



# **CS 412 Intro. to Data Mining**

## **Chapter 6. Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods**

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# Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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- ❑ Basic Concepts
- ❑ Efficient Pattern Mining Methods
- ❑ Pattern Evaluation
- ❑ Summary



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# Pattern Discovery: Basic Concepts

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- What Is Pattern Discovery? Why Is It Important?
- Basic Concepts: Frequent Patterns and Association Rules
- Compressed Representation: Closed Patterns and Max-Patterns

# What Is Pattern Discovery?

- What are patterns?  
□ Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set  
→ កំណត់លក្ខណៈទៅលក្ខណៈដែលមានចំណាំ
- Patterns represent **intrinsic** and **important properties** of datasets
- Pattern discovery: Uncovering patterns from massive data sets
- Motivation examples:  
□ What products were often purchased together?  
□ What are the subsequent purchases after buying an iPad?  
□ What code segments likely contain copy-and-paste bugs?  
□ What word sequences likely form phrases in this corpus?  
Pattern ការណែនាំ Noise ពាមុជ

# Pattern Discovery: Why Is It Important?

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- Finding **inherent regularities** in a data set
- **Foundation** for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

# Basic Concepts: k-Itemsets and Their Supports

- **Itemset**: A set of one or more items  
ັກຕະກຳໂຕເກມເກມໄດ້
- **k-itemset**:  $X = \{x_1, \dots, x_k\}$
- Ex. {Beer, Nuts, Diaper} is a 3-itemset
- **(absolute) support (count)** of  $X$ ,  $\text{sup}\{X\}$ :  
ເຕັກຍົມສິຕະລູໃນໜີ້ support  
ັກຕະກຳລະຫວ່າງ  
ັກຕະກຳທີ່ມີກຳນົດກຳນົດ  
 Frequency or the number of occurrences of an itemset  $X$
- Ex.  $\text{sup}\{\text{Beer}\} = 3$   
ັກຕະກຳທີ່ມີກຳນົດກຳນົດ 4 ໂດຍມີກຳນົດກຳນົດ
- Ex.  $\text{sup}\{\text{Diaper}\} = 4$
- Ex.  $\text{sup}\{\text{Beer, Diaper}\} = 3$
- Ex.  $\text{sup}\{\text{Beer, Eggs}\} = 1$

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

ກຳນົດກຳນົດ

**(relative) support**,  $s\{X\}$ : The fraction of transactions that contains  $X$  (i.e., the probability that a transaction contains  $X$ )

- Ex.  $s\{\text{Beer}\} = 3/5 = 60\%$
- Ex.  $s\{\text{Diaper}\} = 4/5 = 80\%$
- Ex.  $s\{\text{Beer, Eggs}\} = 1/5 = 20\%$

# Basic Concepts: Frequent Itemsets (Patterns)

- An itemset (or a pattern)  $X$  is *frequent* if the support of  $X$  is no less than a *minsup threshold*  $\sigma$  Pattern កំណត់រួចរាល់ គ្មានភាពកិច្ចការណ៍
- Let  $\sigma = 50\%$  ( $\sigma$ : *minsup threshold*)  
For the given 5-transaction dataset
  - All the frequent 1-itemsets:
    - Beer: 3/5 (60%); Nuts: 3/5 (60%)
    - Diaper: 4/5 (80%); Eggs: 3/5 (60%)
  - All the frequent 2-itemsets:
    - {Beer, Diaper}: 3/5 (60%)
  - All the frequent 3-itemsets?
    - None គ្មាន 3 item គឺចងកំណត់រួចរាល់មិនអាចទូទាត់បាន

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



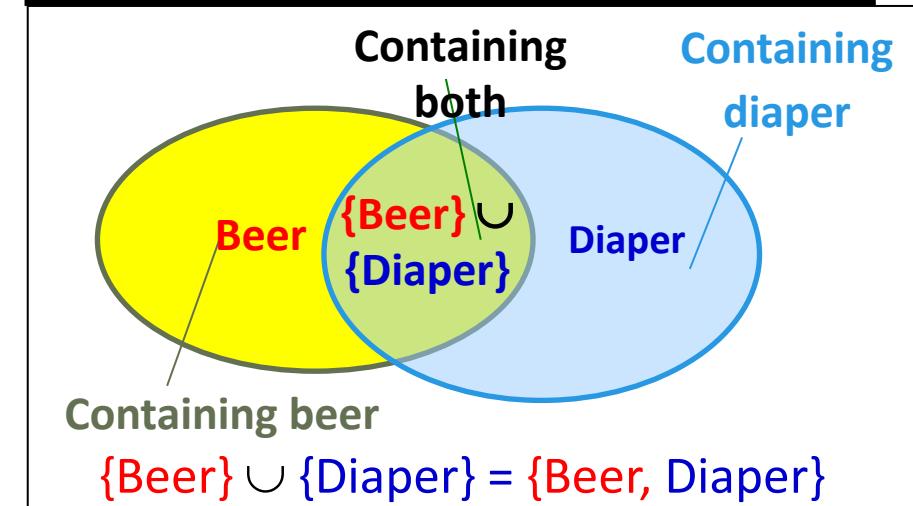
- Why do these itemsets (shown on the left) form the complete set of frequent  $k$ -itemsets (patterns) for any  $k$ ?
- **Observation:** We may need an efficient method to mine a complete set of frequent patterns

# From Frequent Itemsets to Association Rules

ອຍກວ່າກີ່າກວາການນັ້ນທີ່ມີຄວາມມີກົດໝາຍ ຂະໜົດຈົບປົງຫຼືຍຸ່ນ

- Comparing with itemsets, rules can be more telling
  - Ex. *Diaper* → *Beer*
  - *Buying diapers may likely lead to buying beers*
- How strong is this rule? (support, confidence)
  - Measuring association rules:  $X \rightarrow Y$  (s, c)
  - Both  $X$  and  $Y$  are itemsets
  - **Support, s:** The probability that a transaction contains  $X \cup Y$
  - Ex.  $s\{\text{Diaper, Beer}\} = 3/5 = 0.6$  (i.e., 60%)
  - **Confidence, c:** The *conditional probability* that a transaction containing  $X$  also contains  $Y$
  - Calculation:  $c = \frac{\text{sup}(X \cup Y)}{\text{sup}(X)}$
  - Ex.  $c = \text{sup}\{\text{Diaper, Beer}\}/\text{sup}\{\text{Diaper}\} = \frac{3}{4} = 0.75$

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



Note:  $X \cup Y$ : the union of two itemsets

- The set contains both  $X$  and  $Y$

# Mining Frequent Itemsets and Association Rules

## ❑ Association rule mining

- ❑ Given two thresholds:  $minsup$ ,  $minconf$
- ❑ Find **all** of the rules,  $X \rightarrow Y$  (s, c)
  - ❑ such that,  $s \geq minsup$  and  $c \geq minconf$
- ❑ Let  $minsup = 50\%$ 
  - ❑ Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - ❑ Freq. 2-itemsets: {Beer, Diaper}: 3
- ❑ Let  $minconf = 50\%$ 
  - ❑  $Beer \rightarrow Diaper$  (60%, 100%)
  - ❑  $Diaper \rightarrow Beer$  (60%, 75%)

(Q: Are these all rules?)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

## ❑ Observations:

- ❑ Mining association rules and mining frequent patterns are very close problems
- ❑ Scalable methods are needed for mining large datasets

confident នៅក្យា  
តើអ្នកលើចំណាំដឹងអ្នកនៅនេះ  
នៅនេះ

# Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB<sub>1</sub> contain?

