CONTINUOUS RANDOM VARIABLE

$X \in S$

EX $S = [0, 1)$

$S = [-\infty, \infty]$

$S = [0, \infty)$

DENSITY FUNCTION $p_X(x)$

SUCH THAT $\mathbb{P}(A) = \int_A p_X(x)\,dx$

PROBABILITY

FOR ANY $A \subset S$

- $\int_S p_X(x)\,dx = 1$
- $p_X(x)$ HAS UNITS!

CUMULATIVE DISTRIBUTION

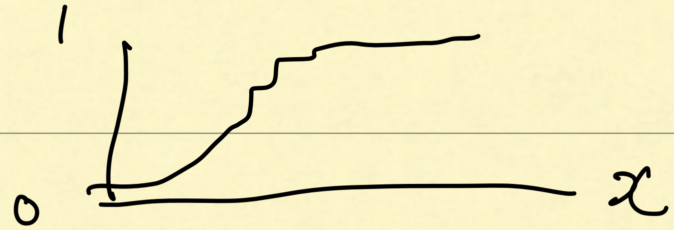
$F_X(x) = \mathbb{P}(X \leq x)$

- $$F_X(x) = \int_{-\infty}^{x} p_X(x)\,dx$$

- $F_X(x) = 0$ AS $x \to -\infty$

  $F_X(x) = 1$ AS $x \to \infty$

  F_X IS NON DECREASING

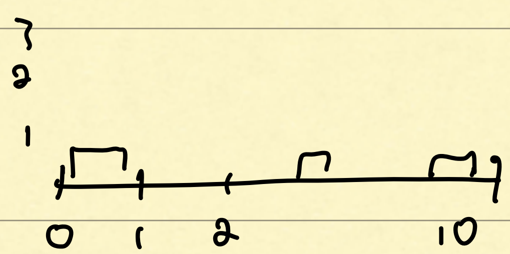


- $$p_X(x) = \frac{d}{dx} F_X(x)$$

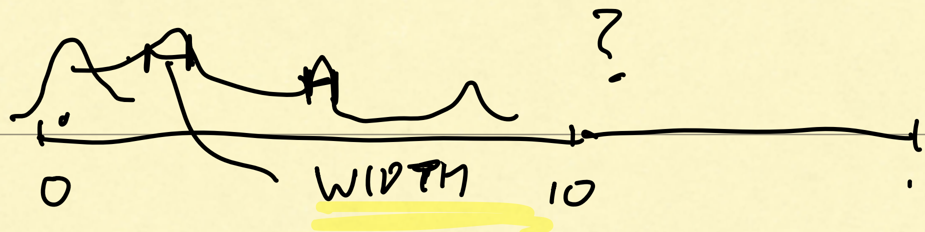
- GIVEN DATA = $\begin{bmatrix} 0.1 \\ 7 \\ 32 \\ \vdots \end{bmatrix}$ $N=10$

