Development - Summary of the take home exam

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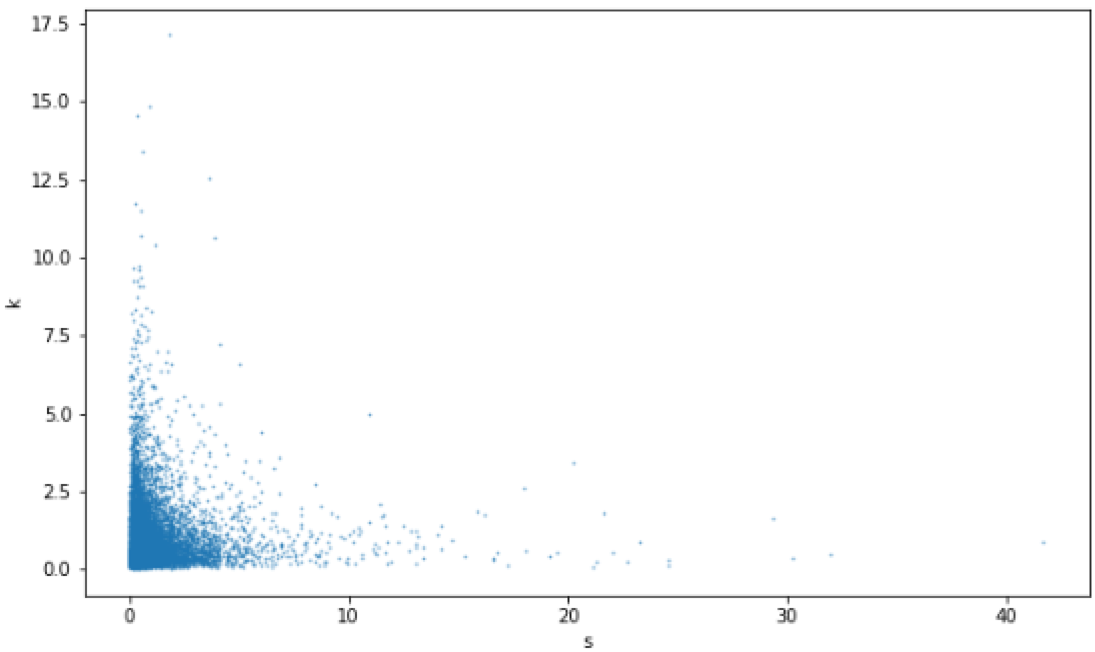

Figure 1: Q1.1. Scatter plot of s and k. Mean value of s and k is 1. Since ln(s) has higher variance, maximum value of s reaches over 40, comparing to k that reaches 17.5

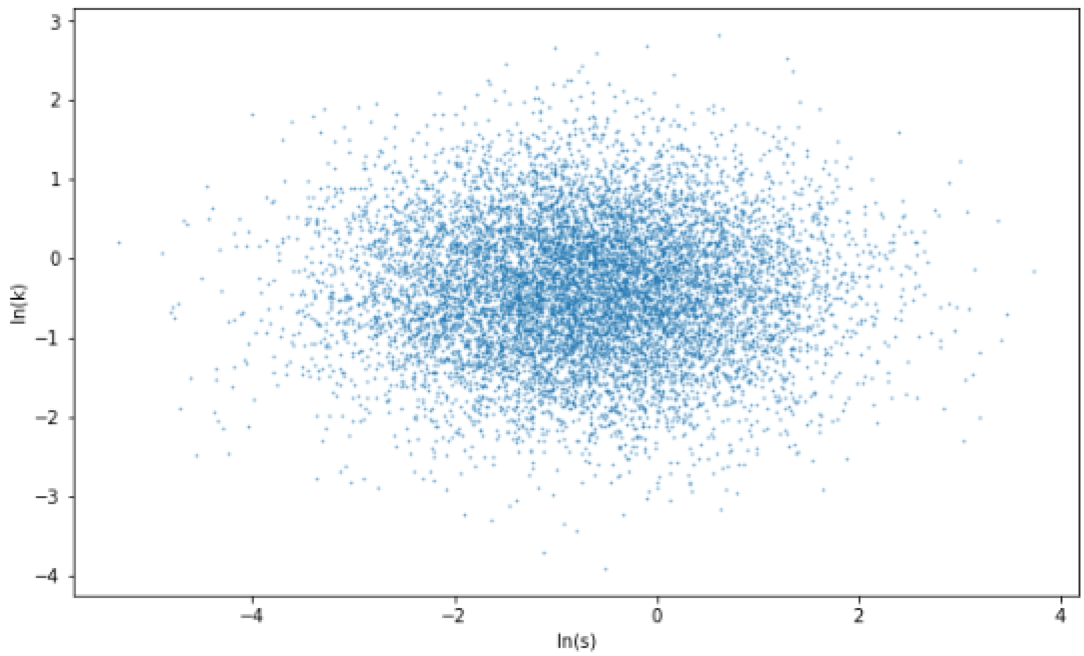

Figure 2: Q1.1. Scatter plot of ln(s) and ln(k). Correlation is zero and mean values are below zero such that mean of s and k is approximately 1.

