SAT-Minesweeper

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Game rule:

遊戲目標:找到盤面上所有非地雷的格子，過程中不點擊任何含有地雷的格子

對任意盤面上的格子其周圍最多有8個地雷且確切周圍的地雷數量會呈現在該格子上

Problem formulation

給定一個盤面，包含一些已經打開的數字和可能會有的一些已經標記的地雷， 是否存在一個地雷分佈使得這個盤面能夠出現？

NP-completeness (NP)

We can check every grid with number and verify that the adjacent mines number is equal to the grid number. Assume we are given n grids,

We then need to check at most 8*n grids. Therefore, Checking the Minesweeper Consistency takes O(n), which is in polynomial time.

NP-completeness (NP-Hard)

Reduce to Minesweeper consistency problem to a well-known NP-hard problem SAT(Boolean satisfiability problem).

Let a_m denote there is a mine at a. $a_i\ for\ 0 \leq i \leq 8$ denote there Is i mines in the neighboring squares around a, similarly for b,c,…,i.

1. $(a_m + a_0 + a_1 + \cdots + a_8) = 1$
2. $(a_m' + a_0')(a_m' + a_1') \ldots (a_7' + a_8') = 1$ totallty C$\binom{10}{2}$ clauses.
3. For k = 0,1,…,8, if e_k is true, then k of $a_m, b_m, c_m, d_m, f_m, g_m, i_m$ are true

Totally 90 variables needed!

a	b	c
d	e	f
g	h	i

Reduction to SAT problem

For k = 0,1,…,8, if e_k is true, then k of $a_m, b_m, c_m, d_m, f_m, g_m, i_m$ are true.

Implication scheme:

$$e_0 \rightarrow\ a_m{}'b_m{}'c_m{}'d_m{}'f_m{}'g_m{}'h_m{}'i_m{}'$$

Let $a_m{}'b_m{}'c_m{}'d_m{}'f_m{}'g_m{}'h_m{}'i_m{}'$ = 0000_0000

$e_1\ \rightarrow 10000000 + 01000000 + 00100000 + 00010000 + 00001000 + 00000100 + 00000010 + 00000001$ C$\binom{8}{1}$

$e_2 \rightarrow 11000000 + \cdots$ C$\binom{8}{2}$

$e_3 \rightarrow 11100000 + \cdots$ C$\binom{8}{3}$

…

Reduction to SAT problem

How to Convert these implication logic form to SOP form?

Brute force -> exponential time and may generate Clauses up to $8^{28} = 1.93 * 10^{25}$, which is not realistic!

Sol: try to combine some intermediate clauses and observe them + think the other way.

Reduction to SAT problem

e_1:

$(e_1' + a_m + b_m + c_m + d_m + f_m + g_m + h_m + i_m)(e_1' + a_m' + b_m')(e_1' + a_m' + c_m') \ldots (e_1' + b_m' + c_m')(e_1' + b_m' + d_m') \ldots (e_1' + h_m' + i_m') = 1$

totally C$\binom{8}{0}$ +C$\binom{8}{2}$

…..

Thinks like the same way

Reduction to SAT problem

Clauses number:

e_0: 8

e_1: C$\binom{8}{0}$ +C$\binom{8}{2}$

e_2: C$\binom{8}{1}$ +C$\binom{8}{3}$

e_3: C$\binom{8}{2}$ +C$\binom{8}{4}$

e_4: C$\binom{8}{3}$ +C$\binom{8}{5}$

e_5: C$\binom{8}{4}$ +C$\binom{8}{6}$

e_6: C$\binom{8}{5}$ +C$\binom{8}{7}$

e_7: C$\binom{8}{6}$ +C$\binom{8}{8}$

Experiment

MineSat.cpp   test1.c

input > test1.txt

```
3 3
1 -2 -2
-2 3 -2
-2 -2 -2
```

```
xt ./output/test1.out
==============================[MINISAT]===============================
| Conflicts |     ORIGINAL      |           LEARNT           | Progress |
|           | Clauses Literals | Clauses Literals  Lit/Cl |          |
=======================================================================
|         0 |   15597    35442 |       0        0    -nan |  0.000 % |
=======================================================================
SAT
1 X X
1 3 3
0 1 X
```