- TDB<sub>1</sub>: T<sub>1</sub>: {a<sub>1</sub>, ..., a<sub>50</sub>}; T<sub>2</sub>: {a<sub>1</sub>, ..., a<sub>100</sub>}

- Assuming (absolute) *minsup* = 1

- Let's have a try

1-itemsets: {a<sub>1</sub>} : 2, {a<sub>2</sub>} : 2, ..., {a<sub>50</sub>} : 2, {a<sub>51</sub>} : 1, ..., {a<sub>100</sub>} : 1,

2-itemsets: {a<sub>1</sub>, a<sub>2</sub>} : 2, ..., {a<sub>1</sub>, a<sub>50</sub>} : 2, {a<sub>1</sub>, a<sub>51</sub>} : 1, ..., ..., {a<sub>99</sub>, a<sub>100</sub>} : 1,

..., ..., ..., ...

99-itemsets: {a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>99</sub>} : 1, ..., {a<sub>2</sub>, a<sub>3</sub>, ..., a<sub>100</sub>} : 1

100-itemset: {a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>100</sub>} : 1

- The total number of frequent itemsets:

$$\binom{100}{1} + \binom{100}{2} + \binom{100}{3} + \dots + \binom{100}{100} = 2^{100} - 1$$

A too huge set for any  
one to compute or store!



# Expressing Patterns in Compressed Form: Closed Patterns

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- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset)  $X$  is **closed** if  $X$  is *frequent*, and there exists *no super-pattern*  $Y \supset X$ , *with the same support* as  $X$ 
  - Let Transaction DB  $TDB_1$ :  $T_1: \{a_1, \dots, a_{50}\}$ ;  $T_2: \{a_1, \dots, a_{100}\}$
  - Suppose  $minsup = 1$ . How many closed patterns does  $TDB_1$  contain?
    - Two:  $P_1: \{\{a_1, \dots, a_{50}\}: 2\}$ ;  $P_2: \{\{a_1, \dots, a_{100}\}: 1\}$
  - **Closed pattern** is a **lossless compression** of frequent patterns
    - Reduces the # of patterns but does not lose the support information!
    - You will still be able to say:  $\{\{a_2, \dots, a_{40}\}: 2\}$ ,  $\{\{a_5, a_{51}\}: 1\}$

# Expressing Patterns in Compressed Form: Max-Patterns

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- Solution 2: **Max-patterns**: A pattern  $X$  is a **max-pattern** if  $X$  is frequent and there exists no frequent super-pattern  $Y \supset X$
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB  $TDB_1$ :  $T_1: \{a_1, \dots, a_{50}\}$ ;  $T_2: \{a_1, \dots, a_{100}\}$
  - Suppose  $minsup = 1$ . How many max-patterns does  $TDB_1$  contain?
    - One:  $P: \{a_1, \dots, a_{100}\}: 1$
  - **Max-pattern is a lossy compression!**
    - We only know  $\{a_1, \dots, a_{40}\}$  is frequent
    - But we do not know the real support of  $\{a_1, \dots, a_{40}\}, \dots$ , any more!
  - Thus in many applications, mining close-patterns is more desirable than mining max-patterns

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# Efficient Pattern Mining Methods

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- ❑ The Downward Closure Property of Frequent Patterns
- ❑ The Apriori Algorithm
- ❑ Extensions or Improvements of Apriori
- ❑ Mining Frequent Patterns by Exploring Vertical Data Format
- ❑ FP-Growth: A Frequent Pattern-Growth Approach
- ❑ Mining Closed Patterns

# The Downward Closure Property of Frequent Patterns

- Observation: From TDB<sub>1</sub>: T<sub>1</sub>: {a<sub>1</sub>, ..., a<sub>50</sub>}; T<sub>2</sub>: {a<sub>1</sub>, ..., a<sub>100</sub>}
    - We get a frequent itemset: {a<sub>1</sub>, ..., a<sub>50</sub>}
    - Also, its subsets are all frequent: {a<sub>1</sub>}, {a<sub>2</sub>}, ..., {a<sub>50</sub>}, {a<sub>1</sub>, a<sub>2</sub>}, ..., {a<sub>1</sub>, ..., a<sub>49</sub>}, ...
    - There must be some hidden relationships among frequent patterns!  
*இது மிகவும் தீவிரமான விவரம்*
  - The **downward closure** (also called “Apriori”) property of frequent patterns
    - If {beer, diaper, nuts} is frequent, so is {beer, diaper}
    - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
    - Apriori: Any subset of a frequent itemset must be frequent
  - Efficient mining methodology
    - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?  A sharp knife for pruning!

# Apriori Pruning and Scalable Mining Methods

ឧបរិប្បានអាជីវកម្ម និងការសកលបន្ថែម

- **Apriori pruning principle:** If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

# Apriori: A Candidate Generation & Test Approach

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- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - **Repeat**
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set  $k := k + 1$
  - **Until** no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

# The Apriori Algorithm (Pseudo-Code)

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$C_k$ : Candidate itemset of size k

$F_k$  : Frequent itemset of size k

$K := 1;$

$F_k := \{\text{frequent items}\}; \quad // \text{frequent 1-itemset}$

**While** ( $F_k \neq \emptyset$ ) **do {**  $\quad // \text{when } F_k \text{ is non-empty}$

$C_{k+1} := \text{candidates generated from } F_k; \quad // \text{candidate generation}$

Derive  $F_{k+1}$  by counting candidates in  $C_{k+1}$  with respect to  $TDB$  at  $\text{minsup}$ ;

$k := k + 1$

**}**

**return**  $\cup_k F_k \quad // \text{return } F_k \text{ generated at each level}$

# The Apriori Algorithm—An Example

ចំណាំទីនេះ ឬ ឈ្មោះជារាយ mean Support នៅរាយទី 1 នៅលើទីនេះ

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

minsup = 2

$C_1$   
1<sup>st</sup> scan

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$F_1$

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

$F_2$

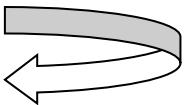
Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

$C_2$

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

$C_2$

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}



$C_3$

Itemset
{B, C, E}

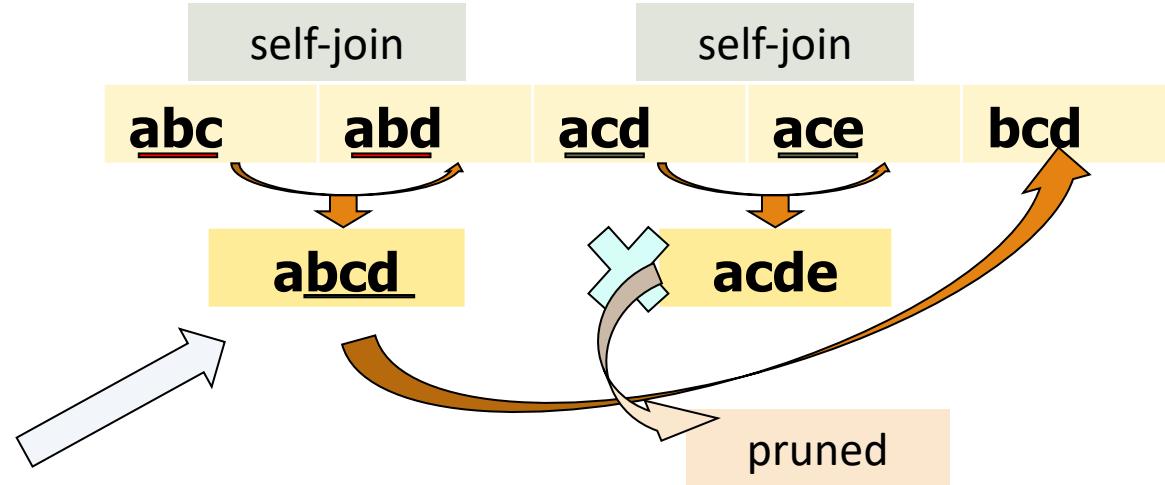
3<sup>rd</sup> scan

$F_3$

Itemset	sup
{B, C, E}	2

# Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining  $F_k$
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining:  $F_3 * F_3$ 
    - $abcd$  from  $abc$  and  $abd$
    - $acde$  from  $acd$  and  $ace$
  - Pruning:
    - $acde$  is removed because  $ade$  is not in  $F_3$
  - $C_4 = \{abcd\}$