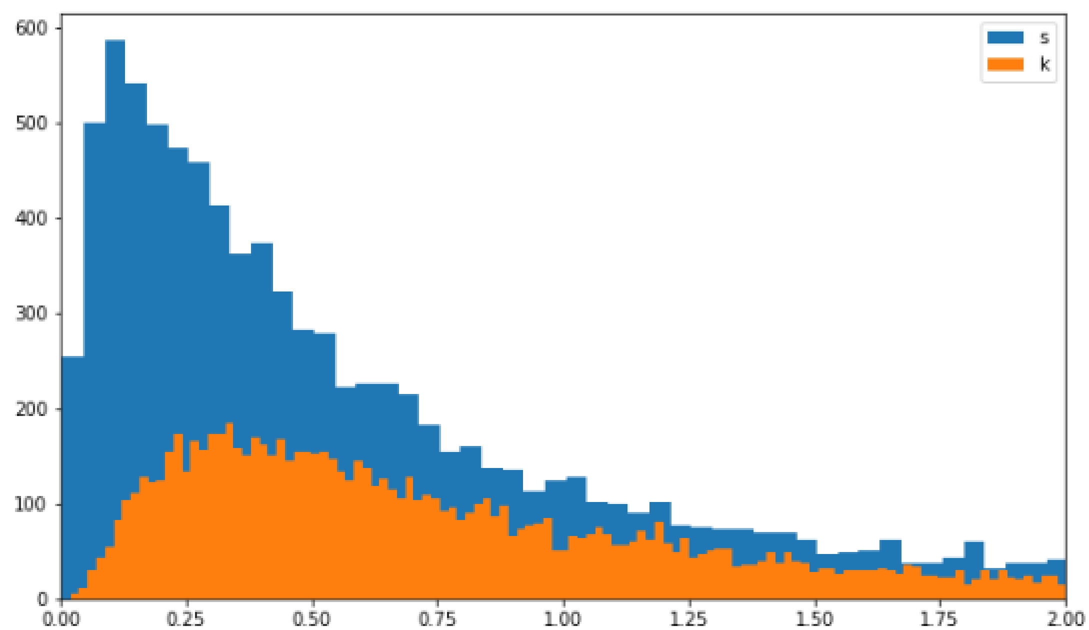

Figure 3: Q1.1. Histogram of s and k. We see that again s has higher variance, this bins and tail are wider.

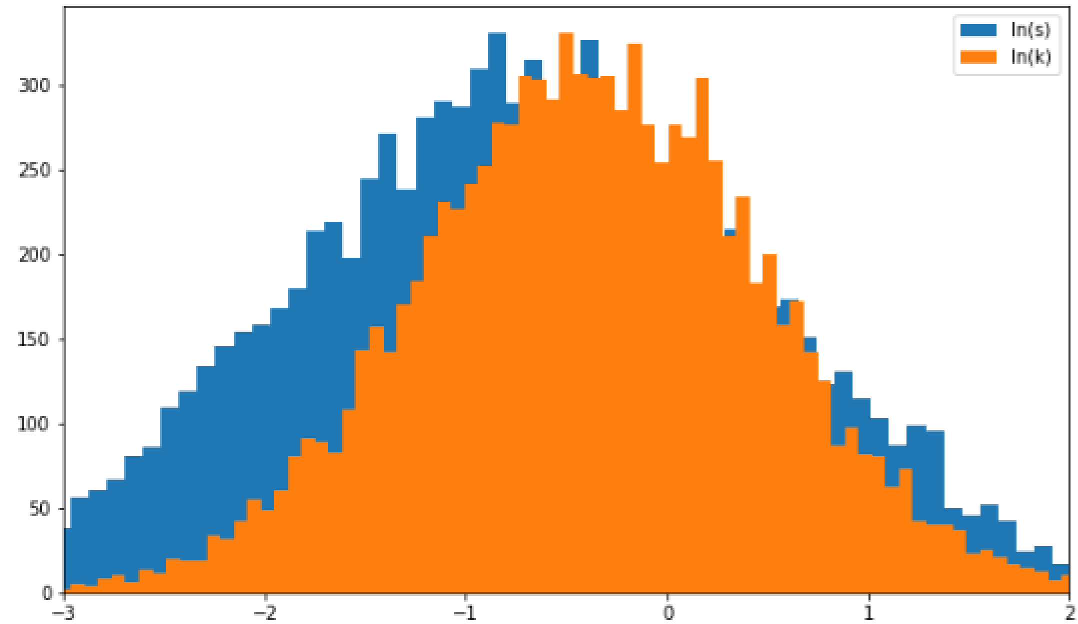

Figure 4: Q1.1. Histogram of ln(s) and ln(k).

