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2017 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Abstract

Urban smart growth plays an increasingly important role in urbanizing the world. Policy makers or shareholders need to adjust relevant growth plans on a regular basis. To make optimal decisions, it is important to evaluate and predict the urban smart growth level. In the basic model, We select 40 attributes from fields of economy, society, ecology, etc. Then we propose a novel classification algorithm by leveraging Pressure-State-Response (PSR) model. In the advanced model, we eliminate magnitude difference and dimensional difference using, Standard Deviation Method and Information Entropy Method. Then, we select 18 attributes and calculate their corresponding weights with 8 cities' data. Furthermore, we conduct an in-depth analysis of Cambridge in Ontario and MelbourneLGA according to their local government growth plans. By leveraging Smart Development Evaluation Metric (SDEM), we can make conclusions that: 1) the two cities' CD values are similar, 2) Cambridge has a higher Current Development Level, underlining a better smart development, 3) MelbourneLGA has a higher Growth Rate, indicating a more dutiful government. Specifically, we use Fitting Method and GM (1, 1) Model to predict urban smart growth. Based on the prediction, we use the Nonlinear Programming Model to calculate future policy improvement intervals. Furthermore, we also make suggestions through a Nonlinear Multi-objective Programming Model with Monte Carlo method and combine the short-term and long-term goals in current growth plans. We can obtain the following evaluation results. First, for Cambridge, they should encourage solid waste recycling (A), cut new housing construction budget (B) and improve fairness by compromising on part of efficiency (C). Second, for MelbourneLGA, they should enhance the quality of wastewater treatment (D), limit minimum male to female workers ratio (E), transfer human and financial resources in the tree planting industry into other industries (F) and cut poor students' tuition (G). Finally, based on the Weighted Increment Method, potential ranking is: A>C>B, E>G>F>D. In our complete policy evaluation model, detailed development plans are made in the short term, the medium term and the long term respectively. Then, we establish a **Dynamic Non**linear Multi-objective Programming Model and analyze the best development strategies in different terms. Finally, we draw a conclusion that: Compared to the original governmentars smart growth plan, our plan can exhibit a similar performance in the short term and medium term, but will outperform it in the long term.

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1 Introduction

1.1 Background

Since the industrial revolution of the 1860s, especially in the early part of the 19th century, the urbanization accelerated. Tenements and luxury apartment buildings replaced brownstones. Skyscrapers came to adorn urban landscapes. According to the World Development Report 1989, the world's developed countries are more than 80% urbanized, including 92% in the UK, 97% in Belgium, 86% in Australia, 88% in the Netherlands and 86% in Denmark. For developing countries, the trend is similar, and it is projected that by 2050, 66 % of the worlds population will be urban.

Dramatically, from the sixth decade of the 20th century, in the process of urbanization in developed countries, there has been a phenomenon of urbanization, also known as suburbanization or anti-urbanization. In the 1960s, six major cities in the United States with more than one million inhabitants reduced their population by 1.4 million and the number of big cities with a population of 50-100 million from 20 to 16. The total population of 16 large cities also decreased by 2.2 million, accounting for the proportion of the population dropped from 12.2% to 9.7%.⁸

It is hard to imagine why the process of urbanization developed reversely. The main reasons are as follows: Firstly, due to the development of economy, the industrial structure of the developed countries has changed greatly. The advanced development of the electronic communication industry and the traffic industry has brought the western developed countries into the information society. In addition, due to a large number of economic and other activities gathered to the city, urban congestion and environmental degradation have been highlighted. So the rich in the big cities first began to leave the city, moving to good conditions and living in the suburbs. Then the middle class and the city center area of some enterprises and institutions also moved to the suburbs. Finally, the dramatic decline in population growth in developed countries is also a reason for urban population decline.

Based on the above points, resource economy, environment preservation, developing harmonious society and resolving the peoples livelihood problems are all ranked as important affairs now. The scale of the city is getting increasingly bigger, which is liable to cause great pressure to the earth. Furthermore, urban environment, economic and social problems in different historical stages have been interacting and gradually accumulating, making the city more fragile. Therefore, in this sense, only the city embarks on the road of sustainable development, will it have national and global sustainable development.

Naturally, in 1990s, Smart Growth Theory came into being, as a way to build cities, towns, and neighborhoods that are economically prosperous, socially equitable, and environmentally sustainable.

1.2 Our Work

In this paper, we address the challenge of modeling urban development by utilizing and ameliorating small growth theories, both qualitatively and quantitatively. We devise the metric in our model, combined with the three E's of sustainability, to measure the current growth plans of two selected cities and propose possible approaches for optimizing them. Finally, we use dynamic nonlinear multi-objective programming model to explain the feasibility and effectiveness of our metric mentioned above in the case that the population

of each city will increase by 50% in 2050.

2 A Model for Sustainability

2.1 The Principle of Index Selection

In order to reflect the city's intelligent growth and provide the adjustment basis for the policy makers or shareholders, we need to select some observational indexes. In the process of selection, we need to choose those indexes that meet the basic principles. Three main principles are:

• Systematic Principle

Systematic principles require the indexes to not only reflect the main features and status of the ecological, economic, and social subsystems, but also reflect their inner relationships. Therefore, when we analyze how to facilitate urban sustainable development, we need to have a holistic perspective rather than considering certain indexes respectively, or it is more apt to lead to lopsided and absolute analysis.

• Comparability Principle

Comparability principles require that the indexes should be caliber consistent and comparable to each other. In this study, the indexes should be universally applicable for different cities even with different cultures, and they can reflect the authentic conditions of urban smart growth. Thus, we select the relative value and the per capita value instead of absolute value and the total value, so as to readily compare the status of the development between cities in different continents.

• Relevance Principle

Relevance principles require the indexes to be linked to the ultimate goal. That is, the government or other shareholders can use the indexes to make relevant program adjustments. In this study, there should be a strong correlation between the indexes and the growth of urban smart. If not, it will be impossible to accurately reflect the real situation of urban smart growth and control these indexes to achieve urban smart growth; even worse, this will result in the growth plan ineffective.