# Candidate Generation: An SQL Implementation

- ❑ Suppose the items in  $F_{k-1}$  are listed in an order

- ❑ Step 1: self-joining  $F_{k-1}$  insert into  $C_k$

select  $p.item_1, p.item_2, \dots, p.item_{k-1}, q.item_{k-1}$

from  $F_{k-1}$  as  $p, F_{k-1}$  as  $q$

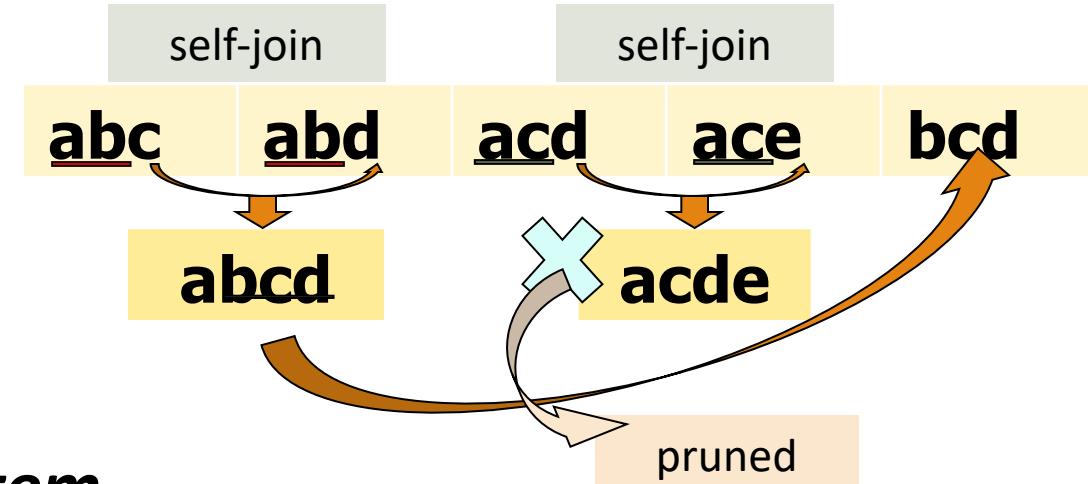
where  $p.item_1 = q.item_1, \dots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$

- ❑ Step 2: pruning

for all  $itemsets c$  in  $C_k$  do

for all  $(k-1)$ -subsets  $s$  of  $c$  do

if ( $s$  is not in  $F_{k-1}$ ) then delete  $c$  from  $C_k$



# Apriori: Improvements and Alternatives

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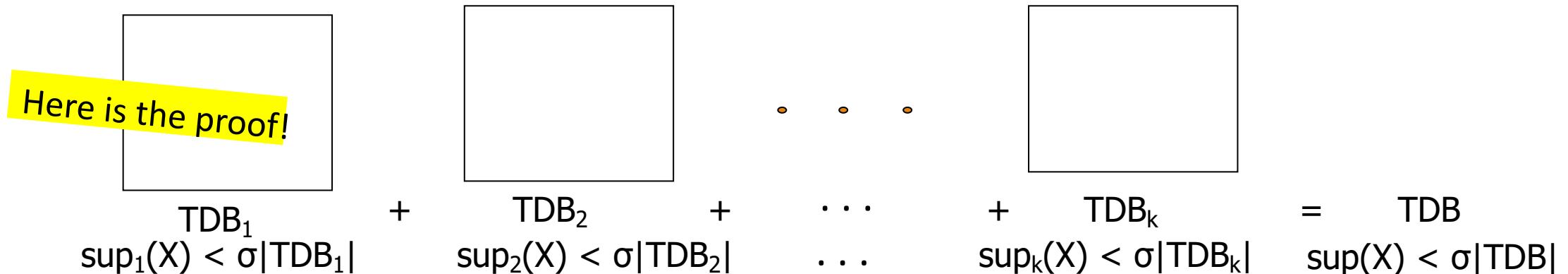
- ❑ Reduce passes of transaction database scans
  - ❑ Partitioning (e.g., Savasere, et al., 1995) 

To be discussed in subsequent slides
  - ❑ Dynamic itemset counting (Brin, et al., 1997)
- ❑ Shrink the number of candidates
  - ❑ Hashing (e.g., DHP: Park, et al., 1995) 

To be discussed in subsequent slides
  - ❑ Pruning by support lower bounding (e.g., Bayardo 1998)
  - ❑ Sampling (e.g., Toivonen, 1996)
- ❑ Exploring special data structures
  - ❑ Tree projection (Agarwal, et al., 2001)
  - ❑ H-miner (Pei, et al., 2001)
  - ❑ Hypcube decomposition (e.g., LCM: Uno, et al., 2004)

# Partitioning: Scan Database Only Twice

- Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*



Here is the proof!

$$TDB_1 + TDB_2 + \dots + TDB_k = TDB$$
$$\sup_1(X) < \sigma|TDB_1| \quad \sup_2(X) < \sigma|TDB_2| \quad \dots \quad \sup_k(X) < \sigma|TDB_k| \quad \sup(X) < \sigma|TDB|$$

- Method: Scan DB twice (A. Savasere, E. Omiecinski and S. Navathe, VLDB'95)
  - Scan 1: Partition database so that each partition can fit in main memory (why?)
    - Mine local frequent patterns in this partition
  - Scan 2: Consolidate global frequent patterns
    - Find global frequent itemset candidates (those frequent in at least one partition)
    - Find the true frequency of those candidates, by scanning TDB; one more time

# Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Hashing: Different itemsets may have the same hash value:  $v = \text{hash}(\text{itemset})$
- 1<sup>st</sup> scan: When counting the 1-itemset, hash 2-itemset to calculate the bucket count
- Observation: A  $k$ -itemset cannot be frequent if its corresponding hashing bucket count is below the  $\text{minsup}$  threshold
- Example: At the 1<sup>st</sup> scan of TDB, count 1-itemset, and
  - Hash 2-itemsets in the transaction to its bucket
    - {ab, ad, ce}
    - {bd, be, de}
    - ...
  - At the end of the first scan,
    - if  $\text{minsup} = 80$ , remove ab, ad, ce, since  $\text{count}\{\text{ab, ad, ce}\} < 80$

Itemsets	Count
{ab, ad, ce}	35
{bd, be, de}	298
.....	...
{yz, qs, wt}	58

**Hash Table**

# Exploring Vertical Data Format: ECLAT

- ❑ ECLAT (Equivalence Class Transformation): A depth-first search algorithm using set intersection [Zaki et al. @KDD'97]
- ❑ Tid-List: List of transaction-ids containing an itemset
- ❑ Vertical format:  $t(e) = \{T_{10}, T_{20}, T_{30}\}$ ;  $t(a) = \{T_{10}, T_{20}\}$ ;  $t(ae) = \{T_{10}, T_{20}\}$
- ❑ Properties of Tid-Lists
  - ❑  $t(X) = t(Y)$ : X and Y always happen together (e.g.,  $t(ac) = t(d)$ )
  - ❑  $t(X) \subset t(Y)$ : transaction having X always has Y (e.g.,  $t(ac) \subset t(ce)$ )
- ❑ Deriving frequent patterns based on vertical intersections
- ❑ Using **diffset** to accelerate mining
  - ❑ Only keep track of differences of tids
  - ❑  $t(e) = \{T_{10}, T_{20}, T_{30}\}$ ,  $t(ce) = \{T_{10}, T_{30}\} \rightarrow \text{Diffset } (ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

