Large Synoptic Survey Telescope and Weak Gravitational Lensing

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1 Introduction

With the inbound construction of the Large Synoptic Survey Telescope (LSST), further study of dark matter can be achieved by using weak gravitational lensing. With this method, the distortion effects are simplified. A glance at the eccentricity provided by the images around a mass distribution allows us to discover the amount of shear on a particular galaxy. The group that I worked with sought to simulate the effects of gravitational lensing and analyze how to find dark matter.

2 Weak Gravitational Lensing

As light travels from a very distant source, it is deflected by a mass distribution as it crosses the boundary of its gravitational potential. The light is bent towards us at a particular angle that allows the study of very distant galaxies, much like using a very long telescope.

Our project was fairly simple. We had a mass distribution perceived to be something like galaxy clusters that caused a *weak* lensing effect on galaxies in the background. The lensing mass caused some distortion and magnified the images to better analyze. Granted, our data was not exactly real data, but it allowed us the opportunity to study this insightful method.

3 Generating Data

The way we created our own data was a process in itself initially. The language chosen to do this was Python. Originally, we only created random points across a Cartesian grid. From there, we moved on to create random circles, then random ellipses with random eccentricities. After that, we kept the positions as random, but excluded a zone around the perceived mass that was not likely to be distorted and added a Gaussian distribution to the eccentricities across the entire board. This data was then used in a few simple formulas that would help us create a simplified simulation. After doing some calculations (see below) we stored our generated data into matrices. The parameters store were the positions, namely x and y, and the components of the complex ellipticity, ϵ_1 and ϵ_2. All of this was then exported to an external file.

4 Computations

From the aforementioned data we made, we used a few simplified calculations. In the weak gravitational lensing case, we did not have to worry about multiple images or the convergence, κ having an enormous effect, since it is much less than one.

From the reduced shear,

$$g = \frac{\gamma}{1-\kappa}$$

As mentioned before, $\kappa << 1$ so $g = \gamma$

Therefore, we obtained the final ellipticity of a lensed image by the following:

$$\epsilon_f = \epsilon^s + \gamma$$

Where ϵ^s is the source ellipticity.

Every ϵ^s generated was done so by using a Gaussian distribution for each of the components for the complex number. For γ, we had to do some math for as well. We know that the polar representation of the complex shear is:

$$\gamma = |\gamma| e^{2\phi}$$

Here, ϕ is the position angle of the shear vector. This angle will have to always be tangential to the position vector. Approximating the dependence of the *magnitude* of the shear to a function inversely proportional to the position from the origin, $1/r$, and using Euler's formula,

$$\gamma = \frac{1}{r}(\cos 2\phi + i \sin 2\phi)$$

$$\gamma = \frac{\cos 2\phi}{r} + i\frac{\sin 2\phi}{r} = \gamma_1 + i\gamma_2$$

This clearly gives me the components for the complex shear. Now, ϕ must change as the radial position from the lensing mass changes. This angle will have to be dependant on θ, which will be the angle of the position vector with respect to the x axis. I know it needs to be tangential to this position vector, so I define:

$$\phi = 90^\circ + \theta$$

Now, I can easily add the source ellipticity and the shear complex number to obtain the final ellipticity of a particular galaxy. This was another obstacle for my group to try and remedy. We had a complex ellipticity, but in Python, we require major and minor axes to define an ellipse object on a plot. We have one equation for the ellipticity,

$$\epsilon_f = \epsilon_{f1} + i\epsilon_{f2}$$

But two unknowns for the major and minor axes. I also know that

$$|\epsilon_f| = \sqrt{\epsilon_{f1}^2 + \epsilon_{f2}^2}$$

and

$$|\epsilon_f| = \frac{1 - \frac{b}{a}}{1 + \frac{b}{a}}$$

Where b is the minor axis and a is the major axis. I will define $b/a = R$. Solving for R, then

$$R = \frac{b}{a} = \frac{1 - |\epsilon_f|}{1 + |\epsilon_f|}$$

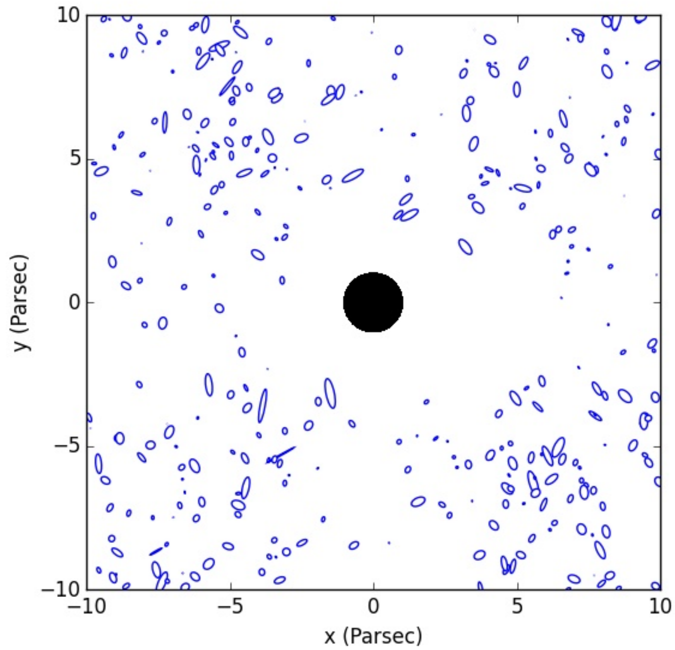
Scientifically, we really only care about the *ratio* of the minor and major axes, but I had to do this due to the parameters required by our code. Since we know ϵ_f, I kept b constant and changed a

$$a = \frac{b}{R}$$

Since I am keeping b essentially unchanged, it was generated for this system using a random number from .1 to .3. a is then calculated using the above formula. Thus, we have our major and minor axes.

With all the parameters needed, I can actually create figures of each galaxy represented as a distorted ellipse after the shear has been added to it. Each galaxy's x position, y position, $\epsilon_f 1$, and $\epsilon_f 2$ is saved into an array and exported onto an external text file.
The outcome of the simulation is show in Figure 1.

Figure 1: This is a snapshot of the simulation where we generated 10^4 galaxies. The dark spot in the middle is some yet unknown mass and the ellipses each represent a galaxy distorted by the lensing mass.



5 Analysis

As mentioned before, there was a file created containing the details of every galaxy in the image. This was our way of essentially mimicking what would have been data received from LSST.

The group was then destined to collect this data, assuming no prior knowledge as to what exactly the shear was due to this mass. Using a least squares fit model, we could put all our data together and find an *average* value for γ. From this, we could ultimately tell what the total mass is of the lensing object.

6 Conclusion

More time will be needed to further analyze the data we created using the aforementioned method to find the average value for the shear. Continuation of this project would lead to appropriately finding the total matter necessary to cause this kind of distortion. Assuming the *luminous* matter can be found, the amount of dark matter can then be calculated.