2.2 Outline of Our Approach

Currently, there have been a lot of authorities establishing empirical of evaluation indexes for urban development. Some of them focus on a certain factor, such as the impact of the natural environment on the development of a city. Some use a strong subjective analysis method such as Analytic Hierarchy Process, in which the data are even derived from people's satisfaction surveys, not excluding ethnic and cultural tendencies and other subjective factors. Others, however, take hundreds of factors into consider. Although it is detailed and concrete, the heavy workload and the fair impact of errors make the general scientific research workers discouraged.

Selecting from many evaluation criteria, we start with the **PSR Environment Evaluation System** as our basic framework. It was first introduced in 1979 by Canadian statisticians David J. Rapport and Tony Friend, and co-developed by the Organization for Economic Cooperation and Development (OECD) and the United Nations Environment Program (UNEP) during the 1980s and 1990s.

PSR,⁷ short of Pressure, State and Response, is a very commonly used evaluation model in ecosystem health assessment sub-discipline in the environmental quality evaluation subject. It is widely used in water resources, land resource and agricultural sustainable development index evaluation system research and research in other fields. The specific meaning of the three angles is as follows:

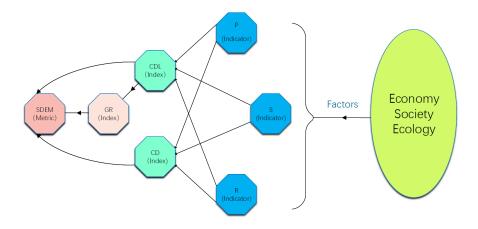


Figure 1: PSR model

• Pressure

Pressure indicators(P) characterize the impacts of human economic and social activities exerted on the environment, which are mostly negative. That is, P always shows the factors in the unsustainable development of human economic activities and consumption patterns.

• State

State indicators(S) characterize the environmental state and environmental changes over a specific period of time, including the status of ecosystems, the natural environment and quality of humanity life and health, etc. To some extent, S represents system state in the process of sustainable development.

• Response

Response indicators(R) characterize how society and individuals act to mitigate, deter, restore and prevent negative impacts of human activities on the environment, and how they make efforts to promote city's sustainable development. In short, R characterizes the response of human beings to the process of sustainable development.

We can calculate some indexes, according to the PSR model, which aim at reflecting different aspects of urban smart growth. These indexes are Current Development Level, Coordinate degree and Growth rate of current development. And furthermore, based on the values of these three indexes, we can calculate the value of our metric, Smart Development Evaluation Metric. Here are the specific meanings of the four indexes and metric:

• Current Development Level

The value of Current Development Level (CDL) reflects the combined effects of P, S and R on the urban smart growth level. Exactly speaking, it is a weighted sum of the values of P,S and R.

• Coordinate Degree

Coordinate degree (CD) reflects the relationship of internal structure of cities in the smart-growth pattern, namely the coordinate degree of P,S,R towards overall level . More

importantly, it plays a vital role in the healthy smart development.

• Growth Rate of Current Development Level

Growth rate of current development level (GR), as the name suggests, is a growth rate based on current development level, which reflects the trend of urban smart development. In fact, it is a guideline for policy makers to make a long-term plan.

• Smart Development Evaluation Metric

Smart Development Evaluation Metric (SDEM) is a comprehensive metric, calculated by CDL, CD, GR. It reflects a city's overall level of smart growth, and is the most representative evaluation metric of our model .

2.3 Assumptions

A myriad of economical, cultural and environmental factors may impact the avail ability of urban growth plan and how cities achieve sustainable development, so we can hardly account for the impact of each of these factors on the urban smart growth. We adopt a number of simplifying assumptions for our model:

- 1. Tiny variables obey the Gaussion Distribution. It is reasonable, because all kinds of external factors or small effect factors (e. g. climate change) seldom have abnormal significant impacts on the model. Based on this assumption, we can easily quantify the external or small effects in our study.
- 2. Little difference in the economic opportunities cities This assumption allows us to arrange for long-term growth plans without regard to the economic uncertainty. We only need to analyze the randomness of economic opportunities from the statistical point of view. As a result, we simplify the model.
- **3.** No serious accidents In reality, serious accidents are uncertain and involve a wide range, which should be considered in complex models one by one. In our model, we will ignore their impacts on smart growth.
- **4. No external assistance**This assumption is reasonable, because a city's external assistance (occasional and random) plays a small role compared to its own development, whose annual cumulative output is almost zero compared to the annual output of the city own development. So, we can ignore it.
- 5. The population growth only brings the net negative effect to the cityThis assumption is not always true. Since the cities we choose are basically in line with this trait, we consider this assumption reasonable within the scope of our study. However, in a more complex model, this assumption will be discarded. We consider both the net negative effect and the net positive effect which may be brought by population growth.
- 6. The development will be strictly in accordance with its government-developed growth plan. There is a strong correlation between city data and growth plans. We can use city data to evaluate the strengths and weaknesses of growth plans.

2.4 The Model

2.4.1 Three factors that affect SDEM

Now we focus on the basis of the evaluation model - a variety of factors, such as urban traffic level, the cultural level of residents, urban natural ecological protection, the city's GDP, the level of civic education and the government's policies. How they affect the value of Smart Development Evaluation Metic(SDEM) in the terms of economy, society and

ecology? First of all, we divide hundreds of thousands of factors into three categories: economic factors, social factors and ecological factors.

- **Economy:** a set of factors in the economic field for given cities to reflect the level of its intellectual development in a short term, such as the city's GDP.
- Society: a set of factors in the urban layout, social science and other fields for given cities to reflect the level of its intellectual development in a short term, such as urban traffic levels and the cultural levels of residents.
- **Ecology:**a set of factors in geography, biome, and ecological sustainability for given cities for given cities to reflect the level of its intellectual development in a short term, such as urban natural ecological protection.

Effectively, there are numerous factors influencing SDEM. But in order to seize the key points of the study, according to relevance principles, we use these three factors mentioned above. Such simplification is reasonable, because any other potential factors affect SDEM by exerting influences on these three ones.

Take education and technology **for instance**. According to Solow growth model introduced by Robert Solow and Dual Sector Model introduced by American economist William Arthur Lewisin in 1954, 5Output growth rate(y)can be expressed as:

y = Population Growth Rate(n) + Technological cadvance rate(g)

g is a function of the proportion of the universities' labor force(u):

$$g = g(u)$$

The main conclusion is that science and technology have an impact on economy, while education contributes to the growth of GDP by promoting the development of science and technology. Thus, we can classify technology and education into economic factors. Other factors can be similarly analyzed.

