Combinatorics

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1 Basic Exercises

1.1 Exercise

How many five-digit even numbers are there in which no digit repeats?

Solution

To form a five-digit even number, the last digit must be one of the even digits. Therefore we have the following choices for the last digit: 0, 2, 4, 6, 8.

There are **5 choices** for the last digit.
However, we must consider the cases where the last digit is 0 and where it is not.

That's because if we choose 0 as the last digit, then the choice doesn't affect the first digit, but if we choose any other even digit, then it reduces the choices for the first digit with one less option.

We split the cases as follows:
$\overline{9} \cdot \overline{8} \cdot \overline{7} \cdot \overline{6} \cdot \overset{0}{\overline{1}} + \overline{8} \cdot \overline{8} \cdot \overline{7} \cdot \overline{6} \cdot \overset{2,4,6,8}{\overline{4}}$

1.2 Exercise

How many different 4-digit safe codes are there with exactly one odd digit in them?

Solution

First of all, the provided description of the problem tells us that's safe codes and not numbers, so we can use 0 as the first digit.

What's more, the problem states that there is exactly one odd digit
in the code, which means that the other three digits must be even.

There are 5 odd and also 5 even digits.
We can choose the position of the odd digit in 4 different ways,
as the code has 4 digits.

Therefore, the total number of different 4-digit safe codes with exactly one odd digit is given by:
$(4 \cdot \overset{odd}{\overline{5}}\ ) \cdot \overset{even}{\overline{5}} \cdot \overset{even}{\overline{5}} \cdot \overset{even}{\overline{5}} = 4 \cdot 5^4$

1.3 Exercise

How many different 4-digit safe codes are there in which there is exactly one odd digit and all the digits are different?

Solution

The situation is similar to the previous exercise,
but now we have the restriction that all digits must be **different**.
$(4 \cdot \overset{odd}{\overline{5}}\ ) \cdot \overset{even}{\overline{5}} \cdot \overset{even}{\overline{4}} \cdot \overset{even}{\overline{3}} = 5^2 \cdot 4^2 \cdot 3$