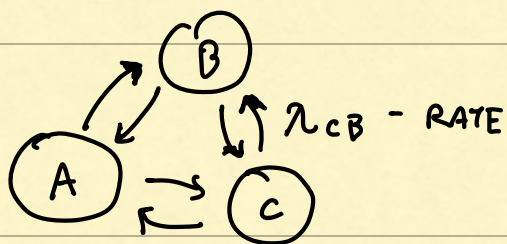


# CONTINUOUS TIME MARKOV CHAINS



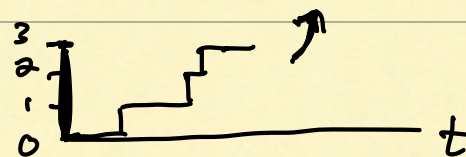
$$p_i(t) = \mathbb{P}(X(t) = i)$$

$$\frac{d}{dt} \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} -\sum \lambda_{21} & \lambda_{N1} \\ \lambda_{12} \\ \lambda_{1N} \end{bmatrix} \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix}$$

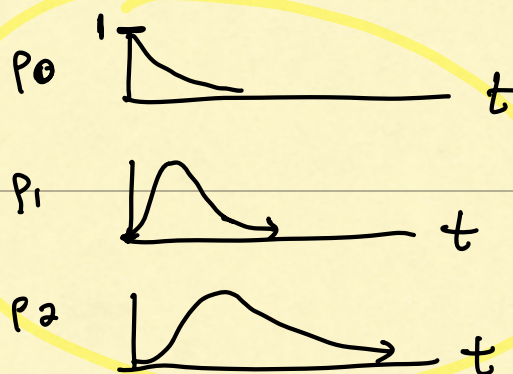
EX

COUNTING POISSON EVENTS

$N(t) = \# \text{ EVENTS up to } t$



$$\begin{bmatrix} -\lambda & 0 & 0 \\ \lambda & -\lambda & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \\ 0 & 0 & 0 & \ddots \\ 0 & 0 & 0 & \ddots \end{bmatrix}$$



EX

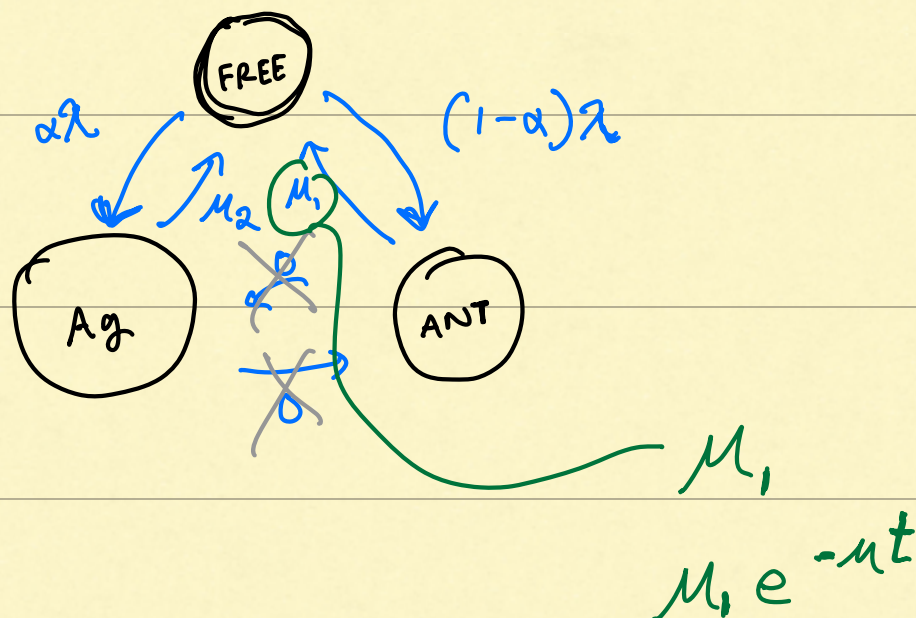
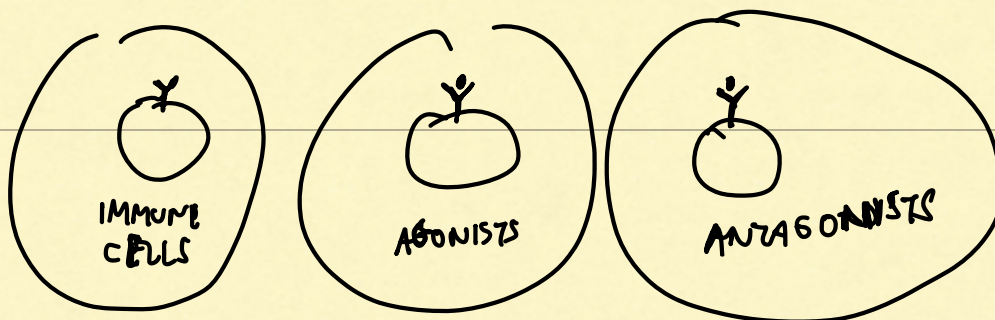
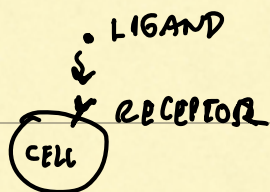
TWO MUTATIONS

$$\begin{matrix} uu \\ um \\ mu \\ mm \end{matrix} \begin{bmatrix} -2\lambda \\ \lambda \\ \lambda \\ 0 \end{bmatrix}$$

TO FIND THE STATIONARY DISTRIBUTION(S), SET

$$\frac{d}{dt} \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

PS 5



FREE AGO ANT

$$\begin{bmatrix} -\lambda & \mu_2 & \mu_1 \\ \alpha\lambda & -\mu_2 & 0 \\ (1-\alpha)\lambda & 0 & -\mu_1 \end{bmatrix} \begin{matrix} \text{FREE} \\ \text{AGG} \\ \text{ANT} \end{matrix}$$

$(1-\alpha)\lambda$

$$P_{\text{FREE}} =$$

$$P_{\text{AGG}} =$$

$$P_{\text{ANT}} =$$

CHECKS

$$\alpha = 0, \quad P_{\text{AGG}} = 0$$

$$\alpha = 1, \quad P_{\text{ANT}} = 0$$

$$\lambda = 0, \quad \begin{matrix} P_{\text{FREE}} = 1 \\ P_{\text{AGG}} = 0 \\ P_{\text{ANT}} = 0 \end{matrix}$$

$$\mu_1 \rightarrow \infty, \quad P_{\text{ANT}} \rightarrow 0$$

$$\mu_1 \rightarrow 0 \text{ and } \mu_2 \neq 0, \quad \begin{matrix} P_{\text{ANT}} \rightarrow 1 \\ \alpha \neq 1 \end{matrix}$$