Galaxy Simulations and the Weak Lensing by a Dark Matter Halo

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1 Abstract

I developed a program that will simulate the effects of a galaxy cluster weakly lensing upon a distribution of background galaxies with intrinsic ellipticities and orientations. From this simulation, then, a catalog of galaxies was created and analyzed using a minimization function. At first, the the hypothetical lensing cluster was created as a simple isothermal sphere and a basic least squares fit was used to analyze the data generated by the simulation in an effort to extract the mean shear. In a later development, the simulated background galaxies were lensed by an Nevarro, Frenk, White (NFW) object instead. A minimization program provided by Professor Lindsay King was used to obtain the virial radius and concentration of the cluster. The two methods are compared and the accuracy of the NFW approach is tested to yield more information about how future surveys, like LSST, can affect the accuracy of measurements.

2 Introduction

It is widely believed that studying distorted galaxies can yield information about the cluster causing the distortion of the images. As light from a distant galaxy travels close to a mass distribution, the path of the photons is deflected by the Newtonian potential and allows the cluster to behave like a lens. The distances measured in this technique - the distances from the Earth to the lens and from the lens to the source - are so large that the the thickness of the mass distribution responsible for the deflection can be considered to be very small and therefore, the situation can be approached with a thin lens approximation. Hence, a method analogous to optics to treat these celestial bodies is applicable, this technique is called gravitational lensing. The Images of gravitationally lensed galaxies are then projected onto the plane of the mass distribution.

The properties from these images are what give crucial information concerning the dark matter profiles of the lensing cluster. Two vital properties found among a group of lensed galaxies are the convergence and shear, of which I elaborate upon later on. The convergence defines how magnified an image is while the shear defines how distorted the galaxy appears to observers. I consider two types of applied lenses used in the simulations.

The initial singular isothermal sphere approach involved the approximation of the reduced shear,

$$g = \frac{\gamma}{1-\kappa}.$$

With γ being the shear and κ the convergence. In this approximation, I had assumed that $\kappa \ll 1$ and so so for this case, $g = \gamma$. The approximation was valid under the weak gravitational lensing limit, where the galaxies are not magnified. The total ellipticity (Bartelmann, Narayan 2008) for a lensed image is determined by

$$\varepsilon_f = \frac{\varepsilon_s + g}{1 + g^* \kappa}. \tag{1}$$

ε_s and ε_f are the intrinsic and final ellipticities respectively. Applying the weak lensing limit, $\kappa \ll 1$, then (1) becomes $\varepsilon_f = \varepsilon_s + \gamma$

The values for the intrinsic ellipticities vome from a Gaussian distribution centered around zero because the *mean* ellipticity, μ, of the galaxies prior to being distorted is expected to be zero. The standard deviation used for this distribution is $\sigma_\varepsilon = 0.2$. Or rather,

$$\mathcal{P}(x) = \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma_\varepsilon^2}}$$

The singular isothermal sphere model predicts that the density of a galaxy cluster should be analogous to the ideal gas able to move only in a spherically symmetric potential:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}.$$

σ_v is the one dimensional velocity dispersion and r is the interior radius of the galaxy cluster.

It has been shown that the singular isothermal sphere model actually overestimates the sizes of the models and the common analysis now involves using NFW profiles to obtain a more accurate measurement of the dark matter halos. (Navarro, Frenk, White 1995) The dark matter NFW density profile can be given by

$$\rho(r) = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2}, \tag{2}$$

in which r_s is the scale radius of the lensing galaxy cluster and is defined by $r_s = r_{200}/c$. r_{200} is the virial radius and c is the concentration parameter. The critical density, $\rho_c = [3H^3(z)]/[8\pi G]$, which depends on the Hubble constant, $H(z)$, at a certain redshift z. The corresponding overdensity depends on the concentration by the following:

$$\delta_c = \frac{200c^3}{3[ln(1+c) - c/(1+c)]}$$

The overdensity and the concentration are both related to the halo mass and could predict the mass dependence at the epoch of dark matter halo formation. The aforementioned virial radius, r_{200}, is defines a dark matter mass profile with a density of $200\rho_c$. Thus, the mass of a halo with radius r_{200} is given as such:

$$M_{200} = \frac{4\pi}{3} 200 \rho_{\text{crit}} r_{200}^3$$

.

