LET $X \sim p_X(x)$ AND $Y = g(X)$

THEN

$$p_Y(y) = \sum_k p_X(x_k(y)) \cdot \left| \frac{dx_k}{dy} \right|$$

WHERE $\{x_k\}$ IS THE PRE-IMAGE

IF $\frac{dg}{dx} \neq 0$ THEN

$$p_Y(y) = \sum_k p_X(g_k^{-1}(y)) \left(\left| \frac{dg}{dx} \right|_{x = x_k} \right)^{-1}$$

IF g IS MONOTONE (INCREASING OR DECREASING) THEN

$$p_Y(y) = p_X(g^{-1}(y)) \left(\left| \frac{dg}{dx} \right|_{x = g^{-1}(y)} \right)^{-1}$$

EX $K \sim \mathrm{UNIF}(1, 10)$

$$p_K(k) = \begin{cases} \frac{1}{9} & 1 \leq k \leq 10 \\ 0 & \text{ELSE} \end{cases}$$

$T = \frac{1}{K}$. WHAT IS $p_T(t)$?

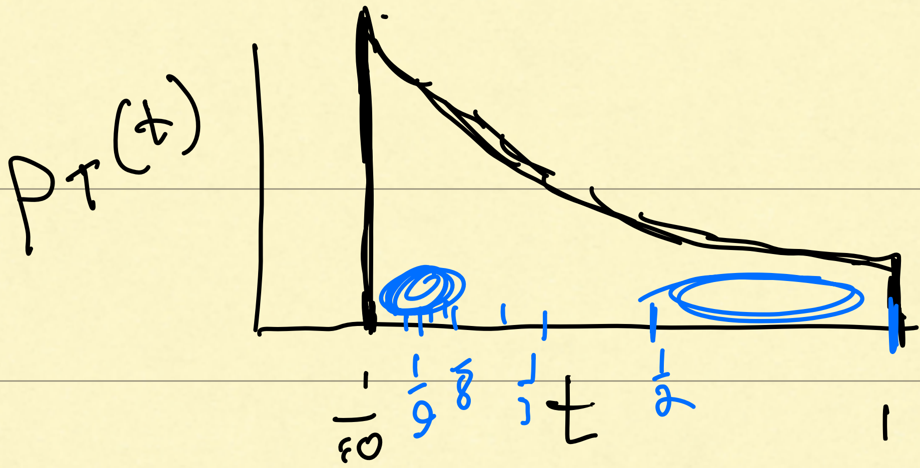
$g(k) = \frac{1}{k}$ $\qquad \frac{dg}{dk} = -\frac{1}{k^2}$

$g^{-1}(t) = \frac{1}{t}$

$$\Rightarrow p_T(t) = \begin{cases} \frac{1}{9} \left| \left(-\frac{1}{k^2} \right) \right|^{-1} & \frac{1}{10} \leq t \leq 1 \\ 0 & \text{ELSE} \end{cases}$$

$$= \begin{cases} \frac{1}{9} k^2 & \frac{1}{10} \leq t \leq 1 \\ 0 & \text{ELSE} \end{cases}$$

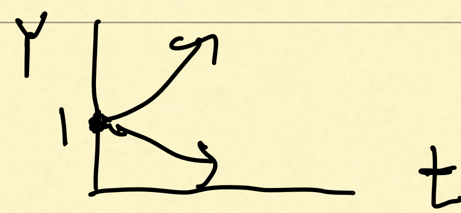
$$= \begin{cases} \frac{1}{9}\,\frac{1}{t^2} & \frac{1}{10} \le t \le 1 \\ 0 & \text{ELSE} \end{cases}$$

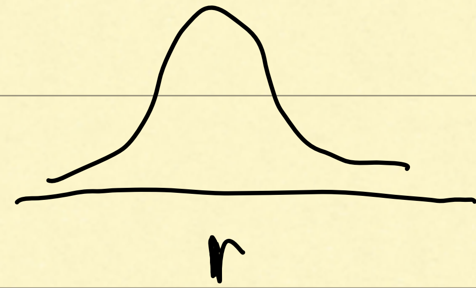


HETEROGENEITY

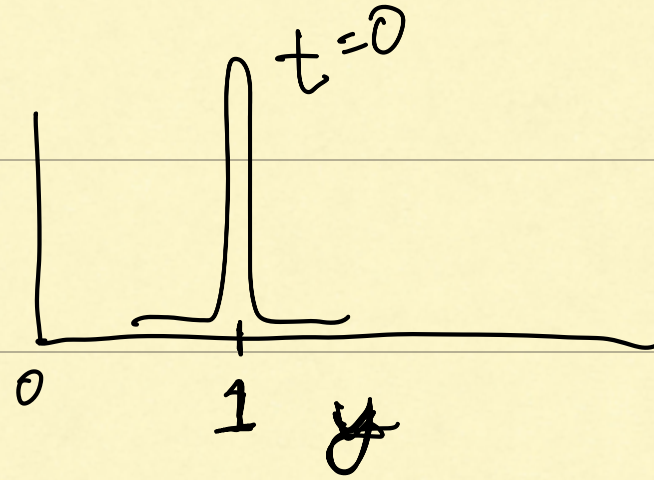
PS 6

$$\frac{dY}{dt} = (R-1)Y, \quad Y(0) = 1$$



$p_R(r) =$ 

$Y(t) \sim p_Y(y;t)$

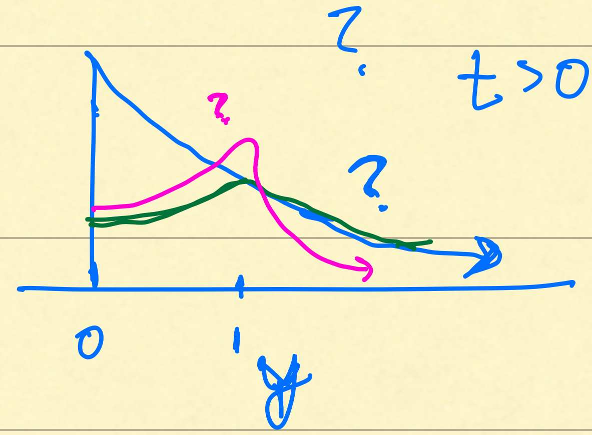
p_Y 

PROTOTYPIC	PS6
$X \sim p_X(x)$	$R \sim p_R(r)$
$Y = g(X)$	$Y = e^{(R-1)t}$
$Y \sim p_Y(y)$	Y

$g(r) = e^{(r-1)t}$

$\mathbb{P}(R > 1) = 0.5$

$\mathbb{P}(R > 1) = 0.5$



$\mathbb{P}_Y(y) = \dots$