Experiment

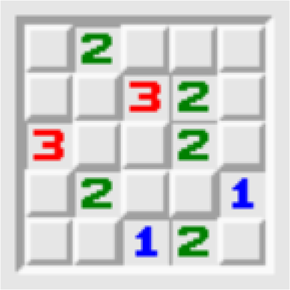
MineSat.cpp   test2.

input > test2.txt

```
3 3
-2 -2 -1
-1 2 -2
1 -2 -1
```

```
xt ./output/test2.out
============================[MINISAT]============================
| Conflicts |          ORIGINAL         |          LEARNT          | Progress |
|           |    Clauses     Literals   |    Clauses Literals Lit/Cl |          |
=================================================================
|         0 |      15597      35442 |        0        0  -nan | 0.000 % |
=================================================================
UNSAT
```

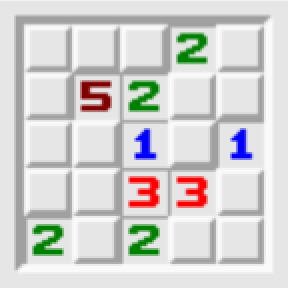
Experiment




```
5 5
-2 2 -2 -2 -2
-2 -2 3 2 -2
3 -2 -2 2 -2
-2 2 -2 -2 1
-2 -2 1 2 -2
```

```
_1.txt ./output/test5_5_1.out
==============================[MINISAT]===============================
| Conflicts |      ORIGINAL      |           LEARNT           | Progress |
|           |  Clauses  Literals |  Clauses  Literals  Lit/Cl |          |
=======================================================================
|         0 |    91053    208218 |        0         0    -nan |  0.000 % |
=======================================================================
SAT
2 2 2 X 2
X X 3 2 X
3 X 3 2 1
1 2 X 2 1
0 1 1 2 X
```

Experiment

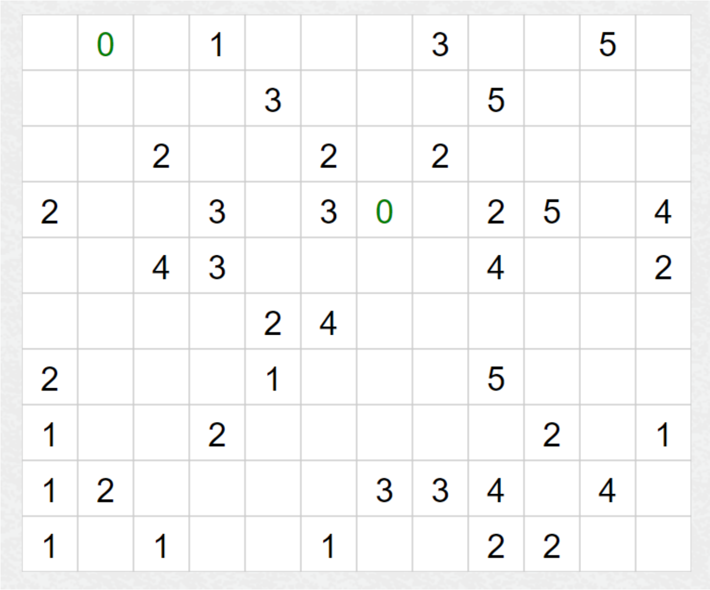



```
test5_5_2.txt
5 5
-2 -2 -2 2 -2
-2 5 2 -2 -2
-2 -2 1 -2 1
-2 -2 3 3 -2
2 -2 2 -2 -2
```

```
_1.txt ./output/test5_5_1.out
==============================[MINISAT]===============================
| Conflicts |     ORIGINAL      |          LEARNT          | Progress |
|           | Clauses  Literals | Clauses  Literals  Lit/Cl |          |
=======================================================================
|         0 |   91053    208218 |       0         0    -nan |  0.000 % |
=======================================================================
SAT
2 2 2 X 2
X X 3 2 X
3 X 3 2 1
1 2 X 2 1
0 1 1 2 X
```