The singular isothermal sphere (SIS) has been simplified further to provide a single straightforward parameter to ensure the simulation worked adequately enough prior to moving on to something more complex. The NFW profile is further developed in section 2. In section 3, the development and outcome of the simulation will be presented. Section 4 will contain the results obtained after applying a minimization program to each of the models. Section 5 will compare the accuracy of our results and elaborate on how treatment of the NFW objects could be improved by LSST.

3 Analysis of the NFW Halo

I have already mentioned the simple parameters used for the SIS model. An NFW object is considerably more complex and befitting of separate analysis. Still within the weak lensing regime, the convergence and shear are derived from the lensing potential for an NFW object.(Wright, Brainerd 2000) Firstly, the critical surface mass density has been defined as:

$$\Sigma_c \equiv \frac{c_l^2}{4\pi G} \frac{D_s}{D_d D_d s}. \tag{3}$$

D_s is the distance from the source to the lens, D_d is the distance to the observer, and D_{ds} is the total distance from observer to source. c_l is notably different here as the speed of light, not the concentration parameter of the galaxy cluster. The surface mass density for a halo can be calculated by the integral

$$\Sigma(R) = 2\int_0^\infty \rho(R, z')dz'$$

z' here is the line of sight axis. R, the projected radius relative the lensing galaxy cluster, depends on the radial vectors in the sky as follows: $R = (\theta_1^2 + \theta_2^2)^{1/2}$. The ratio of the radius and the scale radius will equal to a dimensionless distance. Or rather, $x = R/r_S$. The outcome provided after using this introduced radius in the NFW density profile (2) yields three cases for $x > 1$, $x = 1$, and $x < 1$, each of which will correspond to a the surface mass density Σ_{NFW} (Bartelmann 1996)

The simplest case for $x = 1$ is given by

$$\Sigma_{\text{NFW}}(x) = \frac{2r_s\delta_c\rho_c}{3} \tag{4}$$

Then the other two cases:

For $x < 1$

$$\Sigma_{\text{NFW}}(x) = \frac{2r_s\delta_c\rho_c}{3(x^2-1)}\left[1 - \frac{2}{\sqrt{1-x^2}}\,\text{arctanh}\sqrt{\frac{1-x}{1+x}}\right] \tag{5}$$

For $x > 1$

$$\Sigma_{\text{NFW}}(x) = \frac{2r_s\delta_c\rho_c}{3(x^2-1)}\left[1 - \frac{2}{\sqrt{x^2-1}}\,\text{arctan}\sqrt{\frac{x-1}{1+x}}\right] \tag{6}$$

Both the convergence and the shear of the NFW object can now be determined using the surface mass density.

$$\kappa_{\text{NFW}}(x) = \frac{\Sigma_{\text{NFW}}(x)}{\Sigma_c} \tag{7}$$

$$\gamma_{\text{NFW}}(x) = \frac{\bar{\Sigma}_{\text{NFW}}(x) - \Sigma_{\text{NFW}}(x)}{\Sigma_c} \tag{8}$$

The *mean* surface mass density (Wright, Brainerd 2000) of the halo, $\bar{\Sigma}_{\text{NFW}}$, is given by the integral in the form of

$$\bar{\Sigma}_{\text{NFW}} = \frac{2}{x^2}\int_0^x x'\Sigma_{\text{NFW}}(x')dx'$$

This yields three difference cases again from (4), (5), and (6). Once more, the simplest case is presented first:

For $x = 1$

$$\bar{\Sigma}_{\text{NFW}} = 4r_s\delta_c\rho_c\left[1 + \ln\left(\frac{1}{2}\right)\right] \tag{9}$$

Now the other two cases are considered similarly:

For $x < 1$

$$\bar{\Sigma}_{\rm NFW}(x<1) = \frac{4r_s\delta_c\rho_c}{x^2}\left[\frac{2}{\sqrt{1-x^2}}\,\mathrm{arctanh}\sqrt{\frac{1-x}{1+x}}\right] + \ln\left(\frac{1}{2}\right) \tag{10}$$

For $x > 1$

$$\bar{\Sigma}_{\rm NFW}(x>1) = \frac{4r_s\delta_c\rho_c}{x^2}\left[\frac{2}{\sqrt{x^2-1}}\,\arctan\sqrt{\frac{x-1}{1+x}}\right] + \ln\left(\frac{1}{2}\right) \tag{11}$$

Now both the convergence and shear can be calculated by using equations (7) and (8) with equations (4)-(6) and (10)-(11). The reduced shear can be derived for a lensing galaxy cluster with an NFW dark matter halo by relating it with the shear and convergence as $g = \gamma_{\rm NFW}/(1-\kappa_{\rm NFW})$. The prescription described here was invoked in the program used to simulate and NFW object with specified parameters that are later dissected in the parameterization technique.