# Why Mining Frequent Patterns by Pattern Growth?

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- ❑ Apriori: A *breadth-first search* mining algorithm
  - ❑ First find the complete set of frequent k-itemsets
  - ❑ Then derive frequent (k+1)-itemset candidates
  - ❑ Scan DB again to find true frequent (k+1)-itemsets
- ❑ Motivation for a different mining methodology
  - ❑ Can we develop a *depth-first search* mining algorithm?
  - ❑ For a frequent itemset  $\rho$ , can subsequent search be confined to only those transactions that containing  $\rho$ ?
- ❑ Such thinking leads to a frequent pattern growth approach:
  - ❑ FP-Growth (J. Han, J. Pei, Y. Yin, “Mining Frequent Patterns without Candidate Generation,” SIGMOD 2000)

# Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	$\{f, a, c, d, g, i, m, p\}$	$f, c, a, m, p$
200	$\{a, b, c, f, l, m, o\}$	$f, c, a, b, m$
300	$\{b, f, h, j, o, w\}$	$f, b$
400	$\{b, c, k, s, p\}$	$c, b, p$
500	$\{a, f, c, e, l, p, m, n\}$	$f, c, a, m, p$

After inserting the 1<sup>st</sup> frequent Itemlist: “ $f, c, a, m, p$ ”

1. Scan DB once, find single item frequent pattern:

Let min\_support = 3

$f:4, a:3, c:4, b:3, m:3, p:3$

2. Sort frequent items in frequency descending order, f-list

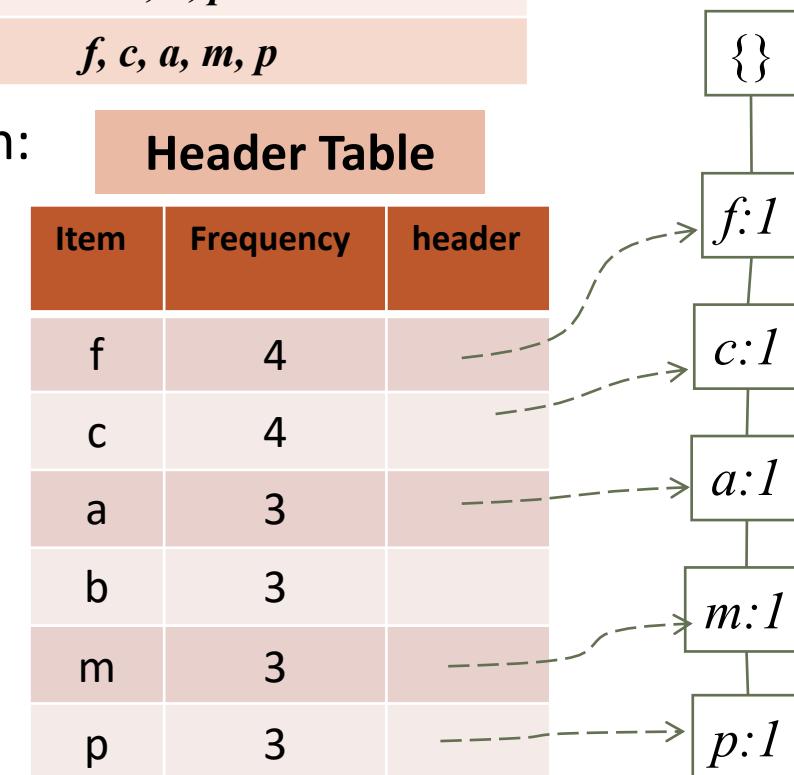
F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

- The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



# Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	$\{f, a, c, d, g, i, m, p\}$	$f, c, a, m, p$
200	$\{a, b, c, f, l, m, o\}$	$f, c, a, b, m$
300	$\{b, f, h, j, o, w\}$	$f, b$
400	$\{b, c, k, s, p\}$	$c, b, p$
500	$\{a, f, c, e, l, p, m, n\}$	$f, c, a, m, p$

After inserting the 2<sup>nd</sup> frequent itemlist “ $f, c, a, b, m$ ”

1. Scan DB once, find single item frequent pattern:

Let min\_support = 3

$f:4, a:3, c:4, b:3, m:3, p:3$

2. Sort frequent items in frequency descending order, f-list

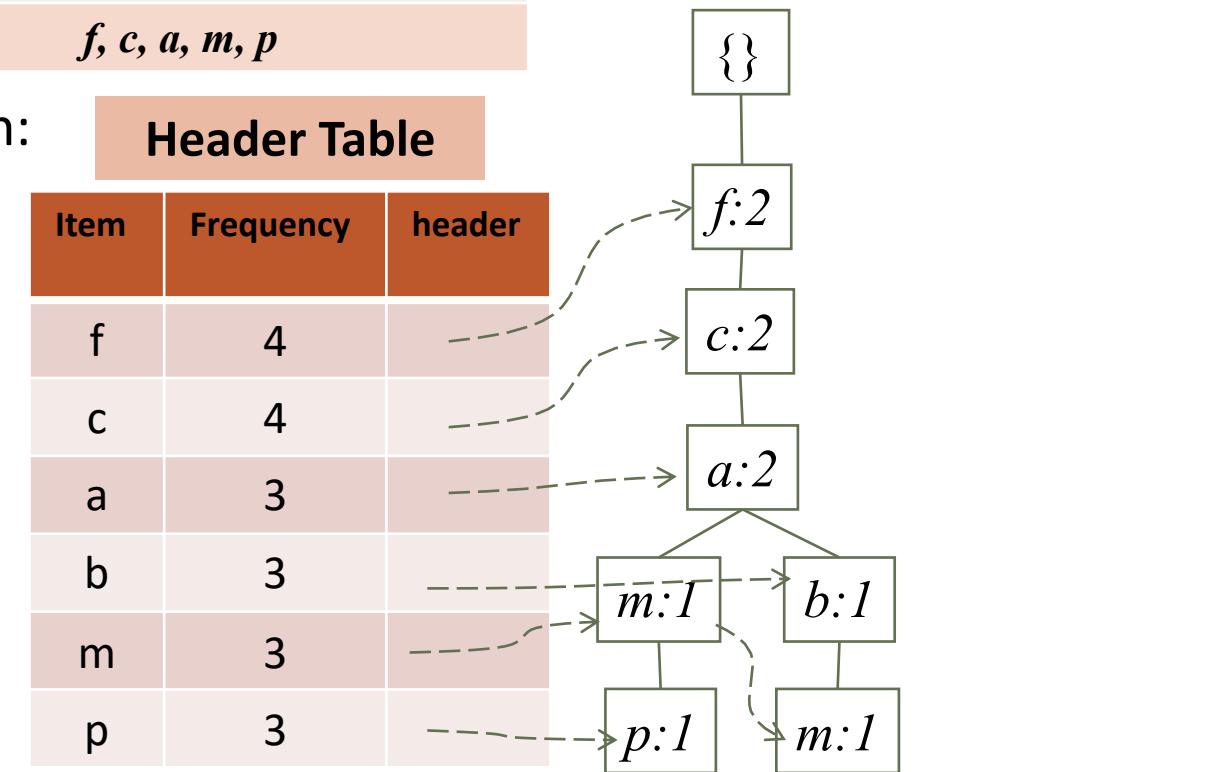
F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



# Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	$\{f, a, c, d, g, i, m, p\}$	$f, c, a, m, p$
200	$\{a, b, c, f, l, m, o\}$	$f, c, a, b, m$
300	$\{b, f, h, j, o, w\}$	$f, b$
400	$\{b, c, k, s, p\}$	$c, b, p$
500	$\{a, f, c, e, l, p, m, n\}$	$f, c, a, m, p$

1. Scan DB once, find single item frequent pattern:

Let min\_support = 3

f:4, a:3, c:4, b:3, m:3, p:3

2. Sort frequent items in frequency descending order, f-list

F-list = f-c-a-b-m-p

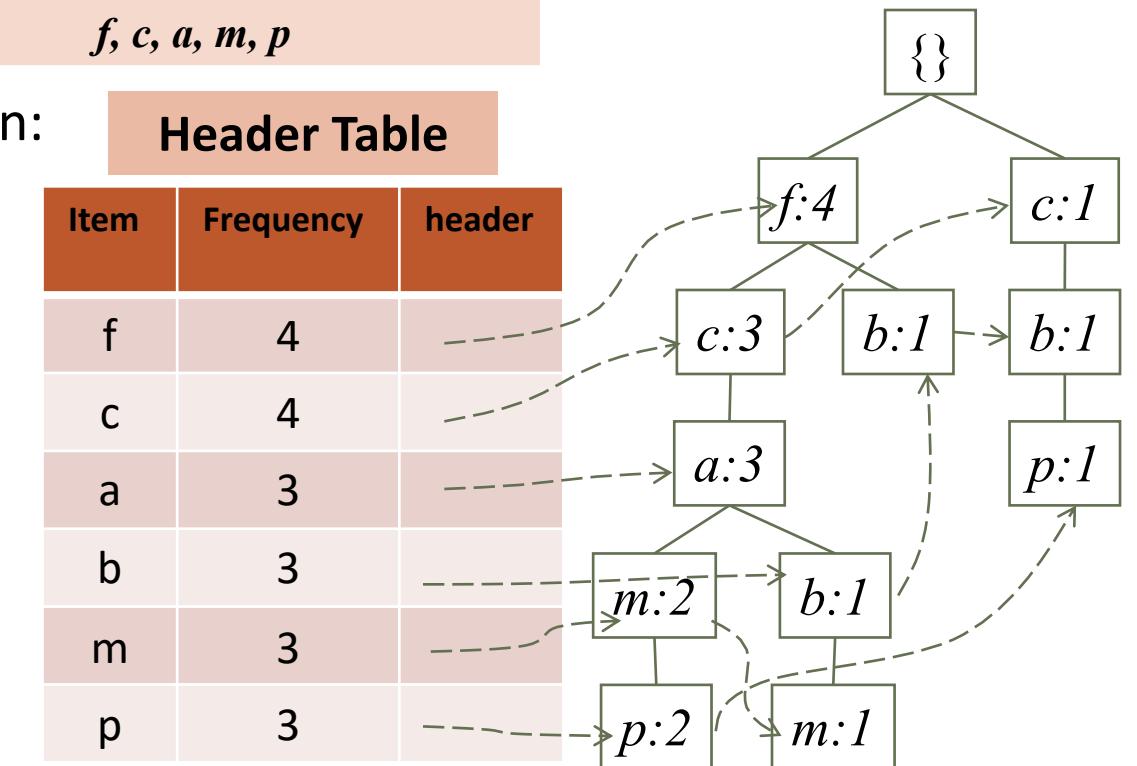
3. Scan DB again, construct FP-tree

The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated

Header Table

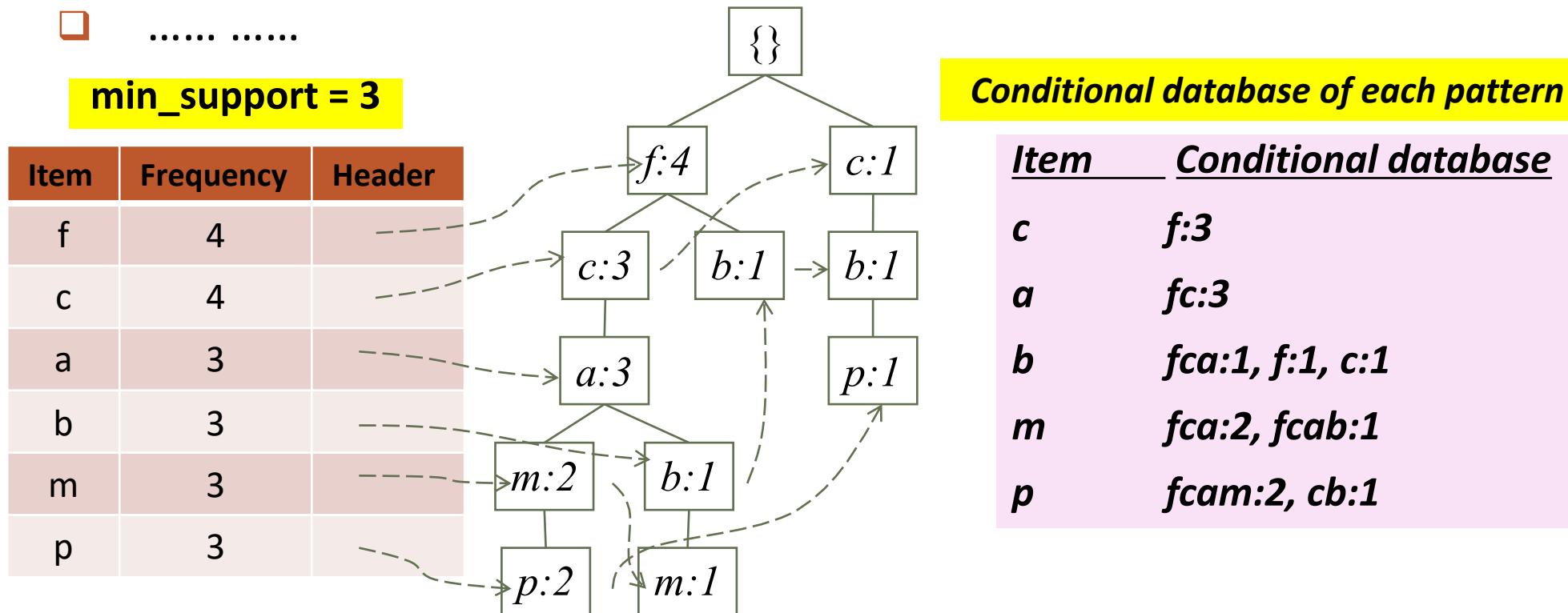
Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

After inserting all the frequent itemlists



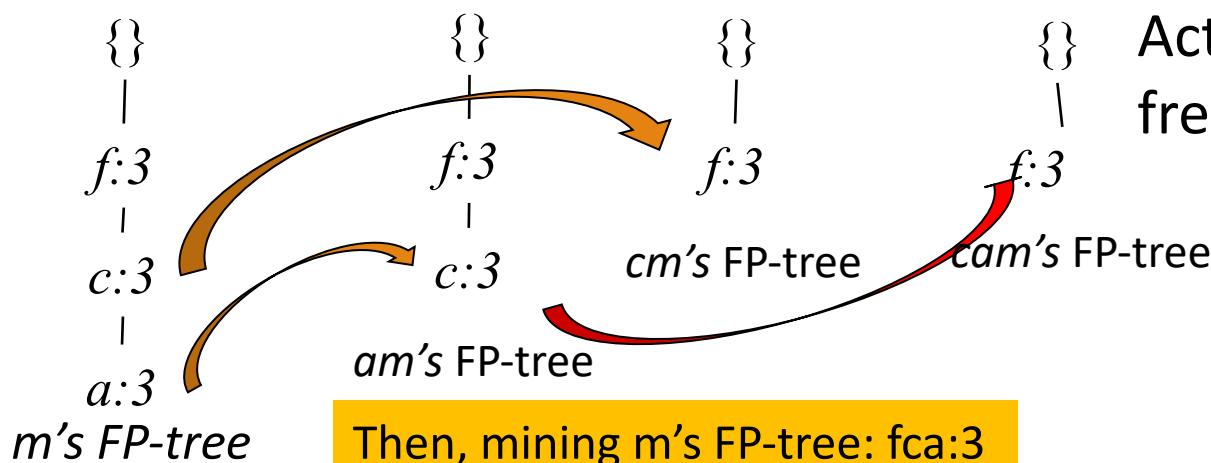
# Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - Patterns containing  $p$ :  $p$ 's conditional database:  $fcam:2, cb:1$
  - $p$ 's conditional database (i.e., the database under the condition that  $p$  exists):
    - transformed prefix paths* of item  $p$
  - Patterns having  $m$  but no  $p$ :  $m$ 's conditional database:  $fca:2, fcab:1$
  - ..... .....