Experiment

Easy




```
sy1.txt ./output/test_easy1.out
============================[MINISAT]============================
| Conflicts |     ORIGINAL       |          LEARNT          | Progress |
|           | Clauses Literals | Clauses Literals  Lit/Cl |          |
=================================================================
|         0 |  704184  1614000 |       0        0    -nan |  0.000 % |
=================================================================
SAT
0 0 1 1 2 X X 3 X X 5 X
0 0 1 X 3 3 3 X 5 X X X
1 1 2 3 X 2 1 2 X 5 X X
2 X 2 3 X 3 0 1 2 5 X 4
3 X 4 3 X 3 2 3 4 X X 2
3 X X 3 2 4 X X X X 3 1
2 X X 3 1 X X 6 5 3 1 0
1 3 X 2 2 3 4 X X 2 2 1
1 2 2 1 1 X 3 3 4 X 4 X
1 X 1 0 1 1 2 X 2 2 X X
```

Experiment

Medium

	4		3						2				2	2
3					3		4			5		5		
		2			1	1		1				5		
1			1		3					3			4	
	0								2	2	3	3		1
	1							1	2					
			2		4		5	3			3			1
		2	1		3					3		3	3	2
2		2	2		4			2				2		
	1				3		1		2			2	2	

```
dium1.txt ./output/medium1.out
==============================[MINISAT]===============================
| Conflicts |         ORIGINAL         |           LEARNT           | Progress |
|           | Clauses Literals | Clauses Literals  Lit/Cl |          |
======================================================================
|        16 |  908898  2083458 |      16       651    40.7 |  0.000 % |
======================================================================
SAT
X 4 X 3 X 3 X X X 2 X X 3 2 2
3 X X 3 1 3 X 4 2 3 5 X 5 X X
X 3 2 1 0 1 1 1 1 2 X X 5 X X
1 1 0 1 2 3 2 1 1 X 3 3 X 4 3
0 0 1 3 X X X 2 1 2 2 3 3 X 1
1 1 1 X X 6 X 3 1 2 X 3 X 4 2
X 2 2 2 2 4 X 5 3 X 2 3 X X 1
3 X 2 1 1 3 X X X 3 3 3 3 3 2
2 X 2 2 X 4 3 4 2 3 X X 2 2 X
1 1 1 2 X 3 X 1 0 2 X X 2 2 X
```

Experiment

Hard

2		4	2		1	2	1			
4				3				2		3
		4				3				
3				4					5	4
	5						2			
			4		3	3	3	3		3
4				2				2		
		3			5				5	
				2						4
	1					2				
1		1	1			2	3			
			0	1				3	2	

```
==============================[MINISAT]==============================
| Conflicts |          ORIGINAL         |          LEARNT          | Progress |
|           |    Clauses      Literals  |  Clauses Literals Lit/Cl |          |
=====================================================================
|        23 |   788880    1808250 |       23       671     29.2 | 0.000 % |
=====================================================================
SAT
2 X 4 2 2 1 2 1 1 1 X
4 X X X 3 X 2 X 2 3 3
X X 4 3 X 3 3 1 2 X X
3 4 3 3 4 X 3 2 3 5 4
X 5 X X 3 X X 2 X X X
X X X 4 3 3 3 3 3 4 3
4 X 5 X 2 3 X 3 2 X 2
2 X 3 2 X 5 X 5 X 5 X
1 2 2 2 2 X X 5 X X 4
0 1 X 1 1 2 2 4 X X X
1 2 1 1 1 1 2 3 X X 3
X 1 0 0 1 X 2 X 3 2 1
```