2.4.2 the Observation Factors of P, S, R

After the definition and analyses of three main factors for SDEM, we will explain how to combine them with PSR environment evaluation system. Actually, it is not contradictory to evaluate the urban sustainable development from these two aspects. The reason is that for each indicator in PSR, it can be divided into three aspects that are economy, society and ecology. Thus, we will get 9 categories, such as, Pressure-Society, Pressure-Economy, Pressure-Ecology and State-Society.⁴

We put them into a table in Appendix , in which the sign after the parentheses indicates the effect on the P, S, and R values, and can be calculated from the positive or negative effects according to the factors selected in the following table, respectively Values of P, S, R.

2.4.3 Calculating CDL, CD, GR

Since SDEM is a function of Current Development Level (CDL), Coordinate Degree (CD), and Growth Rate of Current Development Level (GR), we will calculate the values of CDL, CD, and GR using appropriate algorithms at first, so as to calculate the value of SDEM. According to the outline in **Section 2.1**, the values of CDL, CD and GR are

functions of P, S and R. Thus, we will establish a specific form of the function to calculate the value of SDEM.

$$CD = \left| \frac{P + S + R}{\sqrt{P^2 + S^2 + R^2}} \right|$$

$$CDL = \sum_{i=1}^{TotalP} P_i \cdot W_P^i + \sum_{j=1}^{TotalS} S_j \cdot W_S^j + \sum_{k=1}^{TotalR} R_k \cdot W_R^k$$

$$GR = \left(\sum_{i=1}^{RecentYears} \frac{\Delta CDL_i}{CDL_{i-1}} \right) \cdot \frac{1}{RecentYears}$$

$$SDEM = \sum_{i=1}^{Total} w_i \cdot Attribute_i$$

- The higher the CDL value is, the better the performance of sustainable development will be. That is to say, smaller pressure will contribute to a better urban sustainable development.
- If the CD value is closer to $\sqrt{3}$ (ie 1.732), the coordination of P, S and R will be higher.
- GR value is determined by **the principle of least squares** through the use of MATLAB software to achieve the fitting. If the GR value is larger, cities' sustainable development will accelerate.

2.4.4 Final Confirmation of SDEM

In the formula, w_i represents the weight of the ith kind of indicators, which is obtained by the former research results, v_i is the value of this indicator. SDEM belongs to (0,1), and a larger SDEM represents a better urban smart growth, a higher degree of equilibrium between indicators and a more optimistic perspective of urban development. On the contrary, a smaller SDEM implies a slower city smart development, a larger difference between indicators and a pessimistic expectation of urban development.

2.5 Evaluation of the Model

2.5.1 Strengths

1. PSR logical rigor

PSR answers the question that what happened, why it happened and how we will do. Its strict logic shows in the selection of the index system and the clear idea in the division.

2. Factor selection

The selection of the factors strictly follows systematic principles, comparability principles and relevance principles. As a result, selected factors are both quantifiable and comprehensive, reflecting the citys smart growth precisely.

3. Factor conversion

Adopting a transformation approach, we have rationally classified all kinds of factors into three aspects, economic factors, social factors and ecological factors. For instance,

education and technology can increase the output of knowledge products by promoting the cultivation of knowledge talents, and thus they affect the growth of economy. Such transformation from education and technology to economy makes the economic, social and ecological factors involve a wider range.

2.5.2 Weaknesses

We have developed the index system and established a simple model. But in terms of quantitative calculation, there are still some obvious shortcomings in the simple model: 1. In the process of multi-factor comprehensive evaluation, the indicators cannot be calculated or evaluated directly because of their different magnitudes and different units. So, it is hard to formulate a quantitative measure of urban smart growth 2. We lack an objective and scientific quantitative method in the determination of the weight of each observation factor.

3 Advanced Model

Now, we will use the **Standard Deviation Method** to make factors dimensionless. We also use the **Information Entropy Method** to determine the weight, so as to optimize the model. In order to validate the rationality of the index system and the advanced model, we choose a large number of relevant cities as samples, use the advanced model and the factors to calculate the results, and then sum up the merits and demerits of the model through the analysis.

3.1 Model Building

In this section, in order to make our model fit comparability principles, we firstly standardize the data to eliminate the effect of magnitude and dimensional of different data. Then, we take advantage of Information Entropy Theory to obtain the scientific and authentic weight.

Data standardization contains two aspects, deviation standardization and formal standardization. Considering the readability, we adopt the method of dispersion standardization. Deviation standardization is an effective and accurate calculating method to linearly map the original data into the interval (0, 1) by computing the original data and its extreme value.

As for a positive correlation variable of urban smart growth, its standardized value is shown as the following formula:

$$r_i = \frac{r_i - min_r}{max_r - min_r}$$

As for a pessimistic correlation variable, its standardized value is as follows:

$$r_i = \frac{max_r - r_i}{max_r - min_r}$$

Next, we will use IEM to determine the weight of each factor to the final SDEV. Initially, Claude Elwood Shannon introduced entropy into information theory, whose basic idea is to determine the objective weight according to the variability of factor. In general,

larger information entropy of a certain factor (E_j) indicates a lower degree of variance and the less amount of information provided. It also means the factor plays a less important role in the comprehensive evaluation, and correspondingly, its weight should be smaller.

