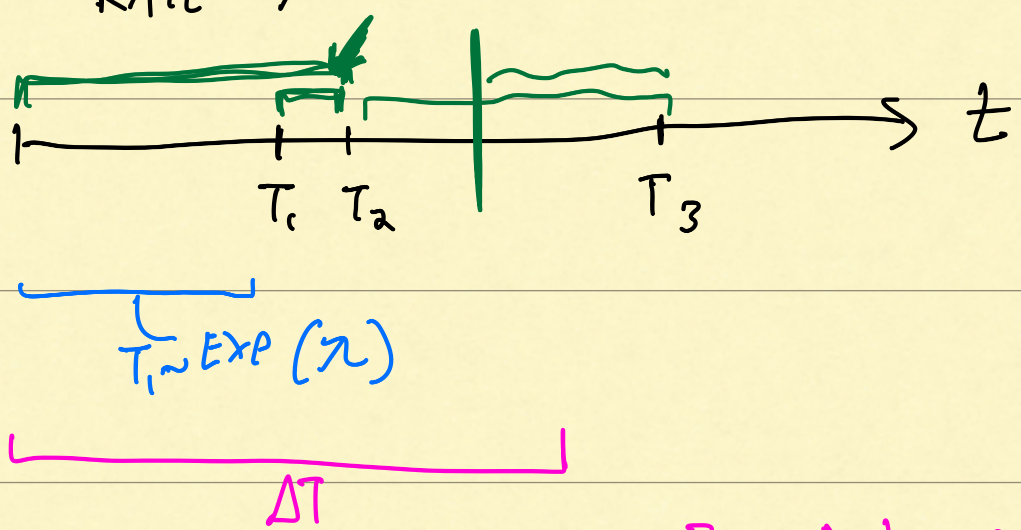
POISSON PROCESS

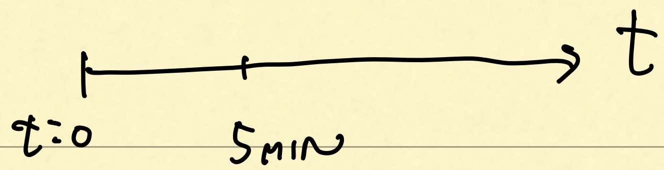
RATE λ



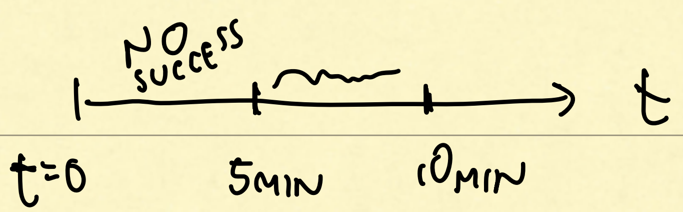
HOW MANY EVENTS? $N \sim \mathrm{POISSON}(\lambda \Delta T)$

PROPERTIES

- WAITING TIME



$$\mathbb{P}(T_1 < 5\,\mathrm{MIN}) = 1 - e^{-\lambda \cdot (5\,\mathrm{MIN})}$$



$$\mathbb{P}(T_1 < 10\,\mathrm{MIN} \mid T_1 > 5\,\mathrm{MIN}) = \left\{ \begin{array}{l} \mathbb{P}(A \mid B) \\ = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \end{array} \right.$$

$$= \mathbb{P}(T_1 < 10\,\mathrm{MIN} \cap T_1 > 5\,\mathrm{MIN})$$

$$\cancel{\mathbb{P}(T_1 < 10\text{MIN})}\ \mathbb{P}(T_1 > 5\text{MIN})$$

$$= \frac{\int_{5\text{MIN}}^{10\text{MIN}} \lambda e^{-\lambda t}\, dt}{1 - \left(1 - e^{-\lambda \cdot 5\text{MIN}}\right)} = \dots$$

$$= 1 - e^{-\lambda \cdot 5\text{MIN}} = \mathbb{P}(T_1 < 5\text{MIN})$$

POISSON PROCESS IS MEMORYLESS:

THE PROBABILITY OF AN EVENT IN THE NEXT ΔT, GIVEN THAT IT HAS NOT HAPPENED YET, IS INVARIANT

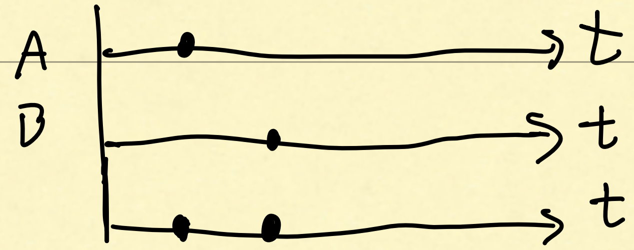
THE POISSON PROCESS IS THE UNIQUE MEMORYLESS PROCESS.