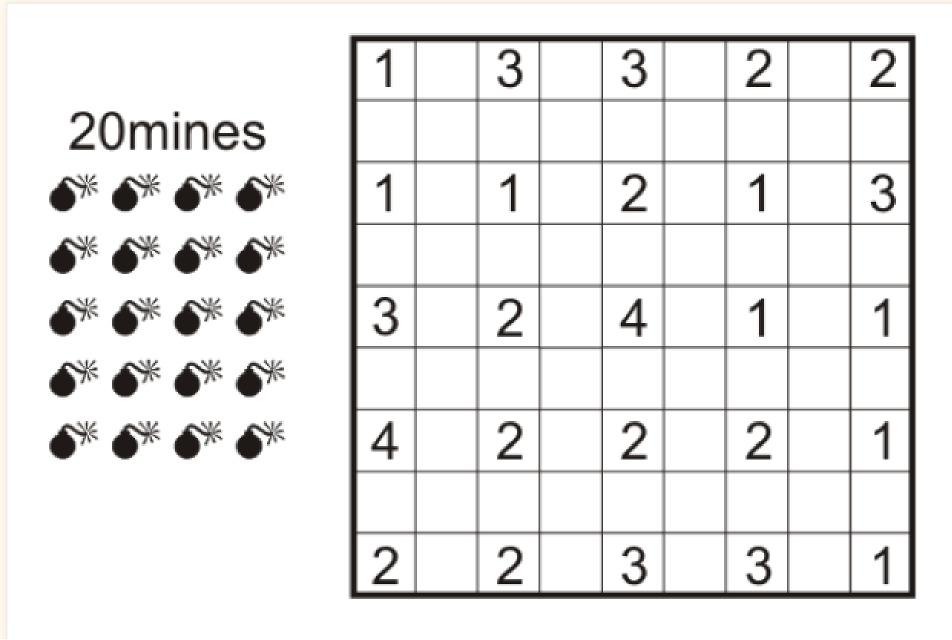
Experiment

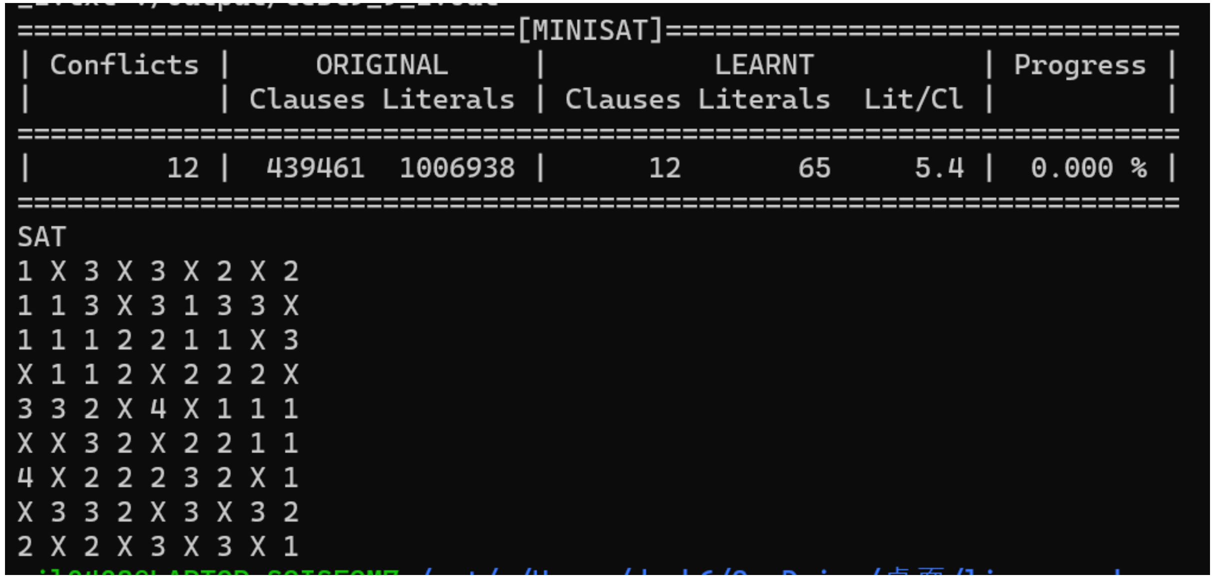
Mixed

											2		
2		4		2		4		3				5	
				2	2			5				3	
3	4					3				3	3	1	
	3			4					4		2	1	
		3	2	3				5		2			
			1		3								
4			1		1	2	3			2			0
				3		2	1		0				1
5		4				5			0				1
				4					2	3			
3			1	2	4	4						2	1
		0					1		2		1		

```
==============================[MINISAT]==============================
| Conflicts |          ORIGINAL        |         LEARNT          | Progress |
|           | Clauses Literals |  Clauses Literals  Lit/Cl |          |
=====================================================================
|         0 | 1144122   2623014 |        0          0    -nan |  0.000 % |
=====================================================================
SAT
0 1 X X 2 2 X 2 2 1 2 2 X X
2 4 4 3 2 X 4 X 3 X 4 X 5 X
X X X 2 2 2 4 X 5 3 X X 3 1
3 4 3 3 X 2 3 X X 4 3 3 1 0
X 3 2 X 4 X 5 5 X 4 X 2 1 1
X X 3 2 3 X X X 5 X 2 2 X 1
X 5 X 1 1 3 X X X 2 2 2 2 1
4 X 3 1 0 1 2 3 2 1 2 X 2 0
X X 4 2 3 2 2 1 1 0 2 X 3 1
5 X 4 X X X 5 X 2 0 2 3 X 1
X X 3 2 4 X X X 3 2 3 X 2 1
3 3 1 1 2 4 4 3 2 X X 2 2 1
X 1 0 1 X 2 X 1 1 2 2 1 1 X
```

Experiment





```
===============================[MINISAT]================================
| Conflicts |       ORIGINAL       |          LEARNT          | Progress |
|           | Clauses  Literals    | Clauses Literals  Lit/Cl |          |
=========================================================================
|        12 |  439461   1006938    |      12      65     5.4  |  0.000 % |
=========================================================================
SAT
1 X 3 X 3 X 2 X 2
1 1 3 X 3 1 3 3 X
1 1 1 2 2 1 1 X 3
X 1 1 2 X 2 2 2 X
3 3 2 X 4 X 1 1 1
X X 3 2 X 2 2 1 1
4 X 2 2 2 3 2 X 1
X 3 3 2 X 3 X 3 2
2 X 2 X 3 X 3 X 1
```

Reduce SAT to Minesweeper problem

Configure the Minesweeper plane to form various logic circuits.

$\mathbf{X} \longrightarrow$

$\cdots$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\cdots$
$\cdots$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\cdots$
$\cdots$	x	1	x'	x	1	x'	x	1	x'	x	1	x'	x	1	x'	x	$\cdots$
$\cdots$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\cdots$
$\cdots$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\cdots$

Figure 3: A wire.