According to the definition of information entropy (E) in information theory, the information entropy of a group of data is:¹⁰

$$E_j = -ln(n)^{-1} \cdot \sum_{i=1}^n p_{ij} ln p_{ij}$$
$$p_{ij} = \frac{Y_{ij}}{\sum_{i=1}^n Y_{ij}}$$

According to the above formula, we can calculate E_j . Then, basing on E_j , we get the weight of each factor:

$$w_i = \frac{1 - E_i}{k - \sum E_i} (i = 1, 2, \dots, k)$$

and we can calculate ω in the basic model in the next part:

3.2 Calculation and Results

We collect the relevant data about eight cities and calculate the value of each factor. Then, we draw the following table:

Results Factor Code Attribute Positive	Population Density(/km2) Factor A State-Society		Percentage of City Population Living In Poverty Factor C Pressure-Society	Primary Education Student/Teacher Ratio Factor D State-Economy	Total Electrical Energy Use Per capita(kwh/year) Factor E Pressure-Economy	Number of Higher Education degrees per 100k population Factor F State-Economy
Weigh Results	4.98%	5.09%	5.26%	4.95%	4.95%	5.96%
Results	NO2(Nitrogen Dioxide) Concentration Measured in Tonnes Per Capita	Percentage of Women Employed In The City	Average Life Expectancy	Suicide Rate per 100k population	Crimes Against Property Per 100k Population	Violent Crime Rate Per 100 000 Population
Factor Code	Factor G	Factor H	Factor I	Factor J	Factor K	Factor L
Attribute	Pressure-Ecology	Response-Society	State-Society	Pressure-Society	Pressure-Society	Pressure-Society
Positive	0	1	1	0	0	0
Weigh Results	4.96%	4.98%	4.71%	5.26%	4.94%	4.84%
Results	Number of Homeless per 100k population	Percentage of the city's	Number of Internet Connetions Per 100k Population	Kilometres of bicycle Paths and lanes per 100 000	Green Area Per 100k Population	Annual Number Of Trees Planted Per 100k Population
Factor Code	Factor M	Factor N	Factor O	Factor P	Factor Q	Factor R
Attribute	Pressure-Society	Response-Ecology	State-Society	State-Ecology	Pressure-Ecology	Response-Ecology
Positive	0	1	1	1	1	1
Weigh Results	5.10%	7.81%	5.31%	6.17%	5.26%	9.45%

Figure 2: Weigh Calculation Results

4 Analysis of the Current Growth Plan

In order to conduct a specialized study on city smart growth, we select two midsized cities (with a population of between 100,000 and 500,000 persons) from samples, Cambridge (Canada) and MelbourneLGA (Australia). We lucubrated their current growth plans and use the metric in **Section 2** to evaluate their smart growth. Then, we validate the rationality and sensitivity of the current growth plans.

4.1 Current Growth Plan

With study of local government development documents, we extract the current growth plan of Cambridge and MelbourneLGA from Cambridge Local Plan¹ and Plan Melbourne 2015 Review⁶ respectively, as shown in the appendix.

For Cambridge, the promotion of the highest possible standard of design in new development reflects the principles of taking advantage of compact building design. The consolidation and improvement of existing residential communities, the promotion of new housing and community facilities in the growth areas are able to create a range of housing opportunities and choices. The careful conservation and enhancement of historic areas, Green spaces and leisure, shopping facilities can not only preserve open space, farmland, natural beauty, and critical environmental areas, but foster distinctive, attractive communities with a strong sense of place. The provision of appropriate infrastructure, in particular transport infrastructure, is a means to support new developments and promote more sustainable living patterns. In order to strengthen and direct development towards existing communities, the promotion of employment growth in sustainable and accessible locations is needed to support the future expansion of education, research, knowledge-based industries and essential services.

Based on the different effects of each factor on the final observation index (the weight of each factor) calculated in the above index system, we analyze those initiatives with a relatively small weight theoretically, and analyze initiatives with a relatively large weight quantitatively. In the Cambridge's growth plan:

- 1. The careful conservation and enhancement of historic areas and green spaces correspond to the factor layer of Response-Ecology.
- 2. Creating a range of housing opportunities and choices exerts an effect on pressure-Society through number of homeless per 100k population.
- **3.** Similarly, the promotion of employment growth, expansion of education, research, knowledge-based industries and essential services correspond to State-Economy.

For MelbourneLGA, integration of transport, social and land use planning require mix land uses. In addition, the report has repeatedly stressed the protection of the green environment, such as better protection of the peri-urban area, protection of food production and agriculture in the green wedges, protect the leafy green heritage suburb protection of waterways and parklands. They all show the sustainable growth principle of preserving open space, farmland, natural beauty, and critical environmental areas. Meanwhile, MelbourneLGAs growth plan attaches great importance to urban sustainable development. For one thing, it tends to increase density around activity centers and transport corridors and make better planning for new suburbs in the growth corridors. For another thing, it focus more on the concept of a 20 minute city in defining neighborhood, fulfilling greater priority for walking and cycling by creating walkable neighborhoods. More importantly, the idea of establishing a fixed urban growth boundary in the growth plan fits the smart growth planning goals (curb continued urban sprawl and reduce the loss of farmland surrounding urban centers) well.

Since the two growth plans contain different objective factors, their focuses are different too. In the MelbourneLGA's growth plan:

- 1. Population growth matches with Pressure-Society, while population corresponds to State-Society. And, they all have pessimistic effects on the final metric.
- 2. Greater priority for walking and cycling can be classified into the factor of kilometres of bicycle Paths and lanes per 100 000 to be quantified, and then, it can affect State-Ecology and the final metric
- 3. The protection of the green environment emphasized in the growth plan can affect Response-Ecology and the final metric through many kinds of factors, such as green area per 100k population and annual number of trees planted per 100k population

4.2 Gross and Structure Analysis

According to the analysis in **Section 4.1**, Cambridge and MelbourneLGA have been restructured according to their current growth plan and develop against primary factors and secondary factors respectively. We collect relevant data of the two cities and calculate the value of P, S, R, CD L, CD, GR and SDEM.

In order to compare the degree of urban smart growth on the basis of the value of SDEM, we introduce SDEM grading evaluation table:

SDEM	SD grade
< 0.2	Very serious unsustainable
0.2~0.3	Unsustainable
0.3~0.4	Critical unsustainable
0.4~0.5	Critical sustainable
>0.5	Great sustainable

Figure 3: SD Grade

To be able to compare the two cities relevant data more intuitively, we plot a box diagram based on calculation results as below.

According to the grading evaluation table, Cambridge is in a better state of intelligent growth, while MelbourneLGA is in an ordinary state of intelligent growth.

We have conducted a macroscopic analysis of the two urban growth plans. Now we continue the micro analysis. In the light of the calculation results, we plot a radar map to better reflect the structural problems of the two cities' growth plans, as is shown in the above figure.

From the chart, It is not difficult to find that compared with Cambridge, MelbourneLGA has a lower value of pressure index and a higher value of response index. They have similar values of state index.