4 Simulation Programs

4.1 *The Singular Isothermal Sphere Simulation*

Python was the chosen programming language used to create the mapping of galaxies across a patch of sky covering a square area of 2 Mpc^2. The purpose of the program was to imitate synthetic data obtained from a telescope by generating galaxies with random orientations and positions and then applying a gravitational potential to distort them. The simulation's birth began with a very simple means of visually mapping galaxies on a Cartesian grid. The coordinates and orientation of the each individual galaxy were randomly generated. At first, the galaxies were assumed to be perfect circles, so the ellipticity was zero. Later, this changed and I pulled the components of the ellipticity from a Gaussian distribution centered around 0 and a width of 0.2. The galaxies were then distorted under the weak gravitational lensing limit to cause tangential stretching. This was accomplished by applying the a very simplified mean shear similar to the singular isothermal sphere:

$$\bar{\gamma}(r) = \frac{A}{R}$$

R is taken as the distance away from the center point of the galaxy cluster. A is an adjustable parameter arbitrarily chosen to be .35 to retain proper scaling and to prevent from accidentally displaying strong lensing effects, which as another application not discussed here. The reason for having generated a random orientation for each independent galaxy was to find a means to obtain the components of the complex shear value given by (Bartelman, Narayan 2000)

$$\gamma_1(\theta) \equiv \bar{\gamma}(R) \cos 2\phi \tag{12}$$

$$\gamma_2(\theta) \equiv \bar{\gamma}(R) \sin 2\phi \tag{13}$$

The angle ϕ is therefore the position angle of the object with respect to the x-axis of the grid, inherited from the randomly generated positions. With a value for the shear and the intrinsic ellipticities, the components of the quantities were added correspondingly using (1) to obtain the final ellipticity of the galaxy image. From this new complex value, ε_f, the magnitudes and angles were each used to obtain the altered orientations and the stretching caused by the shear. Due to high demand in the technical environment of Python, the major and minor axes needed to be calculated. The way these axes are found is by setting the relationship between the axis and the magnitude of the complex number that defines the ellipse in the following manner:

$$|\varepsilon_f| = \frac{1 - \frac{b}{a}}{1 + \frac{b}{a}}$$

. b is the minor axis and a is the major axis. The ratio between the minor and major axis will be $\chi \equiv b/a$. Therefore, the relation between the ratio and the magnitude of the final ellipticity becomes

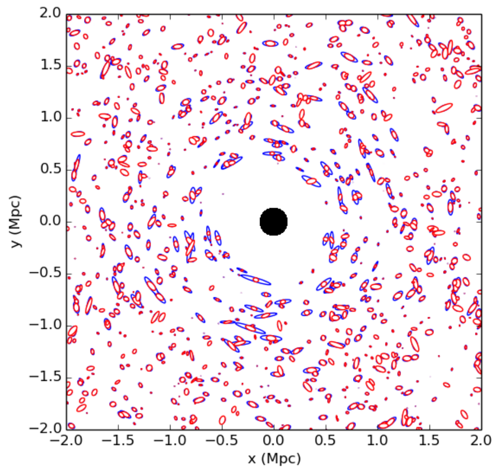
$$|\varepsilon_f| = \frac{1 - \chi}{1 + \chi} \rightarrow \chi = \frac{1 - |\varepsilon_f|}{1 + |\varepsilon_f|} \tag{14}$$

The final ellipticity of the galaxy has been already calculated, so the magnitude is a straightforward calculation. Thus, finding the ratio of the axes for each galaxy is a trivial and a simply repetitive matter across a distribution of galaxies. Because it is the *ratio* that is important to us, the individual major and minor axes give little vital information. One of the minor axis was assigned a random number within a range of 0 and 0.5. The according major axis was found by dividing the minor axis by the ratio of axes. $a = b/\chi$. With all the parameters fulfilled, every galaxy generated is then mapped out on a grid representing a 2 Mpc by 2 Mpc square area. A threshold was established to remove galaxies far too distorted by the lens. Ofthe 1000 galaxies originally generated, only those with a magnitude of shear less than .8 and a distance away from the cluster center greater than .5 were displayed. The parameters for the shear and the distance scales can all easy be adjusted for different desired values.