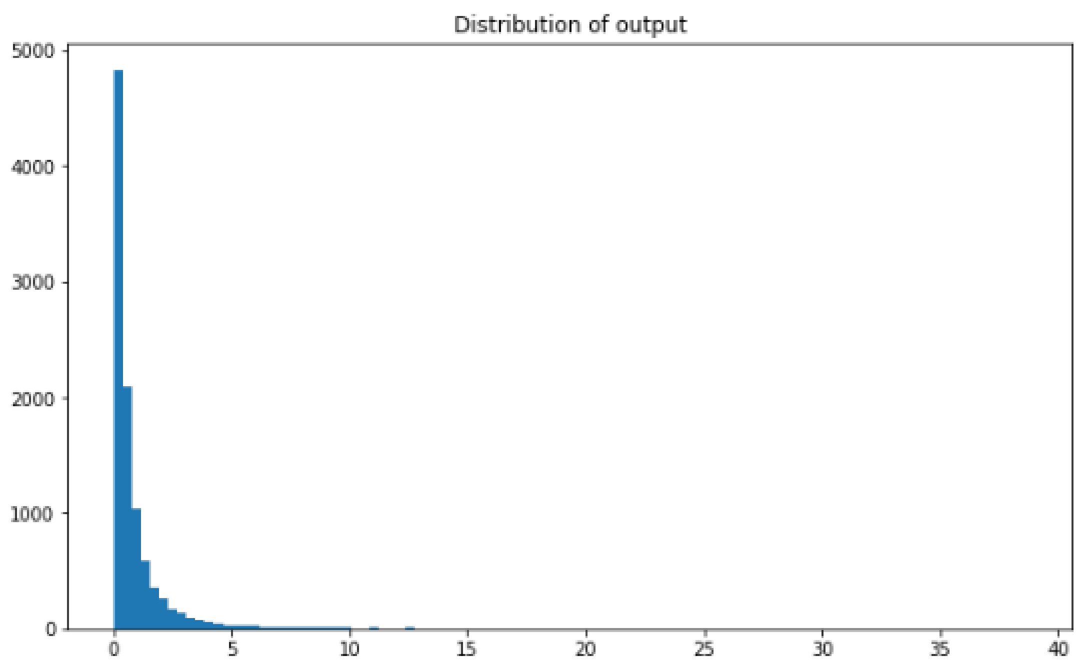

Figure 5: Q1.2. Distribution of original (true) output

Problem:

$$max_{k_i} \sum_i s_i k_i^{\gamma}$$

st.

$$K = \sum_i k_i$$

$$L = \sum_i s_i k_i^{\gamma} + \lambda(K - \sum_i k_i)$$

FOC:

$$[k_i] : s_i \gamma k_i^{\gamma-1} = \lambda$$
$$[k_j] : s_j \gamma k_j^{\gamma-1} = \lambda$$

which implies:

$$\frac{s_i}{s_j}(\frac{k_i}{k_j})^{\gamma-1} = 1$$

so

$$k_j = (\frac{s_i}{s_j})^{\frac{1}{(\gamma-1)}} k_i$$

Therefore,

$$k_i = K - \sum_{j \neg i} k_j$$
$$k_i((1 + \frac{s_i}{s_j})^{\frac{1}{(\gamma-1)}} + (\frac{s_z}{s_z})^{\frac{1}{(\gamma-1)}} + \dots) = K$$
$$k_i = \frac{K}{((1 + \frac{s_i}{s_j})^{\frac{1}{(\gamma-1)}} + (\frac{s_z}{s_z})^{\frac{1}{(\gamma-1)}} + \dots)}$$

