THE ENTIRE LIKELIHOOD LANDSCAPE

EX $T \sim p_T(t) = \lambda e^{-\lambda t}$ $\quad t>0$

EXPONENTIAL

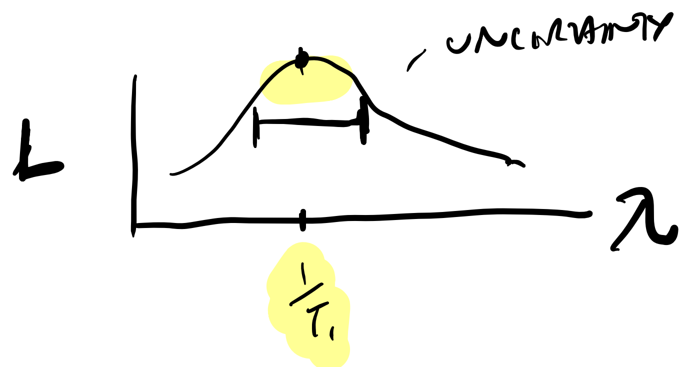
λ - RATE

$= \frac{1}{\tau} e^{-t/\tau}$

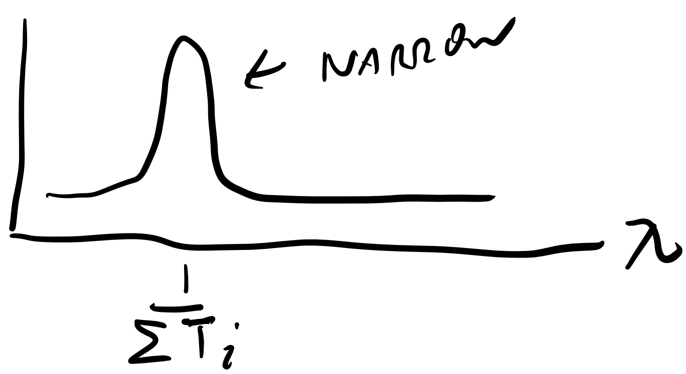
$\tau = \frac{1}{\lambda}$ - MEAN TIME

DATA T_1

$$L(\lambda) = \lambda e^{-\lambda T_1}$$ LIKELIHOOD



DATA $T_1 \dots T_N$

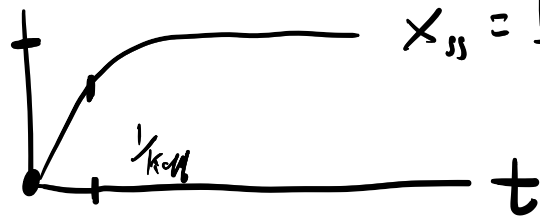


EX $$\frac{dX}{dt} = k_{on} - k_{off}X \qquad X(0)=0$$

$$Y = X + \epsilon$$

ϵ - RANDOM NORMAL WITH STANDARD DEVIATION σ

$\Rightarrow$ $X(t) =$

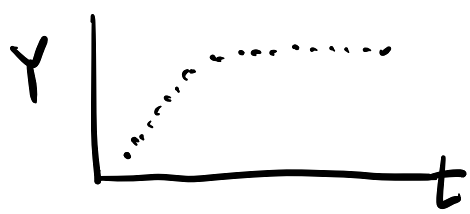


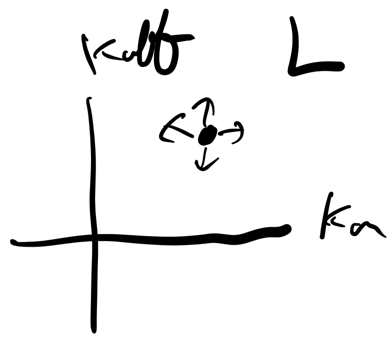
$X_{ss} = \frac{k_{on}}{k_{off}}$

$$X(t) = \frac{k_{on}}{k_{off}}\left(1 - e^{-k_{off}t}\right)$$

$$= X_{ss}\left(1 - e^{-t/\tau}\right)$$

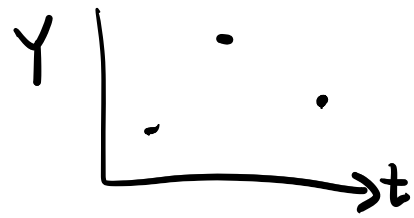
LOW σ, MANY DATA

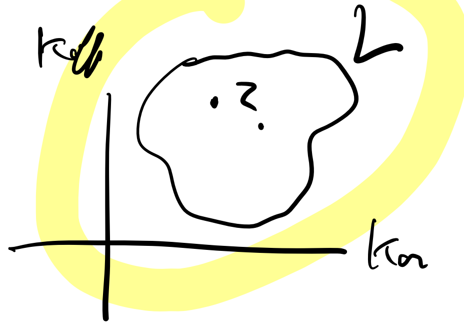




HIGH σ

LOW DATA





LATE DATA

ISSUES WITH LIKELIHOOD LANDSCAPE