Reduce SAT to Minesweeper problem

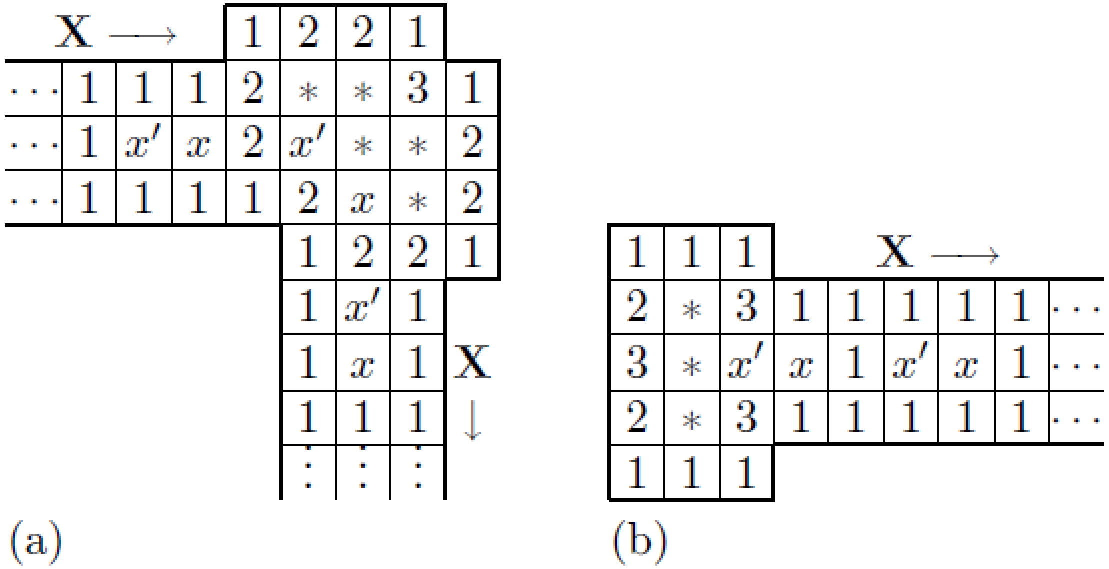


Figure 4: (a) A bent wire. (b) A terminated wire.

Reduce SAT to Minesweeper problem

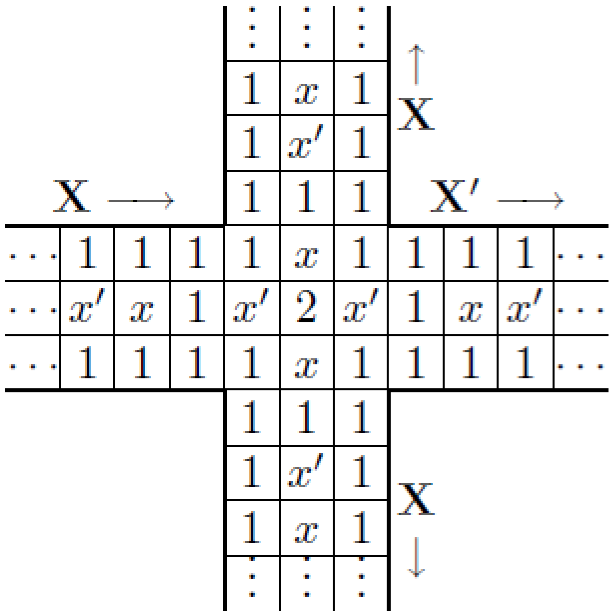


Figure 5: A three-way splitter.

Reduce SAT to Minesweeper problem

		$\mathbf{X} \longrightarrow$				1	1	1			$\mathbf{X}' \longrightarrow$			
$\cdots$	1	1	1	1	1	2	$*$	2	1	1	1	1	1	$\cdots$
$\cdots$	x'	x	1	x'	x	3	x'	3	x	x'	1	x	x'	$\cdots$
$\cdots$	1	1	1	1	1	2	$*$	2	1	1	1	1	1	$\cdots$
						1	1	1						

Figure 6: A NOT gate.