Figure 6: Q1.3 Derivation of optimal capital

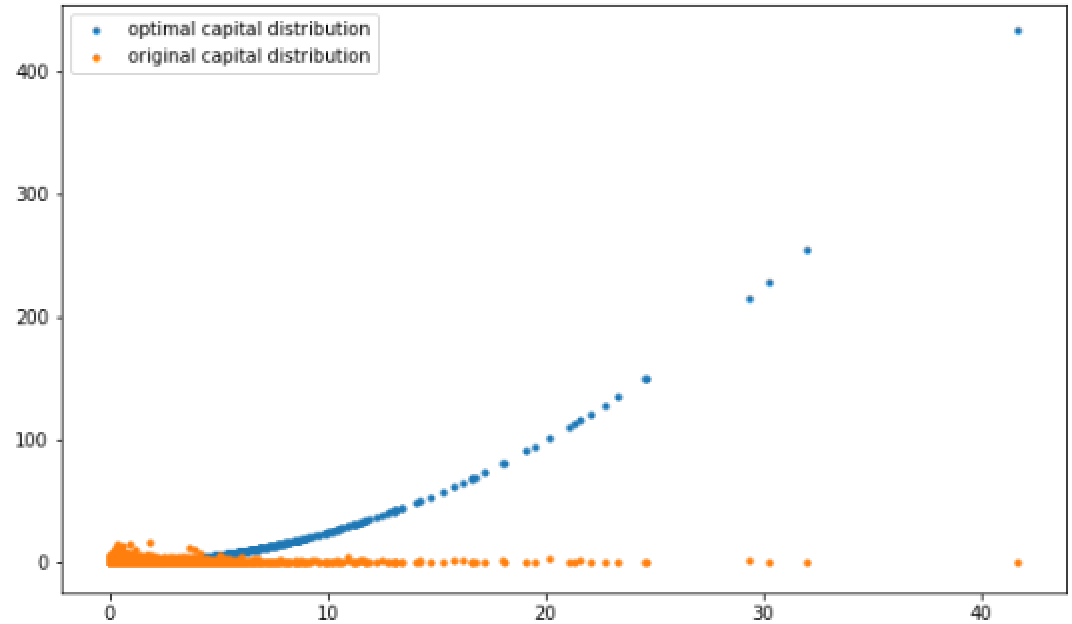

Figure 7: Q1.4 and 1.5. Comparison of optimal capital distribution and original capital distribution. Optimally, more productive agents obtained much more capital than if capital is drawn from normal distribution. **Variance of random capital is 4.2 and variance of optimal output is 1248.7.**
This makes up Output gain equal to 2.3

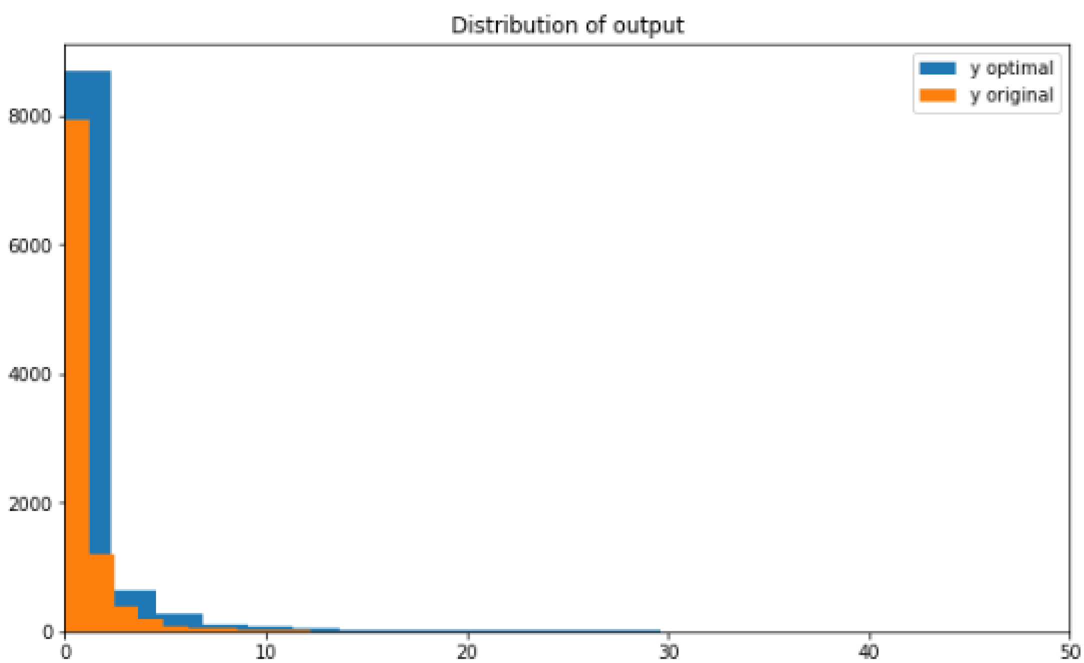

Figure 8: Q1.4 and 1.5. Distribution of output original and optimal. Optimal output is much more unequal