- L - NOT A PROBABILITY DISTRIBUTION
  - NO MEAN, NO STDEV, NO CONFIDENCE INTERVALS
  - DOESN'T INTEGRATE TO 1

BAYESIAN STATISTICS

$$\underbrace{p(\underset{\text{PARAM}}{\theta} \mid \underset{\text{DATA}}{X})}_{\text{POSTERIOR}} = \frac{\overbrace{p(X \mid \theta)}^{=\text{LIKELIHOOD}}\ \underset{\text{PRIOR}}{p(\theta)}}{p(X)}$$

$$\int p(x)\,dx = 1$$

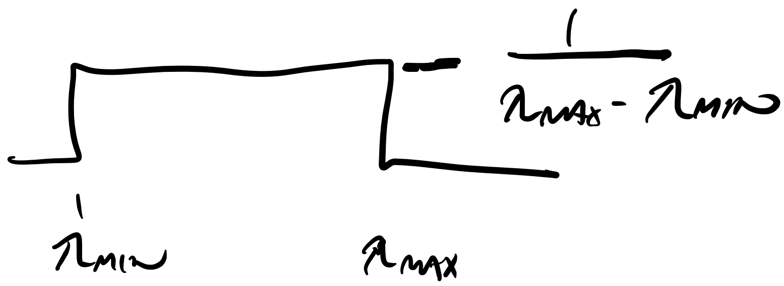
$$= \frac{L(\theta)\,P(\theta)}{P(x)}$$

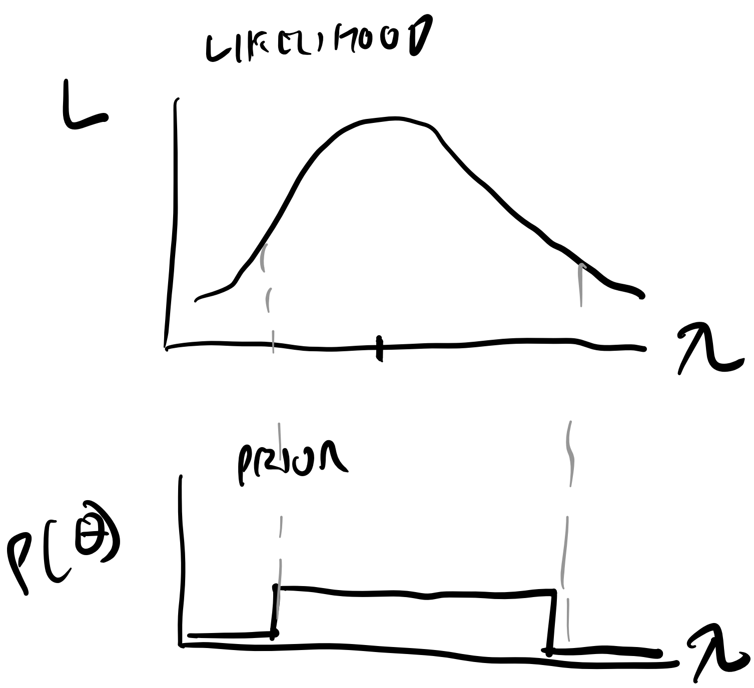
INSTEAD OF $L(\theta)$,

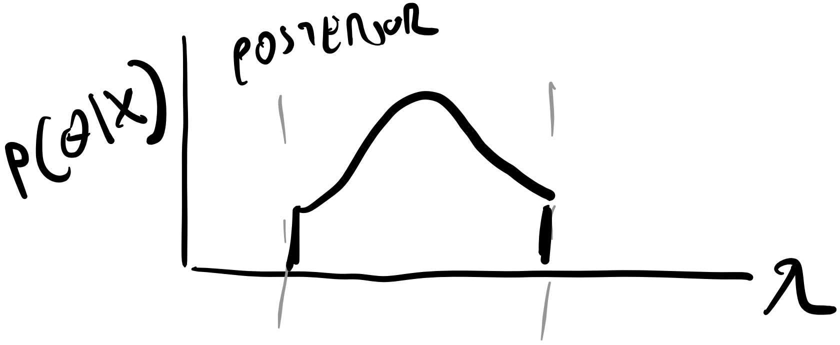
CONSIDER $P(\theta | X)$

EX $\quad T \sim p_T(t) = \lambda e^{-\lambda t} \qquad t > 0$

PRIOR: $\lambda \sim$ UNIFORM IN $(\lambda_{MIN}, \lambda_{MAX})$







NOTES

1) AS $\lambda_{MIN} \to 0$, $\lambda_{MAX} \to \infty$

$$P(\lambda | T) = \frac{\lambda e^{-\lambda T}}{1/T^2}$$

POSTERIOR IS WELL-DEFINED EVEN THOUGH PRIOR (UNIF(0, ∞)) DOES NOT EXIST :