- IF EVENT A IS POISSON WITH λ_A & EVENT B IS POISSON WITH λ_B,

THEN THE NEXT EVENT IS POISSON WITH RATE $\lambda_A + \lambda_B$

IT IS A WITH PROBABILITY $\dfrac{\lambda_A}{\lambda_A + \lambda_B}$

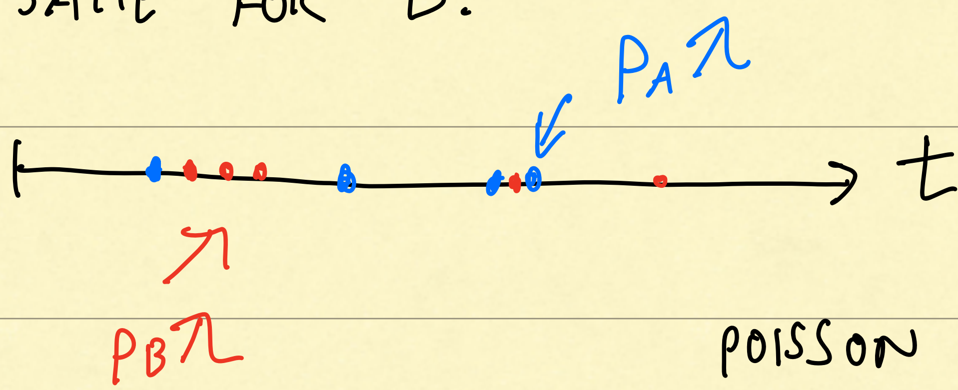
- POISSON RACING



- IF A POISSON PROCESS HAS RATE λ AND IS OF TYPE "A" WITH PROBABILITY p_A AND TYPE "B" WITH PROBABILITY $(1-p_A)$, (INDEPENDENTLY),

THEN TYPE A EVENTS ARE POISSON WITH RATE $\lambda_A = p_A \cdot \lambda$

SAME FOR B.

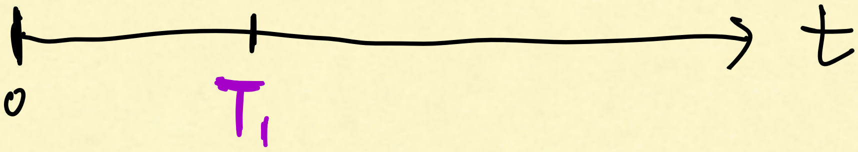


POISSON THINNING

EX. MUTATION

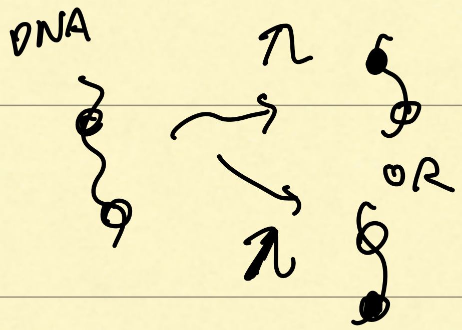
SUPPOSE MUTATIONS ARE POISSON WITH RATE $\lambda = 1\ yr^{-1}$

1 MUTATION $E[T_1] = \frac{1}{\lambda} = 1 \text{ yr}$



2 MUTATIONS, INDEPENDENT, EACH WITH RATE λ.

Q: WHEN DOES THE FIRST MUTATION OCCUR?

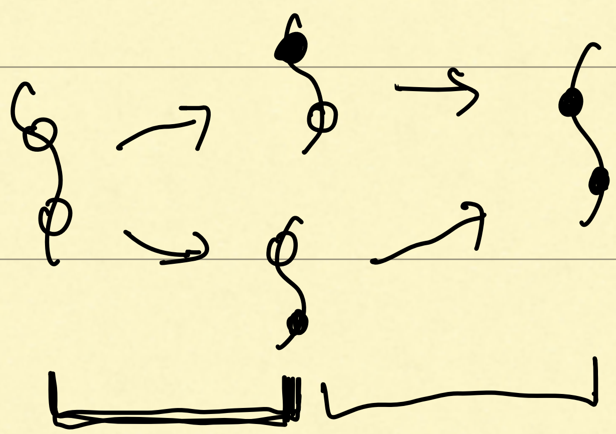


POISSON RACING

POISSON WITH RATE $\lambda + \lambda = 2\lambda$

$$E[T] = \frac{1}{2\lambda} = 0.5 \text{ yr}$$

Q: WHEN IS THE 2ND MUTATION?



$$E[T] = ?$$

$$E[T] = E[T_1] + E[T_2]$$

$$= \frac{1}{2\lambda} + \frac{1}{\lambda} = 1.5 \text{ yrs}$$

PS4

$$E[\text{WAIT}] = E\left[\text{WAIT} \mid \begin{array}{c}\text{1ST}\\ \text{MEETING}\\ \text{LATE}\end{array}\right] \cdot \mathbb{P}\left(\begin{array}{c}\text{1ST}\\ \text{MEETING}\\ \text{LATE}\end{array}\right)$$

$$+ \; E\left[\text{WAIT} \mid \begin{array}{c}\text{1ST}\\ \text{MEETING}\\ \text{NOT}\\ \text{LATE}\end{array}\right] \cdot \mathbb{P}\left(\begin{array}{c}\text{1ST}\\ \text{MEETING IS}\\ \text{NOT LATE}\end{array}\right)$$