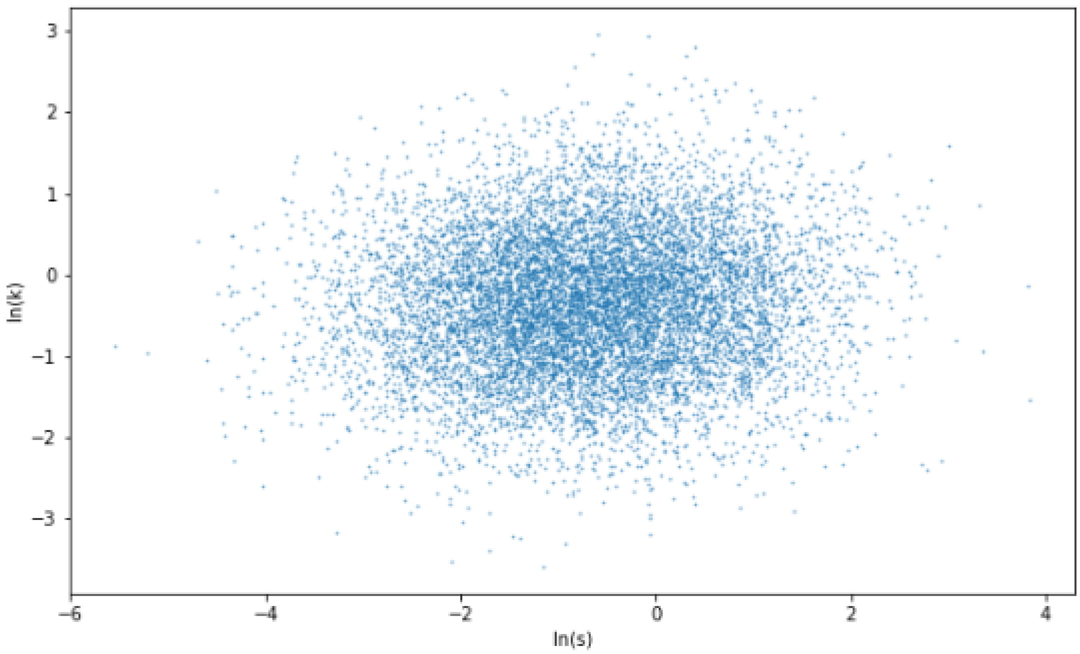

Figure 9: Q1.6. Scatter plot of ln(s) and ln(k) when correlation is 0.25. It is difficult to see the correlation on the plot, therefore I added the next plot to illustrate what kind of relation we would expect if correlation was higher

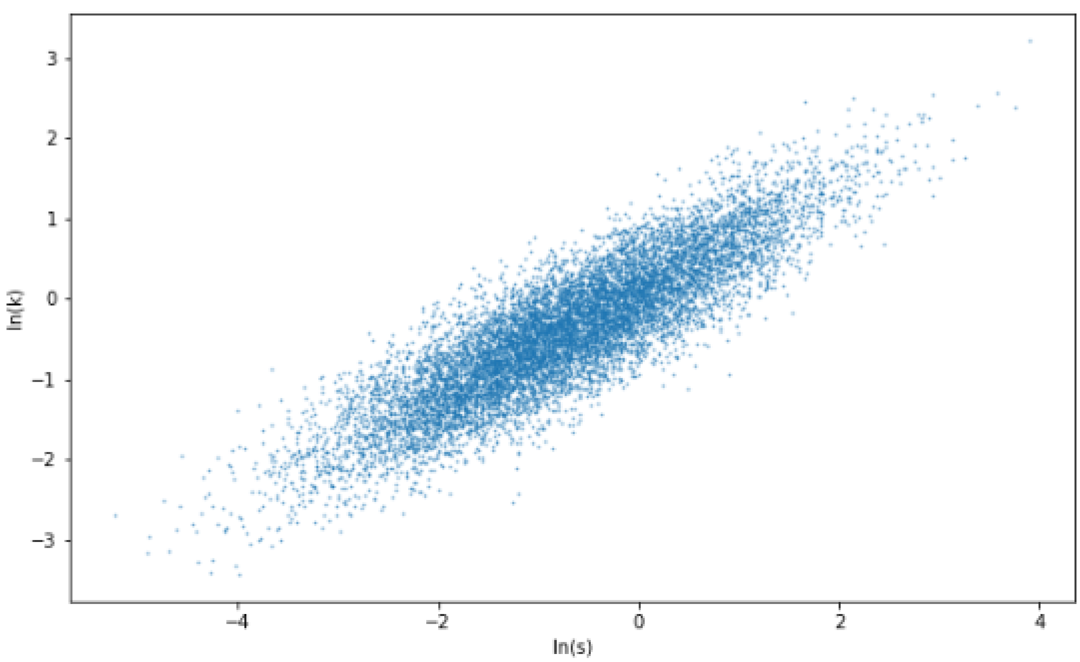

Figure 10: Q1.6. Scatter plot of ln(s) and ln(k) when correlation is 0.95. Example picture that illustrate high correlation of ln(s) and ln(k)