TO GET THE DENSITY

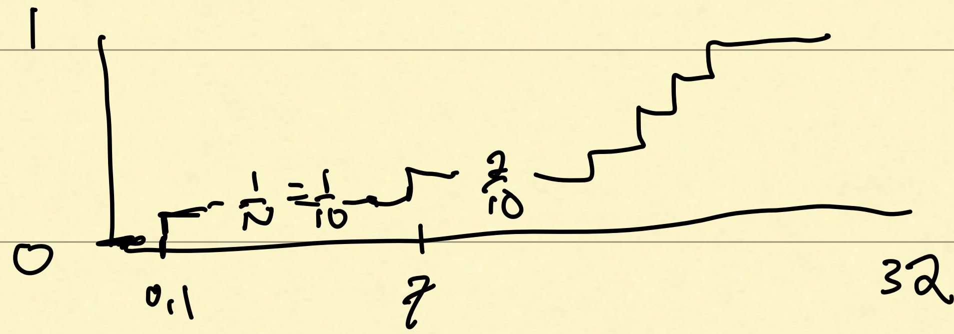
① BIN



② KERNEL SMOOTHING

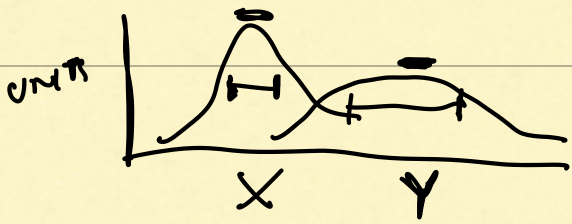


TO GET THE CUMULATIVE



EMPIRICAL CUMULATIVE

- DENSITIES

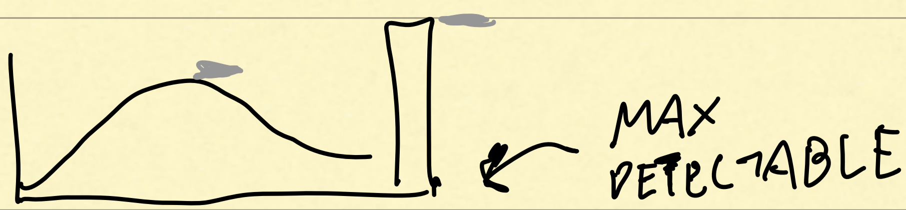


CUMULATIVE

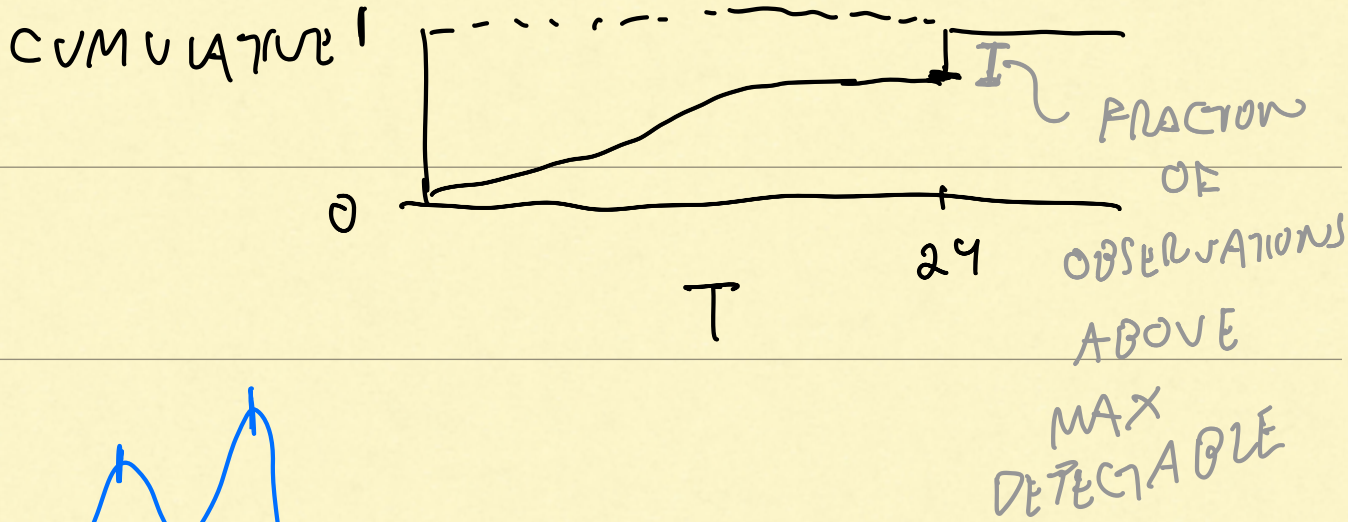
KOLMOGOROV SMIRNOV DISTANCE

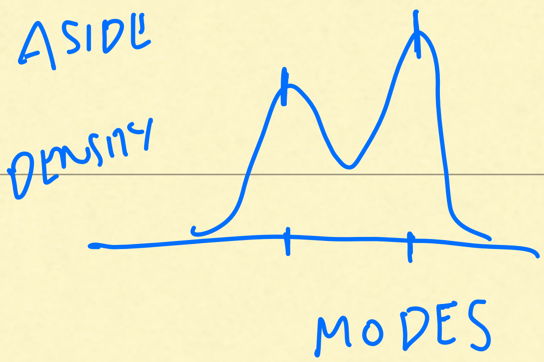
- DATA WITH BOTH CONTINUOUS & DISCRETE OBSERVATIONS

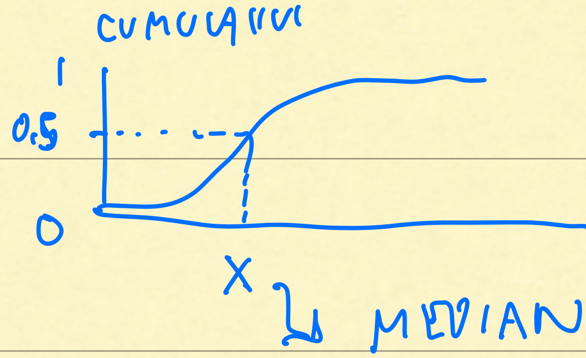
DENSITY



0 T 24



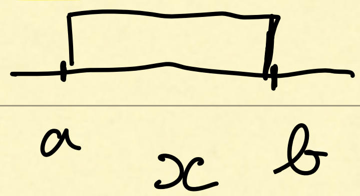




FAMOUS CONTINUOUS RANDOM VARIABLES