⇒ IRREGULAR PRIOR

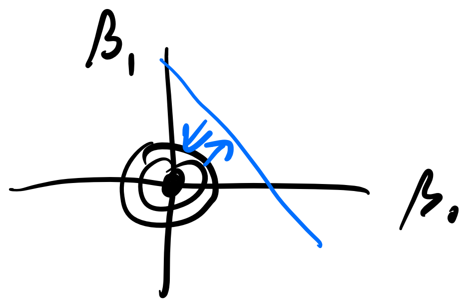
2a) NON-UNIFORM PRIOR

MY DOG WEIGHS 70 lbs

VET SCALE 50 lbs

2b)

RIDGE : PRIOR $p(\beta_i) = e^{-\frac{\sum \beta_i^2}{\lambda}}$



LASSO : PRIOR $p(\beta_i) = e^{-\frac{\sum |\beta_i|}{\lambda}}$

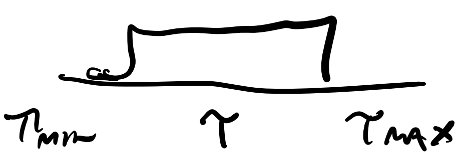
3)

T - EXPONENTIAL

$$p_T(t) = \frac{1}{\tau} e^{-t/\tau} \qquad t > 0$$

τ - PARAMETER

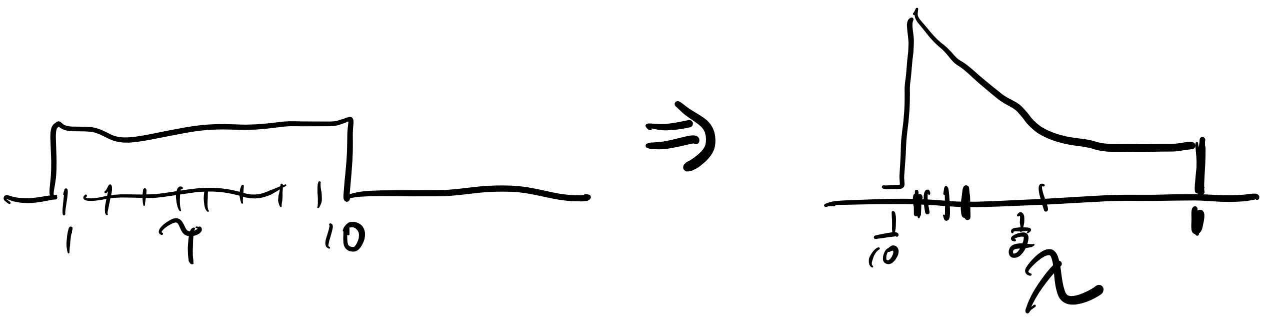
PRIOR $P(\tau) \sim$ UNIF



$$p_T(t) = \lambda e^{-\lambda t} \qquad t>0$$

λ – PARAMETER

$$\lambda = \frac{1}{\tau}$$



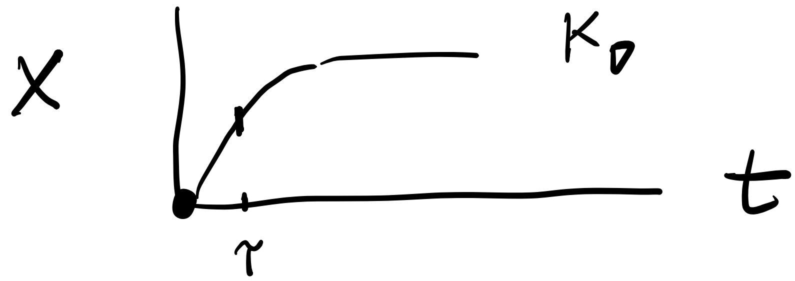
PRIOR IS NONUNIF IN λ

$\Rightarrow$ NO SUCH THING AS A (NONPARAMETRIC) UNIFORM PRIOR

Ex

$$\frac{dX}{dt} = \frac{1}{\tau}\left(K_D - X\right) \qquad X(0)=0$$

CHEMICAL AFFINITY μM^{-1}



$\Leftrightarrow \quad \dfrac{dX}{dt} = k_{on} - k_{off} X \qquad X(0) = 0$

TO FIND THE POSTERIOR

$\Rightarrow$ MARKOV CHAIN MONTE CARLO

- ROBUST
- EFFICIENT
- EASY TO CODE