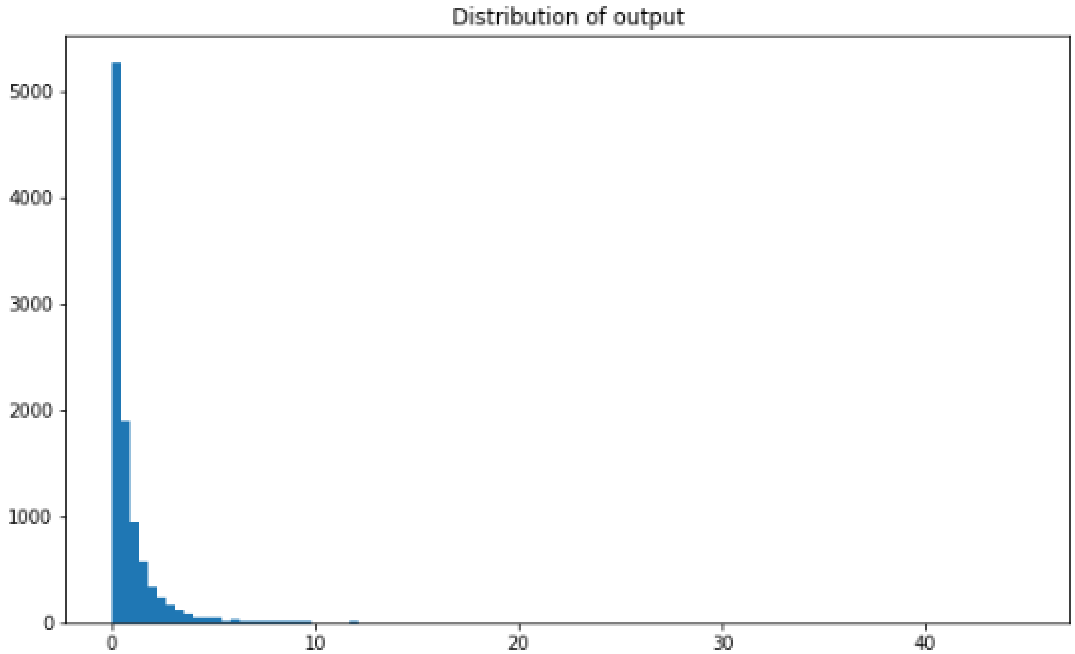

Figure 11: The variance in output is higher when gamma is 0.8 (comparing to gamma 0.5 form previous example)