The value of CD (Coordinate Degree) reflects the degree of coordination between P, S, and R. Therefore, although MelbourneLGA is more stressful than Cambridge, its positive response compensates for this deficiency, resulting in the CD of the two cities being substantially identical. But, the contribution of P, S, R to smart growth is different. As a result, the CDL of the two is still different, but not obvious.

In addition, due to the amplification effect of the time, the value of GR based on timevarying fit magnifies the short-term differences of the value of CDL, resulting in greater

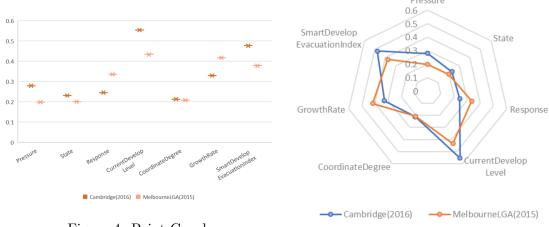


Figure 4: Point Graph

Figure 5: Radar Graph

disparity in the value of GR. Consequently, the value of SDEI derived from above factors are different, but the difference is always kept within a small range.

5 Our Plan and Potential Analysis

We have analyzed Cambridges and Melbourne LGAs strategies in the previous section, and we evaluate them using our model. In this section, we firstly forecast the future development of the two cities following the current growth plans. Secondly, on the basis of the forecast, we use the Multi-objective Non-linear Programming Model in Operations Research. Meanwhile, we propose our growth plan and use our metric to evaluate the success of our smart growth plans, taking into account geographic location, projected growth rates and economic opportunities. In the end, we use the Weighted Incremental Method to analyze each factors potential and rank it.

5.1 Prediction of Key Variables

First of all, we predict the population growth of the two cities. The variable of population growth belongs to Pressure-Society, which is negatively related to the value of SDEM. We then use the **GM** (1,1) **Prediction Model** and take advantage of differential equations. That will help us fully exploit the nature of the system with high precision and less data required. The detail derivation is available in Appendix. By solving the functions above, we can get the graph going as follow:

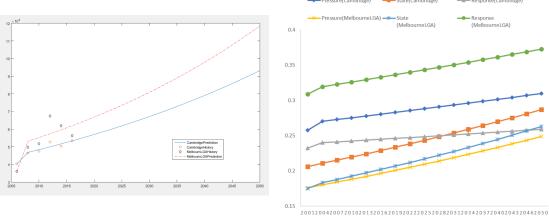


Figure 6: Population

Figure 7: PSR Prediction

We predict every factor and get the prediction of P,S,R.From the figure, it is not difficult for us to find that the P, S, and R values of Cambridge and Melbourne show steady growth in the future. In addition, although Cambridges P and S values are larger than those of Melbourne, Melbournes R value is much larger, resulting in Melbournes P and S values to grow faster due to more positive response measures. So, the future gap of the P and S values between the two cities is shrinking.

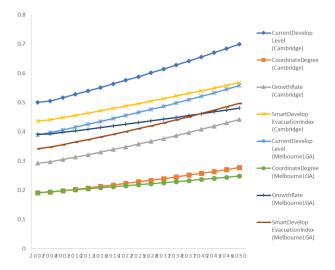


Figure 8: All Metric Prediction

The figure mainly shows that the CDL, CD, GR and SDEM of Cambridge and Melbourne all have a steady growth trend in the future. In addition, at present, there is a certain gap between Melbourne's current development level and Cambridge's. However, Melbourne has greater development potential, and its smart growth is relatively faster. That is to say, there is still a gap between their smart growths, but the gap will shrink.

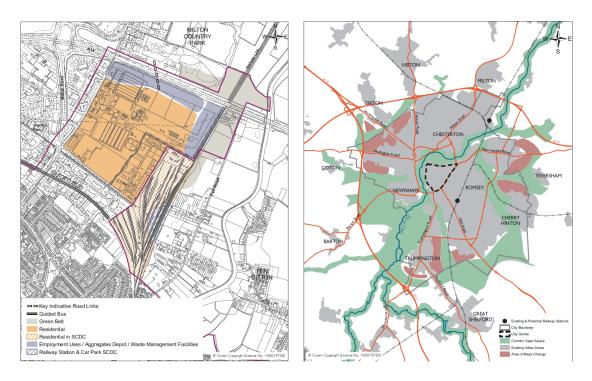


Figure 9: Northern Fringe

Figure 10: Cambridge

5.2 Policy Assessment

Any measure of success must incorporate the demographics, growth needs, and geographic conditions of a city, so we will propose our policy recommendations based on geography, expected growth rates, and economic opportunities of our selected cities. In this topic, we have known the data of the population of Melbourne and Cambridge for the last 5 years, taking them as the data sequence.

5.2.1 Geography

For Cambridge, the detailed initiatives given by Cambridge Local Government cover aspects of construction and utilization of new regions. The area marked by the purple line shows one of the new development areas, Northern Fringe. As illustrated in the lower left corner, the solid orange area will be expected to be used for housing construction (increased state-society values), the solid gray area will be used for employment purposes (reduced pressure-society values) and the dashed gray area will be used for subway stations and parking lot construction (increasing state-society values). Cambridge is typical of the Humid continental climate and its relatively flat terrain with a vast hinterland (the terrain is shown below).

As for MelbourneLGA, specific study of its geographical environment and urban expansion rate leads to following results: MelbourneLGA is near the ocean, east and west facing the expansion of nearby cities. Holocene sand accumulation is to the north of it, and rich agricultural landform needs to be carefully protected.