# Mine Each Conditional Database Recursively

min_support = 3	
Conditional Data Bases	
<u>item</u>	<u>cond. data base</u>
c	f:3
a	fc:3
b	fca:1, f:1, c:1
m	fca:2, fcab:1
p	fcam:2, cb:1



- For each conditional database

- Mine single-item patterns
- Construct its FP-tree & mine it

*p*'s conditional DB: **fcam:2, cb:1**  $\rightarrow$  **c: 3**

*m*'s conditional DB: **fca:2, fcab:1**  $\rightarrow$  **fca: 3**

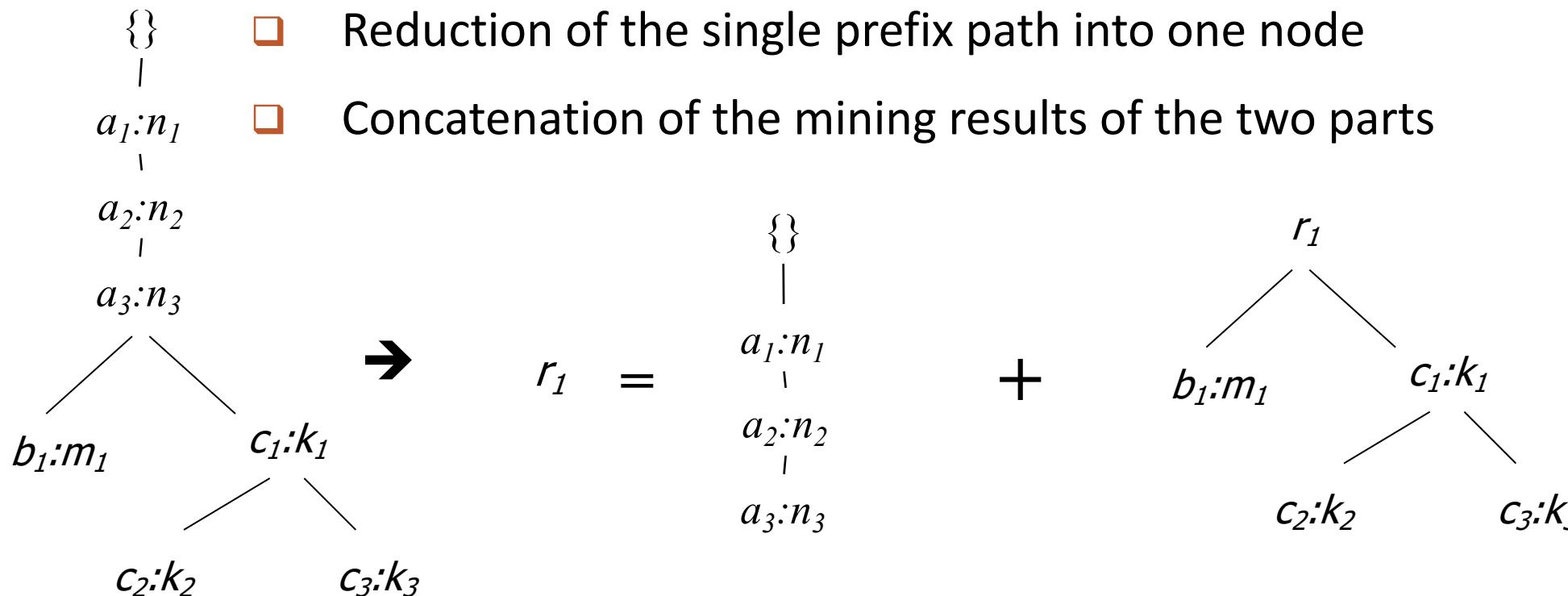
*b*'s conditional DB: **fca:1, f:1, c:1**  $\rightarrow$   $\emptyset$

Actually, for single branch FP-tree, all the frequent patterns can be generated in one shot

**m: 3**  
**fm: 3, cm: 3, am: 3**  
**fcm: 3, fam:3, cam: 3**  
**fcam: 3**

# A Special Case: Single Prefix Path in FP-tree

- ❑ Suppose a (conditional) FP-tree  $T$  has a shared single prefix-path  $P$
- ❑ Mining can be decomposed into two parts



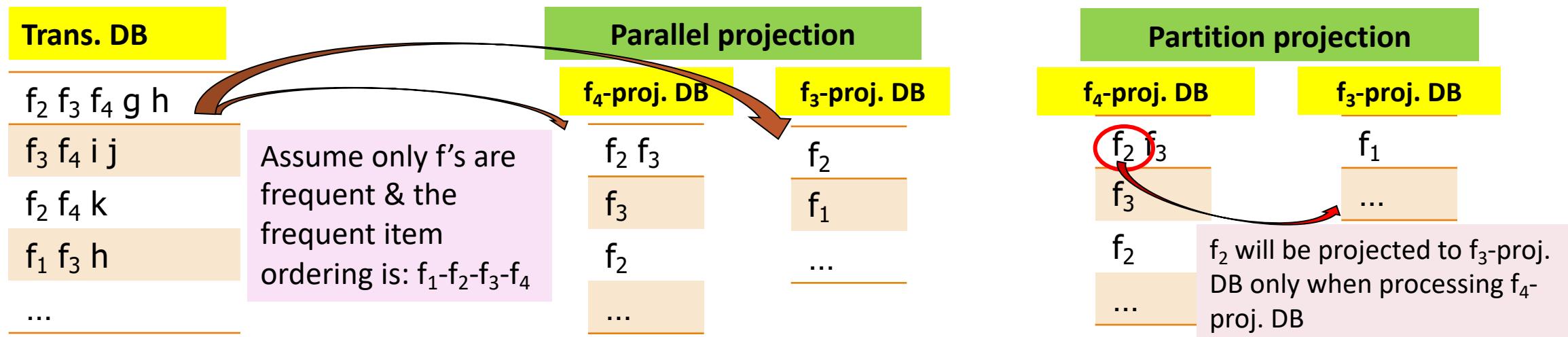
# FPGrowth: Mining Frequent Patterns by Pattern Growth