4.2 *The NFW Object Simulation*

The NFW dark matter density proved to be far more complicated, but the SIS program was successfully modified to account for the shear due to an NFW object. I have already discussed much of the analysis for

Figure 1: This is a snapshot of the simulation where 10^3 galaxies were generated. The dark spot in the middle represents a singular isothermal sphere profile of dark matter that would have an effective mean shear of .35/R. The red ellipses represent source galaxies and the blue ellipses represent gravitationally lensed galaxy images.



the NFW object in section 3. Many of the techniques used in subsection 4.1 were applied here as well. The final ellipticities, the axis ratios, and the shear components were calculated in much the same fashion, but the model for the shear was obviously far more involved. Both (8) and (7) were used to determine the effect the NFW model had upon the background of galaxies. The simulation had two variable parameters: c and r_{200}. The values were fixed at $c = 5$ and $r_{200} = 1.5$ Mpc. The redshift of the lens is $z = .2$ and redshift of the source galaxy is $z = 1.0$. Similar to the SIS case, the patch of sky is a 2 Mpc by 2 Mpc square area with galaxies too close to the mass being excluded. There was an established threshold of the shear that only permitted a value less than .5 to keep strongly lensed galaxies out of the image.

Both simulations created a catalog of the coordinate position and the components of ellipticity for every galaxy displayed. The file containing the information for each of the galaxy is saved onto the machine for future by a minimization program.

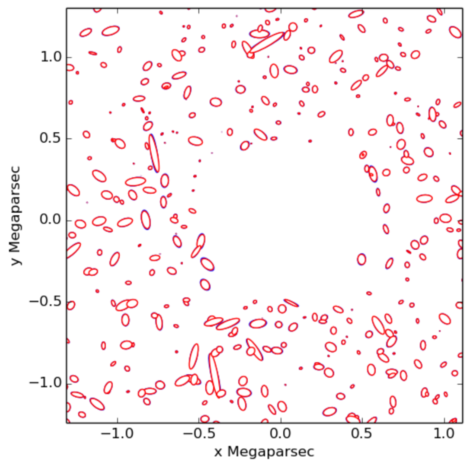
5 Minimization Results

For the SIS model, a very basic least squares method fit was used in an attempt to find the mean shear. The model assumed by the program was one such as

$$\bar{\gamma} = \frac{p}{R}$$

An initial guess of 1 yielded a fit value, p, of .41. Bins were created, beginning at the inner radius of the

Figure 2: Snapshot of the NFW simulation where 10^3 galaxies were generated. The galaxy cluster in the center (not depicted in the simulation) is an NFW object, with a shear and convergence according to equations (7) and (8). Once more, the red ellipses represent source galaxies and the blue ellipses represent gravitationally lensed galaxy images. Note that this is a zoomed in image of the actual 2 by 2 Mpc patch of sky.



allowed weak lensing boundary. Each successive bin was separated by 0.2 Mpc from the previous one. Any galaxies found within the area of the encompassing bin have a corresponding ellipticity accounted for and the mean shear of galaxies in such a bin is calculated. The parameterization fit is plotted below.

For the NFW simulation, the parametrization of an NFW profile (King, Schneider 2000) can be obtained by finding the scale radius, $r_s = r_{200}/c$. The models are in the form of equations (4), (5), (6), and (8). The effects of an NFW profile were parameterized using the the program created by Lindsay King. Three different results were found, each according to the density of galaxies across a patch of sky.(Figure 4) In this model, a trend needed to be established between the accuracy of the minimization and the density of galaxies per arcminute squared.

Figure 3: The red dots represent the average shear found at each of the bins. The blue line represents the least squares fit calculated by the program. The green line is the true curve known from the simulation of the SIS model that should be .35/R. The x-axis label should be in Megaparsec. The y-axis is the magnitude of the shear.