5.2.2 The Expected Growth Rate

The new land is only a small part of the total area of the city, according to the assumption, so it is reasonable that the new land can be transformed from the original function of government programs into another, which implifies the discussion. Based on the government plan, we estimate that Cambridge's annual urban growth rate is 172hectares per year. We think this is a very reasonable growth rate. While MelbourneLGA annual growth rate is about 91.3 ctares per year according to the analysis of the local government report, we believe that the expansion rate is also reasonable and is one of the main contributors to its high Response index. The new land and some of policies to improve the old city will increase the value of smart development by improving the three indicators of Pressure, State and Response. In order to enhance the planning, we introduce the following nonlinear programming model:

$$max \quad SDEM = \sum W_P * (P_{Base} + P_{New}) + W_S * (S_{Base} + S_{New}) + W_R * (R_{Base} + R_{New})$$

$$\begin{cases} P_{New} + S_{New} + R_{New} <= New Developed Resource \\ min\{P_{New}^2, S_{New}^2, R_{New}^2\} >= Lowest Level Required \\ P_{Base} + P_{New} + S_{Base} + S_{New} + R_{Base} + R_{New} <= min\{Budget, CityGrowth\} \\ max\{P_{Base}^2, S_{Base}^2, R_{Base}^2\} <= Limited Best Improvement \end{cases}$$

In the above formula, $W_{pressure}$ is the weight of P indicator. The first formula constrains the utilization degree of the new land. The new land is either used to improve the pressure, state, or response, or temporarily left for future development. The second formula constrains the minimum equilibrium level of development of the new land between the three indicators, so that the model does not use the additional land for only one purpose. The third formula constrains its ability to improve the level of urban smart growth from the perspective of city operators and governments, which will be limited by the minimum between government budget and urban growth rate. The fourth formula describes the three indicators of the original improvement of the limited space, it is impossible to use all the financial resources in one aspect.

5.2.3 Economic Factor

In this subsection, we incorporate economic opportunity factors into the model. Economic opportunities differ from economic growth in that they own volatility, uncertainty, duality, and satisfy certain statistical characteristics. Economic opportunities are likely to be growth opportunities such as the creation of new regional trade organizations or a negative global economic crisis. In the special long run, the frequency of the two is close to the offset of positive and negative effects. Therefore, in the medium and long-term model, we introduce the economic opportunity factors $\varepsilon_{economy} \sim N(0,1)$ and its influence coefficient $\mu_{economy}$

We modify the constraints on economy in the previous section as follows:

min
$$P_1d_{state-society}^- + P_2d_{pressure-ecology}^+ + P_3(d_{SDEM}^+ + d_{SDEM}^-)$$

s.t.

$$P_{New} + S_{New} + R_{New} - d_{pnew}^+ + d_{snew}^- + \cdots - d^+ +_{rnew} + \varepsilon_{economy} <= New Resource$$
...

Through Monte Carlo simulation, take the number of tests N=200 times, and results go as follow:

For Cambridge

With the analysis of the P, S, and R values in the present period, we can see that its lower coordination degree narrows SDEM values growth. If the CDL value is constant, we can increase R value by declining the P and S values, thus changing the coordination between the three. Considering that the R value has a hysteresis effect, in a short term, SDEM change rate will decline when the P and S values decreased. In a long term, the promoting effect of the increased R value on SDEM is greater than the dragging effect of the decreased the P and S values, so the increased rate of SEDM value will be accelerated. In addition, Cambridge's current R value is small, implying future development potential large. In a long time, SDEM values growth curve is like a shape of J. If the P value and S value are increased by decreasing the R value, since they are large at present and will reach saturation soon, the SEDM value will hardly change any more.

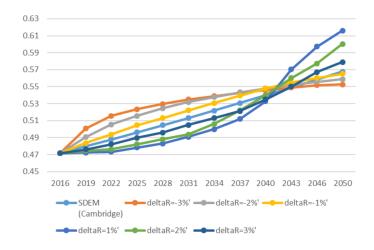


Figure 11: Policies Based on Response

For MelbourneLGA

As shown in the figure, there are similar problems in the coordination degree between MelbourneLGA and Cambridge, and the growth curve of SDEM is the same as that of Cambridge in an earlier stage. The difference is that, MelbourneLGAs R value is large at present. When we can increase the R value by declining the P and S values, the R value will reach the saturation state soon. So, the SDEM value will reach the steady state quickly, which leads to the growth curve of SDEM value is S shape.

In addition to the above characteristics, Cambridge and MelbourneLGA in P and S alternative relationship also have similar characteristics. As shown in the figure, for

Cambridge, its current P value is greater than the S value, implying the growth space of the S value is greater. When the sum of P and S values is constant, increasing the S value and decreasing the P value will cause the SDEM value to rise rapidly in the short term. As time goes on, the P value reach saturated more quickly than the S value does, so that SDEM growth rate decreased until it remains constant.



Figure 12: Policies Based on Response

Figure 13: Policies Based on P and S

On the contrary, MelbourneLGAs P value has a greater potential. When the sum of P and S values is constant, increasing the P value and decreasing the S value for MelbourneLGA can have a similar result with increasing the S value and decreasing the P value for Cambridge.

So far, we have elaborated the calculation principle of the development function of a stage, we define it as develop() function, which is quantified through specific data, based on SDEM evaluation model, multi-objective nonlinear programming algorithm . As for its dependent variables and independent variables, we will explain in section 6.

5.3 Recommend Policies

Aspects 1(Cambridge)

We should strengthen the project investment can enhance the city's responsiveness, especially in Response-Ecology and Response-Society. Cambridge's Response is relatively low compared to its State, Pressure, thus affecting its coordination, which has a pessimistic effect on SDEM.

We find that the improvement of the citys internal environment its resource allocation satisfaction still needs improving even in many years. That may because the government is too close with relevant gainers to arouse public discontent.

Here are some specific and feasible policies:

A: establishing policies to encourage solid waste recycling or to increase its propaganda.

B: establishing policies more liable to sacrifice part of the efficiency to guarantee the relative fairness.

Aspects 2(Cambridge)

We should slow down the investment in Pressure, especially in Pressure-Society. The reason is that its progress in the Pressure is so significant, for the control of population growth rate and the number of homelessness (housing construction), that it consumes

a large amount of government budgets. According to the rules of diminishing marginal effect in economics, the financial resources in this area devoted to improving the State and Response will benefit the city smart growth.

Here are some specific and feasible policies:

C: reducing new housing construction budgets and increasing the population growth moderately.

Aspects 1(MelbourneLGA)

The government should continue increasing investment in Response to balance the higher Response-Ecology and the lower Response-Society currently. One reason is that increased Response represents an improvement in terms of State and Pressure in the short term and a slow growth rate in the long term. Thus, we can see Response as a main factor in the long-term smart growth. Analyzing empirical of data in detail, the city has a low female employment rate and a high annual tree planting rate. We suggest MelbourneLGA should keep a modest balance in both areas.