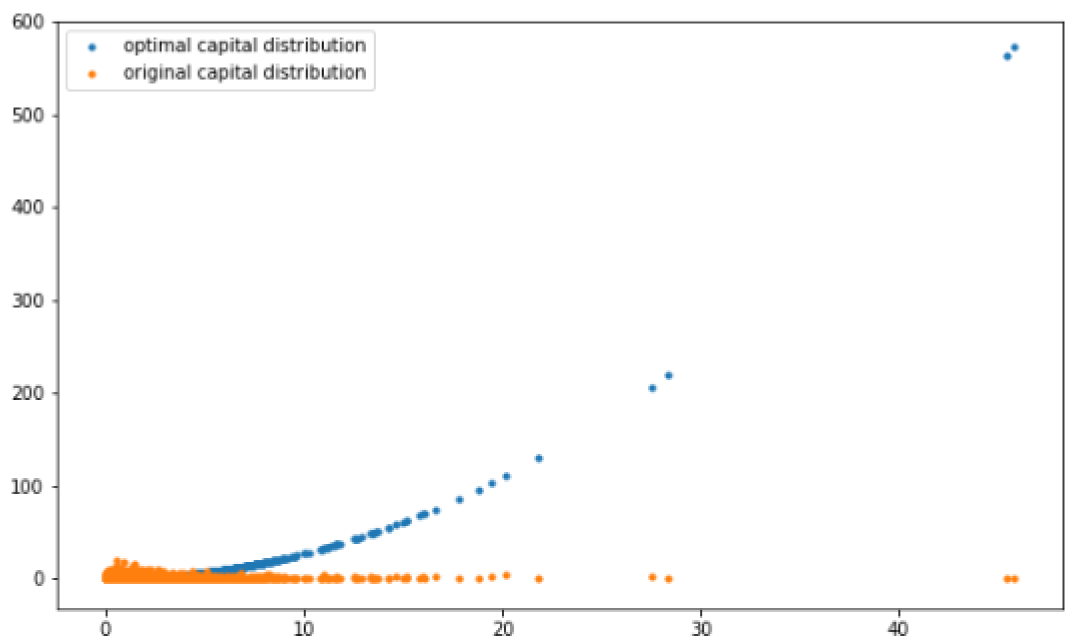

Figure 12: Q1.6 **Output gain is equal to 2.2**

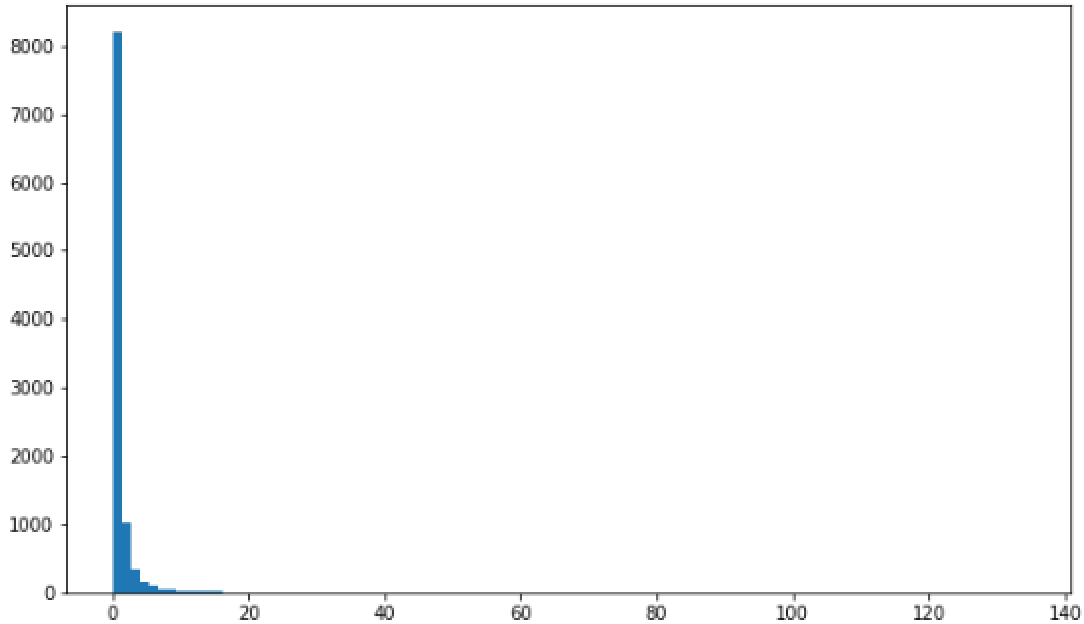

Figure 13: Q2 When gamma is 0.8 the ouptut per agent has higer variance

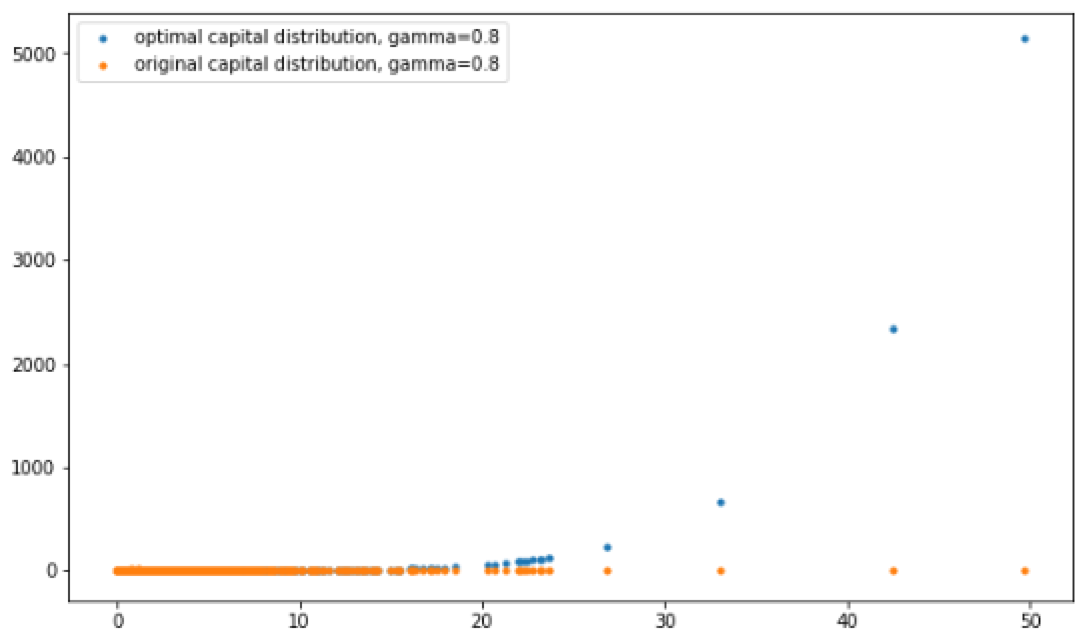

Figure 14: Q2 When gamma is 0.8 the ouptut per agent has higer variance

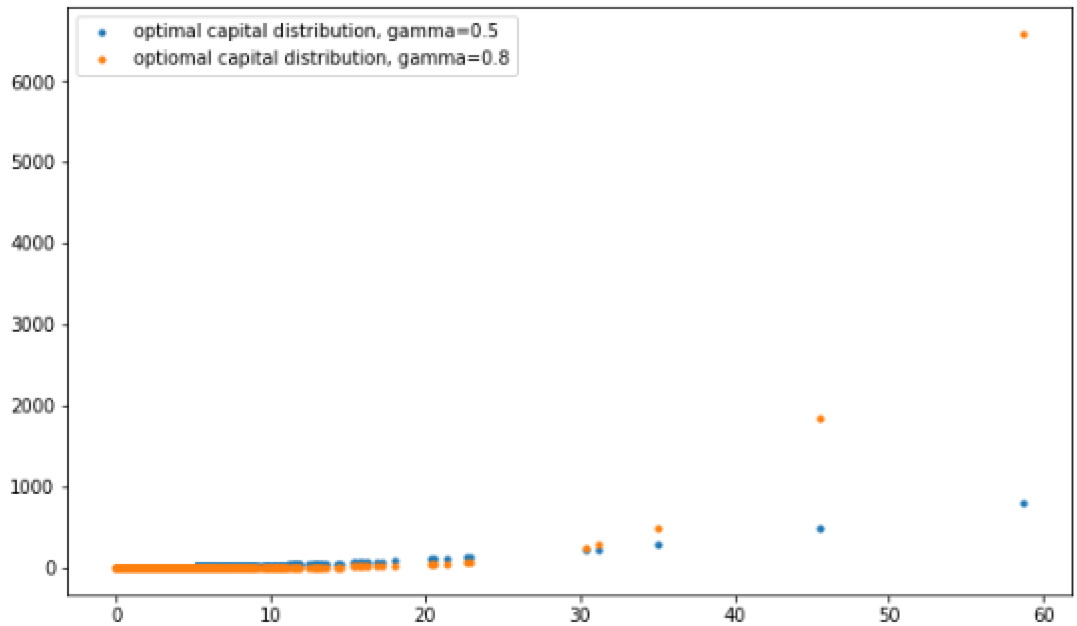

Figure 15: Q2 Comparison of optimal capital when gamma=0.5 and gamma=0.8. Weh gamma increases the more productive agents obtain optimally even more capital, so inequality increases. **However, ouptut gain is still 2.3**

Figure 16: QUESTION 3.
When we optimize the output on the village level, output gain equals 1.6 which is less than under optimization on the aggregate level.

Problem:

$$max_{k_i} \sum_i s_i(a_i, k_i) k_i^{\gamma}$$

st.

$$K = \sum_i k_i$$

if $\sigma = 1$ then CES becomes Cobb-Douglas, so problem can be reformulated as:

$$max_{k_i} \sum_i a_i^{\alpha} k_i^{1-\alpha+\gamma}$$

FOC:

$$[k_i] : (1-\alpha+\gamma) k_i^{\gamma-\alpha} a_i^{\alpha} = \lambda$$
$$[k_j] : (1-\alpha+\gamma) k_j^{\gamma-\alpha} a_j^{\alpha} = \lambda$$

which implies:

$$(\frac{a_i}{a_j})^{-\alpha} (\frac{k_i}{k_j})^{\gamma-\alpha} = 1$$

so

$$k_j = (\frac{a_j}{a_i})^{\frac{-\alpha}{(\gamma-\alpha)}} k_j$$

Therefore,

$$k_i = K - \sum_{j \neg i} k_j$$
$$k_i((1 + \frac{a_i}{a_j})^{\frac{-\alpha}{(\gamma-\alpha)}} + (\frac{a_z}{a_z})^{\frac{-\alpha}{(\gamma-\alpha)}} + \ldots) = K$$
$$k_i = \frac{K}{(1 + (\frac{a_i}{a_j})^{\frac{-\alpha}{(\gamma-\alpha)}} + (\frac{a_z}{a_z})^{\frac{-\alpha}{(\gamma-\alpha)}} + \ldots)}$$

Figure 17: Q4.

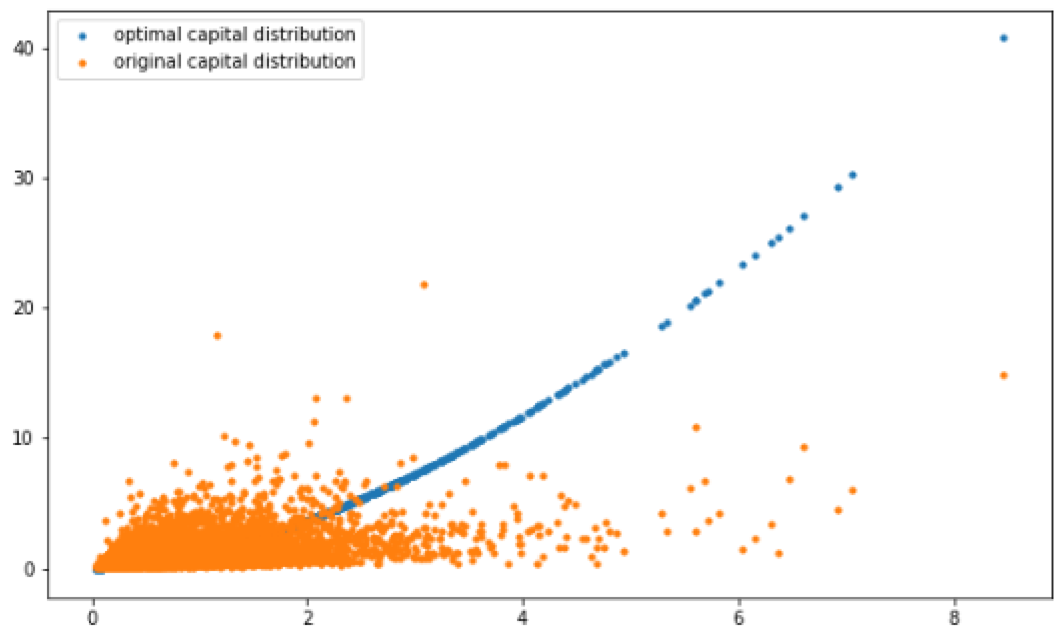

Figure 18: Q4. Optimal vs original capital distribution when sigma=1. Output gain accounts for 1.54