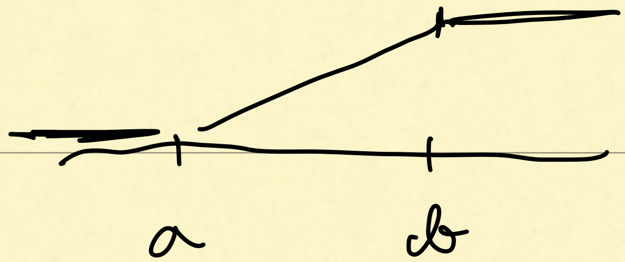
- UNIFORM

$X \sim \text{UNIF}(a, b)$

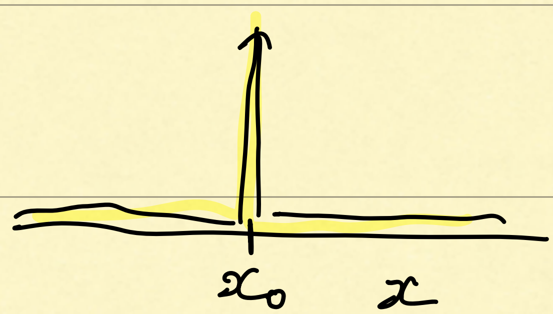


$$p_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{ELSE} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a < x < b \\ 1 & b < x \end{cases}$$



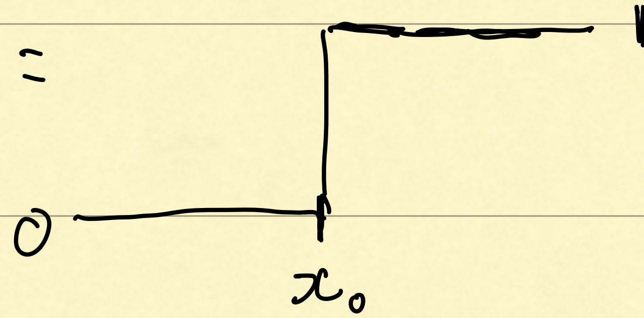
- DELTA, DIRAC



$X \sim \text{DELTA}(x_0)$
$\sim \delta(x_0)$

$p_X(x) = \delta(x - x_0)$

$F_X(x) =$



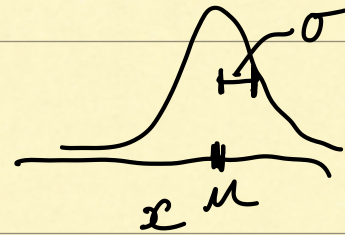
$$= \begin{cases} 0 & x < x_0 \\ 1 & x_0 < x \end{cases}$$