Here are some specific and feasible policies:

D: enhance the quality of waste water treatment and promoting environmental taxes on emissions

E: limiting minimum male to female workers ratio in organizations or companies

F: transferring human and financial resources in the tree planting industry into other industries

Aspects 2(MelbourneLGA)

The government should guarantee a good coordination between Pressure, State and Response It is because MelbourneLGA concentrates more on the long-term growth plan, so its short-term development not so balanced in terms of the three aspects. So the government is better to improve the Pressure and State in the short term to deal with some critics protest.

G: Funding for university education, cutting poor student tuition and making the whole society more fair.

5.4 Potential Mining

This subsection will focus on the discussion and rank of the potential of each item based on the calculated results of the previous section. In addition to subjective analysis and objective evaluation using forecasting data, a new quantitative evaluation method, called **Weighted Delta Method**, will be introduced in this section.

$$potential_i = \sum_{k=1}^{Total Affected Factors} W_k (1 - \frac{\Delta factor_k}{Current Develop Level})$$

 $potential_i$ represents the potential of the i-th policy, $factor_k$ indicates the i-th policy will affect the k-th factor, and it is of positive and negative value according to its positive or negative effects.

By using the data in last section and the Weighted Delta Method, we can draw the conclusion that the most potential ranking is A > C > B, E > G > F > D.

We analyze the results, finding that the most potential policies are those focusing on the improvement of Response for gaining a long-term benefits. This fits our former conclusion very well.

6 Complete Policy Evaluation Model

6.1 Multi - stage Policy Evaluation

In fact, we can divide the period from 2017 to 2050 into more detailed stages of development to develop more accurate development plans. We reference materials and then divided the years into a period from 2017 to 2020(short-term), a period from 2020 to 2035(medium), a period from 2035 to 2050(long-term), and a period of 50 years later (special long-term). It is found that the explicit phase, reasonable state transition condition and non- after - effect are very consistent with the dynamic programming model.

• Step1

Introduces a state group quantity F_{Sort}^{Stage} , which is a set of quantities:

$$\{P, S, R, LastSort, UpdatePolicy\}$$

Stage represents the short, medium, and long phases. Sort represents the ranking of F, in the Stage stage, according to the SDEM values of each F to sort in descending order, that is, in each stage, we will have a number of collections by the current SDEM F_{Sort}^{Stage} . Each F_{Sort}^{Stage} represents the state where the SDEM is ranked as Sort, P, S, R is the current state quantity, LastSort is the last state source of the current state, UpdatePolicy represents what policies is used from the previous state to achieve the existing state.

• Step2

The state transition equation describes the operation and the transfer rule between the two states, which are as follows:

$$F_k^{Stage+1} = the \quad kth \quad Develop(F_i^{Stage}, Policy_j) \\ i = 1, 2, ..., LastStageNumber \\ j = 1, 2, ..., 27 \\ i = 1, 2,$$

There are 27 kinds of policies, combining respectively the three actions of paying more attention on P,S,R, maintaining the original attention on P,S,R, or modest neglecting P,S,R, based on original policies. And plan by changing the objective function priority P1,P2,P3 in Develop function.

• Step3

Set the initial state to F_1^{Start} . After the calculation of each stage, we use the state with the highest SDEM value in the final stage, F_1^{Final} , and then we take it as an example to dig out the status of each stage one by one, namely the transfer strategy through the LastSort and UpdatePolicy:

$$F_1^{Final} \Rightarrow F_{LastSort}^{Final-1} \Rightarrow \cdots \Rightarrow F_1^{Start}$$

• Step4

For each stage we set the evaluation weights $W_{Start}, W_{Start+1}, \cdots, W_{Final}$, we can find $BestPolicy = max\{W_{Start}F_1^{Start} + \cdots + W_{Final}F_i^{Final}\}$ $i = 1, 2, \cdots, TotalFinalNum$

So far, we have completed the calculation of our Monte Carlo dynamic nonlinear multiobjective programming.

0.65 0.6 0.55 0.5 0.45 0.45 0.40 2016 2019 2022 2025 2028 2031 2034 2037 2040 2043 2046 2050 SmartDevelop WeighPlanA WeighPlanB WeighPlanC EvacuationIndex (0.33:0.33:0.34) (0.2:0.3:0.5) (0.1:0.2:0.7)

6.2 Support for Long-term Growth

(Cambridge)

Figure 14: Different Policies Prediction

The above figure shows the optimal SDME development curve based on weight plan A (short term, 0.33: 0.33: 0.34), weight plan B (equilibrium, 0.2: 0.3: 0.5) and weight plan C (special long-term strategy, 0.1: 0.2: 0.7). We can see that in Plan A, the society tends to sacrifice long-term development to obtain short-term rapid growth; in Plan B, the city will have a temporary low growth rate in exchange for long-term substantial progress; In Plan C, with a special long-term perspective, the city will have a super-fast pace of growth around 2050. Although data in 50 years are beyond the scope of our forecast, it is conceivable that Plan C will continue to have a very long-term growth.

Our plan in terms of supporting the 50% growth in population are very similar to that in Plan B, since the policy is actually based only on the SDEM value in 2050, which is 100% weight. Under such an evaluation criteria, we will adopt neither Plan A which seeks for short-term returns ignoring long-term growth, nor Plan C which pursues specially long-term returns and is of highly strategic vision. Our plan will be inferior to the original governments smart growth plan in the short term, but in the medium term will almost catch up it, and in the long term achieve small scale beyond.

In fact, there may be some special factors affecting our theoretical expectation. For example, urban leaders pursuing political achievements change the city's original balanced development plan into short-term plans when the original plans implementation is under half-way, just like president Trump abolished the economic impact of TPP on the city, or the possibility that the city suffered an unpredictable natural disaster, which lead the right curve to drop directly. These are unpredictable factor. Some of them only require us to make a choice between the presents and the futures interest. Some are liable to move in the right direction, while others may be completely devastating disasters.

These stochastic factors cannot be dealt with in this paper. We propose only one possible solution - the introduction of random factors such as economic opportunity, which obeys more complicated statistical laws.

7 Strengths and Weaknesses

• Strengths: The Objectivity of Weight Determination

In this paper, we adopt the **Information Entropy Method** to determine each factors weight in the index system. It is different from the Subjective Weight Determination Method, such as the main level analysis, and it eliminates the interference from the subjective views, making the empowerment of each factor have a strong objectivity and feasibility.