Reduce SAT to Minesweeper problem

	⋮	⋮	⋮																				
U	1	1	1			1	2	2	1		1	1	1		1	1	1						
↓	1	u'	1			2	*	*	3	2	3	*	2	1	2	*	3	2	1				
	1	u	1	1	2	4	*	s	a_1	a_2	a_3	t'	3	t	t'	3	*	*	2				
1	2	2	1	1	*	*	4	*	3	2	3	*	2	1	1	2	t	*	2				
2	*	u'	2	2	4	s'	3	1	1	0	1	1	1	0	0	1	2	2	1		**T** ⟶		
2	*	*	3	u	u'	s	2	1	1	1	1	1	1	1	1	1	t'	1	1	1	1	1	⋯
2	4	5	*	4	*	4	t	t'	1	t	t'	1	t	t'	1	t	2	t	1	t'	t	1	⋯
2	*	*	3	v	v'	r	2	1	1	1	1	1	1	1	1	1	t'	1	1	1	1	1	⋯
2	*	v'	2	2	4	r'	3	1	1	0	1	1	1	0	0	1	2	2	1				
1	2	2	1	1	*	*	4	*	3	2	3	*	2	1	1	2	t	*	2				
	1	v	1	1	2	4	*	r	b_1	b_2	b_3	t'	3	t	t'	3	*	*	2				
	1	v'	1			2	*	*	3	2	3	*	2	1	2	*	3	2	1				
↑	1	1	1			1	2	2	1		1	1	1		1	1	1						
V	⋮	⋮	⋮																				

Figure 9: An AND gate.

Reduce SAT to Minesweeper problem

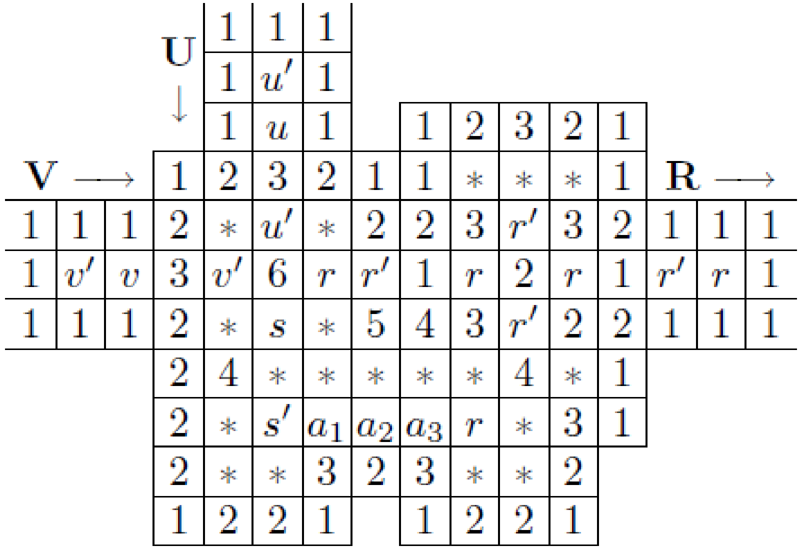


Figure 10: An OR gate

Reduce SAT to Minesweeper problem

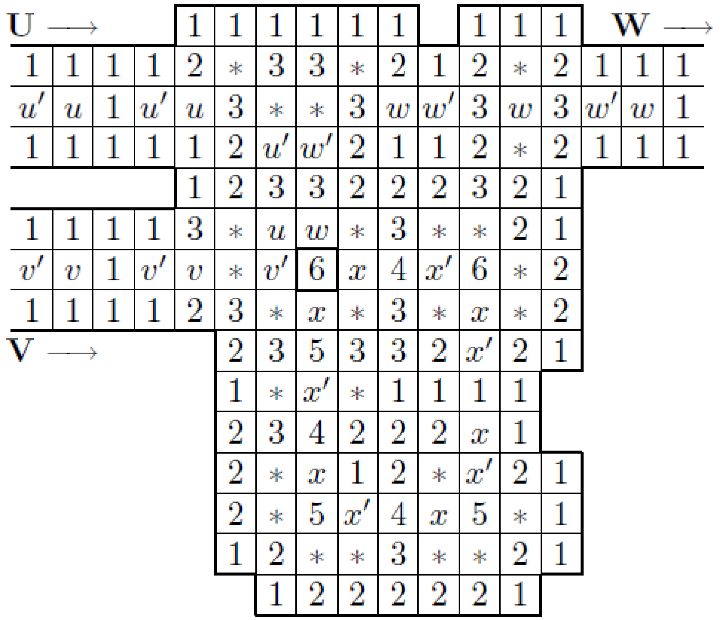


Figure 8: An XOR gate.