- GAUSSIAN/

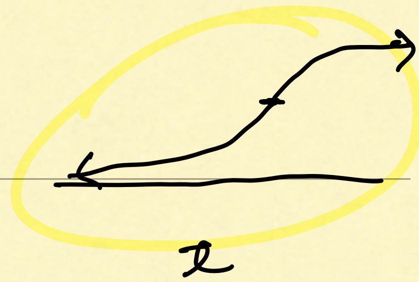
NORMAL

$$X \sim \text{NORMAL}(\mu, \sigma)$$

$$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{\sigma^2}}$$



$$F_X(x) = \text{erf}(x)$$

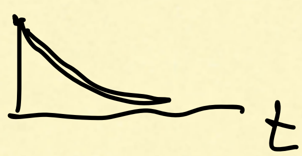


Z - STANDARD NORMAL $\sim \text{NORM}(0,1)$

- EXPONENTIAL $\quad T \sim \text{EXP}(\lambda)$

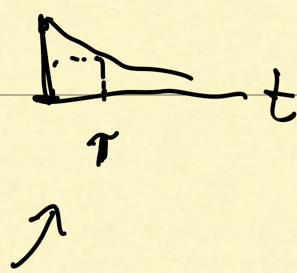
$\sim \text{EXP}(\tau)$

$$p_T(t) = \lambda e^{-\lambda t} \quad t > 0$$



RATE

$$p_T(t) = \frac{1}{\tau} e^{-t/\tau} \quad t > 0$$



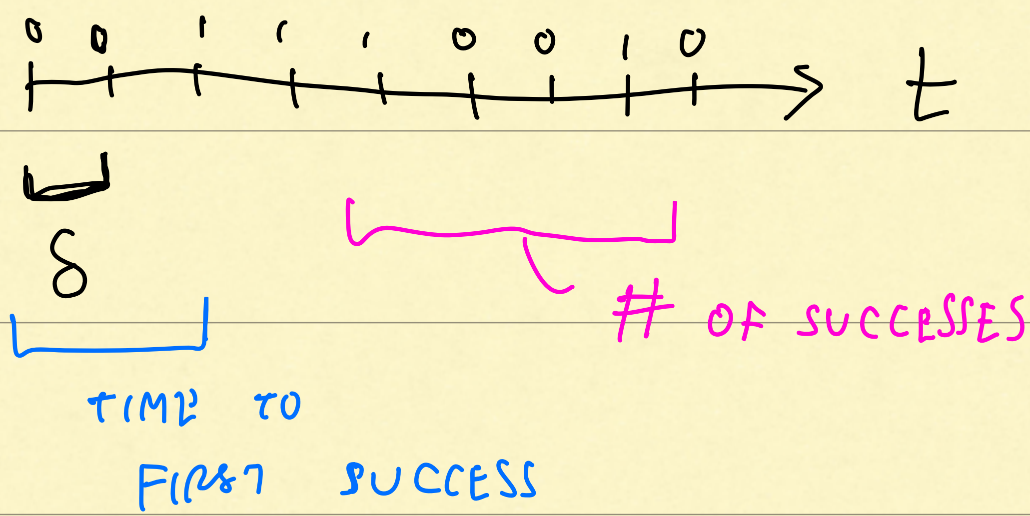
CHARACTERISTIC TIMESCALE

$$F_T(t) = 1 - e^{-t/\tau}$$
$$= 1 - e^{-\lambda t}$$

$$E[T] = \tau = 1/\lambda$$

POISSON PROCESS

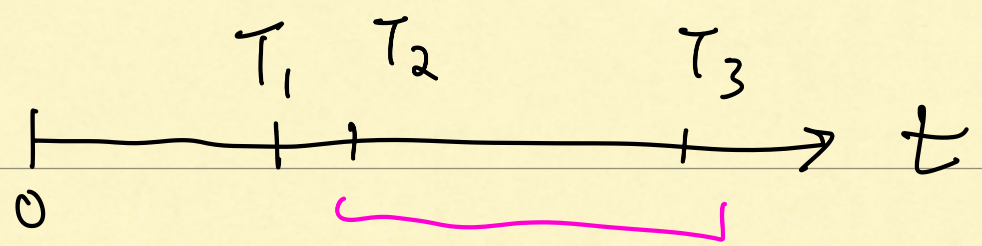
RECALL THE DISCRETE BERNOULLI SEQUENCE



LET $\delta \to 0$ AND $p = \lambda \delta$,

$$\lambda = \frac{p}{\delta} \text{ FIXED}$$

THE POISSON PROCESS IS THE CONTINUOUS TIME STOCHASTIC PROCESS OBTAINED FROM THE LIMIT OF BERNOULLI TRIALS



OF SUCCESSES

TIME TO FIRST SUCCESS

$$T_1 \sim \mathrm{EXP}(\lambda)$$

$$N \sim p_N(i) = \frac{(\lambda \Delta T)^i e^{-\lambda \Delta T}}{i!}$$

$i \geq 0$

POISSON DISTRIBUTION