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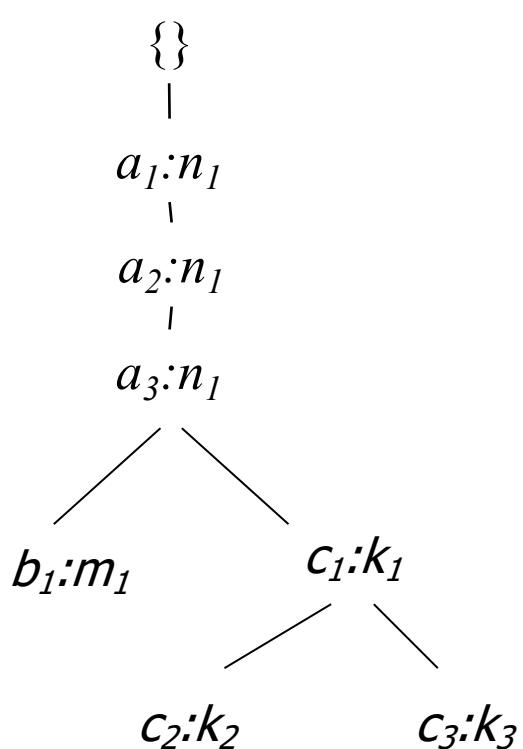
- Essence of frequent pattern growth (FPGrowth) methodology
  - Find frequent single items and partition the database based on each such single item pattern
  - Recursively grow frequent patterns by doing the above for each *partitioned database* (also called the pattern's *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

# Scaling FP-growth by Item-Based Data Projection

- ❑ What if FP-tree cannot fit in memory?—Do not construct FP-tree
  - ❑ “Project” the database based on frequent single items
  - ❑ Construct & mine FP-tree for each projected DB
- ❑ Parallel projection vs. partition projection
  - ❑ Parallel projection: Project the DB on each frequent item
    - ❑ Space costly, all partitions can be processed in parallel
  - ❑ Partition projection: Partition the DB in order
    - ❑ Passing the unprocessed parts to subsequent partitions



# CLOSET+: Mining Closed Itemsets by Pattern-Growth



- ❑ Efficient, *direct* mining of closed itemsets
- ❑ Intuition:
  - ❑ If an FP-tree contains a single branch as shown left
  - ❑ “ $a_1, a_2, a_3$ ” should be merged
- ❑ Itemset merging: If Y appears in every occurrence of X, then Y is merged with X
  - ❑  $d$ -proj. db: {acef, acf}  $\rightarrow$   $acfd$ -proj. db: {e}
  - ❑ Final closed itemset:  $acfd:2$
- ❑ There are many other tricks developed
  - ❑ For details, see J. Wang, et al., “CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets”, KDD'03

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2

a:3, c:3, d:2, e:3, f:3

F-List: a-c-e-f-d

# Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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- ❑ Basic Concepts
- ❑ Efficient Pattern Mining Methods
- ❑ Pattern Evaluation
- ❑ Summary



# Pattern Evaluation

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- Limitation of the Support-Confidence Framework
- Interestingness Measures: Lift and  $\chi^2$
- Null-Invariant Measures
- Comparison of Interestingness Measures

# How to Judge if a Rule/Pattern Is Interesting?

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- ❑ Pattern-mining will generate a large set of patterns/rules
  - ❑ Not all the generated patterns/rules are interesting
- ❑ Interestingness measures: Objective vs. subjective
  - ❑ Objective interestingness measures
    - ❑ Support, confidence, correlation, ...
  - ❑ Subjective interestingness measures:
    - ❑ Different users may judge interestingness differently
    - ❑ Let a user specify
      - ❑ Query-based: Relevant to a user's particular request
    - ❑ Judge against one's knowledge-base
      - ❑ unexpected, freshness, timeliness

# Limitation of the Support-Confidence Framework

- Are  $s$  and  $c$  interesting in association rules: “ $A \Rightarrow B$ ” [ $s, c$ ]?
- Example: Suppose one school may have the following statistics on # of students who may play basketball and/or eat cereal:

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

2-way contingency table

- Association rule mining may generate the following:
  - $play\text{-}basketball \Rightarrow eat\text{-}cereal$  [40%, 66.7%] (higher  $s$  &  $c$ )
  - But this strong association rule is misleading: The overall % of students eating cereal is  $75\% > 66.7\%$ , a more telling rule:
    - $\neg play\text{-}basketball \Rightarrow eat\text{-}cereal$  [35%, 87.5%] (high  $s$  &  $c$ )

$$\frac{350}{1000}$$

$\Rightarrow$  rule ~~not~~  $\neg$

# Interestingness Measure: Lift

- Measure of dependent/correlated events: **lift**

$$lift(B, C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

- Lift(B, C) may tell how B and C are correlated

← ពេក្តែ ធម្មតារីនិត្យតាមសំណង់រួច

- Lift(B, C) = 1: B and C are independent

← មិនមានសំណង់ទេនូវ

- > 1: positively correlated

← មិនមានសំណង់ទេនូវ

- < 1: negatively correlated

- For our example,

$$lift(B, C) = \frac{400/1000}{600/1000 \times 750/1000} = 0.89$$

$$lift(B, \neg C) = \frac{200/1000}{600/1000 \times 250/1000} = 1.33$$

- Thus, B and C are negatively correlated since lift(B, C) < 1;
- B and  $\neg C$  are positively correlated since lift(B,  $\neg C$ ) > 1

Lift is more telling than s & c

	B	$\neg B$	$\Sigma_{\text{row}}$
C	400	350	750
$\neg C$	200	50	250
$\Sigma_{\text{col.}}$	600	400	1000

# Interestingness Measure: $\chi^2$

- Another measure to test correlated events:  $\chi^2$

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- For the table on the right,

$$\chi^2 = \frac{(400 - 450)^2}{450} + \frac{(350 - 300)^2}{300} + \frac{(200 - 150)^2}{150} + \frac{(50 - 100)^2}{100} = 55.56$$

	B	$\neg B$	$\Sigma_{row}$
C	400 (450)	350 (300)	750
$\neg C$	200 (150)	50 (100)	250
$\Sigma_{col}$	600	400	1000

Expected value

Observed value

- By consulting a table of critical values of the  $\chi^2$  distribution, one can conclude that the chance for B and C to be independent is very low (< 0.01)
- $\chi^2$ -test shows B and C are negatively correlated since the expected value is 450 but the observed is only 400
- Thus,  $\chi^2$  is also more telling than the support-confidence framework

# Lift and $\chi^2$ : Are They Always Good Measures?

- ❑ Null transactions: Transactions that contain neither B nor C
- ❑ Let's examine the new dataset D
  - ❑ BC (100) is much rarer than B¬C (1000) and ¬BC (1000), but there are many ¬B¬C (100000)
  - ❑ Unlikely B & C will happen together!
  - ❑ But, Lift(B, C) = 8.44 >> 1 (Lift shows B and C are strongly positively correlated!)
  - ❑  $\chi^2 = 670$ : Observed(BC) >> expected value (11.85)
  - ❑ *Too many null transactions may “spoil the soup”!*



	B	$\neg B$	$\Sigma_{\text{row}}$
C	100	1000	1100
$\neg C$	1000	100000	101000
$\Sigma_{\text{col.}}$	1100	101000	102100

null transactions

Contingency table with expected values added

	B	$\neg B$	$\Sigma_{\text{row}}$
C	100 (11.85)	1000	1100
$\neg C$	1000 (988.15)	100000	101000
$\Sigma_{\text{col.}}$	1100	101000	102100

# Interestingness Measures & Null-Invariance

- **Null invariance**: Value does not change with the # of null-transactions
- A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant?
$\chi^2(A, B)$	$\sum_{i,j} \frac{(e(a_i, b_j) - o(a_i, b_j))^2}{e(a_i, b_j)}$	$[0, \infty]$	No
$Lift(A, B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$Allconf(A, B)$	$\frac{s(A \cup B)}{\max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A, B)$	$\frac{s(A \cup B)}{s(A) + s(B) - s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A, B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A, B)$	$\frac{1}{2} \left( \frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)} \right)$	$[0, 1]$	Yes
$MaxConf(A, B)$	$\max\left\{\frac{s(A \cup B)}{s(A)}, \frac{s(A \cup B)}{s(B)}\right\}$	$[0, 1]$	Yes

$\chi^2$  and lift are not null-invariant

Jaccard, cosine, AllConf, MaxConf, and Kulczynski are null-invariant measures

# Null Invariance: An Important Property

- Why is null invariance crucial for the analysis of massive transaction data?
- Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	<i>milk</i>	$\neg\text{milk}$	$\Sigma_{\text{row}}$
<i>coffee</i>	<i>mc</i>	$\neg\text{mc}$	<i>c</i>
$\neg\text{coffee}$	$m\neg\text{c}$	$\neg m\neg\text{c}$	$\neg\text{c}$
$\Sigma_{\text{col}}$	<i>m</i>	$\neg m$	$\Sigma$

- Lift and  $\chi^2$  are not null-invariant: not good to evaluate data that contain too many or too few null transactions!
- Many measures are not null-invariant!