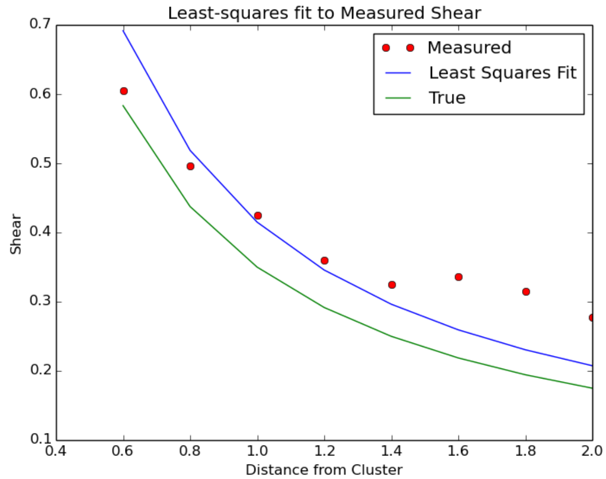
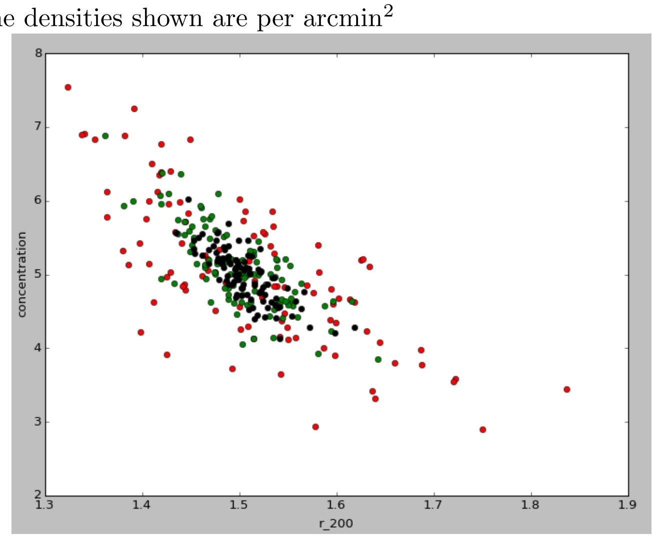


Figure 4: The red dots represent the lowest density at 10, the black dots the highest 100 , and the green in between the two values: 30. It is clear that as athe density is increased, the more condensed the measurements of r_{200} and c become. The densities shown are per arcmin^2



6 Conclusion

In this paper, I have applied two different types of weak lensing signals onto a background of galaxies and simulated the effects to display a visual image. The analogy to optics has allowed crucial analysis to be taken in order to formulate parameters of the dark matter profiles. The detected shear effects could provide further detail to reconstruct the masses of the galaxy clusters. The SIS model provided a good foundation for the early iterations of the simulation program. The idea of creating a synthetic telescope with entirely hypothetical data to catalog proved to be successful in providing a visual aid. The minimization least squares technique used to find the parameters from the data cataloged by the simulation yielded a value of 0.41 where it should have been closer to the value set by the simulation of 0.35. This is a 17% error that gives only a rough estimate of the actual parameters. After some testing, the error vastly decreased as the standard deviation of the Gaussian curve was lowered. For the simplified SIS model presented here, the Gaussian distribution provides a lot of noticeable noise. Some of the errors and different standard deviations are briefly tabulated below.

Standard Deviation σ	Error
0.2	17%
0.15	8.6%
.1	2.9%

The SIS model obviously shows improvement as the standard deviation in the ellipticities of the background galaxies is lowered. The original, intended 0.2 value for the standard deviation was not ideal, but it provides a decent estimation.

The program used to parameterize the values for r_{200} and c for the NFW halo returned adequate values. In figure 4, the spread of the values that were found condensed around a certain region close to the values originally set by the simulation.

6.1 *Large Synoptic Survey Telescope*

The Large Synoptic Survey Telescope (LSST) is a wide-field reflecting survey that will be build in El Piñón, Chile. The predicted year of beginning operations is 2022. One of the primary goals of this survey is to measure gravitational lensing in the deep sky to detect dark matter and dark energy signatures.

It is noteworthy to mention that as the density per arcmin^2 in the parameterization of an NFW object is increased, the more concentrated the values appear to be, as the results from Figure 4 would entail. LSST is presumed to have a density of 30 arcmin^2, which is a good value for ground based telescopes. The NFW model analyzed here (Figure 4) shows that raising the density in a survey would increment the accuracy of the virial radius, r_{200}, and the concentration, c. In the future study for dark matter, more dense surveys could lead to more observations and more data to analyze the elusive properties of dark matter.

7 References

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