DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Master's Thesis in Quantum Computing

Quantum and Classical Generative Modeling for Quantum States Preparation

Wiktor Jurasz

DEPARTMENT OF INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Master's Thesis in Quantum Computing

Quantum and Classical Generative Modeling for Quantum States Preparation

Quantenbasierte und klassische generative Modellierung zur Erzeugung von Quantenzuständen

Author:	Wiktor Jurasz
Supervisor:	Prof. Dr. Christian Mendl
Submission Date:	15.06.2021

I confirm that this master's thesis in quantum computing is my own work and I have documented all sources and material used.

Munich, 15.06.2021 Wiktor Jurasz

Acknowledgments

Abstract

Kurzfassung

Contents

1. Introduction

1.1. Problem Statement

Generative Modeling aims to learn a conditional probability $P(X|Z = z)$, where X is some observable variable and Z is a target variable. With knowledge of this conditional probability, it is possible to generate new observations $\bar{x} \in X$. In general case, one would not try to obtain the probability $P(X|Z)$ exactly, but learn an approximation. To do so a set of samples $x \in X$ is necessary to train a generator function $G : Z \to X$ which given a target variable $z \in Z$ generates new observation $x \in X$.

In the generative framework, the variable X is a multidimensional vector, in particular it can be used to describe an arbitrary quantum state. With this setup, given a finite set of quantum states $\mathcal{Q} = \{|\psi_i\rangle\}, |\psi_i\rangle \in X \forall i$ the generator function G prepares a new quantum state $|\hat{\psi}\rangle$. This new quantum state is expected to come from the same distribution as the samples in the input set $\mathcal{Q}$.

The only missing piece in the above description is the target variable Z. In the context of the function G, generating the quantum states, we can think about Z as a label of the generated state. That is, for a specific $z \in Z$ the function G always generates the same $|\hat{\psi}\rangle$.

In this work we evaluate different approaches to find the probability $P(X|Z = z)$ by learning the function G. We also address the limitations of the existing methods propose a new one that combines quantum and classical generative modeling.

1.2. Previous Work

There exist many different types of generative models. In this work we focus on one particular type, namely Generative Adversarial Networks (to which we refer to as Standard Generative Adversarial Networks - SGANs) [1]. Many different variations of GANs were proposed since their invention [2][3][4]. In context of this work, particularly interesting are Wasserstein GANs (WGANs)[5] which minimize *Earth-Mover* distance between two probability distribution (see Chapter 3) instead of *Jensen–Shannon* divergence (see Chapter 3) as in SGANs.

In recent years there has been an increasing interest in realizing Generative Adversarial Networks in Quantum Computing (QC) realm. Dallaire-Demers et al. proposed QuGANs [6] - Quantum Generative Adversarial Networks where generator and discriminator are parametrized quantum circuits. Similarly Benedetti et al. proposed fully quantum GANs for pure state approximation [7], but with different (more suitable for NISQ [8]) learning method. Hybrid methods were also explored, Zoufal et al. build qGAN [9] - with parametrized quantum circuit as the generator and classical neural network as the discriminator.

De Palma et al. proposed quantum equivalent of Wasserstein distance of order 1 [10] which made the Quantum Wasserstein GANs (QWGANs) [11] possible. This variation of quantum GANs consist of parametrized quantum circuit as the generator and classical linear program as the discriminator.

2. Quantum Computing Introduction

In this chapter we provide a very brief introduction to the key concepts of quantum computing and introduce the notation used in the rest of this paper.

2.1. Parametric Circuits

3. Generative Adversarial Networks (GANs) Introduction

3.1. Standard GANs

3.2. Waserstein GANs (WGANs)

4. Quantum Generative Adversarial Networks

4.1. Standard Quantum GANs (SQGANs)

4.2. Wasserstein Quantum GANs (WQGANs)

5. Unknown Quantum State Generation

5.1. Labeled State Generation

5.2. Unlabeled State Generation

6. Results

7. Conclusions

A. Appendix

B. Figures

List of Figures

List of Tables

Bibliography

[1] I. J. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. *Generative Adversarial Networks*. 2014. arXiv: `1406.2661 [stat.ML]`.

[2] M. Mirza and S. Osindero. *Conditional Generative Adversarial Nets*. 2014. arXiv: `1411.1784 [cs.LG]`.

[3] T. Karras, S. Laine, and T. Aila. *A Style-Based Generator Architecture for Generative Adversarial Networks*. 2019. arXiv: `1812.04948 [cs.NE]`.

[4] A. Radford, L. Metz, and S. Chintala. *Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks*. 2016. arXiv: `1511.06434 [cs.LG]`.

[5] M. Arjovsky, S. Chintala, and L. Bottou. *Wasserstein GAN*. 2017. arXiv: `1701.07875 [stat.ML]`.

[6] P.-L. Dallaire-Demers and N. Killoran. "Quantum generative adversarial networks". In: *Physical Review A* 98.1 (July 2018). ISSN: 2469-9934. DOI: `10.1103/physreva.98.012324`. URL: `http://dx.doi.org/10.1103/PhysRevA.98.012324`.

[7] M. Benedetti, E. Grant, L. Wossnig, and S. Severini. "Adversarial quantum circuit learning for pure state approximation". In: *New Journal of Physics* 21.4 (Apr. 2019), p. 043023. ISSN: 1367-2630. DOI: `10.1088/1367-2630/ab14b5`. URL: `http://dx.doi.org/10.1088/1367-2630/ab14b5`.

[8] J. Preskill. "Quantum Computing in the NISQ era and beyond". In: *Quantum* 2 (Aug. 2018), p. 79. ISSN: 2521-327X. DOI: `10.22331/q-2018-08-06-79`. URL: `http://dx.doi.org/10.22331/q-2018-08-06-79`.

[9] C. Zoufal, A. Lucchi, and S. Woerner. "Quantum Generative Adversarial Networks for learning and loading random distributions". In: *npj Quantum Information* 5.1 (Nov. 2019). ISSN: 2056-6387. DOI: `10.1038/s41534-019-0223-2`. URL: `http://dx.doi.org/10.1038/s41534-019-0223-2`.

[10] G. D. Palma, M. Marvian, D. Trevisan, and S. Lloyd. *The quantum Wasserstein distance of order 1*. 2020. arXiv: `2009.04469 [quant-ph]`.

[11] B. T. Kiani, G. D. Palma, M. Marvian, Z.-W. Liu, and S. Lloyd. *Quantum Earth Mover's Distance: A New Approach to Learning Quantum Data*. 2021. arXiv: `2101.03037 [quant-ph]`.