite set of quantum states $\mathcal{Q} = \{|\psi_i\rangle\}, |\psi_i\rangle \in X \forall i$ the generator function G prepares a new quantum state $|\hat{\psi}\rangle$. This new quantum state is expected to come from the same distribution as the samples in the input set $\mathcal{Q}$.

The only missing piece in the above description is the target variable Z. In the context of the function G, generating the quantum states, we can think about Z as a label of the generated state. That is, for a specific $z \in Z$ the function G always generates the same $|\hat{\psi}\rangle$.

In this work we evaluate different approaches to find the probability $P(X|Z = z)$ by learning the function G. We also address the limitations of the existing methods propose a new one that combines quantum and classical generative modeling.

1.2. Previous Work

There exist many different types of generative models. In this work we focus on one particular type, namely Generative Adversarial Networks (GANs). First version of GANs was proposed bu Goodfellow et al. [1] (to which we refer as Standard GANs - SGANs), since then many different variations of GANs were invented [2][3][4]. In context of this work, particularly interesting are Wasserstein GANs (WGANs)[5] which minimize *Earth-Mover* distance between two probability distribution (see Chapter 3) instead of *Jensen–Shannon* divergence (see Chapter 3) as in SGANs.

In recent years there has been an increasing interest in realizing Generative Adversarial Networks in Quantum Computing (QC) realm. Dallaire-Demers et al. proposed QuGANs [6] - Quantum Generative Adversarial Networks where generator and discriminator are parametrized quantum circuits. Similarly Benedetti et al. proposed fully quantum GANs for pure state approximation [7], but with different (more suitable for NISQ [8]) learning method. Hybrid methods were also explored, Zoufal et al. build qGAN [9] - with parametrized quantum circuit as the generator and classical neural network as the discriminator.

De Palma et al. proposed quantum equivalent of Wasserstein distance of order 1 [10] which made the Quantum Wasserstein GANs (QWGANs) [11] possible. This variation of quantum GANs consist of the parametrized quantum circuit as the generator and the classical linear program as the discriminator.

1.3. Our Contribution

There has been a substantial effort in the direction of bringing GANs into the quantum realm. Nevertheless, this is still very early stage and many more routs are yet to be explored. In this work we focus on building quantum GANs that can generate new, unseen before, quantum states. Majority of models proposed so far are only able to generate the states the has been a part of the training data. Only some architectures [6] account for random noise in the input that allows generate unseen states. However, as we discuss later, those are mostly theoretical and do not seem to work well in practice.

2. Quantum Computing Introduction

In this chapter we provide a very brief introduction to the key concepts of quantum computing and introduce the notation used in the rest of this paper.

2.1. Parametric Circuits

3. Generative Adversarial Networks (GANs) Introduction

3.1. Standard GANs

3.2. Waserstein GANs (WGANs)

4. Quantum Generative Adversarial Networks

4.1. Standard Quantum GANs (SQGANs)

4.2. Wasserstein Quantum GANs (WQGANs)

5. Unknown Quantum State Generation

5.1. Labeled State Generation

5.2. Unlabeled State Generation

6. Results

7. Conclusions

A. Appendix

B. Figures

List of Figures

List of Tables

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