• Strengths: A Wide Range of Intermediate Indicators

A step-by-step approach is adopted in this paper. We establish various factors, three main indexes, and the final metric, among which index CDL, CD, GR have strongly representative and typical characteristics, and can be sensitive to reflect intelligent growth structure of different aspects

• Strengths:Data Authenticity

The data query and application follow the principles of accuracy, authority and prudence. This paper has reliable and definite data sources, which ensures authenticity, and the basic credibility of the model calculation results.

• Weaknesses:Hypothetical simplification

Reasonable assumptions are often helpful to simplify the complex problems, but some assumptions in this paper, such as populations net negative effect are too idealized. They do not fully consider the complexity in the reality. Such simplification narrows our models practical scope.

8 Appendix

8.1 Three Principles and Factor Data

According to systematic principles, we need to establish a comprehensive evaluation index system, and introduce multifaceted variables involved in political, economy, population, transportation, natural geography and many other aspects of urban smart growth. Then, according to comparability principles, we need to establish scientific and objective evaluation criteria, eliminate the effect of the magnitude and dimensional of each variable on the results. Furthermore, according to relevance principles, we need to discard the external variables or small effect variables in the model, analyze the main contradictions and constraints, and master the core of the problem, so that we are able to establish the evaluation criteria that can provide practical improvement strategies.

CityName	Factor A	Factor B	Factor C	Factor D	Factor E	Factor F
Cambridge(2016)	134900.00	112.80	1195.92	6.40	12.00	16.90
Eindhoven(2016)	224788.00	88.84	2530.26	8.30	10.90	17.75
MelbourneLGA(2015)	122207.00	37.70	3088.78	5.03	11.02	14.71
Minna(2014)	255631.00	72.00	3550.43	20.00	20.00	19.00
Porto(2016)	224894.00	41.42	5365.80	17.60	18.70	12.25
Shawinigan(2015)	49428.00	737.00	67.10	8.20	8.40	15.82
Surrey(2016)	526293.00	316.00	1481.01	7.93	15.32	18.17
Vaughan(2015)	317889.00	273.52	1054.00	6.70	11.70	15.57
Factor G	Factor H	Factor I	Factor J	Factor K	Factor L	Factor M
10285.90	37236.00	7.00	47.00	81.80	9.50	3882.40
19721.00	37058.00	24.30	51.50	81.80	13.00	0.08
27083.77	46630.72	15.87	38.81	83.50	8.54	10721.97
3000.00	18000.00	3.00	41.87	54.00	0.39	188.94
5478.00	22494.00	29.80	59.00	80.70	8.00	4023.23
5685.70	18000.00	46.00	24.00	82.41	0.10	400.00
6136.56	39900.64	9.00	55.37	82.59	7.03	7013.15
12765.38	45093.50	24.25	37.60	85.00	3.40	2049.46
Factor N	Factor O	Factor P	Factor Q	Factor R	Factor S	Factor T
1038.70	870.00	53.00	89390.00	92.82	687.17	740
1181.00	227.77	51.00	96000.00	204.64	869.00	2536
2849.26	395.05	20.94	53631.95	147.42	451.93	2640.48
39.12	190.00	61.00	74000.00	20.00	430.31	19559.44
1287.00	609.00	17.23	41243.00	6.41	133.22	59.49
20.00	400.00	342.00	52106.00	2.00	521.80	304
1119.47	79.40	71.62	86000.00	104.10	297.12	960.04
587.94	642.00	57.00	80011.55	68.04	349.17	495.46

8.2 GM(1,1) Derivation

We have known the data of the population of MelbourneLGA and Cambridge in the past 5 years, which are the data series $x^{(0)} = (x^{(0)}(1)andx^{(0)}(2), ..., x^{(0)}(5))$ respectively. One-time cumulative generation sequence, (1-AGO), is defined as follows:

$$\begin{split} x^{(1)} &= (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(5)) \\ &= (x^{(0)}(1), x^{(0)}(1) + x^{(0)}(2), ..., \sum_{i=1}^{5} x^{(0)}(i)) \\ z^{(1)} &= (z^{(1)}(2), ..., z^{(1)}(5)) \\ z^{(1)}(k) &= 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, 4, 5 \end{split}$$

We build functions:

$$gdp^{(0)}(k) + az^{(1)}(k) = b, k = 2, 3, 4, 5$$
$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b$$

8.3 Factors Analyse

To calculate the value of SDEM accurately and quantitatively, which reflects the level of urban smart growth, we need to calculate the value of P, S and R respectively. Since P, S and R are determined by economic, social and environmental factors, we need to categorize specific observation factors of the three indicators of PSR in detail.

From the economic perspective, observation factors that probably affect the value of PSR include per capita GDP, CPI index, unit GDP energy consumption, disposable income of urban residents, per capita income, per capita education funding, R & D expenditure as a proportion of GDP, the proportion of added value of tertiary industry to GDP, the proportion of the three industries, value - added of Service Industry, fossil energy/primary energy consumption ratio, industrial water recycling rate, agriculture value added per worker, crop production index, etc.³

From the social perspective, observation factors that probably affect the value of PSR include population density (people per sq. km of land area), proportion of urban and rural population, proportion of medical insurance population, illiteracy rates above 15 years of age, the Engel coefficient of urban and rural residents, urban/rural income ratio, public transportation rate, urban registered unemployment rate, per capita energy consumption, social Security Coverage, the rate of public reporting of social and ecological damage, the number of buses every ten thousand city people have, environmentally friendly building/civil construction ratio, compliance rate of good classification of household waste and so on.¹¹

From the ecology perspective, include the environmental quality index, the proportion of cultivated land to the total land area, the area occupied per capita of arable land, the effective utilization coefficient of agricultural irrigation water, the per capita water resource, the forest coverage rate, industrial waste water discharge compliance rate, the number of sudden environmental pollution events, emissions of chemical oxygen demand, area of soil and water loss control, energy consumption elasticity coefficient, industrial waste water discharge, carbon dioxide emissions, sulfur dioxide emissions, the proportion of environmental investment in GDP, dependence on foreign oil, per capita afforestation area and so on.

9 Reference

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