Null-transactions  
w.r.t. m and c

Data set	<i>mc</i>	$\neg\text{mc}$	$m\neg\text{c}$	$\neg m\text{c}$	$\chi^2$	<i>Lift</i>
$D_1$	10,000	1,000	1,000	100,000	90557	9.26
$D_2$	10,000	1,000	1,000	100	0	1
$D_3$	100	1,000	1,000	100,000	670	8.44
$D_4$	1,000	1,000	1,000	100,000	24740	25.75
$D_5$	1,000	100	10,000	100,000	8173	9.18
$D_6$	1,000	10	100,000	100,000	965	1.97

# Comparison of Null-Invariant Measures

- ❑ Not all null-invariant measures are created equal
- ❑ Which one is better?
  - ❑  $D_4 - D_6$  differentiate the null-invariant measures
  - ❑ Kulc (Kulczynski 1927) holds firm and is in balance of both directional implications

2-variable contingency table

	<i>milk</i>	$\neg\text{milk}$	$\Sigma_{\text{row}}$
<i>coffee</i>	<i>mc</i>	$\neg\text{mc}$	<i>c</i>
$\neg\text{coffee}$	<i>m</i> $\neg\text{c}$	$\neg\text{m}$ $\neg\text{c}$	$\neg\text{c}$
$\Sigma_{\text{col}}$	<i>m</i>	$\neg\text{m}$	$\Sigma$

All 5 are null-invariant

Data set	<i>mc</i>	$\neg\text{mc}$	<i>m</i> $\neg\text{c}$	$\neg\text{m}$ $\text{c}$	<i>AllConf</i>	Jaccard	<i>Cosine</i>	<i>Kulc</i>	<i>MaxConf</i>
$D_1$	10,000	1,000	1,000	100,000	0.91	0.83	0.91	0.91	0.91
$D_2$	10,000	1,000	1,000	100	0.91	0.83	0.91	0.91	0.91
$D_3$	100	1,000	1,000	100,000	0.09	0.05	0.09	0.09	0.09
$D_4$	1,000	1,000	1,000	100,000	0.5	0.33	0.5	0.5	0.5
$D_5$	1,000	100	10,000	100,000	0.09	0.09	0.29	0.5	0.91
$D_6$	1,000	10	100,000	100,000	0.01	0.01	0.10	0.5	0.99

Subtle: They disagree on those cases

# Analysis of DBLP Coauthor Relationships

- DBLP: Computer science research publication bibliographic database
  - > 3.8 million entries on authors, paper, venue, year, and other information

ID	Author A	Author B	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	Cosine	Kulc
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilbert Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related?
- Use Kulc to find Advisor-advisee, close collaborators

# Imbalance Ratio with Kulczynski Measure

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications:

$$IR(A, B) = \frac{|s(A) - s(B)|}{s(A) + s(B) - s(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets  $D_4$  through  $D_6$ 
  - $D_4$  is neutral & balanced;  $D_5$  is neutral but imbalanced
  - $D_6$  is neutral but very imbalanced

Data set	<i>mc</i>	$\neg mc$	<i>m</i> $\neg c$	$\neg m$ <i>c</i>	Jaccard	<i>Cosine</i>	<i>Kulc</i>	IR
$D_1$	10,000	1,000	1,000	100,000	0.83	0.91	0.91	0
$D_2$	10,000	1,000	1,000	100	0.83	0.91	0.91	0
$D_3$	100	1,000	1,000	100,000	0.05	0.09	0.09	0
$D_4$	1,000	1,000	1,000	100,000	0.33	0.5	0.5	0
$D_5$	1,000	100	10,000	100,000	0.09	0.29	0.5	0.89
$D_6$	1,000	10	100,000	100,000	0.01	0.10	0.5	0.99

# What Measures to Choose for Effective Pattern Evaluation?

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- ❑ Null value cases are predominant in many large datasets
  - ❑ Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers; .....
- ❑ *Null-invariance* is an important property
- ❑ Lift,  $\chi^2$  and cosine are good measures if null transactions are not predominant
  - ❑ Otherwise, *Kulczynski + Imbalance Ratio* should be used to judge the interestingness of a pattern
- ❑ Exercise: Mining research collaborations from research bibliographic data
  - ❑ Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
  - ❑ Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
  - ❑ Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

# Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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- ❑ Basic Concepts
- ❑ Efficient Pattern Mining Methods
- ❑ Pattern Evaluation
- ❑ Summary



# Summary

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- ❑ Basic Concepts
  - ❑ What Is Pattern Discovery? Why Is It Important?
  - ❑ Basic Concepts: Frequent Patterns and Association Rules
  - ❑ Compressed Representation: Closed Patterns and Max-Patterns
- ❑ Efficient Pattern Mining Methods
  - ❑ The Downward Closure Property of Frequent Patterns
  - ❑ The Apriori Algorithm
  - ❑ Extensions or Improvements of Apriori
  - ❑ Mining Frequent Patterns by Exploring Vertical Data Format
  - ❑ FP-Growth: A Frequent Pattern-Growth Approach
  - ❑ Mining Closed Patterns
- ❑ Pattern Evaluation
  - ❑ Interestingness Measures in Pattern Mining
  - ❑ Interestingness Measures: Lift and  $\chi^2$
  - ❑ Null-Invariant Measures
  - ❑ Comparison of Interestingness Measures

# Recommended Readings (Basic Concepts)

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- R. Agrawal, T. Imielinski, and A. Swami, “Mining association rules between sets of items in large databases”, in Proc. of SIGMOD'93
- R. J. Bayardo, “Efficiently mining long patterns from databases”, in Proc. of SIGMOD'98
- N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, “Discovering frequent closed itemsets for association rules”, in Proc. of ICDT'99
- J. Han, H. Cheng, D. Xin, and X. Yan, “Frequent Pattern Mining: Current Status and Future Directions”, Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

# Recommended Readings (Efficient Pattern Mining Methods)

---

- R. Agrawal and R. Srikant, “Fast algorithms for mining association rules”, VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, “An efficient algorithm for mining association rules in large databases”, VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, “An effective hash-based algorithm for mining association rules”, SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, “Integrating association rule mining with relational database systems: Alternatives and implications”, SIGMOD'98
- M. J. Zaki, S. Parthasarathy, M. Ogihsara, and W. Li, “Parallel algorithm for discovery of association rules”, Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, “Mining frequent patterns without candidate generation”, SIGMOD'00
- M. J. Zaki and Hsiao, “CHARM: An Efficient Algorithm for Closed Itemset Mining”, SDM'02
- J. Wang, J. Han, and J. Pei, “CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets”, KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, “Frequent Pattern Mining Algorithms: A Survey”, in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

# Recommended Readings (Pattern Evaluation)

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- C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
- T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371-397, 2010