Polynomial time mapping from logic proposition to Minesweeper plane

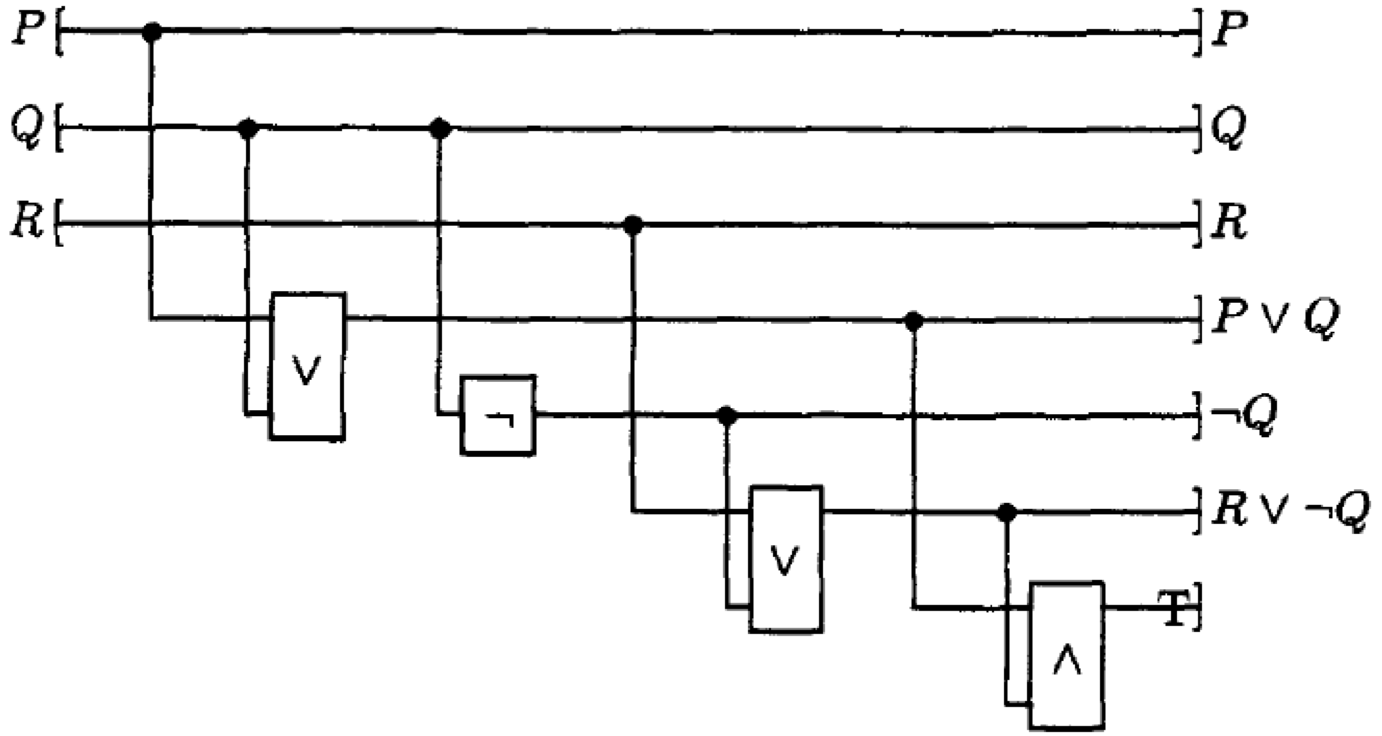


Figure 14. A Minesweeper circuit for $(P \vee Q) \wedge (R \vee \neg Q)$

Reference

1.http://web.math.ucsb.edu/~padraic/ucsb_2014_15/ccs_problem_solving_w2015/NP3.pdf

2. http://web.mat.bham.ac.uk/R.W.Kaye/minesw/ordmsw.htm

3. http://web.mat.bham.ac.uk/R.W.Kaye/minesw/minesw.pdf

Thank you for listening!